Article:

Fernández, E. and Sgalambro, A. orcid.org/0000-0002-0052-4950 (2020) On carriers collaboration in hub location problems. European Journal of Operational Research, 283 (2). pp. 476-490. ISSN 0377-2217

https://doi.org/10.1016/j.ejor.2019.11.038

European Journal of Operational Research 283 (2020) 476–490

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

On carriers collaboration in hub location problems

Elena Fernández [a,*], Antonino Sgalambro [b,c]

[a] Department of Statistics and Operations Research, Universidad de Cádiz, Spain
[b] Sheffield University Management School, Sheffield, United Kingdom
[c] Istituto per le Applicazioni del Calcolo "Mauro Picone", National Research Council, Italy

ARTICLE INFO

Article history:
Received 6 October 2018
Accepted 18 November 2019
Available online 21 November 2019

Keywords:
Location
Hub location
Mixed integer programming
Carriers collaboration

ABSTRACT

This paper considers a hub location problem where several carriers operate on a shared network to satisfy a given demand represented by a set of commodities. Possible cooperative strategies are studied where carriers can share resources or swap their respective commodities to produce tangible cost savings while fully satisfying the existing demand. Three different collaborative policies are introduced and discussed, and mixed integer programming formulations are provided for each of them. Theoretical analyses are developed in order to assess the potential savings of each model with respect to traditional non-collaborative approaches. An empirical performance comparison on state-of-art sets of instances offers a complementary viewpoint. The influence of several diverse problem parameters on the performance is analyzed to identify those operational settings enabling the highest possible savings for the considered collaborative hub location models. The number of carriers and the number of open hubs have shown to play a key role; depending on the collaborative strategy, savings of up to 50% can be obtained as the number of carriers increases or the number of open hubs decreases.

1. Introduction

Hub location models address joint location and routing decisions in network systems. Location decisions focus on the selection of nodes to place hubs whereas the routing is to find paths for serving the existing demand between pairs of nodes, via the selected hubs. One essential characteristic of these networks is that direct connections between origin/destination pairs are substituted by fewer indirect, but privileged connections using the hubs. Typical applications include parcel delivery, air transportation, and telecommunications. Hub location is nowadays one of the most studied areas within locational analysis, because of its wide range of practical applications, and due to the high economic impact of the decisions encompassed in these problems. The seminal work (Campbell, 1992; 1994; O'Kelly, 1986; 1987a) has led to a very rich research field. Topics of current interest include, among others, the use of more general cost structures (Alibeyg, Contreras, & Fernández, 2016; 2018; Puerto, Ramos, Rodríguez-Chía, & Sánchez-Gil, 2016; Taherkhani & Alumur, 2019), competitive settings (Lüer-Villagra & Marianov, 2013; Sasaki, Campbell, Ernst, & Krishnamoorthy, 2009), solution procedures (Contreras, Cordeau, & Laporte, 2011a; 2011b; Contreras, Díaz, & Fernández, 2011c; Martins de Sá, Contreras, & Cordeau, 2015), and the use of special topologies for the inter-hub network (Contreras & Fernández, 2012; Contreras, Fernández, & Marín, 2010; Contreras, Tanash, & Vidyarthi, 2017). The reader is addressed to Campbell and O'Kelly (2012); Contreras (2015) for overviews of recent challenging developments in the area.

It often arises in practice that multiple carriers operating within the same underlying network compete for capturing customers when the same type of service is offered. This motivated the study of hub location models where carriers aim at maximizing the demand captured within a given coverage radius (Campbell & O'Kelly, 2012; Lowe & Sim, 2012), aim at maximizing the net profit (Alibeyg et al., 2016; 2018), or compete with other carriers to maximize their market share (Eiselt & Marianov, 2009; Marianov, Serra, & ReVelle, 1999). In Adler and Smilowitz (2007) global alliances and mergers in the airline industry under competition are developed within a game theoretical framework for a stylized hub-and-spoke model. Carriers may however be motivated to adopt a collaborative perspective for the design of a hub location system, as an alternative to the competitive one. This is the focus of this paper as we explain below.

* Corresponding author.
E-mail addresses: e.fernandez@upc.edu, elena.fernandez@uca.es (E. Fernández), a.sgalambro@sheffield.ac.uk (A. Sgalambro).

https://doi.org/10.1016/j.ejor.2019.11.038

The benefits of collaboration in logistic systems have received increasing attention especially in the last two decades. Different business conditions as well as external regulations may push carriers to consider collaborative strategies, in view of the huge savings and environmental improvements that may occur. Therefore, different collaboration modes at different levels of logistic systems have been proposed and studied in the literature (see, e.g., Crainic & Laporte, 1997; Okdinawati, Simatupang, & Sunitiyoso, 2015). The literature on this topic is certainly very large and an extensive review of the state of the art is outside the scope of this paper. We thus refer the interested reader to Verdonck, Caris, Ramaekers, and Janssens (2013) and Simatupang and Sridharan (2008) for discussions on the related literature.

The collaboration that takes place between agents that operate at the same level of the supply chain is known as horizontal cooperation (Cruijssen, Cools, & Dullaert, 2007; Pomponi, Fratocchi, Tafuri, & Palumbo, 2013). This type of collaboration is particularly suitable when different carriers establish alliances for improving the performance of logistic systems, especially service to common customers. Several optimization models for horizontal carrier collaboration have been studied in the last years in fields related to transportation and logistics (see, for instance, Defryn, Sörensen, & Dullaert, 2019; Fernández, Fontana, & Speranza, 2016; Fernández, Roca-Riu, & Speranza, 2018; Quintero-Araujo, Gruler, Juan, & Faulin, 2019). In the area of hub location we are not aware of any previous work dealing with collaboration, except for Hernández, Unnikrishnan, and Awale (2012) that considers a hybrid centralized hub location system shared by all the carriers. The proposed model is *centralized* as it merges the collaborative activities of all carriers within a unified hub-and-spoke system, but is *hybrid* in the sense that direct shipment of commodities is allowed, and decided by each carrier on a one-to-one basis. The objective mixes costs of both the centralized system and direct shipments of all carriers. In this paper we follow a different approach, analyzing and comparing several collaborative policies, encompassing uncentralized systems as well. Direct shipments are excluded, allowing to better focus on the collaborative aspects of the studied systems.

Despite the little work developed on the topic, examples of this type of collaborative approaches abound in classical hub location application fields, like air and ground transportation networks. Indeed, business opportunities may encourage partners of airline alliances to search for hub transportation networks that better suit their joint interests. Moreover, cooperation may also be partly due to regulations restricting the ability of carriers to operate flights to locations in a foreign country beyond the primary airport that the carrier uses to facilitate international connections. Thus, cooperation between partners based in different countries allows carriers greater access to potential passengers in locations where they are not allowed to operate their own flights. Mail and parcel delivery are also classical application areas for hub location, where carrier collaboration may improve systems performance. The highly negative effect of heavy traffic in urban areas related to numerous simultaneous trips with low load factors has motivated local authorities to encourage carrier collaboration in order to reduce not only operational costs, but also traffic and space occupancy.

In this work we assume that multiple carriers operate on a shared network and make optimal decisions on the location of their respective hubs and the routing of their demands through the network. We propose three alternative collaborative policies, each of them linked to a different optimization model, which are contrasted against the non-collaborative setting. The first model reflects the maximum possible collaboration in the sense that carriers operate jointly, sharing their resources and serving the overall demand as if they had merged into a single carrier. Instead, in the other two models each carrier operates exclusively on a backbone network induced by its own hubs. While in the second model each commodity is served by the carrier that offers the most efficient routing, in the third model all the commodities with the same origin are routed by the same carrier. The characteristics of each model are illustrated with a small numerical example, which is also used to highlight their differences. We show theoretically that all three collaborative models may produce arbitrarily large savings with respect to the non-collaborative situation. Furthermore, mixed integer linear programming formulations are proposed in each case. Extensive computational experiments have been carried out in order to analyze the empirical performance of each of the models, using a set of benchmark instances adapted from the literature. We give results on the computational effort required to optimally solve each formulation with a commercial solver, and compare the performance of the different policies, under several parameters settings. Useful managerial insight is gathered from the obtained numerical results.

Our theoretical results apply to general networks, not necessarily complete, allowing for set-up costs for the elements that are activated. However, for the sake of clarity in the presentation, the formulations we propose for each of the collaboration policies are first developed over fundamental p-hub location models (Marín, Cánovas, & Landete, 2006; O'Kelly, 1987a), under the assumption that the input graph is complete, and ignoring all set-up costs. In that case there exist optimal solutions in which the routing paths for the commodities consist of three edges at the most. This simplifies the presentation, and allows to concentrate on the role of the collaboration policies. Still, the difficulty of such formulations should not be understimated, since instances with up to twenty nodes involve over 400,000 variables already for two carriers, and more than 6.5 million variables when the number of carriers raises to ten. For the above reasons, nearly all our computational experiments have been carried out with the formulations over the fundamental models, whose numerical results already highlight their empirical difficulty, particularly as the number of carriers increases. When the model involves set-up costs on the network design decisions the maximum number of edges used in optimal routing paths cannot be set in advance so the resulting formulations are notably more involved. Then, possible alternatives are, for instance, to impose the modeling assumption that routing paths contain at most one inter-hub connection, or to allow arbitrary long inter-hub paths. The first alternative preserves in essence the structure of the formulations at the expense of losing some generality, even if new binary decision variables and additional constraints are still needed. The second alternative gains generality at the expense of producing more involved formulations requiring further decision variables and constraints. In this work we opt for the first alternative, and develop some extensions of the basic formulations that include set-up costs for the activated elements while still restricting routing paths to contain at most one inter-hub connection. We have carried out a set of additional computational experiments in order to gain further insight on the tradeoff between the alternative collaboration policies with these extended formulations.

The main contributions of this paper are the following:

(i) We introduce hub location models dealing with carrier collaboration, based on three alternative potential policies for the collaboration agreement. To the best of our knowledge this is the first time that collaboration among carriers, encompassing uncentralized systems, is studied in the context of hub location.

(ii) We develop a theoretical worst-case analysis to quantify the potential savings that each collaboration policy may produce relative to the non-collaborative setting. This analysis applies to general networks, not necessarily complete, allowing for set-up costs for the elements that are activated.

(iii) For each collaboration policy, mixed-integer mathematical programming formulations are computationally tested in terms of: their difficulty for being optimally solved with a commercial solver, their sensitivity to several diverse problem parameters, their relation to the other formulations, and the structure of the solution networks they produce.

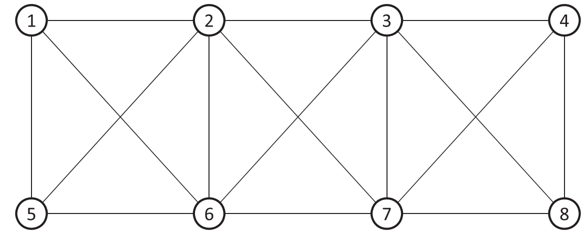
The remainder of the paper is organized as follows. In Section 2 we briefly introduce the notation, before presenting in detail the three different carrier collaboration models. Sections 3, 4 and 5 are respectively devoted to the model with full collaboration, the model where commodities can be transferred to other carriers and the model where all commodities with the same origin are routed by the same carrier. All three policies are compared theoretically by analyzing in each case the potential savings obtained with respect to a non-collaborative behavior, and mixed integer programming formulations are presented. Section 6 is dedicated to the computational experience we have carried out: the computational environment and sets of benchmark instances we have used are described, and the obtained numerical results summarized and analyzed, deriving some managerial insights. The paper ends with some conclusions and a discussion on promising avenues for future research.

2. Notation and modeling assumptions

Consider a set of carriers T that operate on the same network $G = (V, A)$, where $V = \{1, 2, \dots, n\}$. For each carrier $t \in T$, $c^t_{ij} \geq 0$ is the unit transportation cost through arc $(i, j) \in A$, not necessarily symmetric. We assume that this cost function satisfies the triangle inequality. We further assume that $c^t_{ii} = 0$, for all $i \in V$, $t \in T$. For $t \in T$, let $p^t \leq n$ be a parameter indicating the number of hubs that carrier t must locate (open), $H^t \subseteq V$ the set of potential locations for the hubs of t, f^t_k the set-up cost for opening a hub at node $k \in H^t$, and $h^t_{k,k'}$ the set-up cost for carrier t activating the inter-hub arc $(k, k') \in A$. Each carrier $t \in T$ has a service demand given by a set of commodities, indexed in a set R^t. Let $\mathcal{D}^t = \{(o_r, d_r, w_r) : r \in R^t\}$ denote the set of commodities of carrier t, where the triplet (o_r, d_r, w_r) indicates that an amount of flow $w_r > 0$ must be routed from origin $o_r \in V$ to destination $d_r \in V$. The origin/destination pair associated with a given commodity will also be referred to as its OD pair. We denote by $W^t_i = \sum_{r \in R^t : o_r = i} w_r$ the overall flow of carrier t with origin at $i \in V$. Finally, we denote by $R = \bigcup_{t \in T} R^t$ the index set of the joint set of commodities $\mathcal{D} = \bigcup_{t \in T} \mathcal{D}^t$.

As usual in hub location models, the flows between OD pairs (o_r, d_r), $r \in R$, must be routed via *feasible paths* of the form $(o_r, k_1, \dots k_{i_r}, d_r)$ where k_{i_s}, $s = 1 \dots, i_r$, are open hubs. Throughout we assume multiple allocation of commodities to open hubs. That is, it is possible that two commodities with the same origin are routed using a different first hub. Let $0 < \alpha \leq 1$ denote the discount factor that is applied to the transportation costs of the flows routed through inter-hub arcs, which we assume independent of the carrier. The routing cost of commodity $r \in R^t$, $t \in T$ via path $P = (o_r, k_1, \dots k_{i_r}, d_r)$ is thus $C^r_P = w_r(c^t_{o_r k_1} + \alpha \sum_{s=1}^{i_r - 1} c^t_{k_s k_{s+1}} + c^t_{k_{i_r} d_r})$.

We will denote by *non-collaborative* **p**-*hub location problem* (**p**-NCHLP) the problem arising when all carriers operate independently, and thus without any type of collaboration. In the **p**-NCHLP each carrier $t \in T$ must make the following decisions: (*i*) Select a subset of hubs $S^t \subset H^t$, with $|S^t| = p^t$; and, (*ii*) route all commodities in D^t via selected hubs. The objective is to minimize the sum of the set-up costs of the hubs activated by each carrier, plus the set-up costs of the inter-hub connections used by each carrier, plus the overall routing costs over all the commodities. Indeed, the above problem can be decomposed in $|T|$ independent p^t-hub location subproblems, one for each carrier.



(a) Input graph.

Fig. 1. Illustrative example for two carriers with $p^1 = p^2 = 2$. $\mathcal{D}^1 = \{(5, 4, M), (5, 2, \varepsilon), (1, 4, \varepsilon), (1, 8, \varepsilon)\}$, $\mathcal{D}^2 = \{(1, 8, M), (1, 6, \varepsilon), (5, 4, \varepsilon), (5, 8, \varepsilon)\}$.

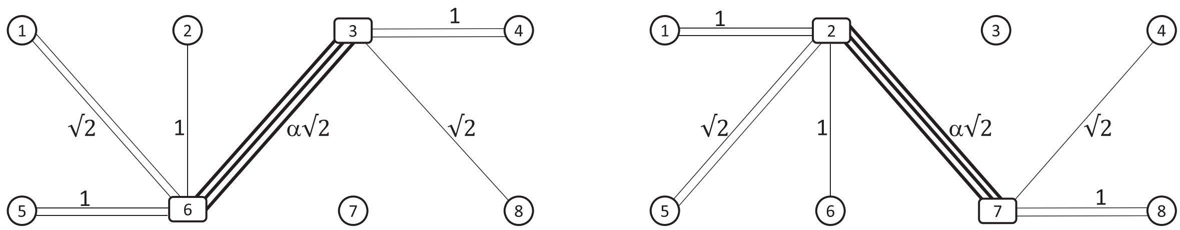
3. The unrestricted collaboration hub location problem

The **p**-*unrestricted collaboration hub location problem* (**p**-UCHLP) assumes full collaboration among carriers. That is, we assume that all carriers make a (unique) joint decision referring to both the location of the hubs and the routing of the commodities, as if they had merged into one single company. Hence the **p**-UCHLP reduces to select $p = \sum_{t \in T} p^t$ hubs from $H = \bigcup_{t \in T} H^t$ and to route the joint set of commodities $\mathcal{D}$ via the selected hubs so as to minimize the overall set-up plus routing costs. Therefore, the **p**-UCHLP is an uncapacitated multiple allocation hub location problem on the given input graph where $\Sigma_{t \in T} p^t$ hubs must be opened.

Example 1. Consider the eight node network depicted in Fig. 1, where two carriers operate with the same transportation costs. Each horizontal or vertical link has a unit transportation cost $c_{ij} = 1$, whereas *diagonal* links have unit transportation costs $c_{ij} = \sqrt{2}$. For ease of presentation let us assume that there are no set-up costs, neither for the facilities nor for inter-hub arcs, so only the routing costs affect the quality of the solutions. Assume also that $\mathcal{D}^1 = \{(5, 4, M), (5, 2, \varepsilon), (1, 4, \varepsilon), (1, 8, \varepsilon)\}$, $\mathcal{D}^2 = \{(1, 8, M), (1, 6, \varepsilon), (5, 4, \varepsilon), (5, 8, \varepsilon)\}$, $H^1 = H^2 = V$ and $p^1 = p^2 = 2$. Let us finally assume that the discount factor α is such that $1 + \alpha < \sqrt{2}$ and $\varepsilon \ll M$.

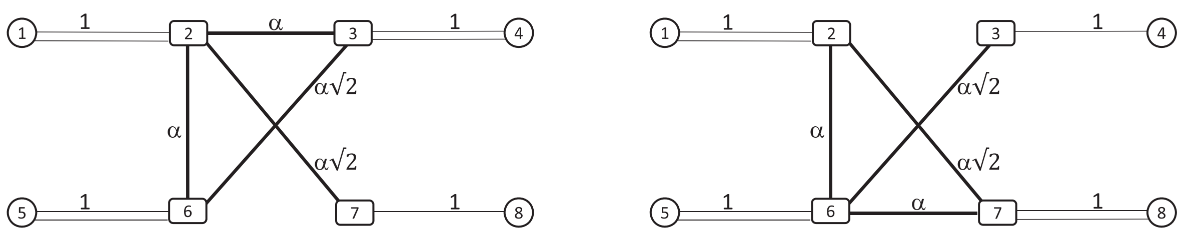
Fig. (2a) and (2b) depict optimal solutions for the **p**-NCHLP, for carriers 1 and 2, respectively, when both carriers operate independently. Each arc is replicated as many times as it is used in the solution. Thick lines indicate inter-hub connections. Carrier 1 opens hubs at nodes 3 and 6. The commodities of $\mathcal{D}^1$ are routed as follows: commodity (5, 4, M) via path $5-6-3-4$; commodity (5, 2, ε) via path $5-6-2$; commodity (1, 4, ε) via path $1-6-3-4$; and commodity (1, 8, ε) via path $1-6-3-8$. Carrier 2 opens hubs at nodes 2 and 7. The routing of its commodities is: commodity (1, 8, M) via path $1-2-7-8$; commodity (1, 6, ε) via path $1-2-6$; commodity (5, 4, ε) via path $5-2-7-4$; and commodity (5, 8, ε) via path $5-2-7-8$. The routing cost is the same for both carriers. Each of them has a routing cost of $M(2 + \sqrt{2}\alpha) + \varepsilon[3 + 3\sqrt{2} + 2\alpha\sqrt{2}]$.

Fig. 3 depicts an optimal solution to the **p**-UCHLP, where carriers 1 and 2 fully collaborate, for the instance of Fig. 1. Now the $p_1 + p_2$ open hubs are located at nodes 2, 3, 6 and 7, which are connected with inter-hub discounted edges. Optimal routings for the commodities of carriers 1 and 2 are shown in Figs. (3a) and (3b), respectively (note that alternative optimal routings exist). Again carrier 1 routes commodity (5, 4, M) via path $5-6-3-4$, and (5, 2, ε) via path $5-6-2$ (now the discount factor is applied to arc (6, 2), since both 6 and 2 are hubs). However, (1, 4, ε) is now routed via path $1-2-3-4$, and commodity (1, 8, ε) via path $1-2-7-8$. Carrier 2 uses the same routing paths as without collaboration for commodities (1, 8, M) and (1, 6, ε), $1-2-7-8$ and $1-2-6$, respectively, although in the latter the discount factor is applied to arc (2, 6) that connects hubs 2 and 6. Commodity



(a) Solution for carrier 1 (b) Solution for carrier 2

Fig. 2. Solution of the non-collaborative model for the illustrative example of Fig. 1.



(a) Solution of p^1p^2-UCHLP for carrier 1 (b) Solution p^1p^2-UCHLP for carrier 2

Fig. 3. Solution for the example of Fig. 1 for the p^1p^2-UCHLP with full collaboration.

(5, 4, ε) is now routed via path 5 – 6 – 3 – 4; and commodity (5, 8, ε) via path 5 – 6 – 7 – 8. The routing cost of carrier 1, which is the same as that of carrier 2, is $M(2+\sqrt{2}\alpha)+\varepsilon[5+(2+\sqrt{2})\alpha]$.

3.1. Worst-case analysis

In this section we make a worst-case comparison between the full collaboration **p**-UCHLP and the non-collaboration model **p**-NCHLP. Let us denote by $z^*(UCHLP)$ the optimal value of the **p**-UCHLP and by $z^*(NCHLP)$ the optimal value of the **p**-NCHLP on the same given instance. As we see below the costs savings that can be obtained with the **p**-UCHLP with respect to the **p**-NCHLP can be arbitrarily large. Fig. 4 illustrates one such example, which we will use in the proof of Theorem 1.

Theorem 1.

(i) There exists no finite upper bound for the ratio $z^*(NCHLP)/z^*(UCHLP)$.
(ii) $1 \le z^*(NCHLP)/z^*(UCHLP)$ and the bound is tight.

Proof.

(i) To prove the result it is enough to show that there is at least one instance for which the ratio is arbitrarily large. Let us show one such instance.
Consider the input network $G=(V,A)$ depicted in Fig. 4 where two carriers operate. For both carriers, the unit transportation cost through each horizontal arc is M (arbitrarily large), through each vertical arc ε (arbitrarily small), and through each *diagonal* arc $\sqrt{M^2+\varepsilon^2}$. Let us assume that both carriers have the same set-up costs for the hubs and for inter-hub arcs, and that, in each case, these costs are all the same, i.e. for $t\in\{1,2\}$, $f_k^t=\bar{f}$, for all $k\in V$, and $h_{k,k'}^t=\bar{h}$, for all $(k,k')\in A$. Let us also assume that both carriers have the same demand $\mathcal{D}^1=\{(1,4,1),(2,3,1),(5,8,1),(6,7,1)\}$, $\mathcal{D}^2=\{(1,4,1),(2,3,1),(5,8,1),(6,7,1)\}$, $H^1=H^2=V$ and $p^1=p^2=2$. Finally, let us assume a discount factor α.
In the non-collaboration model **p**-NCHLP both carriers locate their hubs at nodes 9 and 10, and activate the inter-hub arc (9, 10). The routing of the commodities is the same for both carriers and is indicated in Table 1. The total cost is $z^*(NCHLP)=2[2\bar{f}+\bar{h}]+8\left[M+\sqrt{M^2+\varepsilon^2}+\varepsilon\alpha\right]$.
In the full collaboration model **p**-UCHLP, carrier 1 locates the hubs at nodes 2 and 3 and activates the inter-hub arc (2, 3) while carrier 2 at nodes 6 and 7 and activates the inter-hub arc (6, 7) (or vice-versa). Carrier 1 routes all the commodities with origin at nodes 1 and 2, whereas carrier 2 routes the commodities with origin at nodes 5 and 6. The routing of the commodities is also indicated in Table 1 and the total cost of the solution is $z^*(UCHLP)=2[2\bar{f}+\bar{h}]+8\varepsilon[1+\alpha]$. Therefore

$$\frac{z^*(NCHLP)}{z^*(UCHLP)}=\frac{2[2\bar{f}+\bar{h}]+8\left[M+\sqrt{M^2+\varepsilon^2}+\varepsilon\alpha\right]}{2[2\bar{f}+\bar{h}]+8\varepsilon[1+\alpha]},$$

which tends to ∞ when $M\to\infty$.

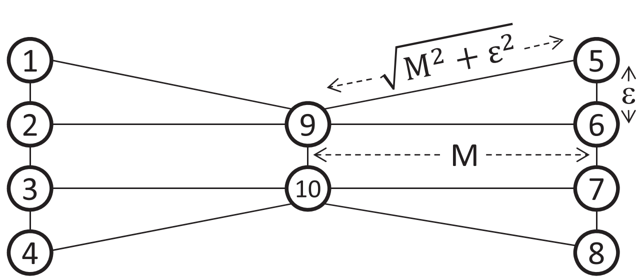
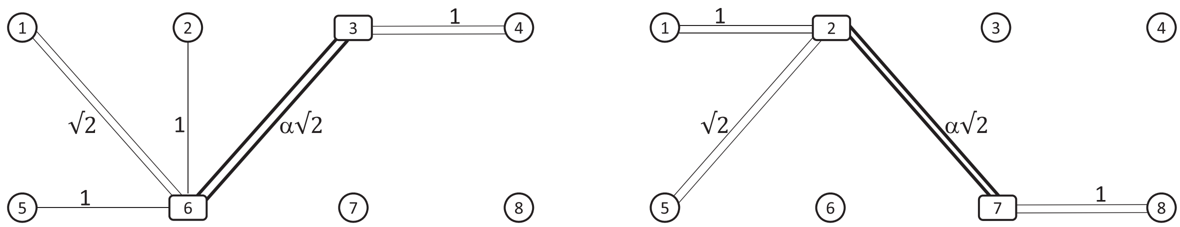


Fig. 4. Input graph to illustrate potential savings of **p**-UCHLP relative to **p**-NCHLP for two carriers.

Table 1
Routing paths of commodities in worst-case construction.

	No coll.		**p**-UCHLP	
	Path	Routing cost	Path	Routing cost
(1, 4, 1)	1 – 9 – 10 – 4	$2\sqrt{M^2+\varepsilon^2}+\alpha\varepsilon$	1 – 2 – 3 – 4	$2\varepsilon+\varepsilon\alpha$
(2, 3, 1)	2 – 9 – 10 – 3	$2M+\alpha\varepsilon$	2 – 3	$\varepsilon\alpha$
(5, 8, 1)	5 – 9 – 10 – 8	$2\sqrt{M^2+\varepsilon^2}+\alpha\varepsilon$	5 – 6 – 7 – 8	$2\varepsilon+\varepsilon\alpha$
(6, 7, 1)	6 – 9 – 10 – 7	$2M+\alpha\varepsilon$	6 – 7	$\varepsilon\alpha$



(a) Solution of p^1p^2-CCHLP for carrier 1 (b) Solution p^1p^2-CCHLP for carrier 2

Fig. 5. Solution for the example of Fig. 1 for the **p**-CCHLP with commodity transfer.

(ii) To see that $1 \le z^*(NCHLP)/z^*(UCHLP)$ note that $z^*(UCHLP) \le z^*(NCHLP)$ since any feasible solution for the **p**-NCHLP is also feasible for the **p**-UCHLP.
To see that the bound is tight we observe that it is also possible to build examples where collaboration produces no savings. One such example is to consider again the instance of Fig. 4 with $H^1 = H^2 = V$ and $p^1 = p^2 = 2$, but assume that the carriers demand is $\mathcal{D}^1 = \{(1,5,1), (2,6,1), (3,7,1), (4,8,1)\}$, $\mathcal{D}^2 = \{(1,5,1), (2,6,1), (3,7,1), (4,8,1)\}$. □

Despite the potential savings that the **p**-UCHLP may produce with respect to the non-collaboration model, it can be argued that, even if the overall number of hub nodes opened in the unrestricted collaboration model **p**-UCHLP is the same as when no collaboration exists, in practice the cost for the design of the network where carriers will operate will be higher, as more inter-hub edges will be activated. This will happen especially when there are no inter-hub set-up costs, or such costs are small in comparison to the other costs. In particular, in the example of Fig. 1 the non-collaborative model uses only two inter-hub edges (edge (3, 6) for carrier 1 and edge (2, 7) for carrier 2) while the solution to **p**-UCHLP uses five inter-hub edges, namely (2, 3), (2, 6), (2, 7), (3, 6), and (6, 7).

In the next sections we develop two alternative collaboration models where each carrier activates its own hub nodes so the maximum overall number of activated inter-hub edges is the same as in the unrestricted collaboration model. In all such models the following assumptions hold: common hubs for different carriers are allowed (i.e. multiple carriers might choose the same hub), and, shipment routes cannot share networks of different carriers, that is, all the intermediate hubs and inter-hub links used in a given shipment route must be activated by the same carrier.

4. The commodity-transfer collaboration hub location problem

For the *commodity-transfer collaboration hub location problem* (**p**-CCHLP) we assume the following agreement among the carriers. Each carrier selects p^t locations for the hubs to open, among its candidate set H^t. Then, the routing of each commodity can be *transferred* among carriers, so a commodity of $\mathcal{D}^t$, $t \in T$ can be routed by a different carrier $t' \in T$, $t' \neq t$, in the network induced by the hubs selected by carrier t', and vice-versa. The objective is to minimize the sum of the routing costs of all the commodities. Fig. 5 shows an optimal solution to the **p**-CCHLP for the instance of Fig. 1. Like in the problem without collaboration, carrier 1 opens hubs at nodes 3 and 6, and carrier 2 opens hubs at nodes 2 and 7. Now the routing of commodity $(1,8,\varepsilon) \in \mathcal{D}^1$ is transferred to carrier 2 who offers the smallest routing cost $(2+\sqrt{2}\alpha$ instead of $2\sqrt{2}+\sqrt{2}\alpha)$. Similarly, the routing of commodity $(5,2,\varepsilon) \in \mathcal{D}^1$ is transferred to carrier 2. Furthermore, the routing of commodities $(5,4,\varepsilon) \in \mathcal{D}^2$ and $(1,6,\varepsilon) \in \mathcal{D}^2$ is transferred to carrier 1. All other commodities are routed by their original carriers.

The networks of both carriers and the commodities routed by each of them are shown in Fig. (5a) and (5b), respectively. Now carrier 1 routes a flow $M+\varepsilon$ corresponding to commodities $(5,4,M) \in \mathcal{D}^1$ and $(5,4,\varepsilon) \in \mathcal{D}^2$ via path $5-6-3-4$. Furthermore, carrier 1 also routes commodity $(1,4,\varepsilon) \in \mathcal{D}^1$ via path $1-6-3-4$, and commodity $(1,6,\varepsilon) \in \mathcal{D}^2$ via the direct path $1-6$. Carrier 2 routes a flow $M+\varepsilon$ corresponding to commodities $(1,8,M) \in \mathcal{D}^2$ and $(1,8,\varepsilon) \in \mathcal{D}^1$, via the same path as before $(1-2-7-8)$. Furthermore, carrier 2 also routes commodity $(5,2,\varepsilon) \in \mathcal{D}^1$ via the direct path $5-2$, and commodity $(5,8,\varepsilon) \in \mathcal{D}^2$ via path $5-2-7-8$. The routing cost of each of the carriers is $M(2+\sqrt{2}\alpha)+\varepsilon\left[3+2\sqrt{2}+2\sqrt{2}\alpha\right]$.

Practical settings where the **p**-CCHLP may be of interest include potential collaboration agreements both in air transportation and parcel deliveries. Nowadays it is common that several air companies offer a shared service of origin/destination trips, which are *operated* by one single line but offer transport to customers of other companies in the same alliance. It is also common in parcel delivery that, depending on the area, service is carried out by a company different from the one that was actually hired.

The following properties hold for the **p**-CCHLP:

Remark 1. *P1* Feasible solutions for the **p**-CCHCP are also feasible for the **p**-UCHLP. Clearly, the reverse is not true. Thus the optimal value of the unrestricted collaboration model gives a lower bound on the optimal value of the commodity-transfer collaboration model.

Worst-case analysis indicates that the commodity-transfer collaboration model **p**-CCHLP may also produce arbitrarily large savings in relation to the non-collaboration model **p**-NCHLP. Observe that the **p**-UCHLP solution built in item (*i*) of the proof of Theorem 1, which illustrates potential arbitrarily large savings, is also a feasible solution to the **p**-CCHLP on that instance. On the other hand, the **p**-UCHLP solution built in item (*ii*) of the proof of Theorem 1 is feasible for the **p**-CCHLP on that instance as well. We therefore have the following result:

Theorem 2. *Let $z^*(CCHLP)$ denote the optimal value of the* **p**-*CCHLP for a given instance and by $z^*(NCHLP)$ the optimal value of the same instance for the* **p**-*NCHLP. Then,*

(i) There exists no finite upper bound for the ratio $z^*(NCHLP)/z^*(CCHLP)$.
(ii) $1 \le z^*(NCHLP)/z^*(CCHLP)$ and the bound is tight.

4.1. Mathematical programming formulation for the **p**-*CCHLP*

Here we develop mathematical programming formulations for some particular cases of the **p**-CCHLP. We start with the case when the underlying hub location model is defined over a complete graph and without any set-up costs, that we denote **p**-CCHLP$_0$, which allows to concentrate on the role of the collaboration policy.

Remark 2.

- Property P2: There is an optimal solution to the $\mathbf{p}$-CCHLP$_0$ where routing paths use one inter-hub edge at the most. The assumption that the input graph is complete and that routing costs satisfy the triangle inequality, together with the fact that there are no set-up costs, indicate that if an optimal solution routed some flow via a path with more than one inter-hub edge, such a path could be substituted by a direct arc connecting the first and last hubs in the path without deteriorating the value of the solution.
- As opposed to the $\mathbf{p}$-UCHLP, which can be decomposed in $|T|$ independent subproblems, the $\mathbf{p}$-CCHLP$_0$ (and its extensions) cannot be decomposed in independent subproblems, since the actual set of commodities that will eventually be served by each of the carriers is not known in advance. That is, the problem incorporates one additional level to the decision-making process, corresponding to the allocation of each commodity of the global set R to one of the carriers; recall that such allocation can be different from the carrier the commodity originally corresponded to. The decision variables that we use in the formulation below, reflect this additional decision-making level by adding one additional index to both the location and the routing variables of the classical 4-index formulation for the p-hub location problem. Hence, the formulation below is, in fact, a 5-index formulation, which can be seen as an extension of the 4-index formulation of Marín et al. (2006) for the $\mathbf{p}$-UCHLP. To the best of our knowledge, the formulation we present is new, as it simultaneously (re)-partitions the global set of commodities among the carriers, and builds $|T|$ p-hub location networks for serving the commodities allocated to each of the carriers.

As a consequence of property P2, we can restrict our attention to routing paths of the form (o_r, k, l, d_r) where k, l are open hubs and it is possible that $k = l$. When both o_r and d_r are hubs these paths reduce to (o_r, o_r, d_r, d_r) and consist of one single arc. Otherwise, if o_r is not a hub then $k \neq o_r$; similarly, $l \neq d_r$ if d_r is not a hub. Moreover, the unit routing cost through path (o_r, k, l, d_r) reduces to $C^r_{kl} = w_r(c^t_{o_r k} + \alpha c^t_{kl} + c^t_{ld_r})$.

Based on Remark 2, we can build a valid formulation for the $\mathbf{p}$-CCHLP$_0$ using the following decision variables:

- Binary variables z^t_k, $k \in H^t$, $t \in T$. $z^t_k = 1$ if and only if a hub is opened at vertex k for carrier t.
- Binary variables x^{rt}_{kl}, $r \in R$, $t \in T$. $x^{rt}_{kl} = 1$ if and only if commodity r is routed by carrier t via hubs k and l.

The formulation is as follows:

$$\mathbf{p}-CC_0: \quad \min \sum_{r\in R}\sum_{t\in T}\sum_{k,l\in H^t} C^r_{kl} x^{rt}_{kl} \tag{1a}$$

$$s.t. \quad \sum_{k\in H^t} z^t_k = p^t \qquad t \in T \tag{1b}$$

$$\sum_{t\in T}\sum_{k,l\in H^t} x^{rt}_{kl} = 1 \qquad r \in R \tag{1c}$$

$$x^{rt}_{kk} + \sum_{l\in H^t:l\neq k} x^{rt}_{kl} + \sum_{l\in H^t:l\neq k} x^{rt}_{lk} \leq z^t_k \qquad r \in R, t \in T, k \in H^t \tag{1d}$$

$$x^{rt}_{kl} \geq 0 \qquad r \in R, k, l \in H^t, t \in T \tag{1e}$$

$$z^t_k \in \{0, 1\} \qquad k \in H^t, t \in T \tag{1f}$$

Constraints (1b) impose that the number of hubs selected by each carrier is correct. Constraints (1c) guarantee that each commodity is routed by exactly one carrier, whereas (1d) guarantee that the intermediate nodes used to serve a commodity are open as hubs by the carrier that routes it. Finally, (1e)–(1f) define the domain of the variables. Note that, as it happens with other uncapacitated hub location formulations, it is enough to impose the integrality of the z variables.

Formulation (1a)–(1f) has $\Sigma_{t\in T}|H^t|$ binary variables z and $\Sigma_{t\in T}|H^t|^2$ continuous variables x. Its number of constraints is $|T| + |R|\left(1 + \sum_{t\in T}|H^t|\right)$.

Below we extend formulation (1a)–(1f) to deal with set-up costs on the design decisions, under the modeling assumption that routing paths use one inter-hub link at the most. We refer to this problem as $\mathbf{p}$-CCHLP$_{\text{Setup}}$. Recall that, in the presence of set-up costs, property P2 no longer holds, so it is needed to impose it explicitly.

Since, feasible routing paths for $\mathbf{p}$-CCHLP$_{\text{Setup}}$ are the same as those of $\mathbf{p}$-CCHLP$_0$, the *only* change in the formulation affects the objective function, where the terms corresponding to the activated hubs and inter-hub links must be included. Using the design variables z the set-up costs of the activated hubs can be written as:

$$\sum_{t\in T}\sum_{k\in H^t} f^t_k z^t_k. \tag{2a}$$

Unfortunately, with the above decision variables it is not possible to express the set-up costs of the inter-hub arcs used by the different carriers. For this, we define an additional binary variable X^t_{kl} associated with each carrier $t \in T$ and potential pair of intermediate hubs k, $l \in H^t$, whose value is one if and only if carrier t activates the inter-hub link (k, l). Then the set-up cost for inter-hub arcs can be expressed as:

$$\sum_{t\in T}\sum_{\substack{k,l\in H^t\\k<l}} h^t_{kl} X^t_{kl}. \tag{3a}$$

To guarantee the correct activation of the inter-hub arcs that are used, the routing variables x used for routing flows through inter-hub arcs must be related to the new design variables X. This can be done by means of the set of constraints:

$$\left(x^{rt}_{kl} + x^{rt}_{lk}\right) \leq X^t_{kl} \qquad r \in R, t \in T, k, l \in H^t, k < l. \tag{4a}$$

In addition to the variables and constraints of $\mathbf{p}$-CCHLP$_0$, formulation $\mathbf{p}$-CCHLP$_{\text{Setup}}$ has $|T||H^t|^2$ additional binary variables X and $|R|\Sigma_{t\in T}|H^t|^2$ additional constraints.

5. The origin-allocation collaboration hub location problem

Similarly to the $\mathbf{p}$-CCHLP, in the $\mathbf{p}$-*origin-allocation collaboration hub location problem* ($\mathbf{p}$-OCHLP) each carrier selects p^t locations for the hubs to open, among its own candidate set H^t, and the collaboration agreement still allows commodity transfer among carriers. Though, in this case it is imposed that all commodities with origin in the same node must be routed by the same carrier. Note that this restriction is not imposed to destinations, i.e., in the $\mathbf{p}$-OCHLP, even if as an origin each node is allocated to one single carrier, as a destination it may receive flows from several carriers.

The rationale for this assumption refers to circumstances where there are very high installation and operational costs derived from administrative or manipulation operations of flows at their origins, so these costs can be substantially decreased when restricting these operations to at most one carrier per origin. Thus, there is an allocation of origin nodes to carriers, although a given node may be recipient of commodities routed by different carriers. As before, the objective is to minimize the sum of the routing costs of all the commodities.

Fig. 6 shows an optimal solution to the $\mathbf{p}$-OCHLP for the instance of Fig. 1. Like in the previous models, for this instance carrier 1 opens hubs at nodes 3 and 6, and carrier 2 opens hubs at nodes 2 and 7. Now, origin node 5 is allocated to carrier 1 whereas origin node 1 is allocated to carrier 2. Hence, carrier 1 routes commodities $(5, 4, M) \in \mathcal{D}^1$, $(5, 4, \varepsilon) \in \mathcal{D}^2$, $(5, 2, \varepsilon) \in \mathcal{D}^1$,

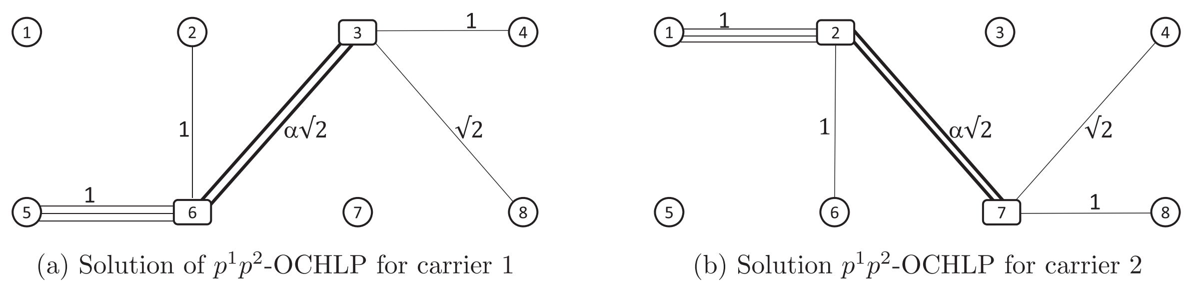


Fig. 6. Solution for the example of Fig. 1 for the p^1p^2-OCHLP with origin-allocation to carriers.

and $(5,8,\varepsilon) \in \mathcal{D}^2$, whereas carrier 2 routes commodities $(1,4,\varepsilon) \in \mathcal{D}^1$, $(1,8,\varepsilon) \in \mathcal{D}^1$, $(1,8,M) \in \mathcal{D}^2$, and $(1,6,\varepsilon) \in \mathcal{D}^2$. The optimal networks for carriers 1 and 2 are shown in Fig. (6a) and (6 b), respectively. The overall routing cost of each carrier is $M(2+\sqrt{2}\alpha) + \varepsilon\left[5+\sqrt{2}+2\sqrt{2}\alpha\right]$.

Observe that feasible solutions to **p**-UCHLP may not be feasible for **p**-OCHLP. Hence a property similar to P1 of Remark 1 no longer holds for **p**-OCHLP in relation to **p**-UCHLP. Thus, $z^*(OCHLP)$ does not necessarily yield a valid lower bound on $z^*(NCHLP)$. Still, we can analyze the worst-case for the savings that **p**-OCHLP may produce relative to the non-collaboration model. In particular, since the solution to **p**-UCHLP used in item (i) of the proof of Theorem 1 is feasible for **p**-OCHLP as well, we have the following result.

Theorem 3. *Let $z^*(OCHLP)$ denote the optimal value of the **p**-OCHLP for a given instance and $z^*(NCHLP)$ the optimal value of the same instance for the **p**-NCHLP. Then, there exists no finite upper bound for the ratio $z^*(NCHLP)/z^*(OCHLP)$.*

5.1. Mathematical programming formulation for the **p**-OCHLP

Here we develop mathematical programming formulations for some particular cases of the **p**-OCHLP. Again we start with the case when the underlying hub location model is defined over a complete graph and without any set-up costs, that we denote by **p**-OCHLP$_0$. Since Property P2 remains valid for the **p**-OCHLP$_0$, we can use the following sets of decision variables:

- Binary variables s_i^t, $i \in O$, $t \in T$. $s_i^t = 1$ if and only if all the commodities with origin at vertex i are routed by carrier t.
- Binary variables z_k^t, $k \in H^t$, $t \in T$. $z_k^t = 1$ if and only if a hub is opened at vertex k for carrier t.
- Binary variables x_{kl}^{rt}, $r \in R$, k, $l \in H^t$, $t \in T$. $x_{kl}^{rt} = 1$ if and only if commodity r is routed by carrier t via hubs k and l.

The formulation for the **p**-OCHLP$_0$ is as follows:

$$\mathbf{p}-OC_0: \quad \min \sum_{r\in R}\sum_{t\in T}\sum_{k,l\in H^t} C_{kl}^r x_{kl}^{rt} \tag{5a}$$

$$s.t. \quad \sum_{k\in H^t} z_k^t = p^t \qquad t \in T \tag{5b}$$

$$\sum_{t\in T} s_i^t = 1 \qquad i \in O \tag{5c}$$

$$\sum_{k,l\in H^t} x_{kl}^{rt} = s_{o_r}^t \qquad r \in R, t \in T \tag{5d}$$

$$x_{kk}^{rt} + \sum_{l\in H^t: l\neq k} x_{kl}^{rt} + \sum_{l\in H^t: l\neq k} x_{lk}^{rt} \le z_k^t \qquad r \in R, t \in T, k \in H^t \tag{5e}$$

$$x_{kl}^{rt} \ge 0 \qquad r \in R, k, l \in H^t, t \in T \tag{5f}$$

$$s_i^t \in \{0,1\} \qquad i \in O, t \in T \tag{5g}$$

$$z_k^t \in \{0,1\} \qquad k \in H^t, t \in T \tag{5h}$$

As before, constraints (5b) impose that the number of hubs selected by each carrier is correct. Constraints (5c) guarantee that each vertex that is the origin of some commodity is allocated to one carrier, (5d) that each commodity is routed by exactly one carrier, and (5e) that the intermediate nodes used to serve a commodity are open as hubs by the carrier that routes it. Finally, (5f)–(5h) define the domain of the variables.

Formulation (5a)–(5h) has $\Sigma_{t\in T}|H^t|$ binary variables z, $|O||T|$ binary variables s, plus $\Sigma_{t\in T}|H^t|^2$ continuous variables x. Its number of constraints is $|T| + |R|\left(|T| + \sum_{t\in T}|H^t|\right)$.

6. Computational experiments

In this Section a thorough computational test of the collaborative models presented in this paper is reported and discussed. The design of the experiments was devised to fulfill different goals, as follows:

- perform a proof-of-concept of the collaborative models introduced throughout the paper;
- analyze the computational effort required by the adopted formulations for the collaborative models as compared with those for the original non-collaborative model;
- gather managerial insight on the effects of the different collaborative policies presented in this paper.

In the remainder of this section we will describe the considered testbed and the computational framework, present the outputs of the experiments, and discuss the main findings. The section is divided in several parts, each of them focusing on one specific aspect. In particular, Section 6.1 describes the characteristics of the benchmark instances we have used and the computational environment, as well as the number of decision variables of the formulations on complete graphs without set-up costs, and the tables summarizing the numerical results obtained with such formulations for the instances with up to 20 nodes. These formulations involve over 400,000 variables already for two carriers and more than 6.5 million variables when the number of carriers raises to ten. In Section 6.2 we analyze the obtained results in terms of the computational burden required by each of the formulations, and also relative to the characteristics of the solutions produced by each of them. Section 6.3 summarizes the results we have obtained with a set of larger instances with up to 50 nodes and two carriers. We have carried out a final series of experiments on formulations that include set-up costs for the activated inter-hub arcs, whose results are presented in Section 6.4.

6.1. Testbed generation and computational framework

With the aim of assessing the potential impact of hub location collaborative schemes in the context of different application settings we considered a computational testbed consisting of two

Table 2
Number of commodities at varying nodes (n) and carriers ($|T|$).

n	Number of carriers					
	1	2	4	6	8	10
5	25	32	44	63	84	103
10	100	128	179	258	359	407
15	225	287	438	584	797	909
20	400	511	768	1036	1411	1626

main sets adapted from datasets commonly used in the literature. To analyze collaborative models in the field of airline transport we have used the Civil Aeronautics Board (CAB) instances (O'Kelly, 1987b), which are based on the airline passenger interactions between cities in the United States of America as evaluated by the Civil Aeronautics Board and are available at O'Kelly (1987b). The Australian Post instances, also referred to as the AP dataset, introduced by Ernst and Krishnamoorthy (Ernst & Krishnamoorthy, 1996), were used for assessing the implications in the field of logistics. In these instances nodes represent postcode districts, described along with their coordinates, and flow volumes (mail flow).

The complete set of commodities was considered for each instance, interpreting for each OD pair the amount of demand w_{ij} as the overall demand to be served by all carriers altogether. The distribution of the overall demand among carriers was computed by adopting the following procedure. For each commodity $r \in R$ an integer number k_r of active carriers was drawn from a uniform distribution $U(1, |T|)$. This means that, the total number of commodities generated is, on average, $|R|(|T|+1)/2$, so the mean number of commodities originally allocated to each carrier is $|R|(|T|+1)/(2|T|)$. Then, to identify the set of active carriers T^r, k_r indices were drawn, again from a uniform distribution $U(1, |T|)$. A weight coefficient λ^t was drawn from $U[0, 100]$ for each active carrier $t \in T^r$, and all weight coefficients were finally normalised to produce weighted demand values $w^t_{ij} = w_{ij} \frac{\lambda^t}{\sum_{t' \in T^r} \lambda^{t'}}$, $t \in T^r$, such that eventually $\sum_{t \in T} w^t_{ij} = w_{ij}$.

Experiments were run for varying values of the network size n, the number of carriers $|T|$, the value of the discount factor α and the number p of hubs to be located, as follows:

- $n \in \{5, 10, 15, 20\}$.
- $\alpha \in \{0.25, 0.50, 0.75\}$.
- $|T| \in \{2, 4, 6, 8, 10\}$.
- $p^t \in \{1, 2, 3, 4, 5\}$.

Based on the above described procedure and on the size of the considered instances, the actual number of commodities considered in the instances for each value of nodes and carriers is represented in Table 2, including the case with only one carrier to allow a comparison with traditional hub location problems. As can be observed, the number of involved commodities rapidly grows as the number of nodes increases. For $n = 20$, the instances with the largest number of carriers ($|T| = 10$) have over 1600 commodities, which is approximately the number of commodities of classical models with $n = 40$.

The information of Table 2 is complemented with that of Table 3, where we show the number of z and x decision variables involved in each of the proposed formulations, for the considered values in the number of nodes (n) and carriers ($|T|$). As can be seen with just two carriers ($|T| = 2$) the instances with 20 nodes involve up to 40 and 408,800 binary and continuous variables, respectively, depending on the formulation. The largest instances with 20 nodes and 10 carriers involve 400 binary variables and over six million five-hundred thousand continuous variables. Formulation for the OC model includes $n \times |T|$ additional binary variables s.

By testing each instance from both CAB and AP testbeds on all of the different 4 formulations, an overall amount of 2400 runs were executed. Experimental results were computed by using the IBM Ilog Cplex 12.6 solver, running on an 64 bit Intel Xeon CPU at 2.80 gigahertz with 64 gigabyte Ram and Linux Ubuntu 14.04 as an Operating System. All tests were executed with an absolute gap tolerance for optimality of 10^{-3} and allowing a maximum computing time of 7200 seconds.

The obtained numerical results are summarized in Tables 4–7. Each table analyzes the effect of a parameter on one of the studied performance measures. All tables have the same structure. The heading is followed by two blocks of rows: one for the *CAB instances* and one for *AP instances*. In each table the first two columns indicate the testbed (CAB or AP) and the parameter that is analyzed. This is followed by four blocks of columns, one for each of the tested formulations: the non-collaborative model (*NC-HLP*), the unrestricted collaboration model of Section 3 (*UC-HLP*), the commodity-transfer model of Section 4 (*CC-HLP*), and the origin-allocation model of Section 5 (*OC-HLP*). Each block consists of three columns: the first one (*cpu*) gives average computing times in seconds, the second one (*%Opt*) the percentage of instances of the group that are solved to optimality, and the third one (*%G*) the average percentage optimality gap at termination. The results presented in the tables represent mean values among subsets of experiments with the instances of the testbed shown in the first column where the parameter analyzed in the table was fixed to the value given in the second column, for varying values of all other parameters. The last row in each block (with entry *Avrg* in the column of the parameter) gives the averages over all the tested values of the parameter, i.e., the averages of the values of the previous rows of the block.

6.2. Discussion and main findings from the computational results

We discuss here the results and the main findings arising from this first large batch of computational experiments, organized in two main parts: first, for each set of benchmark instances and combination of the relevant parameters, we analyze and compare the computational effort required to solve the different collaborative formulations, as well as the optimality gaps obtained within the given time limit. Then, the operational performance secured by different collaboration options is compared, to gather useful managerial insight from the computational experience.

6.2.1. Analysis of computational effort

A first general evidence can be observed by analyzing the entries corresponding to *cpu* in the last line of each table, where the mean computing time required to solve an instance over all the experiments on the full testbed is presented. The formulation for the unrestricted collaborative model turns out to be the most easily solvable, together with the non-collaborative model. A much stronger computational effort is required on the average by the formulations for collaborative models p-CCHLP and p-OCHLP, where for the largest instances computing times can be substantially longer than those required by the previously mentioned formulations. Such observation is confirmed by looking at the optimality gaps results on the last line of tables: here, it turns out that the time limit suffices to find optimal solutions on all instances for p-NCHLP and p-UCHLP, while strictly positive optimality gaps over 20% are still present on the average on the p-CCHLP and p-OCHLP formulations after the allowed 7200 seconds.

6.2.2. Performance comparison

Next we direct our attention to managerial insights that can be derived from the analysis of the computational results. To this aim

Table 3
Number of z and x variables for the different formulations at varying nodes (n) and carriers ($|T|$).

	$\|T\|$											
	1		2		4		6		8		10	
n	z_k^t	x_{kl}^{rt}	z_k^t	x_{kl}^{rt}	z_k^t	x_{kl}^{rt}	z_k^t	x_{kl}^{rt}	z_k^t	x_{kl}^{rt}	z_k^t	x_{kl}^{rt}
5	5	625	10	1600	20	4400	30	9450	40	16,800	50	25,750
10	10	10,000	20	25,600	40	71,600	60	154,800	80	287,200	100	407,000
15	15	50,625	30	129,150	60	394,200	90	788,400	120	1,434,600	150	2,045,250
20	20	160,000	40	408,800	80	1,228,800	120	2,486,400	160	4,515,200	200	6,504,000
20	20	160,000	40	408,800	80	1,228,800	120	2,486,400	160	4,515,200	200	6,504,000

Table 4
Mean CPU times and percent optimality gaps at varying number of nodes.

	n	NCHLP			UCHLP			CCHLP			OCHLP		
		cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G
CAB	5	0.7	100.00	0.00	0.7	100.00	0.00	1.7	100.00	0.00	2.3	100.00	0.00
	10	22.4	100.00	0.00	32.2	100.00	0.00	1156.7	100.00	0.07	59.4	100.00	0.00
	15	225.3	100.00	0.00	323.9	100.00	0.00	3785.2	100.00	3.46	2834.9	77.33	0.76
	20	1355.4	100.00	0.00	1799.9	100.00	0.00	5042.7	94.67	22.00	4968.5	43.84	16.24
	Avrg	400.9	100.00	0.00	359.2	100.00	0.00	2488.0	98.67	6.33	1956.2	80.54	4.21
AP	5	0.6	100.00	0.00	0.7	100.00	0.00	1.8	100.00	0.00	1.9	100.00	0.00
	10	20.6	100.00	0.00	28.8	100.00	0.00	2166.8	73.33	2.68	384.1	100.00	0.00
	15	228.6	100.00	0.00	401.7	100.00	0.00	4424.3	42.67	11.62	4291.4	45.33	7.14
	20	1286.2	100.00	0.00	1690.9	100.00	0.00	5209.1	36.49	23.27	5323.3	29.73	22.80
	Avrg	384.0	100.00	0.00	526.6	100.00	0.00	2942.9	63.21	9.35	2490.7	68.90	7.43

Table 5
Mean CPU times and percent optimality gaps at varying number of carriers.

	$\|T\|$	NCHLP			UCHLP			CCHLP			OCHLP		
		cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G
CAB	2	35.8	100.00	0.00	49.3	100.00	0.00	54.0	100.00	0.00	56.5	100.00	0.00
	4	134.6	100.00	0.00	182.3	100.00	0.00	1520.3	91.67	0.51	1040.5	98.33	0.02
	6	315.9	100.00	0.00	409.4	100.00	0.00	3091.8	65.00	4.39	2187.7	80.00	2.81
	8	592.1	100.00	0.00	850.8	100.00	0.00	3903.3	58.33	11.77	3231.4	66.67	7.12
	10	926.3	100.00	0.00	1204.2	100.00	0.00	3894.3	50.85	15.12	3287.2	56.90	11.22
	Avrg	400.9	100.00	0.00	539.2	100.00	0.00	2488.0	73.24	6.33	1956.2	80.54	4.21
AP	2	41.9	100.00	0.00	58.3	100.00	0.00	70.4	100.00	0.00	82.9	100.00	0.00
	4	141.8	100.00	0.00	186.8	100.00	0.00	2453.1	71.67	2.60	2360.3	70.00	2.21
	6	287.6	100.00	0.00	407.1	100.00	0.00	3808.1	51.67	9.91	3088.9	63.33	6.01
	8	569.2	100.00	0.00	815.5	100.00	0.00	4368.2	43.33	15.81	3418.0	55.00	13.23
	10	879.5	100.00	0.00	1176.3	100.00	0.00	4033.0	49.15	18.57	3520.7	55.93	15.86
	Avrg	384.0	100.00	0.00	526.6	100.00	0.00	2942.9	63.21	9.35	2490.7	68.90	11.04

Table 6
Mean CPU times and percent optimality gaps at varying number of hubs.

	p	NCHLP			UCHLP			CCHLP			OCHLP		
		cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G
CAB	1	431.0	100.00	0.00	590.1	100.00	0.00	1035.5	95.00	0.97	1150.7	90.00	3.22
	2	399.4	100.00	0.00	546.4	100.00	0.00	2542.5	75.00	9.35	2053.4	76.67	6.38
	3	393.6	100.00	0.00	511.8	100.00	0.00	3184.4	63.33	9.22	2198.8	79.66	4.76
	4	389.6	100.00	0.00	539.6	100.00	0.00	3065.6	66.67	7.08	2210.7	76.67	4.03
	5	391.0	100.00	0.00	508.1	100.00	0.00	2614.3	66.10	5.02	2171.1	79.66	2.64
	Avrg	400.9	100.00	0.00	539.2	100.00	0.00	2488.0	73.24	6.33	1956.2	80.54	4.21
AP	1	422,1	100.00	0.00	567.8	100.00	0.00	1033.8	96.67	0.15	408.5	90.00	3.09
	2	365,0	100.00	0.00	518.4	100.00	0.00	3285.4	58.33	13.56	715.1	70.00	9.18
	3	427,4	100.00	0.00	634.1	100.00	0.00	3884.9	50.00	13.71	742.1	63.33	9.99
	4	347,0	100.00	0.00	494.4	100.00	0.00	3632.2	50.00	12.31	822.4	60.00	8.01
	5	358,6	100.00	0.00	416.8	100.00	0.00	2877.2	61.02	6.97	844.1	61.02	6.89
	Avrg	384.0	100.00	0.00	526.6	100.00	0.00	2942.9	63.21	9.35	706.4	68.90	7.43

Table 7
Mean CPU times and percent optimality gaps at varying discount factor.

	α	NCHLP			UCHLP			CCHLP			OCHLP		
		cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G	cpu	%Opt	%G
CAB	0.25	109.4	100.00	0.00	536.9	100.00	0.00	2657.2	72.00	9.28	2026.7	78.00	6.45
	0.50	533.7	100.00	0.00	524.8	100.00	0.00	2418.2	74.00	5.98	1959.4	82.00	3.95
	0.75	559.7	100.00	0.00	555.9	100.00	0.00	2387.7	73.74	3.70	1881.8	81.63	2.21
	Avrg	400.9	100.00	0.00	539.2	100.00	0.00	2488.0	73.24	6.33	1956.2	80.54	4.21
AP	0.25	126.6	100.00	0.00	515.3	100.00	0.00	3035.3	60.00	14.59	2553.5	67.00	11.25
	0.50	509.9	100.00	0.00	504.1	100.00	0.00	2942.0	65.00	8.50	2479.6	69.00	7.23
	0.75	515.6	100.00	0.00	560.9	100.00	0.00	2850.6	64.65	4.90	2438.6	79.71	3.79
	Avrg	384.0	100.00	0.00	526.6	100.00	0.00	2942.9	63.21	9.35	2490.7	66.67	7.43

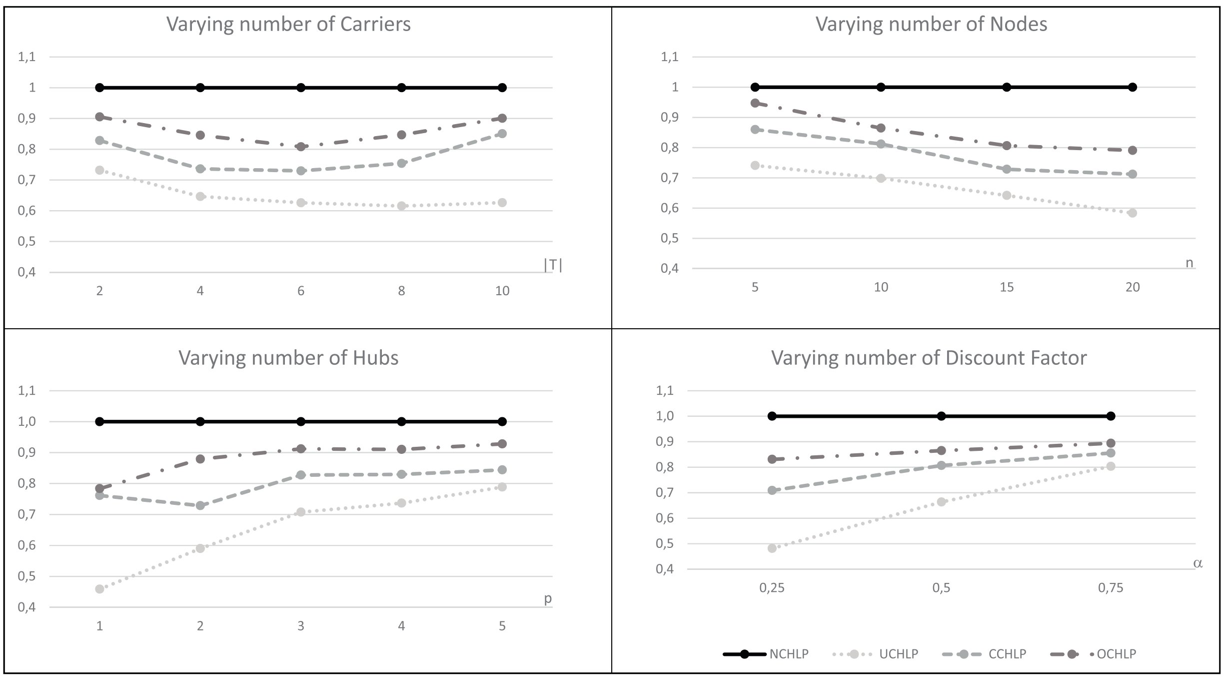


Fig. 7. Performance comparison for CAB instances.

we focus on Figs. 7 and 8, which respectively depict average saving ratios produced by collaborative models relative to the non-collaborative model *p*-NCHLP, at varying values of the considered parameters, for the CAB and AP benchmark instances optimally solved within the allowed computing time. That is, for a given collaborative model, the saving ratio of each optimally solved instance is calculated as the ratio between the objective function values of the collaborative model and that of the non-collaborative one on that instance. This explains why the values of the series labeled *NC-HLP* are exactly one.

Concerning the proof-of-concept for collaborative policies in hub location, a very general evidence can be derived from these pictures where, for each model and set of instances substantial reductions can be attained. The unrestricted collaboration model always permits the most efficient allocation of commodities on the network, allowing a minimization of the overall cost which constantly outperforms all the other models. The commodity-transfer collaboration model still guarantees a very good performance as compared to the other two models. As can be observed, the non-collaborative model is constantly the less cost-effective one throughout the whole considered testbed.

Beyond the proof-of-concept from the above observation, one can derive a basic evidence with important implications from an application point of view: designing joint managerial policies enhancing a coordinated use of resources, both for the selection of hubs to activate and for the allocation of the flows to serve, it is possible to satisfy the customers demand at a lower overall logistics cost, hence producing substantial savings for the overall set of carriers. Such savings can be then redistributed according to different strategies, in such a way that the situation of each individual carrier also improves. This is an important issue that deserves further attention, although a thorough analysis is outside of the scope of this paper.

Moreover, the figures quantify the effect of the different parameters in the savings of the alternative models, and confirm that the relative behavior of collaborative models is similar for the two distinct testbeds and, independently of the number of nodes and commodities, is as follows:

$$z^*_{UCHLP} < z^*_{CCHLP} < z^*_{OCHLP} < z^*_{NCHLP}.$$

By exploring more in detail the results in the figures, it is possible to observe the influence of each individual parameter on the saving profile associated with each collaborative model. The figures reporting results at varying number of carriers are instrumental in understanding how the overall savings associated with the use of collaborative models improve as the number of carriers progressively increases from 2 to 10 carriers. Such pattern is common on both the testbeds, with the improvement being more regular and

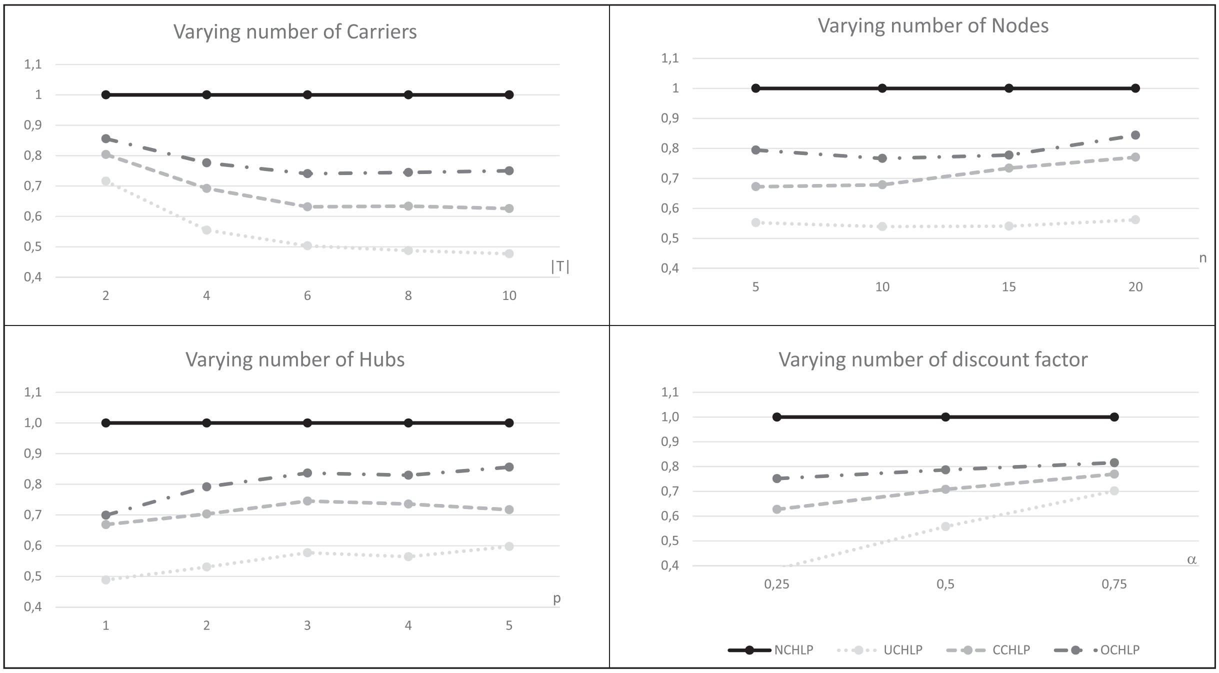


Fig. 8. Performance comparison for AP instances.

intense for the Australian Post testbed. This is particularly interesting if one considers the current trends in logistics management where a number of private carriers are entitled to act on the same transportation network with shared facilities, for example in the use of different classes of distribution centers in urban areas according to multi-tier city logistics paradigms (Crainic & Sgalambro, 2014). By enabling practical and collaborative patterns in commodity demand distribution, performance improvement would benefit from the increasing number of actors, with substantial advantages for the stakeholders of this logistics process.

The figures reporting the savings at varying number of hubs suggest how the relative performance gain in adopting a collaborative policy is more intense when the number of hubs to be allocated decreases, in particular for results on unrestricted collaboration and origin-allocation collaboration models, with a similar pattern on both sets of instances.

While Fig. 7 suggests that for the CAB instances the benefits of collaboration increase with instance size, the same trend cannot be appreciated for the AP instances in Fig. 8. As for the influence of the discount factor, both figures indicate that, as could be expected, benefits increase as α decreases.

Finally, Fig. 9 visualizes the effects of collaborative models **p**-CCHLP and **p**-OCHLP in terms of commodity transfer, based on the number of active carriers. More in detail, the two bar charts on the top of the figure show the average percentage of commodities that are transferred between two different carriers for the two distinct collaborative models, for CAB and AP instances, respectively, whereas the bar charts in the bottom of the figure show the average percentage of demand involved in the transfer of commodities between carriers. This figure confirms the growth in the impact of the collaborative policies as the number of carriers increases, ranging quite regularly from around 50% of the commodities in the case of 2 carriers until around 90% for the experiments with 10 carriers, without apparent differences for the two considered sets of instances. The same numbers apply roughly for the percentage transferred demand.

6.3. Computational experiments with larger instances

In order to analyze the scalability of the proposed formulations we have run additional computational tests with CAB and AP instances with up to fifty nodes, with two carriers ($|T| = 2$), three hubs ($p = 3$) and a discount factor $\alpha = 0.75$. Now we have not set a limit on the maximum computing time, so all instances have been solved to optimality. The obtained results are summarized in Table 8, which contains two blocks of rows, one for the CAB instances and another one for the AP instances. Columns under n and $|R|$ show the number of nodes and commodities of the instances, respectively. Columns under z_k^t and x_{kl}^{rt} respectively show the number of z and x variables of the collaborative formulations, with **p**-OCHLP model utilising $2n$ additional binary origin allocation variables. As can be seen, the two 50 node instances involve over fifteen million continuous variables. The table contains four additional blocks, one for each of the tested models (*NC-HLP*, *UC-HLP*, *CC-HLP*, and *OC-HLP*). Each block consists of two columns, the first one (under *cpu*) with the computing time required to solve the instance to proven optimality and the second one (under $v(\cdot)/v(NC)$) with the saving ratio of the corresponding model relative to the non-collaborative setting, given by the ratio of the optimal value of the tested model over the optimal value of the model without collaboration. As could be expected, the computing times rapidly increase with the size of the instances and the difficulty of the formulation. Formulation *CC-HLP* with the 50 node CAB and AP instances consumed over 13 and 17 hours, respectively. The computing times consumed by the same two instances with formulation *OC-HLP* are over 15 and 17 hours, respectively.

The reader may observe that the savings that are shown in Table 8 are, in general, smaller than the ones displayed in Figs. 7–8. This is not surprising, as the table corresponds to experiments for the smallest number of carriers ($|T| = 2$), which, as mentioned, produces the smallest savings.

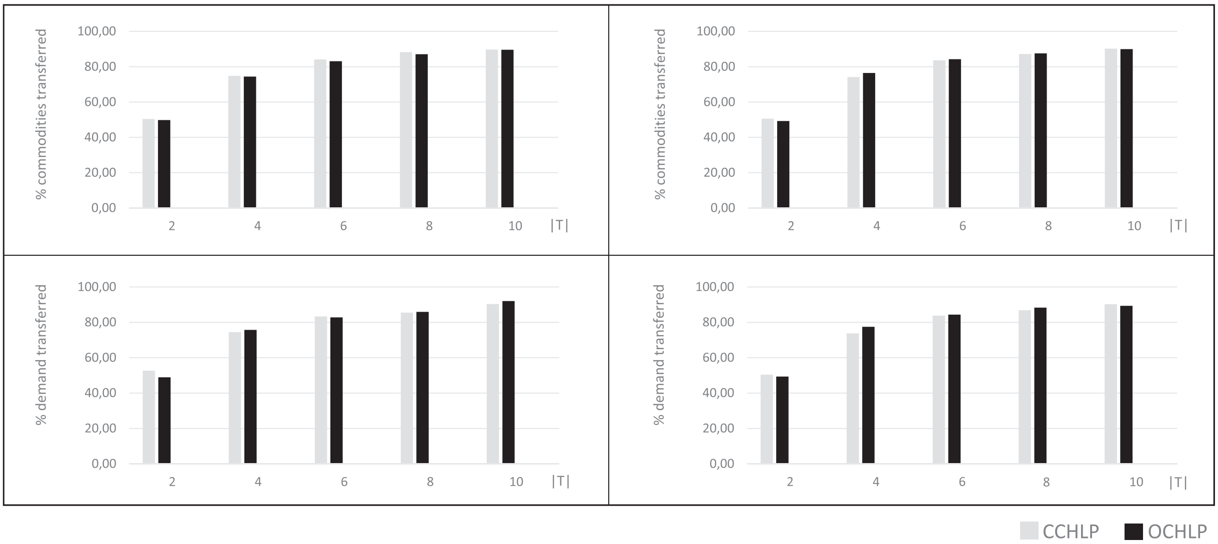


Fig. 9. % of transferred commodities and demand at varying number of carriers.

Table 8
Summary of results with larger instances: dimensions, computing times, and saving ratios.

	n	$\|R\|$	z_k^t	x_{kl}^{rt}	NCHLP		UCHLP		CCHLP		OCHLP	
					cpu	$v(NC)/v(NC)$	cpu	$v(UC)/v(NC)$	cpu	$v(CC)/v(NC)$	cpu	$v(OC)/v(NC)$
CAB	20	511	40	408,800	162	1.00	168	0.88	184	0.89	173	0.93
	25	791	50	988,750	180	1.00	633	0.88	638	0.90	634	0.97
	30	1121	60	2,017,800	640	1.00	1844	0.82	1900	0.84	1809	0.90
	35	1529	70	3,746,050	1833	1.00	4662	0.84	4720	0.86	4753	0.92
	40	2001	80	6,403,200	4687	1.00	10500	0.82	10902	0.85	10193	0.92
	45	2520	90	10,206,000	50564	1.00	20548	0.79	20657	0.85	23796	0.92
	50	3115	100	15,575,000	23392	1.00	38912	0.74	47484	0.79	56130	0.89
AP	20	511	40	408,800	169	1.00	173	0.86	196	0.88	210	0.91
	25	791	50	988,750	183	1.00	616	0.85	744	0.88	776	0.90
	30	1121	60	2,017,800	645	1.00	1956	0.87	2043	0.89	2287	0.91
	35	1529	70	3,746,050	4536	1.00	4518	0.86	5537	0.88	6147	0.91
	40	2001	80	6,403,200	4850	1.00	10685	0.87	13635	0.89	15684	0.92
	45	2520	90	10,206,000	10910	1.00	20553	0.88	26883	0.89	31902	0.92
	50	3115	100	15,575,000	25102	1.00	39578	0.87	60890	0.89	61867	0.92

6.4. Computational experiments with models with set-up costs for inter-hub arcs

We finally summarize the results of the last series of experiments that we have run, where we have considered fixed set-up costs for the activated interhub arcs. For these experiments the tested formulations include binary decision variables X_{kl}^t that represent the interhub arcs that are activated, the additional set of constraints (4a) that force the activation of inter-hub arcs when they are used for sending flows, and the additional term (3a) in the objective function to compute the total set-up cost of the activated arcs. We have used CAB and AP instances with $n \in \{10, 15, 20\}$, $p = 3$, $|T| = 2$, and $\alpha = 0.75$. Furthermore, we have considered set-up costs for the interhub arcs independent from the carriers and the arcs, i.e., $h_{kl}^t = F$ for all $k, l \in H^t$, $t \in T$. The tested values are $F = 50,000 \times s$ with $s \in \{0, 1, \ldots, 10\}$, namely 0, 50,000, 100,000, 150,000, 200,000, 250,000, 300,000, 350,000, 400,000, 450,000, and 500,000. Again a time limit of two hours has been set for each run.

Results of these experiments are presented in Tables 9 and 10. Table 9 contains two blocks of rows, corresponding to CAB and AP instances, respectively. Each block contains one row per tested value of F, plus a final row with the averages over the tested values of F. The type of instance (CAB or AP) is given in the first column, and the values of F in the second column. The remaining columns show, for each HLP model, mean values over all the instances of the block with the tested value of F, of the following indicators: number of activated inter-hub arcs (*#IA*); total cost for interhub arc activation, in percentage of the overall routing plus set-up cost (*%IAC*); total cost for the routing of flows through non-inter-hub arcs, in % of the overall routing plus set-up cost (*%NRC*); total cost for the routing of flows through inter-hub arcs, in % of the overall routing plus set-up cost (*%IRC*); and mean amount of flow routed through each activated inter-hub arc (*Flow*). Table 10 presents the same organization for rows, while the columns show, for each HLP model, average values of the following indicators: saving ratios (*SR*) produced by collaborative models relative to the non-collaborative model (as defined in Section 6.2.2), percentage of commodities transferred between carriers (*%TC*) and percentage of demand transferred between carriers (*%TD*), the last two being only applicable for **p**-CCHLP and **p**-OCHLP models.

The numerical results in Tables 9 and 10 show how a progressive increase in the set-up costs yields a rather regular decrease in the activation of inter-hub arcs, which is fully consistent with the expectations, as the savings secured by the presence of the discount factor become less prominent with respect to the activation costs. In general, the contribution of the set-up cost term

Table 9
Results on instances with set-up costs: impact on the use of inter-hub arcs.

	F	NCHLP					UCHLP					CCHLP					OCHLP				
		#*IA*	%IAC	%NRC	%IRC	Flow	#*IA*	%IAC	%NRC	%IRC	Flow	#*IA*	%IAC	%NRC	%IRC	Flow	#*IA*	%IAC	%NRC	%IRC	Flow
CAB	0	6.00	0.00	55.07	44.93	8713.09	28.00	0.00	24.96	75.04	3183.00	6.00	0.00	39.75	60.25	10715.44	6.00	0.00	46.66	53.34	10846.50
	50,000	5.67	0.67	55.44	43.89	8915.09	19.33	2.30	25.82	71.88	4389.88	5.67	0.71	39.56	59.72	11062.02	5.33	0.68	46.73	52.59	12193.72
	100,000	5.33	1.16	55.99	42.86	9189.70	15.67	3.31	28.81	67.88	5328.35	5.33	1.36	39.88	58.76	11783.27	4.67	1.12	47.38	51.50	13299.33
	150,000	5.00	1.64	56.00	42.36	9843.29	12.67	3.88	31.60	64.51	6758.15	5.33	2.01	39.66	58.32	11783.27	4.67	1.67	47.14	51.18	13299.33
	200,000	4.25	2.12	54.59	43.29	9288.36	8.75	3.69	37.45	58.86	6844.45	4.75	3.14	36.43	60.43	12013.28	4.00	2.16	46.47	51.37	11862.67
	250,000	5.50	1.63	61.31	37.06	12365.62	11.50	3.55	35.74	60.70	10282.89	4.50	1.45	46.97	51.58	17788.15	4.50	1.51	53.09	45.40	19655.33
	300,000	4.33	2.26	58.13	39.61	11039.55	7.67	3.54	41.88	54.58	10284.60	4.33	3.16	41.37	55.47	13831.93	3.67	2.53	49.19	48.29	16341.94
	350,000	4.00	2.45	57.66	39.88	12744.56	7.00	3.87	42.65	53.48	10571.17	4.00	3.08	43.26	53.65	14380.54	3.33	2.36	51.08	46.56	17261.67
	400,000	4.00	2.79	57.47	39.74	12744.56	6.67	4.27	42.86	52.86	10877.55	3.67	2.85	45.78	51.37	15020.77	3.33	2.68	50.91	46.40	17261.67
	450,000	4.00	3.13	57.27	39.60	12744.56	6.00	4.49	45.05	50.46	11581.46	3.67	3.20	45.63	51.18	15020.77	3.33	3.01	50.75	46.24	17261.67
	500,000	3.33	3.18	59.44	37.39	13850.13	6.00	4.96	44.84	50.20	11581.46	3.67	3.54	45.47	51.00	15020.77	3.33	3.33	50.59	46.08	17261.67
	Average	4.64	1.93	56.92	41.15	10946.61	11.67	3.45	36.57	59.99	8230.62	4.64	2.28	41.84	55.88	13317.75	4.18	1.93	48.89	49.18	14904.36
AP	0	6.00	0.00	87.25	12.75	6.06	29.00	0.00	58.27	41.73	3.52	6.00	0.00	75.94	24.06	10.44	6.00	0.00	80.94	19.06	8.94
	50,000	6.00	0.46	86.84	12.70	6.06	25.67	2.39	57.53	40.07	3.86	6.00	0.54	75.53	23.93	10.44	6.00	0.52	80.52	18.95	8.94
	100,000	5.33	0.81	86.92	12.27	6.39	19.00	3.49	60.03	36.48	4.92	6.00	1.07	75.13	23.80	10.44	5.67	0.98	80.35	18.67	9.23
	150,000	4.33	0.99	87.17	11.84	7.21	15.00	4.09	62.27	33.64	5.67	5.67	1.52	75.01	23.47	11.27	5.67	1.47	79.96	18.57	9.23
	200,000	4.33	1.31	86.89	11.80	7.21	11.67	4.21	65.78	30.01	6.12	5.33	1.91	75.30	22.80	11.00	5.67	1.95	79.57	18.48	9.23
	250,000	3.67	1.39	87.55	11.06	7.65	10.33	4.62	67.29	28.10	6.19	5.00	2.23	75.20	22.57	11.53	5.33	2.29	79.73	17.98	9.83
	300,000	3.67	1.67	87.30	11.03	7.65	8.00	4.31	71.31	24.38	6.70	4.67	2.49	75.66	21.85	11.67	5.00	2.57	80.01	17.42	9.91
	350,000	3.33	1.77	87.80	10.43	7.72	6.33	3.99	74.90	21.11	7.32	4.33	2.67	76.17	21.16	12.00	4.00	2.39	81.68	15.93	10.89
	400,000	3.00	1.82	88.44	9.74	7.72	4.67	3.28	79.03	17.69	7.69	4.00	2.81	76.55	20.64	12.22	3.33	2.27	82.77	14.97	11.22
	450,000	2.33	1.57	90.07	8.36	8.24	3.67	2.91	82.19	14.90	7.04	3.33	2.63	78.79	18.58	12.56	3.00	2.31	83.86	13.84	10.89
	500,000	1.67	1.26	92.22	6.52	7.57	2.67	2.32	85.77	11.91	7.15	3.33	2.91	78.57	18.52	12.56	2.67	2.28	85.39	12.33	11.00
	Average	3.97	1.19	88.04	10.77	7.22	12.36	3.24	69.49	27.27	6.02	4.88	1.89	76.17	21.94	11.47	4.76	1.73	81.34	16.93	9.94

Table 10
Results on instances with set-up costs: impact on savings and carrier collaboration.

	F	UCHLP	CCHLP			OCHLP		
		SR	*SR*	*%TC*	*%TD*	*SR*	*%TC*	*%TD*
CAB	0	0.882	0.923	48.58	40.20	0.944	49.58	53.14
	50,000	0.900	0.923	50.28	59.15	0.944	47.50	39.16
	100,000	0.912	0.923	48.93	48.70	0.945	51.79	59.97
	150,000	0.921	0.924	49.16	39.07	0.945	52.50	60.84
	200,000	0.941	0.933	50.50	59.59	0.952	50.86	56.56
	250,000	0.901	0.910	48.86	43.83	0.932	52.19	55.78
	300,000	0.932	0.929	47.32	43.64	0.946	47.95	40.73
	350,000	0.934	0.930	47.80	43.50	0.946	49.44	46.68
	400,000	0.935	0.930	50.47	50.12	0.946	47.24	39.85
	450,000	0.937	0.930	49.70	51.91	0.946	50.56	53.32
	500,000	0.939	0.930	52.29	59.10	0.946	50.56	53.32
	Average	0.922	0.927	49.49	49.46	0.945	49.97	50.87
AP	0	0.813	0.855	52.96	53.42	0.885	48.34	47.64
	50,000	0.831	0.856	50.78	51.71	0.886	49.45	49.30
	100,000	0.844	0.856	50.98	51.75	0.887	51.66	52.36
	150,000	0.853	0.858	48.02	47.36	0.888	51.66	52.36
	200,000	0.860	0.859	52.30	52.87	0.889	48.34	47.64
	250,000	0.865	0.861	47.14	46.17	0.891	49.38	49.15
	300,000	0.870	0.862	53.99	54.84	0.892	51.59	52.20
	350,000	0.873	0.863	53.44	54.38	0.893	48.41	47.80
	400,000	0.875	0.864	48.26	48.00	0.894	52.18	52.76
	450,000	0.877	0.865	47.60	46.95	0.895	50.09	50.45
	500,000	0.878	0.866	53.61	54.04	0.895	50.03	50.29
	Average	0.858	0.861	50.83	51.05	0.891	50.10	50.18

%IAC is small for all models, and the trend for the variation of *%IRC* and *%NRC* at increasing values of *F* shows no substantial differences among models.

Columns under *Flow* of Table 9 show that increasing the value of *F* produces a regular increase on the mean amount of flow circulating per activated inter-hub arc. This confirms the capability of the considered collaborative policies for modeling economies of scale. Note the high differences in the values for the entries of such columns between the CAB and AP instances, which are due to the very different demand values on the adopted benchmarks which, on average, are 141379.33 and 241.67, respectively.

By observing the results in Table 10, it appears how the cost savings enabled by the collaborative models present different behaviours when the set-up costs increase. In fact, the unrestricted collaboration model shows a regular and substantial decrease in the cost savings, whereas the other two collaborative policies turn out to be less sensitive to set-up cost increase. For the latter, no regular trend can be appreciated for the savings or the percentage of transferred commodities and demand. On the other hand, while UCHLP shows a relative advantage with respect to CCHLP and OCHLP in terms of cost savings for low set-up values, such advantage tends to disappear as the fixed costs increase.

7. Conclusions

In this paper we have introduced several hub location models that address the potential gains that can be attained when collaborative policies among carriers that operate on the same network are implemented. To the best of our knowledge this is the first time that collaboration among carriers is studied in the context of hub location, encompassing uncentralized systems as well. Three different collaborative policies have been proposed. Theoretical analyses have shown that in all cases arbitrarily large savings can be attained with respect to traditional non-collaborative approaches. For each model a mixed integer programming formulation has been proposed and computationally tested on two sets of testbed instances adapted from the literature. The obtained numerical results confirm empirically the advantages of each of the models and the influence of problem parameters on their performance.

In our opinion this paper opens several avenues of research. On the one hand, our results indicate that the formulations proposed for the considered collaborative models are in practice very demanding computationally for available off-the-shelf solvers and only instances based on reduced size networks can be solved to proven optimality. This is particularly true for the *p*-OCHLP formulation, which incorporates the allocation variables s_i and the origin-allocation constraints (5d). Therefore *ad hoc* solution methods for these models, exact or heuristic, able to produce optimal or near-optimal optimal solutions of larger instances in small computing times would allow to validate and extend the potential of our findings in logistics management.

On the other hand, even if our theoretical analysis applies to general graphs and more general models, our computational experiments have been carried out for *p*-HLPs with instances defined over complete input graphs; in some cases we have considered inter-hub set-up costs, but so far we have ignored set-up costs for the hubs that are activated. Indeed the trend nowadays is to consider more general settings, with not necessarily complete networks, set-up costs for all the elements that are activated and allowing for longer commodity routing paths with possibly more than one inter-hub arc. While from a model-building point of view this offers no particular difficulty, the requirements of the formulations tested in our computational experiments indicate that off-the-shelf software will no longer be useful for solving similar formulations for more general models, and specific sophisticated methodologies will be required.

A similar observation applies with respect to discount factors. To alleviate notation we have assumed that the discount factor for inter-hub arcs is the same for all carriers, although all our developments remain totally valid also when these discount factors are not necessarily the same. The reader may note however that the same could apply to each individual carrier, who could possibly have a heterogeneous fleet so the discount factor would not necessarily be the same for all vehicles. To the best of our knowledge this (really interesting issue) has not yet been addressed in the literature for classical models with one single carrier.

Finally, the issue of how to distribute the obtained savings among carriers is a closely related topic, which deserves specific attention. In this paper we have shown that substantial savings can be attained by applying collaborative policies. It is thus clear that these savings can be redistributed among the carriers participating in the collaboration in such a way that each of them benefits from part of this savings. Still, the redistribution can be carried out according to different redistribution criteria. A thorough analysis of this topic from a game-theoretical perspective could be therefore an appropriate complement to the optimization viewpoint we have followed. Such a methodology has already been applied in Skorin-Kapov (1998) for the redistribution of the costs among demand points in a classical *p*-median hub location problem. A different perspective would be however needed in our case, not only because several carriers operate simultaneously, but also because savings must be redistributed among carriers (instead of redistributing costs among demand points).

Acknowledgments

This research was partially funded by the Spanish Ministry of Economy and Competitiveness and ERDF funds (Grant MTM2015-63779-R (MINECO/FEDER). This support is gratefully acknowledged. The authors are grateful to the referees for their constructive comments.

References

Adler, N., & Smilowitz, K. (2007). Hub-and-spoke network alliances and mergers:price-location competition in the airline industry. *Transportation Research Part B, 41*, 394–409.

Alibeyg, A., Contreras, I., & Fernández, E. (2016). Hub network design problems with profits. *Transportation Research Part E: Logistics and Transportation Review, 96*, 40–59.

Alibeyg, A., Contreras, I., & Fernández, E. (2018). Exact solution of hub network design problems with profits. *European Journal of Operational Research, 266*(1), 57–71.

Campbell, J. (1992). Location and allocation for distribution systems with transshipments and transportion economies of scale. *Annals of Operations Research, 40*(1), 77–99.

Campbell, J. (1994). A survey of network hub location. *Studies in Locational Analysis, 6*, 31–49.

Campbell, J., & O'Kelly, M. (2012). Twenty-five years of hub location research. *Transportation Science, 46*(2), 153–169.

Contreras, I. (2015). Hub location problems. In *Location science*. Springer.

Contreras, I., Cordeau, J.-F., & Laporte, G. (2011a). Benders decomposition for large-scale uncapacitated hub location. *Operations Research, 59*(6), 1477–1490.

Contreras, I., Cordeau, J.-F., & Laporte, G. (2011b). Stochastic uncapacitated hub location. *European Journal of Operational Research, 212*(3), 518–528.

Contreras, I., Díaz, J., & Fernández, E. (2011c). Branch and price for large-scale capacitated hub location problems with single assignment. *INFORMS Journal on Computing, 23*(1), 41–55.

Contreras, I., & Fernández, E. (2012). General network design: A unified view of combined location and network design problems. *European Journal of Operational Research, 219*(3), 680–697.

Contreras, I., Fernández, E., & Marín, A. (2010). The tree of hubs location problem. *European Journal of Operational Research, 202*(2), 390–400.

Contreras, I., Tanash, M., & Vidyarthi, N. (2017). Exact and heuristic approaches for the cycle hub location problem. *Annals of Operations Research, 258*(2), 655–677.

Crainic, T., & Sgalambro, A. (2014). Service network design models for two-tier city logistics. *Optimization Letters, 8*(4), 1375–1387.

Crainic, T. G., & Laporte, G. (1997). Planning models for freight transportation. *European Journal of Operational Research, 97*, 409–438.

Cruijssen, F., Cools, M., & Dullaert, W. (2007). Horizontal cooperation in logistics: opportunities and impediments. *Transportation Research Part E: Logistics and Transportation Review, 43*, 129–142.

Defryn, C., Sörensen, K., & Dullaert, W. (2019). Integrating partner objectives in horizontal logistics optimisation models. *Omega, 82*(C), 1–12.

Eiselt, H., & Marianov, V. (2009). A conditional p-hub location problem with attraction functions. *Computers & Operations Research, 36*(12), 3128–3135.

Ernst, A., & Krishnamoorthy, M. (1996). Efficient algorithms for the uncapacitated single allocation p-hub median problem. *Location Science, 4*(3), 139–154.

Fernández, E., Fontana, D., & Speranza, M. (2016). On the collaboration uncapacitated arc routing problem. *Computers & Operations Research, 67*, 120–131.

Fernández, E., Roca-Riu, M., & Speranza, M. (2018). The shared customer collaboration vehicle routing problem. *European Journal of Operational Research, 265*(3), 1078–1093.

Hernández, S., Unnikrishnan, A., & Awale, S. (2012). Centralized carrier collaboration multihub location problem for less-than-truckload industry: Hybrid hub-and-spoke network. *Transportation Research Record, 2269*(1), 20–28.

Lowe, T., & Sim, T. (2012). The hub covering flow problem. *Journal of the Operational Research Society, 64*(7), 973–981.

Lüer-Villagra, A., & Marianov, V. (2013). A competitive hub location and pricing problem. *European Journal of Operational Research, 231*(3), 734–744.

Marianov, V., Serra, D., & ReVelle, C. (1999). Location of hubs in a competitive environment. *European Journal of Operational Research, 114*(2), 363–371.

Marín, A., Cánovas, L., & Landete, M. (2006). New formulations for the uncapacitated multiple allocation hub location problem. *European Journal of Operational Research, 172*(1), 274–292.

Martins de Sá, E., Contreras, I., & Cordeau, J.-F. (2015). Exact and heuristic algorithms for the design of hub networks with multiple lines. *European Journal of Operational Research, 246*(1), 186–198.

Okdinawati, L., Simatupang, T., & Sunitiyoso, Y. (2015). Modelling collaborative transportation management: Current state and opportunities for future research. *Journal of Operations and Supply Chain Management, 8*, 96–119.

O'Kelly, M. (1986). Activity levels at hub facilities in interacting networks. *Geographical Analysis, 18*(4), 343–356.

O'Kelly, M. (1987a). A quadratic integer program for the location of interacting hub facilities. *European Journal of Operational Research, 32*(3), 393–404.

O'Kelly, M. (1987b). A quadratic integer program for the location of interacting hub facilities. *European Journal of Operational Research, 32*(3), 393–404.

Pomponi, F., Fratocchi, L., Tafuri, S., & Palumbo, M. (2013). Horizontal collaboration in logistics: a comprehensive framework. *Research in Logistics and Production, 3*, 243–254.

Puerto, J., Ramos, A., Rodríguez-Chía, A., & Sánchez-Gil, M. (2016). Ordered median hub location problems with capacity constraints. *Transportation Research Part C: Emerging Technologies, 70*, 142–156.

Quintero-Araujo, C., Gruler, A., Juan, A., & Faulin, J. (2019). Using horizontal cooperation concepts in integrated routing and facility-location decisions. *ITOR, 26*(2), 551–576.

Sasaki, M., Campbell, J., Ernst, A., & Krishnamoorthy, M. (2009). Hub arc location with competition. *Technical Report*. Technical Report of the

Simatupang, T., & Sridharan, R. (2008). Design for supply chain collaboration. *Business Process Management Journal, 14*, 401–418.

Skorin-Kapov, D. (1998). Hub network games. *Networks, 31*(4), 293–302.

Taherkhani, G., & Alumur, S. (2019). Profit maximizing hub location problems. *Omega, 86*, 1–15.

Verdonck, L., Caris, A., Ramaekers, K., & Janssens, G. (2013). Collaborative logistics from the perspective of road transportation companies. *Transport Reviews, 33*, 700–719.