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Roads to Prosperity without Environmental Poverty: The Role of Impatience

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October 14, 2019

Abstract

This paper advances the hypothesis that impatience negatively depends on environmental quality and aims to explain why some countries stagnate in an ‘environmental and economic poverty trap’. For low levels of environmental quality, advancements in productivity lead impatient agents to direct income increases to consumption (rather than savings), depleting further the environment. Given that productivity increases do not help such economies to escape the trap (contrary to perceived notions), policies should focus on the implementation of behavioral changes.

JEL classification: D90, E21, E70, O44, Q56.

Keywords: Time preference, economic growth, environmental poverty traps, economic poverty traps.

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1 Introduction

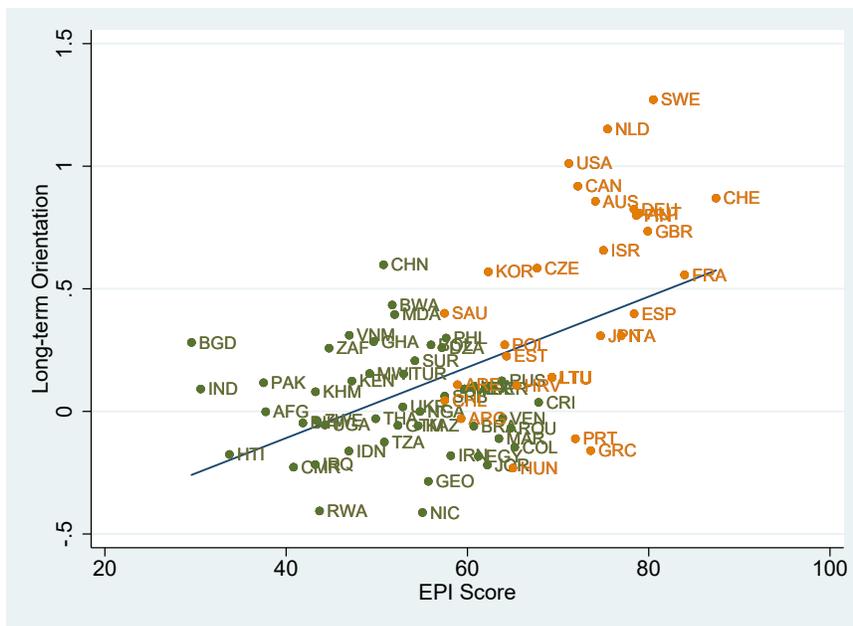
Higher economic growth is widely perceived to result in greater environmental degradation through higher pollution in the early stages of development, which tends to get reversed beyond a point. Following Grossman and Krueger (1995), this inverse U-shaped relation between growth and pollution is commonly referred to as the Environmental Kuznets Curve (EKC). However, countries are often stuck in ‘environmental and economic poverty traps’ characterized, at the same time, by environmental degradation and low growth, without ever reaching the turning point of the EKC (see, e.g., Prieur, 2009, Varvarigos, 2014). This paper proposes a framework that can explain economic and environmental stagnation through a *behavioral* mechanism.

A distinguishing feature of our paper is endogenous discounting or a non-constant rate of time preference. From a historical perspective, Galor and Ozak (2016) show that in societies where the ancestral population experienced a higher crop yield (for a given crop growth cycle), the rewarding experience of agricultural investment set in motion the traits for higher long-term orientation among the descendants of individuals who resided in such geographical regions during that period. In drawing a parallel with what could be expected in an environmental context, better nurturing and protection of the environment in a particular era could result in higher long-term orientation (and hence, a lower rate of time preference) among members of subsequent generations as environmental quality improves. Figure 1 displays the positive relationship between the contemporary rate of time preference (RTP) from Falk et al. (2019) against the Environmental Performance Index (EPI) score for 2018, vindicating this argument.¹

The contribution of our paper is twofold. First, we extend the literature on growth, environment and endogenous discounting (see, among others, Pittel, 2002; Yanase, 2011; Vella et al., 2015;). We model environmental resources as stock, which enables us to capture the existence of environmental and economic poverty traps through the behaviour of agents due to low environmental quality. Second, we shed light on the emergence of policies toward behavioural changes (for instance, through focused educational programs) since improvements in productivity are not enough for a country to escape an environmental

¹The EPI (produced by Yale University and Columbia University) ranks countries on performance indicators across ten issue categories covering environmental health and ecosystem vitality (EPI, 2018).

Figure 1: **Long term orientation vs. EPI.** The graph shows the std. deviation from the world mean of the rate of time preference against the EPI score for 2018 for 90 countries (developed – orange, developing – green, based on the UN classification); better environmental quality is associated with higher long-term orientation. The solid line shows the linear regression fit. Sources: EPI (2018); RTP, Falk et al. (2019)



poverty trap. While, with productivity gains, people become more productive and enjoy higher incomes, they spend a higher proportion of their incomes to consumption because of their low long-term orientation (that increases pollution), rather than savings (that enhance resources for abatement policies).

2 The model

2.1 Firms and Households

The production function of the single good in this economy is given by:

$$Y = AK^a(K_gL)^{1-a}, \tag{1}$$

where Y denotes output, $a \in (0, 1)$ denotes the share of physical capital, K , in the production function, K_g refers to the public capital stock (e.g. infrastructure), funded by the government, and A represents TFP. Labor is constant and normalized to unity ($L = 1$).

The firm maximizes profits, $\pi = (1 - \tau)Y - (r + \delta_K)K - w$, where $\tau \in (0, 1)$ is a tax rate on output, $r \in (0, 1)$ is the economy-wide interest rate and $\delta_K \in (0, 1)$ the depreciation of the private capital stock; $r + \delta_K$ is the rental cost of capital. The first-order conditions are given by:

$$r = Aa(1 - \tau) \left(\frac{K}{K_g} \right)^{a-1} - \delta_K, \quad (2)$$

$$w = A(1 - a)(1 - \tau) \left(\frac{K}{K_g} \right)^a K_g. \quad (3)$$

The representative household maximizes her lifetime utility:

$$\int_0^\infty \ln(C) \exp \left[- \int_0^t \rho(N_v) dv \right] dt, \quad (4)$$

where C is consumption and N is the stock of economy-wide natural resources, interpreted as an index for environmental quality. In turn, $\rho(N) \geq 0$ denotes the endogenous rate of time preference (RTP), which depends negatively on environmental quality, i.e. $\rho_N \leq 0$.

Households are the owners of private capital. Besides the return on their assets at a rate r , they receive labor income, w , and dividends, π . The dynamic budget constraint reads:

$$\dot{K} = rK + w - C + \pi, \quad \text{given} \quad K(0) > 0. \quad (5)$$

Household maximization of (4) s.t. (5) leads to the familiar Euler equation:

$$\frac{\dot{C}}{C} = r - \rho(N). \quad (6)$$

2.2 Motion of environmental quality

Following Jouvét et al. (2005) and Angelopoulos et al. (2013), the stock of environmental quality evolves over time according to:

$$\dot{N} = (1 - \delta_N)(\bar{N} - N) - D, \quad \text{given} \quad N(0) > 0, \quad (7)$$

where \bar{N} denotes environmental quality without degradation, $D > 0$, and $\delta_N \in (0, 1)$ is the degree of environmental persistence. Environmental degradation is a positive function of

polluting emissions, P , and a negative function of public abatement expenditures, E :

$$D = D(P, E) = \frac{P}{\theta E}, \quad (8)$$

where $\theta > 0$ denotes the endogenous efficiency of the abatement technology in alleviating environmental degradation, which we define below. We further assume (among others, Andreoni and Levinson, 2001) that P occurs as a by-product of consumption:

$$P = sC, \quad (9)$$

with $s > 0$ denoting the emissions intensity.

In the same vein as Andreoni and Levinson (2001) and Quaas (2007), the efficiency of abatement expenditures depends on the level of infrastructure over GDP, $\frac{K_g}{Y}$:

$$\theta \equiv \theta \left(\frac{K_g}{Y} \right) = \xi \frac{K_g}{Y}. \quad (10)$$

$\xi > 0$ is a scaling parameter. Intuitively, higher investment in public infrastructure complements public expenditures on abatement and makes it possible to clean the environment in a more efficient way (e.g. public infrastructure, such as roads, is important for the impact of government policies on environmental protection).

2.3 Government budget constraint

The government spends G on infrastructure and E on environmental policy, and collects revenues through a tax on output, $\tau \in (0, 1)$. Assuming a balanced budget, we can write $G + E = \tau Y$. Equivalently, this can be written as:

$$G = b\tau Y \quad \text{and} \quad E = (1 - b)\tau Y, \quad (11)$$

where $b \in (0, 1)$ is the fraction of tax revenue used to finance infrastructure and $1 - b$ is the fraction that finances environmental investment. Thus, government policy can be summarized by the two policy instruments, τ and b . The law of motion for the public

capital stock is given by:

$$\dot{K}_g = G - \delta_{K_g} K_g, \quad \text{given} \quad K_g(0) > 0, \quad (12)$$

where δ_{K_g} denotes the depreciation rate.

3 Decentralized competitive equilibrium

By combining (1)-(12), assuming without loss of generality that $\delta_K = \delta_{K_g} = \delta$, and defining the auxiliary stationary variables, $\omega \equiv \frac{C}{K}$ and $z \equiv \frac{K}{K_g}$, the dynamics of the economy, are provided by:

$$\frac{\dot{\omega}}{\omega} = A(a-1)(1-\tau)z^{a-1} - \rho(N) + \omega, \quad (13)$$

$$\frac{\dot{z}}{z} = A(1-\tau)z^{a-1} - Ab\tau z^a - \omega, \quad (14)$$

$$\dot{N} = (1-\delta_N)(\bar{N} - N - \Xi\omega z), \quad (15)$$

with $\Xi \equiv s/(\xi\tau(1-\delta_N)(1-b))$, and the transversality condition $\lim_{t \rightarrow \infty} \frac{K(t)}{C(t)} \exp \left[- \int_0^t \rho(N_s) ds \right] = 0$. It follows that on the Balanced Growth Path (BGP) $\frac{\dot{\omega}}{\omega} = \frac{\dot{z}}{z} = \frac{\dot{N}}{N} = 0$. From (13)-(15) at the steady-state, the long-run value of \hat{z} is determined by:²

$$\Phi(\hat{z}) \equiv -Ab\tau\hat{z}^a + Aa(1-\tau)\hat{z}^{a-1} - \rho(\hat{N}(\hat{z})) = 0, \quad (16)$$

with $\hat{N}(\hat{z}) \equiv \bar{N} - \Xi[(1-\tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}]$. Provided there exists a solution $\hat{z} > 0$ in (16), the steady state values for growth and consumption-to-capital ratio are determined by $g = r(\hat{z}) - \rho(\hat{N}(\hat{z}))$, with $r(\hat{z})$ from (2), and $\hat{\omega}(\hat{z}) = A(1-\tau)\hat{z}^{a-1} - Ab\tau\hat{z}^a$, respectively. We can now prove the existence and uniqueness of multiple equilibria.

Proposition 1 (*Existence and Uniqueness*) *Under endogenous RTP, there exist parameter values where two stable equilibria arise, with different growth rates ranked $g_1 < g_2$ where $\hat{\rho}_1 > \hat{\rho}_2$, $\hat{\omega}_1 > \hat{\omega}_2$, $\hat{z}_1 < \hat{z}_2$, $\hat{N}_1 < \hat{N}_2$.*

Proof. See Supplementary Appendix A .

²We use hats to denote steady-state values.

Following Proposition 1 our model solves for two stable equilibria: a low-(high-) growth one with low (high) environmental quality, a high (low) RTP, a high (low) consumption-capital ratio and a low (high) physical-to-public capital ratio. In the former case, the propensity to consume is larger, and this generates more pollution, a lower environmental quality and, in turn, ties in with a high value for the degree of impatience. The higher degree of impatience leads to a lower growth rate reinforcing a vicious cycle of lower environmental quality and low growth propagates to keep the economy in an “environmental and economic poverty trap”. The opposite occurs for countries in the good equilibrium.

3.1 Productivity, growth and environmental quality

We now study the effect of a change in productivity (TFP) on environmental quality and growth.

Proposition 2 *If the response of the RTP to environmental quality is sufficiently high, then for the low growth, bad environment equilibrium (g_1, \hat{N}_1) , an increase in productivity, A , has a negative effect on steady-state environmental quality, $\frac{\partial \hat{N}_1}{\partial A} < 0$, the long-run economic growth rate, $\frac{\partial g_1}{\partial A} < 0$, and the physical to public capital ratio, $\frac{\partial \hat{z}_1}{\partial A} < 0$, while it has a positive effect on the consumption to physical capital ratio, $\frac{\partial \hat{\omega}_1}{\partial A} > 0$.*

Proof. See supplementary Appendix B. ■

Intuitively, if the environmental quality is low, individuals’ long-term orientation is weak and, in turn, their propensity to save is low. Then, while with an increase in productivity, their income initially increases (first order effect), agents increase their consumption proportionally more than their savings, $\frac{\partial \omega_1}{\partial A} > 0$. Higher consumption increases pollution and lowers savings, leading to lower income and growth in the future. Subsequently, lower growth results in a lower tax base (for a given tax rate), and the resources for abatement become insufficient to restore the environmental damage (second order effect). If the response of the RTP on the environment is strong, the first order – positive – effect of productivity on income (static) is outweighed by the second order –negative – effect of increasing consumption and, in turn, pollution (dynamic). This results in lower environmental quality and growth as Proposition 2 formally addresses. Figure 2 provides a numerical example

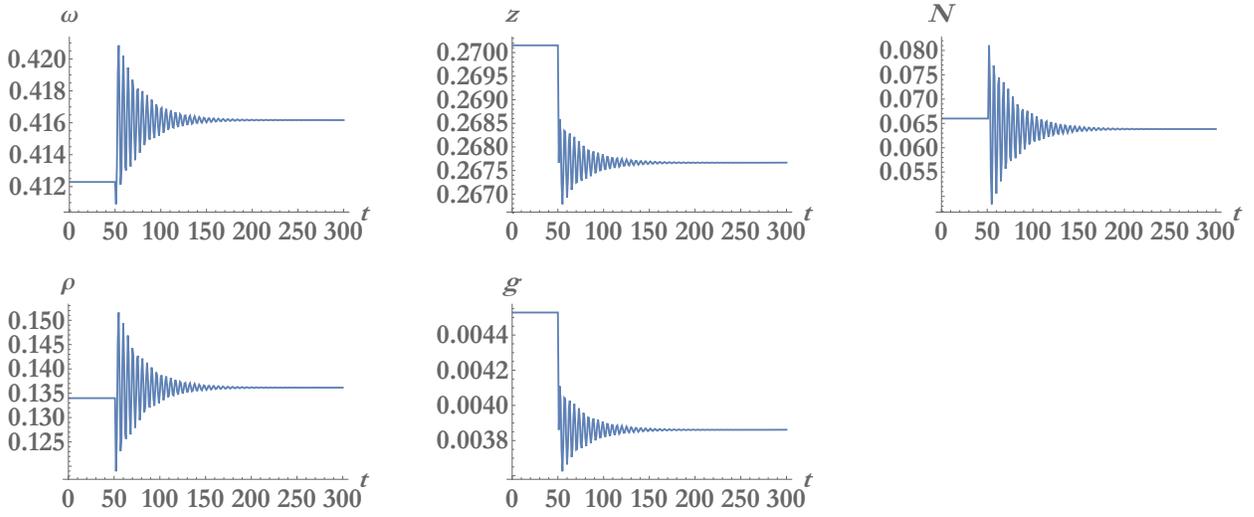
and justifies Proposition 1 (multiple, stable equilibria with non-monotonic dynamics as provided in Appendix A) and Proposition 2 (different responses of the two equilibria to a productivity shock).

4 Conclusion

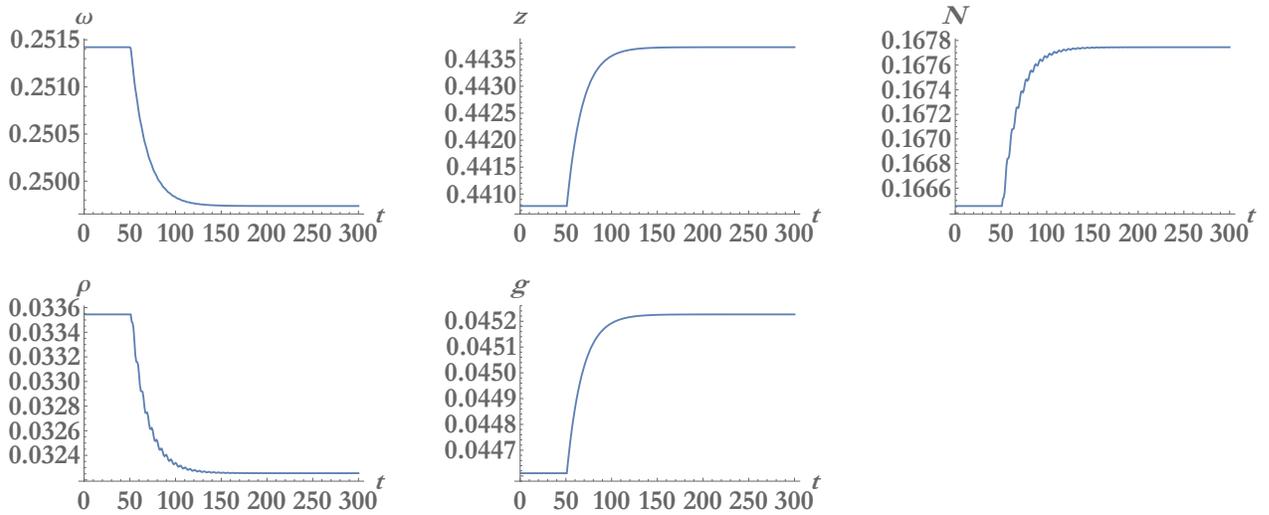
In this paper we provided a new behavioral mechanism behind the fact that some countries often stagnate in environmental and economic poverty traps. In contrast to conventional wisdom, we showed that productivity increases may lead to lower growth and lower environmental quality when the latter shapes individuals' views for the future. This outcome opens up a research route towards the implementation of potential behavioural changes to address the issue.

Figure 2: **Dynamics for the bad and the good steady state.** A small productivity increase at $t = 50$ (from $A = 0.659$ to $A = 0.66$) improves both environmental and development prospects in the good equilibrium (a); it worsens both in the bad equilibrium (b). $\rho(N) = \check{\rho} - \gamma N$, $\alpha = 0.5, \delta = 0.14, \delta_N = 0.9, s = 1, \xi = 0.4, \bar{N} = 20, \tau = 0.561, b = 0.751, \check{\rho} = 0.2, \gamma = 1$.

(a) Bad Equilibrium



(b) Good Equilibrium



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Roads to Prosperity without Environmental Poverty:

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Supplementary Appendices

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Appendix A: Proof of Proposition 1

Existence of multiple equilibria: The method will be to separate function $\Phi(\hat{z}) \equiv Aa(1 - \tau)\hat{z}^{a-1} - Ab\tau\hat{z}^a - \rho(\bar{N} - \Xi[(1 - \tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}])$ in two functions and find their intersection to solve it. Let $\Gamma(\hat{z}) \equiv a(1 - \tau)\hat{z}^{a-1} - b\tau\hat{z}^a$ and $\Lambda(\hat{z}) \equiv \frac{1}{A}\rho(\bar{N} - \Xi[(1 - \tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}])$ be these two functions. Both $\Gamma(\hat{z})$ and $\Lambda(\hat{z})$ are continuous in \hat{z} . First, let us find the domain of z where there cannot be a fixed point. We know that any equilibrium must ensure $\rho(\hat{N}(\hat{z})) = A\Lambda(\hat{z}) \geq 0$. Then it must also hold that $\Gamma(\hat{z}) \geq 0$.

Equation $\Gamma(\hat{z})$ has the following properties:

1. $\Gamma'(\hat{z}) < 0$, $\Gamma''(\hat{z}) > 0$,
1. $\lim_{\hat{z} \rightarrow 0} \Gamma(\hat{z}) = +\infty$, $\lim_{\hat{z} \rightarrow \frac{a(1-\tau)}{b\tau}} \Gamma(\hat{z}) = 0$.

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From the properties of $\Gamma(\hat{z})$ it follows that it is a strictly decreasing and convex function which starts from $+\infty$ and ends at 0 in the domain of feasible equilibria $(0, \bar{z}]$, $\bar{z} \equiv \frac{a(1-\tau)}{b\tau}$. That is, if there is an equilibrium \hat{z} , it will be for $0 < \hat{z} \leq \bar{z}$, because for $\hat{z} > \bar{z}$ then $\Gamma(\hat{z}) < 0$. Also, since $\bar{z} < \frac{1-\tau}{b\tau}$ (because $a < 1$) then for any $z < \bar{z}$ it follows that $\hat{\omega} > 0$ (from $\hat{\omega}(\hat{z}) = A(1-\tau)\hat{z}^{a-1} - Ab\tau\hat{z}^a$).

Equation $\Lambda(\hat{z})$ has the following properties:

1. $\Lambda(\hat{z})$ has a maximum at $z_{max} = \frac{1}{1+a}\bar{z}$,
2. $\lim_{\hat{z} \rightarrow 0} \Lambda(\hat{z}) = \lim_{\hat{z} \rightarrow \frac{1-\tau}{b\tau}} \Lambda(\hat{z}) = \rho(\bar{N})/A$.

Since $\rho'(N) < 0$, we have $\Lambda'(\hat{z}) > 0$ for $a(1-\tau)\hat{z}^{a-1} - b(1+a)\tau\hat{z}^a > 0 \implies \hat{z} < \frac{1}{1+a}\bar{z}$ and $\Lambda'(\hat{z}) < 0$ for $\hat{z} > \frac{1}{1+a}\bar{z}$. Thus, $\Lambda(\hat{z})$ has a maximum at $z_{max} = \frac{1}{1+a}\bar{z}$. Note also that $z_{max} < \bar{z}$ as $a > 0$. Additionally, since Λ starts and ends at $\rho(\bar{N})/A$ it follows that it is an inverse U-shaped curve with $z_{max} \in (0, \bar{z})$. With regards to $\rho(\bar{N})$ we have the following options: $\rho(\bar{N}) \geq 0$ and $\rho(\bar{N}) < 0$. If $\rho(\bar{N}) \geq 0$, since $\bar{z} < \frac{1-\tau}{b\tau}$ and $\lim_{\hat{z} \rightarrow 0} \Gamma(\hat{z}) = +\infty$, Λ crosses Γ once and there is a unique equilibrium. For $\rho(\bar{N}) < 0$ and since $\Lambda(\hat{z})$ is an inverse U-shaped function, Λ crosses Γ at most twice.

Therefore, the sufficient conditions for two equilibria are $\rho(\bar{N}) < 0$ and $\Lambda(z_{max}) > \Gamma(z_{max})$, the latter excluding the possibility of no equilibrium, or the trivial case of the single, tangency, equilibrium. A graphical illustration is provided in Figure 1 of this Appendix.

Ranking of Equilibria: Let $\hat{z}_1 < \hat{z}_2$ be the two equilibria for which $\Phi(\hat{z}_1) = \Phi(\hat{z}_2) = 0$. To find the ranking of the corresponding $\hat{\omega}_1$ and $\hat{\omega}_2$ note that $\hat{\omega}'(\hat{z}) = (a-1)(1-\tau)A\hat{z}^{a-2} - abA\tau\hat{z}^{a-1} < 0$. Thus, $\hat{\omega}$ is a strictly decreasing function of \hat{z} , $\hat{z}_1 < \hat{z}_2 \implies \hat{\omega}_1 > \hat{\omega}_2$. In the same way $g'(\hat{z}) = b\tau a\hat{z}^{a-1} > 0$, so $\hat{z}_1 < \hat{z}_2 \implies g_1 < g_2$. The ranking for the rate of time preference comes from the analysis above. Then, $\hat{z}_1 < \hat{z}_2 \implies \Lambda(\hat{z}_1) > \Lambda(\hat{z}_2) \implies \rho_1 > \rho_2$. Also for $\hat{z}_1 < \hat{z}_2 \implies \hat{\omega}_1 > \hat{\omega}_2$ and environmental quality, $N_1 < N_2$. So, in case of two balanced growth rates with low growth, g_1 , and high growth, g_2 , the endogenous variables are ranked as $\rho_1 > \rho_2$, $\hat{\omega}_1 > \hat{\omega}_2$, $\hat{z}_1 < \hat{z}_2$, $N_1 < N_2$.

Stability analysis: To examine stability, we compute the Jacobian Matrix of the three-dimensional dynamical system (13)-(15) of the main text. Around a steady state $\{\hat{\omega}, \hat{z}, \hat{N}\}$

we get the linearized version of the dynamic system as:

$$\begin{bmatrix} \dot{\omega} \\ \dot{z} \\ \dot{N} \end{bmatrix} = J \begin{bmatrix} \omega - \hat{\omega} \\ z - \hat{z} \\ N - \hat{N} \end{bmatrix},$$

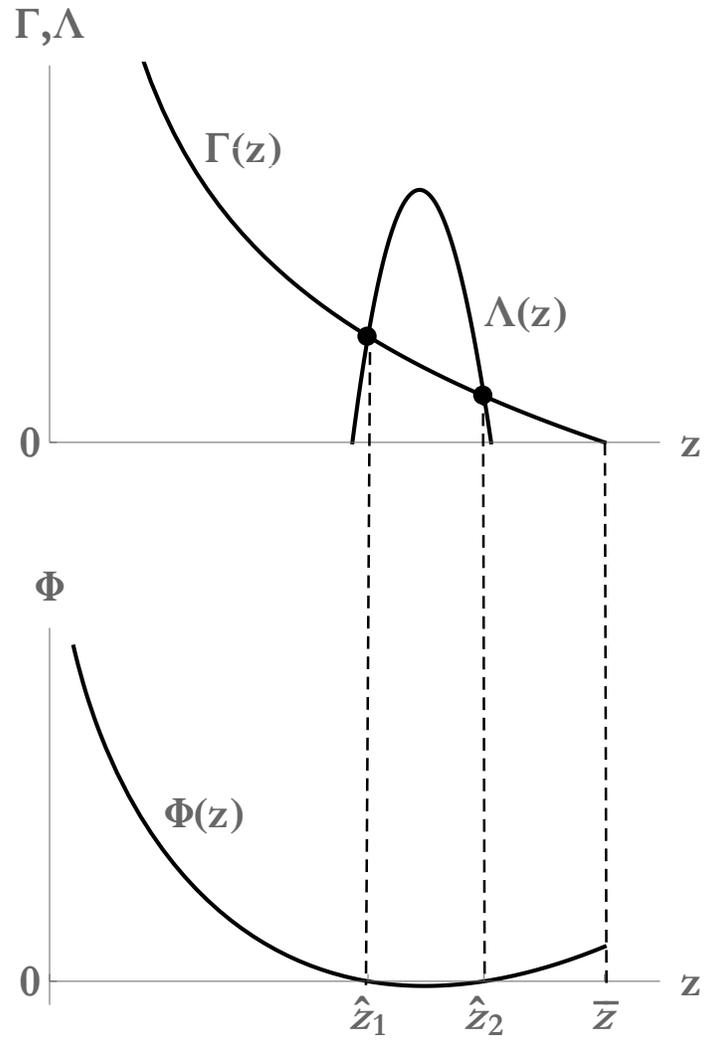
where $J \equiv \begin{bmatrix} J_{\omega\omega} & J_{\omega z} & J_{\omega N} \\ J_{z\omega} & J_{zz} & J_{zN} \\ J_{N\omega} & J_{Nz} & J_{NN} \end{bmatrix}$ the Jacobian evaluated at the steady state. The elements of the matrix are given by:

$$\begin{aligned} J_{\omega\omega} &= \hat{\omega}, J_{\omega z} = A(1 - \tau)(a - 1)^2 \hat{z}^{a-2} \hat{\omega}, J_{\omega N} = -\rho'(\hat{N})\hat{\omega}, \\ J_{z\omega} &= -\hat{z}, J_{zz} = -\frac{A}{\Xi} \frac{\Lambda'(\hat{z})}{\rho'(\hat{N})} - \hat{\omega}, J_{zN} = 0, \\ J_{N\omega} &= -\Xi \hat{z}, J_{Nz} = -\Xi \hat{\omega}, J_{NN} = -(1 - \delta_N). \end{aligned}$$

The local dynamics of the economy are nontrivial and analytically intractable. For that reason we resort to numerical simulations, to (i) compute the two equilibria, (ii) solve for the characteristic equation and compute the eigenvalues of J for those equilibria.

In particular, assuming $\rho(N) = \bar{\rho} - \gamma N$ and using the set of parameter values used in the paper ($\alpha = 0.5$, $A = 0.659$, $\delta = 0.14$, $\tau = 0.561$, $b = 0.751$, $\delta_N = 0.9$, $\xi = 0.4$, $s = 1$, $\bar{N} = 20$, $\gamma = 1$, $\bar{\rho} = 0.2$), we obtain for the low-growth and bad-environment equilibrium ($g_1 = 0.004$, $\hat{N}_1 = 0.066$) the following eigenvalues: $\varepsilon_1 = -0.054 + 1.380i$, $\varepsilon_2 = -0.054 - 1.380i$, $\varepsilon_3 = 0.069$, and for the high-growth and good-environment equilibrium ($g_2 = 0.044$, $\hat{N}_2 = 0.166$) the following eigenvalues: $\varepsilon_1 = -0.051 + 1.389i$, $\varepsilon_2 = -0.051 - 1.389i$, $\varepsilon_3 = -0.058$. Given that there exist two predetermined and one jump variables in our dynamical system, it follows that both equilibria are stable. For the good equilibrium this is straightforward since all real parts are negative. For the bad equilibrium this follows from the fact that the complex eigenvalues are dominant over the third one and have negative real parts. Thus, there exist parameter values such that the multiple (two) equilibria derived in Proposition 1 are both stable and, hence, meaningful for examining changes in the structural and policy parameters of the model.

Figure 1: Multiple equilibria



Appendix B: Proof of Proposition 2

The task is to calculate the effect of an increase of A on the steady state variables $\hat{\omega}$, \hat{z} , \hat{N} and subsequently on $\rho(\hat{N})$ and $g(\hat{N})$, in the presence of multiple equilibria.

To ease exposition we repeat here that for the two equilibria with $\hat{z}_1 < \hat{z}_2 < \bar{z}$ that solve $\Phi(\hat{z}) \equiv \Gamma(\hat{z}) - \Lambda(\hat{z}) = 0$, with $\Gamma(\hat{z}) \equiv b\tau\hat{z}^{a-1}(\bar{z} - \hat{z})$ and $\Lambda(\hat{z}) \equiv \frac{1}{A}\rho(\bar{N} - \Xi Ab\tau\hat{z}^a(\frac{1-\tau}{b\tau} - \hat{z}))$ holds $\hat{z}_1 < z_{max}$ for the bad equilibrium, and for the good $\hat{z}_2 > z_{max}$. Moreover, it follows from the above and $\rho'(\cdot) < 0$ that $\hat{z} < (>)z_{max} \implies \Lambda'(\hat{z}) > (<)0$, while $\Gamma'(\hat{z}) < 0$; see also Figure 1 of Appendix A. Additionally, it follows from eq. (14) of the main text that $\partial\hat{\omega}/\partial\hat{z} < 0$.

The effect of a productivity increase on functions $\Gamma(\hat{z})$ and $\Lambda(\hat{z})$ is given by:

1. $\frac{\partial\Gamma(\hat{z})}{\partial A} = 0$, i.e. function $\Gamma(\hat{z})$ does not shift with A ,
2. $\frac{\partial\Lambda(\hat{z})}{\partial A} = -\frac{1}{A^2}\rho(\hat{N})\left[1 + \epsilon_{\rho N}\left(\frac{\bar{N}}{\hat{N}} - 1\right)\right]$, with $\epsilon_{\rho N} \equiv \rho'(\hat{N})\hat{N}/\rho(\hat{N}) < 0$.

We get from 2. above that if $\epsilon_{\rho N} < -(\bar{N}/\hat{N} - 1)^{-1} < 0 \implies \partial\Lambda(\hat{z})/\partial A > 0$, i.e. the function shifts up. Provided that the subjective discount rate ρ responds sufficiently to a change in environmental quality, as measured by the elasticity $\epsilon_{\rho N}$, function $\Lambda(\hat{z})$ shifts up and since $\Gamma(\hat{z})$ stays unchanged the two equilibria z_1 and z_2 move further apart from each other, i.e. $\partial z_1/\partial A < 0$ and $\partial z_2/\partial A > 0$; see Figure 2 of this Appendix.

With regards to the consumption-capital ratio, we can write from eq. (14) of the main text $\frac{\partial\hat{\omega}}{\partial A} = \frac{\hat{\omega}}{A} + \frac{\partial\hat{\omega}}{\partial\hat{z}}\frac{\partial\hat{z}}{\partial A}$. For the bad equilibrium we have $\partial z_1/\partial A < 0$, while $\partial\hat{\omega}/\partial\hat{z} < 0 \implies \partial\omega_1/\partial A > 0$. From eq. (13) we get that $\frac{\partial\omega_1}{\partial A} = \rho'(\cdot)\frac{\partial N_1}{\partial A} + (1-a)(1-\tau)z_1^{a-1} - A(1-a)^2(1-\tau)z_1^{a-2}\frac{\partial z_1}{\partial A}$. Since $\partial\omega_1/\partial A > 0$, $\rho'(\cdot) < 0$, $\partial z_1/\partial A < 0 \implies \partial N_1/\partial A < 0$. To study the effect of an increase in productivity on growth for the bad equilibrium, notice that, from eq. (6) of the main text, $\frac{\partial g}{\partial A} = \frac{\partial r(\hat{z})}{\partial A} - \rho'(\cdot)\frac{\partial\hat{N}(\hat{z})}{\partial A}$. Using eq. (2), (6) and the definition of $\epsilon_{\rho N}$ from above, we get that $\frac{\partial g}{\partial A} = \frac{g+\delta}{A} + \frac{\rho(\cdot)}{A}\left[1 + \epsilon_{\rho N}\left(\frac{\bar{N}}{\hat{N}} - 1\right)\right] + [-a(1-a)(1-\tau)\hat{z}^{a-1} + \rho'(\cdot)\Xi b\tau(1+a)\hat{z}^a(z_{max} - \hat{z})]\frac{\partial\hat{z}}{\partial A}\frac{A}{\hat{z}}$. Since $\partial z_1/\partial A < 0$ and $z_1 < z_{max}$ for the bad equilibrium, the previous condition of $\epsilon_{\rho N} < -(\bar{N}/\hat{N} - 1)^{-1} < 0$ is a necessary condition for $\partial g_1/\partial A < 0$. Accordingly, if the response of the RTP to a change in environmental quality is sufficiently high to ensure $\frac{\partial r(\hat{z})}{\partial A} < -\rho'(\cdot)\frac{\partial\hat{N}(\hat{z})}{\partial A}$, then $\partial g_1/\partial A < 0$.

Finally, as illustrated in Figure 2 of the main text, there exist parameter values such that a small increase in productivity from $A = 0.659$ to $A = 0.66$, reduces g_1 , N_1 , z_1 , while increases ω_1 and $\rho(N_1)$. The opposite holds true for the variables in the good equilibrium.

Figure 2: **Effect of productivity increase on \hat{z}**

