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Simulation Model of Speed Control for the Moving-Block Systems under ERTMS Level 3

Ronghui Liu

Institute for Transport Studies, University of Leeds, Leeds, UK

R.Liu@its.leeds.ac.uk

Abstract— The moving-block system under ERTMS Level 3 represents a continuous detection of train positions, and continuous infrastructure-to-train communication. This allows trains to follow one another in smaller space and time headways than possible in fixed-block systems. Trains form platoons with undisturbed headways and uniform speeds in moving-block systems, with improved safety and increased capacity. The dynamics of such train platooning are likely to be critical, as this can give rise to several practical effects, including unrealistic performance demand on braking and traction forces, unstable following and incessant speed fluctuations. Dynamic simulation models can be used to test train control algorithms and design solutions to these problems. This paper presents a discrete-time simulation model of a moving-block railway system. The train control algorithm within the system is formulated as a train-following model. We present a constraint train-following model, based on the concept of an optimal velocity and desired following distance. We provide analysis on the performance of the model, in terms of safety and flow stability.

Keywords— ERTMS; Moving-Block; Speed Control; Flow Stability; Simulation.

I. INTRODUCTION

Advancing from the current systems of track-based detection and line-side signalling to train-borne detection and direct system-to-train communication, the European Rail Traffic Management Systems (ERTMS) Level 3 offers the infrastructural and communicational potential to provide a railway system with enhanced efficiency, improved safety and increased capacity. However, the radically different methods of operations mean that successful implementation of ERTMS Level 3 (and more advanced systems) depends on key traffic management and control challenges being addressed [1-3].

With ERTMS Level 3, the locations of the trains continuously monitored and transmitted to the control centre, and the movement authorities (MA) directly communicated to the trains. The individual trains themselves become effective moving blocks (hence the term moving block (MB) rail system). Trains can follow one another in smaller (space and time) headways than possible in fixed-block systems. MB allows the formation of platoons of trains with undisturbed headways and uniform speeds, thus improved safety and capacity. Trains travelling in a platoon, or ‘virtually coupled’ mode, also have the potential on energy savings due to reduced running resistance [4-5].

Provision of a distinct and individual train-based moving block system, traffic management increases the challenges for Level 3. The control of the speed and spacing between trains, as well as managing the network-wide paths of trains in real time, are clearly central to the success of the system.

A. Key Challenges for Modelling ERTMS Level 3

Moving from track-based detection and line-side signalling of the current system, to train-borne detection and communication, ERTMS Level 3 (and above) offers the potentials for rapid response to changes in network and traffic conditions and for enhanced capacity and performance.

With the MB system, the train now only has to follow the speed limits and maintain a safe distance from the train in front, which increases the railway capacity. Also with the real-time and detailed train running information, the control centre now has the opportunity to arrange the movement authorities more sophisticatedly, which introduces challenges for the real-time scheduling and control algorithms. It is also possible for the control centre to provide sophisticated speed and/or acceleration profiles for the trains to follow, which is helpful for energy saving since the energy consumption is related to the detailed running status of the trains but is considered locally in the current train operation [4-7].

A key challenge to the success of the ERTMS Level 3 system therefore lies in the development of network-wide intelligent traffic management and control strategies. Models representing ERTMS at a network level are required to help design and test the proposed traffic management strategies.

At the individual vehicle level, the dynamics of train-following and platooning afforded by ERTMS Level 3 are likely to be critical, as this can give rise to several practical effects, including unrealistic performance demand on braking and traction forces, unstable following and incessant speed fluctuations [8]. The control of the speed and spacing between trains is an integral part of the traffic management for the ERTMS Level 3 MB system.

This paper addresses the challenge in the design of control strategies for the platooning and following of trains in a MB system. Section 2 of the paper introduces the general concept and approaches in modelling train running dynamics, and presents a specific train control algorithms. Section 3 analyses the performance of the proposed train-following model. We

focus on the safety and flow stability aspect of the control algorithm, and present numerical and simulation results to illustrate the performance of the control method. We discuss the practical implications of these results in Section 4, and highlight the future research issues in the design of control strategies for the ERTMS Level 3 MB systems.

II. TRAIN DYNAMICS MODELLING

As acclaimed by Wickens [9], “a train running along a track is one of the most complicated dynamical systems in engineering”. The dynamic behaviour of a train is influenced not only by the train’s mechanical designs (e.g. its size, weight, acceleration forces), but also the interaction between the train and the infrastructure (e.g. the wheel-rail interface, the friction forces) (see an introduction to the railway vehicle dynamics in Iwnicki [10]). In a railroad network, the train movements are controlled by commands sent via trackside signals (as in most of the current railway systems) or directly to the train via GSM-R communication as in ERTMS.

In this paper, we consider a train as a rigid body and focus on the train running dynamics as controlled/influenced by the traffic management commands. More specifically, we model the train’s continuous space-time trajectories/movements as they traverse a railway line and network.

In traditional railway systems (such as the current fixed-block systems), train movement is realised by following an optimal velocity curve. Mathematical methods, such as optimal control [5,6] and simulation method [11], have been used to utilised to obtain the optimal velocity curves. The velocity curves may be optimised to reduce energy consumption and/or enhance comfort, whilst ensuring that the train stops safely by the end of the section and/or arrives at the station on schedule [12,13]. Through advanced technology, such as the Communication-Based Train Control (CBTC) system [14,15], a continuous adaptation of the optimal speed profile is possible to ensure safe headway keeping between trains.

The technology embedded within ERTMS Level 3 allows trains to follow each other on the same track. The trains’ movements are thus continuously adjusted not only according to its own optimal velocity profile but also responding to the movement state of the train(s) in front. We formulate such constraint movements as a controlled train-following problem, with the control mechanisms designed to mimic the movement command in a MB ERTMS Level 3 system.

Vehicle-following models have been widely used in road transport to study the behaviour of cars and the dynamics in traffic flows [16, 17]. These are mathematical models formulated to represent the inter-vehicular dynamics in a single stream of traffic. See reviews on the developments of car-following models in Brackstone and McDonald [18], the choices of behavioural parameter values in Bonsall et al. [19], and the principles in calibration in Hollander and Liu [20].

Two types of vehicle-following models are commonly formulated: safety-distance models and stimulus-response. The safety distance models control the speed of the vehicle such as it can always bring to a safe stop should its proceeding vehicle

brakes to a sudden stop. The Gipps car-following model is one based on the safety distance concept [21], which has been applied in many road traffic simulation software package such as DRACULA [22-24] and AIMSUM [25].

The stimulus-based models formulate the response of a following vehicle (typically in terms of its acceleration/deceleration) in response to a stimulus it received from its preceding vehicle and its own sensitivity. The stimulus may include the velocity of the considered vehicle, the headway and the velocity difference to the proceeding vehicle [26]. The General Motors car-following models are some of the best-known and the earliest vehicle-following modelled developed [27, 28]. The optimal velocity model developed by Bando et al. [29] is another well studied stimulus-response model, in which, the stimulus is the difference between and optimal velocity and the vehicle’s current velocity.

Li and colleagues [11, 30, 31] adapted the optimal velocity car-following model to simulate the train movement on a single-track railway section and use which to design optimal train trajectory.

In this paper, we propose a new train-following model to jointly control both the speed of a train and its spatial separation to the train in front. We term it a dual-controlled train-following model.

A. A Dual-Control Train-Following Model

Consider a simple train-following situation as illustrated in Fig. 1, where train n follows train $n-1$ on a single track. The variable $x_n(t)$ denotes the position of train n as measured from an arbitrary starting point, whilst $v_n(t)$ is its velocity at time t . L_{n-1} is the ‘effective’ size of train $n-1$, which includes the physical length of the train plus a safe margin.

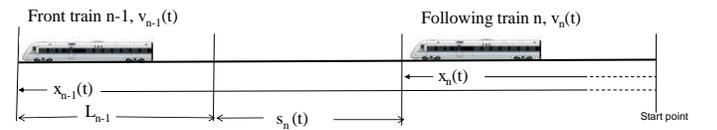


Fig. 1. Definition of a basic train following situation.

Let $v_n(t)$ denotes the velocity of a train n at time t , and $s_n(t)$ the distance gap behind the train in front. We formulate a controlled train-following model to predict the acceleration of the train at the next time instance time $t + \Delta t$:

$$\frac{dv_n(t + \Delta t)}{dt} = \alpha[V^*(s_n(t - t_d)) - v_n(t)] + \beta[s_n(t) - s_n^{des}] \quad (1)$$

where $V(\cdot)$ is an optimal velocity for a train n following at a distance gap s_n behind the train in front, and s_n^{des} is a desired headway for train n .

Eq. (1) is adapted from a multi-anticipatory model proposed by Chen and Liu [32] to model car following behaviour. It is an extension of a strand of well-studied car-following models based on the concept of an ‘optimal velocity’. The concept was first proposed by Bando et al. [29]; the difference between the optimal velocity and the velocity of the considered vehicle is assumed to be a stimulus for driver’s actions. Chen and Liu [32] consider the desired following distance as another explicit stimulus and formulate the following vehicle’s acceleration as a linear function of the optimal velocity and the desired distance.

The first term on the RHS of Eq. (1) represents a control mechanism for train n to reach its optimal speed $V(\cdot)$, while the second term models a control mechanism for train n to keep to its desired separation s_n^{des} to the train in front. α and β are sensitivity parameters for the speed difference term and the space gap term respectively. A higher value of α (or β) means that the train movement control is more sensitive to the speed difference (or to the space difference), than with a lower parameter value.

We adopt also from Chen and Liu [32] the following functions to model the optimal velocity in relation to the space gap:

$$V^*(s) = V_1 + V_2 \tanh(C_1 s - C_2) \quad (2)$$

and the desired following distance in relation to the current running speed and a desired following time T :

$$s^{\text{des}}(t) = s_0 + T v(t) \quad (3)$$

where the constants in (8) are set as: $V_1 = 13$ m/s, $V_2 = 20$ m/s, $C_1 = 0.005$ m⁻¹ and $C_2 = 0.1$. The parameter s_0 in (3) is the minimum space gap between trains and T represents a desired following time headway.

Eq. (2) models the optimal velocity as a function of the space gap, while Eq. (3) represents that the desired following gap varies with the current running speed of the train with a constant time headway. In the next Section, we analyse the stability property of this control mechanism for the ERTMS Level 3 system.

III. STABILITY ANALYSIS OF THE DOUBLY CONTROLLED TRAIN-FOLLOWING MODEL

The controlled train-following model provides a mechanism for the MB ERTMS Level 3 system to control and given movement command to trains following one another in the same direction on a track. At any given point in time, if a train’s speed and its space gap to the train in front in the

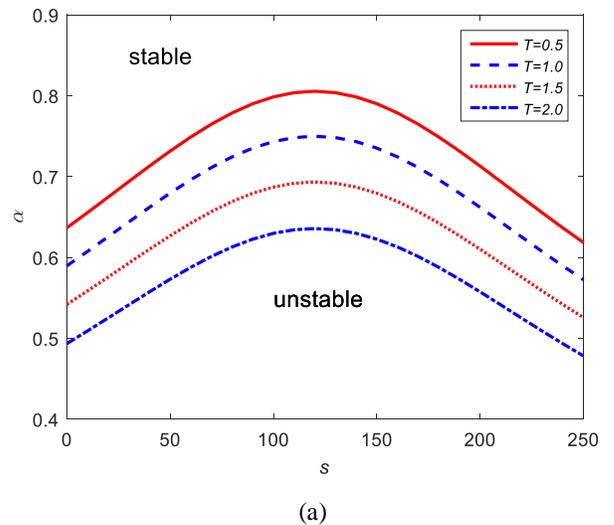
previous time instance are known, the acceleration of the following trains can be calculated using Eqs. (1) – (3).

However, the choice of the parameter values used in these equations can have a significant impact on the performance of the control mechanism in terms of safety and the resulting capacities. In this section, we discuss the train-following parameter values and their effect on stable headways.

A. Theoretical stability conditions

Chen and Liu [32] derived the theoretical condition for linear stability of the dual control model of Eq. (1) - (3). We illustrate below the sensitivity of some of the model parameter values on the stability of the traffic flow systems. Set the default parameter values as: $\beta = 0.05$, $V_1 = 13$ m/s, $V_2 = 20$ m/s, $C_1 = 0.005$ m⁻¹ and $C_2 = 0.1$, Fig. 2 shows the neutral stability lines in the space (α, s) for different values of the desired following time heading T in Eq. (3) and the system response time t_d in Eq. (1).

The area above each of the stability lines in Fig. 2 is the ‘stable region’ which means that, under those parameter values, the control mechanism will result in a stable train following at a safe following distance between two trains and smooth transition in speeds (with small acceleration and deceleration rates). The area below the stability lines represent the ‘unstable region’ where controls with such parameter settings would result in trains over-responding with significantly large acceleration or deceleration rates, causing unstable traffic flows.



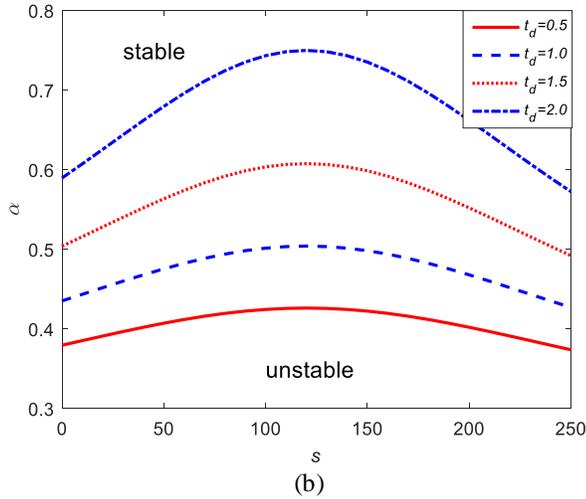


Fig. 2. Stability conditions in the space of (α, s) for different values of: (a) safe following time T , and (b) response time t_d .

Furthermore, results in Fig. 2a show that the stable region increases with increasing desired following time gap. For example, with a desired following time gap $T = 0.5$ sec, a control based on a speed sensitivity parameter $\alpha > 0.8$ is required to yield a stable train following. Whilst, with a longer following time gap, say $T = 2.0$ sec, most of the area above $\alpha > 0.6$ is stable.

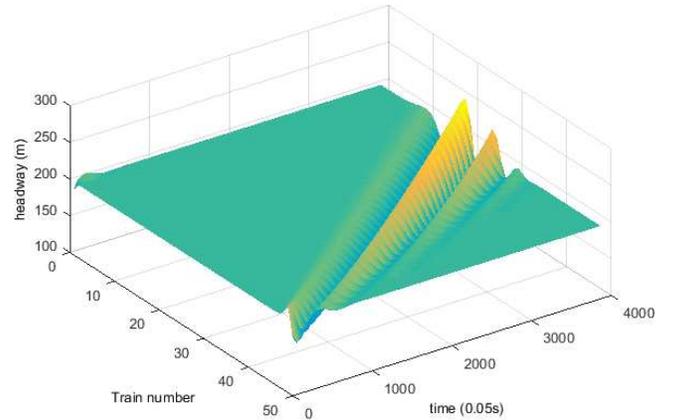
The model parameter t_d represents a delay time for the system (train-driver unit) to respond (to an incident, a speed changes, etc). Fig. 2b shows that the stable following regions are larger with smaller t_d values, implying that the traffic flow will be more stable when the system responds faster.

B. Performance of the dual controlled MB system in a closed boundary condition

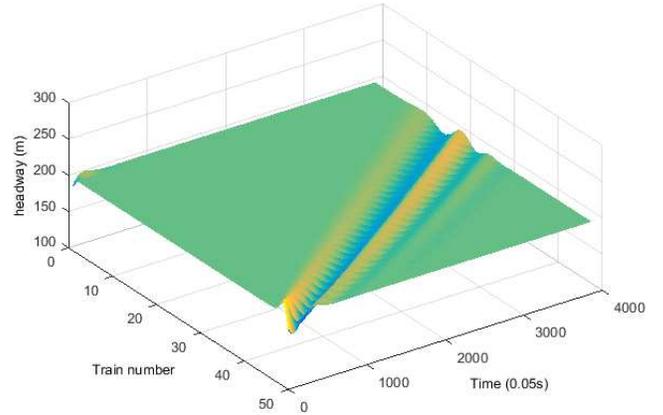
We perform numerical simulations of the controlled following model (1) – (3) under a closed boundary condition. We consider 50 trains travelling on a single-track circular route of length 10 km long. Initially, the trains are placed homogeneously on the track, with a uniform separation of 200m (including 100m train length) between adjacent trains. A small disturbance was introduced to the system. We show below how the disturbance propagates through the stream of trains and how the system stability is affected by the choice of control parameter values.

Fig. 3 shows how the headways between trains vary over trains and through time. In Fig. 3a, we see a disturbance in headways started at around 50sec between train numbers 49 and 50. This disturbance propagates and is amplified through time and space (to other trains). The original homogeneous flow evolves into stop-and-go congestion during the period from 50 sec to 200 sec.

With a different choice of control parameter value $\beta = 0.1$, Fig. 3b shows that the same initial disturbance can be better managed through the control mechanism.



(a)



(b)

Fig. 3. Space-time evolution of the following headways for: (a) $\beta = 0.05$, and (b) $\beta = 0.1$.

C. Performance of the dual controlled MB system in an open network

The dual controlled train-following model of (1) – (3) was discretized and implemented in the TrackULA simulation model. TrackULA is an individual train-based, network-wide, simulation of railway systems. It was adapted from the road network simulation package, code named DRACULA (for Dynamic Route Assignment Combining User Learning and microsimulation) developed at University of Leeds [22-24]. The railway version of the simulation model, TrackULA, was first developed to represent the signal controls at railway junctions under fixed-block operating system [33]. In this paper, the model is further extended to represent the MB system and the train control command under ERTMS Level 3.

We apply this simulation model to evaluate the performance of the dual control mechanisms proposed in Section 2. We set up simulation experiments to model the MB system on a single unidirectional track section with four stations as illustrated in Fig. 4. Trains enter at node A and exit at node D. Nodes B and C are station nodes where, depending on the schedule, trains may stop at one or both stations. The platforms at stations B and C are on the mainline carriageway. When a train stops at such stations, no other trains can by pass them. Therefore no overtaking is possible on this network. The three sections of the lines are each of 20km long.



Fig. 4. The test network for two different designs for the platforms at stations B and C.

Two types of trains are modelled: a fast train and slow train. The fast trains have a maximum speed of 200 km/hr, are 250m long and with a 200 m safety distance headway. The slow trains' maximum speed is 120 km/hr; they are 75 m long and have a 100m safety distance headway.

A total of 16 trains are scheduled to traverse the line from A to D, with one fast train followed by one slow train to depart from A. The fast trains do not stop, while the slow trains are scheduled to stop at stations B and C for 1 minute each.

The departure headway is 6 minutes for each of the two types of trains, with the first fast train departs at time 0 and the first slow train at time 1.5 minutes. Thus the departure times for all 16 trains are: {F0, S1.5, F6, S7.5, F12, S13.5, F18, S19.5, F24, S25.5, F30, S31.5} minutes, where letters F and S stands for fast and slow trains respectively.

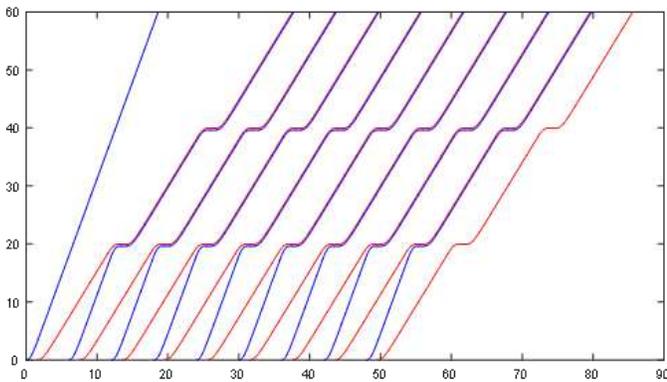


Fig. 5. Space-time trajectories of a group of 16 fast and slow trains traversing the network Fig. 4.

Fig. 5 shows the train trajectories simulated. We can see clearly that, except the first fast train who traverse through the entire network freely, all the other fast trains were obstructed by slow trains running in front of them.

Fig. 6 shows the close-following of trains 3 (a fast train) with the slow train 2 ahead. The closed following headway between the two trains was around 350m when their speeds were at unity.

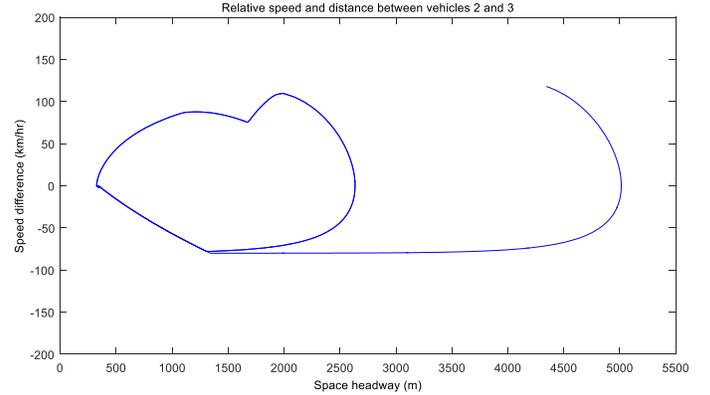


Fig. 6. Relative speed and space distance between the slow train 2 and its following fast train 3.

IV. DISCUSSIONS AND CONCLUSIONS

This paper presented a control algorithm to determine the train movement authorities, giving the location and speed of the train and its proceeding train on the same track in front. The algorithm is a dual-aspect in that it controls and balances the desire for the train to travel as an 'optimal velocity' and for it to keep a safe (desired) following distance to the train in front.

The generic formulation in Eq. (1) allows different 'weights' to be put to the two different aspects of the control, via the parameters α and β . A high value of α (or β) means that the train movement control is a more sensitive to the speed difference (or to the space difference); slight deviation to the optimal speed (or the desired space headway) would result in a large acceleration or deceleration move to the train.

For this control algorithm, we are able to derive theoretically the stability condition: where for a small disturbance of long wavelength, whether a uniform traffic flow would become stable or whether the disturbance would be amplified and the traffic flow never returns to its original uniform state.

This theoretical stability analysis and the example numerical results shown in Fig. 2 provide the train controller with a powerful method to determine the appropriate α value to choose, in order to manage a smooth traffic flow. Similar numerical results can be sought for the β value. The simulation results presented in Fig. 3 demonstrate how a small perturbation can lead to a larger (Fig. 3a), or a smaller (Fig. 3b), disturbances in the traffic flow depending on the choice of the parameter value β .

The practical implication of these results suggest that the train-following formulation and parameter values is critical to the success of the MB Level 3 control systems.

The performance of the control mechanism is illustrated in the simulation model TrackULA, developed as a tool to test the dynamical effect of traffic management and control strategies at a network level. The simulation test shows that the TrackULA implementation of the control algorithm is

The results presented in Figs. 5 and 6 show that the TrackULA simulation model captures the train movements in response to control on track and scheduled stoppings at stations accurately, and demonstrate the role simulation models can play in designing and testing good train control algorithms.

The model of Eq. (1) – (3) is just one form of a control algorithm which makes use of the technology afforded by ERTMS Level 3. More research is needed to develop other control mechanisms for the railway systems, and to investigate the trade-offs between traffic flow stability (and safety) with capacity.

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