Proceedings Paper:
Gautam, PR and Zhang, L (2020) Hybrid SLNR Precoding for Multi-user Millimeter Wave MIMO Systems. In: Proceedings of the 2019 22nd International Symposium on Wireless Personal Multimedia Communications (WPMC). 2019 22nd International Symposium on Wireless Personal Multimedia Communications (WPMC), 24-27 Nov 2019, Lisbon, Portugal. IEEE . ISBN 978-1-7281-5419-0

https://doi.org/10.1109/WPMC48795.2019.9096182

Hybrid SLNR Precoding for Multi-user Millimeter Wave MIMO Systems

Prabhat Raj Gautam
School of Electronic and Electrical Engineering
University of Leeds, Leeds, UK
Email: elprg@leeds.ac.uk

Li Zhang
School of Electronic and Electrical Engineering
University of Leeds, Leeds, UK
Email: l.x.zhang@leeds.ac.uk

Abstract—**It is difficult to implement a fully digital precoding in millimeter wave (mmWave) massive multiple-input multiple-output (MIMO) systems, owing to the huge cost and power consumption of radio frequency (RF) chains. Hybrid precoding that uses the combination of analog beamforming, together with digital precoding, offers a feasible solution to this problem where far lesser number of RF chains are employed compared to the number of antennas. We consider a downlink multi-user massive MIMO scenario and propose a hybrid precoding scheme that maximizes signal-to-leakage-and-noise ratio (SLNR). Each mobile station (MS) determines its analog combiner first and then the base station (BS) constructs the analog beamformer and the digital precoder of the hybrid precoder in two separate stages to maximize the SLNR for each user. We show, through simulations, that the proposed hybrid SLNR-based precoder exhibits a performance close to the fully digital SLNR-based precoder despite using only a few number of RF chains.**

I. INTRODUCTION

There has been an ever-growing demand for larger bandwidth and higher data rates in the communication systems. Increasing the number of antennas to a very high number, in order of few hundreds, in multiple-input multiple-output (MIMO) helps attain the goal of higher throughput [1], [2]. Such a MIMO system is commonly termed as massive MIMO or large-scale antenna systems. Millimeter wave (mmWave) communication, on the other hand, offers a very large and mostly unsullied bandwidth. Moreover, smaller wavelengths of mmWave signals allow a massive number of antennas be accommodated in a small area [3]. The mmWave communication together with massive MIMO, known as mmWave massive MIMO, has thus revealed itself as the most compelling prospect for future wireless communication technology like 5G cellular systems [4], [5]. The mmWave signals suffer from huge attenuation due to smaller wavelengths. However, the large array gain due to huge number of antennas in massive MIMO compensates for the attenuation.

In typical MIMO systems, precoding is performed digitally at baseband to adjust power and phases of the transmit signals. This procedure demands at least one radio frequency (RF) chain per antenna element [3], [6]. But with today's semiconductor technology, dedicating a single RF chain for each antenna in mmWave massive MIMO would escalate the cost and power consumption [7]. This has prompted to embrace hybrid precoding as a viable means of preprocessing in mmWave communication. Hybrid precoding combines baseband or digital precoding realized through a limited number of RF chains, and the analog or RF precoding achieved usually with the use of phase shifters. The works on precoding in mmWave massive MIMO in [3], [6], [8]–[11] have all adopted hybrid precoding. [3], [6], [8] have proposed hybrid precoding techniques for single user MIMO, whereas [9]–[11] have developed hybrid precoding solutions for multi-user MIMO (MU-MIMO) environment.

A two stage hybrid precoding method is proposed in [9] where analog beamforming vectors are selected from a beamsteering codebook and digital precoding is realized by using the RF chains which are equal in number to the users. The analog combiner and beamformer are jointly chosen to maximize received power for each user, and zero-forcing (ZF) precoding is performed on the equivalent baseband channel by inverting the equivalent channel. In [10], hybrid block diagonalization precoding-combining is developed for multi-stream MU-MIMO downlink communication. The RF combiners for all users are chosen from discrete Fourier transform (DFT) basis set. RF beamforming matrix at the BS is constructed by obtaining the phases of the Hermitian transpose of the aggregate channel that includes the RF combiners. And finally block diagonalization precoding-combining is performed on the equivalent channel. In [11], two minimum mean squared error (MMSE)-based hybrid precoding-combining algorithms are presented. The first method is an orthogonal matching pursuit (OMP)-based algorithm to minimize the sum mean squared error (sum-MSE) and the second is an iterative method to minimize weighted sum-MSE.

The hybrid ZF precoder [9] necessitates that the equivalent channel matrix remains well-conditioned. Furthermore, hybrid ZF precoder and hybrid block diagonalization precoder [10] both do not take into account the impact of noise during the design. Thus, in this paper, we design hybrid precoder by maximizing signal-to-leakage-and-noise ratio (SLNR) which also considers the influence of noise and has proven to outperform zero-forcing solutions in traditional MU-MIMO [12]. We consider downlink mmWave MU-MIMO communication where the maximum number of users supported is equal to the number of RF chains at the BS but only as many RF chains are utilized as there are the number of users, similar to [9]. Each mobile station (MS) is assumed to have a single RF

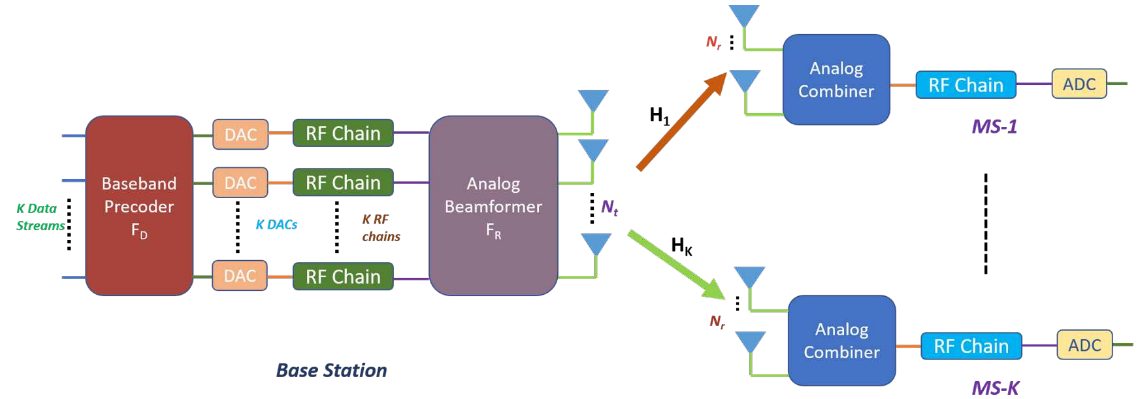

Fig. 1: System diagram showing mmWave MU-MIMO system with hybrid precoding at the BS and analog-only combining at the MS's.

chain and only analog-combining is considered at the MS. The performance of the proposed hybrid precoding-analog combining scheme is compared with the fully digital precoding counterpart and the hybrid ZF precoding. Simulation results reveal that the proposed precoding method, with a smaller number of RF chains, achieves very good spectral efficiency which is better than the hybrid ZF precoding and comparable to the unconstrained digital SLNR precoding.

Throughout the paper we follow mathematical notations as: z is a scalar; $\mathbf{z}$ is a vector; $\mathbf{Z}$ is a matrix; the i-th column of matrix $\mathbf{Z}$ is represented by $\mathbf{z}_i$; $\mathbf{z}^{(i)}$ is the i-th element of vector $\mathbf{z}$; $\mathbf{Z}^{(i,j)}$ is the (i,j)-th entry of matrix $\mathbf{Z}$; $|\mathbf{z}|$ is the 2-norm of $\mathbf{z}$; $(.)^T$, $(.)^*$, $(.)^{-1}$ and $(.)^H$ denote transpose, conjugate, inverse and Hermitian transpose respectively; $\mathbf{I}_N$ is the $N \times N$ identity matrix; $diag\{a_1, a_2, ..., a_n\}$ is a diagonal matrix with $a_1, a_2, ..., a_n$ as its diagonal elements; $\mathbb{E}\{.\}$ denotes expectation operator; $\sim$ means "has the probability distribution of"; $\mathcal{CN}(\mathbf{z}, \mathbf{Z})$ is a complex Gaussian vector with mean $\mathbf{z}$ and covariance matrix $\mathbf{Z}$.

II. System Model

A. Multi-user MIMO System and Signal Model

We consider a mmWave massive MIMO downlink system having a base station (BS) equipped with N_t transmit antennas and M_t RF chains, serving K users or mobile stations (MS's) simultaneously. We assume $M_t \geq K$, however, only K out of M_t RF chains would be used while communicating with K MS's. Each MS has N_r receive antennas and only one RF chain to support single data stream. BS multiplexes K data streams, one data stream each for the K MS's. We consider fully-connected hybrid architecture in which each RF chain is connected to all the antennas through phase shifters.

At the BS, transmit signal is first passed through a $K \times K$ baseband precoder $\mathbf{F}_\mathrm{D}$. $\mathbf{f}_{\mathrm{D}_k} \in \mathbb{C}^{K\times 1}$, the k^{th} column of $\mathbf{F}_\mathrm{D}$ is the digital precoder for the k^{th} MS. Then RF precoder $\mathbf{F}_\mathrm{R} \in \mathbb{C}^{N_t\times K}$, achieved through analog phase shifters, is applied to the transmit signal. The phase shifters impose a constant amplitude constraint on each element of $\mathbf{F}_\mathrm{R}$, *i.e.*, $|\mathbf{F}_\mathrm{R}^{(i,j)}| = \frac{1}{\sqrt{N_t}}$. $\mathbf{f}_k = \mathbf{F}_\mathrm{R}\mathbf{f}_{\mathrm{D}_k}$, $\mathbf{f}_k \in \mathbb{C}^{N_t\times 1}$ is the hybrid precoder for the k^{th} MS and has unit norm in order to satisfy the total power constraint.

Similar to [9]–[11], we take up narrow-band block-fading channel model. Received signal at the k^{th} MS is

$$\mathbf{r}_k = \mathbf{H}_k\mathbf{F}_\mathrm{R}\mathbf{F}_\mathrm{D}\mathbf{s} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{s} = [s_1\ s_2\ ...\ s_K\ ]^T$, $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the transmit signal and s_k is the symbol intended for the k^{th} MS. $\mathbf{H}_k \in \mathbb{C}^{N_r\times N_t}$ is the channel from the BS to the k^{th} MS and $\mathbf{n}_k \sim \mathcal{CN}\left(\mathbf{0}, \sigma_k^2\mathbf{I}_{N_r}\right)$ is the $N_r \times 1$ complex noise vector. We assume that transmit symbols for all MS's are independent of each other, *i.e.*, $\mathbb{E}\left[s_k s_k^*\right] = P_k$, $\mathbb{E}\left[s_i s_j^*\right] = 0$ for $i \neq j$, where P_k is the average power transmitted to the k^{th} user. Since we are considering equal power distribution among different users, $P_k = \frac{P}{K}$ where P is the average total transmit power.

The expression in (1) can be written as

$$\mathbf{r}_k = \mathbf{H}_k\mathbf{f}_k s_k + \mathbf{H}_k \sum_{\substack{m=1\\m\neq k}}^{K} \mathbf{f}_m s_m + \mathbf{n}_k. \tag{2}$$

At the receiver of the k^{th} MS, the received signal $\mathbf{r}_k$ is acted upon by RF combiner $\mathbf{w}_{r_k} \in \mathbb{C}^{N_r\times 1}$ to produce the processed received symbol y_k which is given by

$$y_k = \mathbf{w}_{r_k}^H\mathbf{H}_k\mathbf{f}_k s_k + \mathbf{w}_{r_k}^H\mathbf{H}_k \sum_{\substack{m=1\\m\neq k}}^{K} \mathbf{f}_m s_m + \mathbf{w}_{r_k}^H\mathbf{n}_k. \tag{3}$$

Each element of the analog combiner, $\mathbf{w}_{r_k}$ which is realized through analog phase shifters, is constrained to satisfy $|\mathbf{w}_{r_k}^{(i)}| = \frac{1}{\sqrt{N_r}}$. The second term on the right-hand side of (3), $\mathbf{w}_{r_k}^H\mathbf{H}_k\sum_{m=1,m\neq k}^{K}\mathbf{f}_m s_m$ is interference due to other users and known as co-channel interference (CCI).

B. mmWave Channel Model

A geometric channel model based on extended Saleh Valenzuela model is considered to model mmWave channel so as to represent its limited scattering nature where we assume each scatterer to contribute a single propagation path [6]. The narrowband downlink channel between BS and the k^{th} MS is given by

$$\mathbf{H}_k = \sqrt{\frac{N_t N_r}{L}} \sum_{\ell=1}^{L} \alpha_\ell^k \mathbf{a}_r^k(\theta_\ell^k, \beta_\ell^k) \mathbf{a}_t^{k^H}(\phi_\ell^k, \psi_\ell^k), \qquad (4)$$

where L is the number of propagation paths, α_ℓ^k is the complex gain of the ℓ^{th} path, $\phi_\ell^k \left(\psi_\ell^k\right)$ and $\theta_\ell^k \left(\beta_\ell^k\right)$ are azimuth (elevation) angles of departure (AoDs) of the BS and azimuth (elevation) angles of arrival (AoAs) of the k^{th} MS respectively. $\mathbf{a}_r^k(\theta_\ell^k, \beta_\ell^k)$ and $\mathbf{a}_t^k(\phi_\ell^k, \psi_\ell^k)$ are the antenna array response vectors of the MS and the BS respectively. We consider that uniform planar arrays (UPA) are used at both the BS and the MS. The antenna array response vector of the BS can be written as

$$\mathbf{a}_t^k(\phi_\ell^k, \psi_\ell^k) = \frac{1}{\sqrt{N_t}} \Big[1 \ \dots \ e^{jpd\left(m \sin(\phi_\ell^k)\sin(\psi_\ell^k) + n\cos(\psi_\ell^k)\right)} \ \dots \ e^{jpd\left((X-1)\sin(\phi_\ell^k)\sin(\psi_\ell^k) + (Y-1)\cos(\psi_\ell^k)\right)}\Big]^T, \qquad (5)$$

where $p = (2\pi/\lambda)$, λ is the carrier wavelength, d is the distance between antenna elements which we will take as half the wavelength in the simulations. X and Y are the number of antenna elements along the *width* and the *height* of the UPA respectively so that $XY = N_t$, and $0 \le m < X$ and $0 \le n < Y$ are the antenna element indices. We can write the antenna array response vector of the MS in a similar fashion.

III. PROBLEM FORMULATION

We define signal-to-interference-and-noise ratio (SINR) for the k^{th} MS as

$$\text{SINR}_k = \frac{\left|\mathbf{w}_{\text{r}_k}^H \mathbf{H}_k \mathbf{f}_k s_k\right|^2}{\sum\limits_{m=1, m\neq k}^{K} \left|\mathbf{w}_{\text{r}_k}^H \mathbf{H}_k \mathbf{f}_m s_m\right|^2 + \left|\mathbf{w}_{\text{r}_k}^H \mathbf{n}_k\right|^2}. \qquad (6)$$

The achievable rate for user k when Gaussian symbols are used is given by

$$R_k = \log_2(1 + \text{SINR}_k). \qquad (7)$$

Then the sum-rate or sum spectral efficiency of the system is $R_{sum} = \sum_{k=1}^{K} R_k$. In MU-MIMO, one of the fundamental motivations of using a precoder is to maximize sum-rate of the system which means maximizing SINR for each MS. In light of achieving this, precoding methods like zero-forcing (ZF) precoding and block-diagonalization aim to make CCI terms zero. This is because finding a precoder to maximize SINR_k for each MS_k, $k = 1, 2, ..., K$ poses a serious challenge due to K coupled vectors $\{\mathbf{f}_k\}_{k=1}^K$ [12], [13]. The authors of [12], in view of circumventing this issue, introduced another figure of merit called signal-to-leakage-and-noise ratio (SLNR) to find precoder. SLNR for the k^{th} MS is defined as

$$\text{SLNR}_k = \frac{\left|\mathbf{H}_k \mathbf{f}_k s_k\right|^2}{\sum\limits_{m=1, m\neq k}^{K} \left|\mathbf{H}_m \mathbf{f}_k s_k\right|^2 + \left|\mathbf{n}_k\right|^2}, \qquad (8)$$

where $\sum_{m=1, m\neq k}^{K} \left|\mathbf{H}_m \mathbf{f}_k s_k\right|^2$ is the total power leaked from user k to all other users and is called *leakage* [12]. Thus, SLNR_k indicates the ratio of total useful power for user k to the sum of its interfering power on other users and noise. We can see that the expression for SLNR_k only involves precoding vector for the k^{th} MS unlike the case of SINR_k. Hence maximizing SLNR provides an elegant solution to the problem of determining the precoding vector as it is easy to arrive at a closed form solution.

IV. PROPOSED HYBRID SLNR PRECODING

In this paper, we choose SLNR as an optimizing criterion to determine precoding matrix at the BS. At first, each MS_k chooses its RF combiner $\mathbf{w}_{\text{r}_k}$ from MS beamforming codebook $\mathcal{W}_k$ that maximizes its received power, *i.e.*,

$$\mathbf{w}_{\text{r}_k}^\star = \underset{\mathbf{w}_{\text{r}_k} \in \mathcal{W}_k}{\arg\max}\, \mathbf{w}_{\text{r}_k}^H \mathbf{H}_k \mathbf{H}_k^H \mathbf{w}_{\text{r}_k}. \qquad (9)$$

As in [3], [9], we use beamsteering codebook at the MS. The MS beamforming codebook is characterized by (B_θ, B_β) quantization bits. B_θ (B_β) bits quantify the uniform quantization of the azimuth (elevation) angles. That is, $\mathcal{W}_k$ contains the MS array response vectors $\mathbf{a}_r(\theta, \beta)$ for all combinations of θ and β , where $\theta \in \left\{\frac{2\pi}{2^{B_\theta+1}},\ \frac{3.2\pi}{2^{B_\theta+1}}, ...,\ 2\pi - \frac{2\pi}{2^{B_\theta+1}}\right\}$ and $\beta \in \left\{-\frac{\pi}{2} + \frac{\pi}{2^{B_\beta+1}},\ -\frac{\pi}{2} + \frac{3.\pi}{2^{B_\beta+1}}, ...,\ \frac{\pi}{2} - \frac{\pi}{2^{B_\beta+1}}\right\}$ [3]. With the analog combiner for the k^{th} MS determined, we now define an equivalent channel $\mathbf{h}_{\text{e}_k} \in \mathbb{C}^{1\times N_t}$ for each MS as

$$\mathbf{h}_{\text{e}_k} = \mathbf{w}_{r_k}^H \mathbf{H}_k,\ k = 1, 2,\ \dots\ , K \qquad (10)$$

so that the processed received symbol y_k in (3) can be equivalently written as

$$y_k = \mathbf{h}_{\text{e}_k} \mathbf{f}_k s_k + \mathbf{h}_{\text{e}_k} \sum_{\substack{m=1 \\ m\neq k}}^{K} \mathbf{f}_m s_m + z_k, \qquad (11)$$

where $z_k = \mathbf{w}_{r_k}^H \mathbf{n}_k$ is the complex scalar representing the processed noise. Now at the BS, we seek to determine hybrid precoder for each MS by maximizing SLNR for the effective channel for each of the K MS's. We can now define SLNR for the equivalent channel to the k^{th} MS as

$$\text{SLNR}_k = \frac{\frac{P}{K}\left|\mathbf{h}_{\text{e}_k}\mathbf{f}_k\right|^2}{\frac{P}{K}\left|\tilde{\mathbf{H}}_{\text{e}_k}\mathbf{f}_k\right|^2 + \left|z_k\right|^2}, \qquad (12)$$

where $\frac{P}{K}$ is the power of the k^{th} symbol, *i.e.*, $|s_k|^2 = \frac{P}{K}$ and $\tilde{\mathbf{H}}_{\text{e}_k} = \left[\mathbf{h}_{\text{e}_1}^T\ \mathbf{h}_{\text{e}_2}^T\ \dots\ \mathbf{h}_{\text{e}_{k-1}}^T\ \mathbf{h}_{\text{e}_{k+1}}^T\ \dots\ \mathbf{h}_{\text{e}_K}^T\right]^T$ is $(K-1)\times N_t$ extended channel matrix that only excludes $\mathbf{h}_{\text{e}_k}$. Noting

$|z_k|^2 = \sigma_k^2$ and $|\mathbf{f}_k|^2 = 1$, and defining $\gamma_k = \frac{P}{K\sigma_k^2}$, we may simplify the expression for SLNR as

$$\text{SLNR}_k = \frac{\mathbf{f}_k^H \left(\mathbf{h}_{\mathrm{e}_k}^H \mathbf{h}_{\mathrm{e}_k}\right) \mathbf{f}_k}{\mathbf{f}_k^H \left(\tilde{\mathbf{H}}_{\mathrm{e}_k}^H \tilde{\mathbf{H}}_{\mathrm{e}_k} + \frac{1}{\gamma_k}\mathbf{I}_{N_t}\right) \mathbf{f}_k}. \tag{13}$$

If we find $\mathbf{f}_k$ directly by maximizing the expression in (13), it would give us the fully digital precoder which would require N_t RF chains. The precoder $\mathbf{f}_k$ that we want to determine is the combination of analog beamforming matrix $\mathbf{F}_{\mathrm{R}}$ and the digital precoder $\mathbf{f}_{\mathrm{D}_k}$ implemented using only K RF chains. Thus we split the task of designing precoder into two different stages, *viz.*, finding $\mathbf{F}_{\mathrm{R}}$ and finding $\mathbf{f}_{\mathrm{D}_k}$. Similar to the MS, the BS employs a beamsteering codebook $\mathcal{F}$ with (B_ϕ, B_ψ) quantization bits and containing array response vectors of the BS for the uniformly quantized azimuth and elevation angles. The BS chooses the columns of RF beamformer $\mathbf{F}_{\mathrm{R}}$, $\mathbf{f}_{\mathrm{R}_k}$ from the beamforming codebook $\mathcal{F}$ to maximize SLNR for the k^{th} equivalent channel, *i.e.*,

$$\mathbf{f}_{\mathrm{R}_k}^\star = \underset{\mathbf{f}_{\mathrm{R}_k}\in\mathcal{F}}{\arg\max} \frac{\mathbf{f}_{\mathrm{R}_k}^H \left(\mathbf{h}_{\mathrm{e}_k}^H \mathbf{h}_{\mathrm{e}_k}\right) \mathbf{f}_{\mathrm{R}_k}}{\mathbf{f}_{\mathrm{R}_k}^H \left(\tilde{\mathbf{H}}_{\mathrm{e}_k}^H \tilde{\mathbf{H}}_{\mathrm{e}_k} + \frac{1}{\gamma_k}\mathbf{I}_{N_t}\right) \mathbf{f}_{\mathrm{R}_k}}, k = 1, 2, ..., K. \tag{14}$$

Since we want to design a hybrid precoder to maximize SLNR, it is only natural that we choose columns of RF beamformer from $\mathcal{F}$ which maximize SLNR. The BS, then, sets $\mathbf{F}_{\mathrm{R}} = \left[\mathbf{f}_{\mathrm{R}_1}^\star \ \mathbf{f}_{\mathrm{R}_2}^\star \ ... \ \mathbf{f}_{\mathrm{R}_K}^\star\right]$. To design $\mathbf{f}_{\mathrm{D}_k}$ we first expand the expression for SLNR as

$$\text{SLNR}_k = \frac{\mathbf{f}_{\mathrm{D}_k}^H \left(\mathbf{F}_{\mathrm{R}}^H \mathbf{h}_{\mathrm{e}_k}^H \mathbf{h}_{\mathrm{e}_k} \mathbf{F}_{\mathrm{R}}\right) \mathbf{f}_{\mathrm{D}_k}}{\mathbf{f}_{\mathrm{D}_k}^H \mathbf{F}_{\mathrm{R}}^H \left(\tilde{\mathbf{H}}_{\mathrm{e}_k}^H \tilde{\mathbf{H}}_{\mathrm{e}_k} + \frac{1}{\gamma_k}\mathbf{I}_{N_t}\right) \mathbf{F}_{\mathrm{R}} \mathbf{f}_{\mathrm{D}_k}}. \tag{15}$$

In the above expression, we replace $\tilde{\mathbf{H}}_{\mathrm{eq}_k} = \tilde{\mathbf{H}}_{\mathrm{e}_k}\mathbf{F}_{\mathrm{R}}$, $\mathbf{h}_{\mathrm{eq}_k} = \mathbf{h}_{\mathrm{e}_k}\mathbf{F}_{\mathrm{R}}$, and $\tilde{\mathbf{H}}_{\mathrm{eq}_k} \in \mathbb{C}^{(K-1)\times K}, \mathbf{h}_{\mathrm{eq}_k} \in \mathbb{C}^{1\times K}$ so that the SLNR expression becomes

$$\text{SLNR}_k = \frac{\mathbf{f}_{\mathrm{D}_k}^H \left(\mathbf{h}_{\mathrm{eq}_k}^H \mathbf{h}_{\mathrm{eq}_k}\right) \mathbf{f}_{\mathrm{D}_k}}{\mathbf{f}_{\mathrm{D}_k}^H \left(\tilde{\mathbf{H}}_{\mathrm{eq}_k}^H \tilde{\mathbf{H}}_{\mathrm{eq}_k} + \frac{1}{\gamma_k}\mathbf{F}_{\mathrm{R}}^H \mathbf{F}_{\mathrm{R}}\right) \mathbf{f}_{\mathrm{D}_k}}. \tag{16}$$

The above expression for SLNR is the expression for *generalized Rayleigh quotient* of the two Hermitian matrix pair $(\mathbf{h}_{\mathrm{eq}_k}^H \mathbf{h}_{\mathrm{eq}_k})$ and $(\tilde{\mathbf{H}}_{\mathrm{eq}_k}^H \tilde{\mathbf{H}}_{\mathrm{eq}_k} + \frac{1}{\gamma_k}\mathbf{F}_{\mathrm{R}}^H\mathbf{F}_{\mathrm{R}})$, and hence determining $\mathbf{f}_{\mathrm{D}_k}$ boils down to a generalized eigenvalue problem. The digital part of the hybrid precoder for the k^{th} MS, $\mathbf{f}_{\mathrm{D}_k}$ is thus given by the generalized eigenvector corresponding to the largest generalized eigenvalue of the matrices $(\mathbf{h}_{\mathrm{eq}_k}^H \mathbf{h}_{\mathrm{eq}_k})$ and $(\tilde{\mathbf{H}}_{\mathrm{eq}_k}^H \tilde{\mathbf{H}}_{\mathrm{eq}_k} + \frac{1}{\gamma_k}\mathbf{F}_{\mathrm{R}}^H\mathbf{F}_{\mathrm{R}})$. It is written in condensed form as

$$\mathbf{f}_{\mathrm{D}_k} \propto \text{max generalized eigenvector} \left(\mathbf{h}_{\mathrm{eq}_k}^H \mathbf{h}_{\mathrm{eq}_k}, \left(\tilde{\mathbf{H}}_{\mathrm{eq}_k}^H \tilde{\mathbf{H}}_{\mathrm{eq}_k} + \frac{1}{\gamma_k}\mathbf{F}_{\mathrm{R}}^H\mathbf{F}_{\mathrm{R}}\right)\right). \tag{17}$$

The hybrid precoder $\mathbf{f}_k$ needs to be of unit norm to satisfy total power constraint. Hence the obtained $\mathbf{f}_{\mathrm{D}_k}$ from (17) is normalized so that $|\mathbf{F}_{\mathrm{R}}\mathbf{f}_{\mathrm{D}_k}|^2 = 1$. If $(\tilde{\mathbf{H}}_{\mathrm{eq}_k}^H \tilde{\mathbf{H}}_{\mathrm{eq}_k} + \frac{1}{\gamma_k}\mathbf{F}_{\mathrm{R}}^H\mathbf{F}_{\mathrm{R}})$ is invertible, (17) reduces to a standard eigenvalue problem.

V. Performance Analysis in Single Path Channels

Analyzing the spectral efficiency of hybrid precoding is non-trivial as it involves the combination of analog beamformer and digital precoder. In this section, we will analyze the performance of hybrid SLNR precoding in a single path channel with large number of BS antennas. Because mmWave channel is sparse and large antenna arrays are required for enough received power [9], this case holds significance. We assume perfect channel knowledge at MS and the perfect effective channel knowledge at the BS. We further assume that infinite resolution codebooks are available at both the BS and the MS, *i.e.*, analog beamformer and combiner can choose beamsteering vectors with continuous angles.

The channel from the BS to the k^{th} MS is given by

$$\mathbf{H}_k = \sqrt{N_t N_r}\alpha^k \mathbf{a}_r^k(\theta^k) {\mathbf{a}_t^k}^H(\phi^k). \tag{18}$$

Since it is a single path channel, we have dropped the subscript ℓ from all the parameters. For the sake of simplicity, we have assumed ULAs at the BS and the MS, and hence considered only the azimuth AoAs and AoDs. At first, MS_k chooses analog combiner to maximize received power solving (9).

Given the single path channel $\mathbf{H}_k$ and the choice of $\mathbf{w}_{\mathrm{r}_k}$ to take beamsteering vector of any angle, it is obvious that when $\mathbf{w}_{\mathrm{r}_k} = \mathbf{a}_r^k(\theta^k)$ the received power is maximized. Hence MS chooses $\mathbf{a}_r^k(\theta^k)$ as $\mathbf{w}_{\mathrm{r}_k}$. The effective channel to the k^{th} MS is given by

$$\mathbf{h}_{\mathrm{e}_k} = \sqrt{N_t N_r}\alpha^k {\mathbf{a}_t^k}^H(\phi^k). \tag{19}$$

Now, the BS decides upon the $\mathbf{f}_{\mathrm{R}_k}$, the k^{th} column of analog beamformer $\mathbf{F}_{\mathrm{R}}$ by solving (14). If the number of antennas used (UPA or ULA) at the BS is large, we can write from [10]

$$\lim_{N_t\to\infty} \sum_{\substack{m=1\\ m\neq k}}^{K} {\mathbf{a}_t^k}^H(\phi^k)\mathbf{a}_t^m(\phi^m) = 0. \tag{20}$$

If $\mathbf{a}_t^k(\phi^k)$ is chosen as $\mathbf{f}_{\mathrm{R}_k}$, it will maximize the numerator in (14). At the same time, it will minimize the denominator in (14) following the relation in (20). Therefore, it is fair to say $\mathbf{a}_t^k(\phi^k)$ is the optimal $\mathbf{f}_{\mathrm{R}_k}$. Hence, the BS sets $\mathbf{F}_{\mathrm{R}} = \left[\mathbf{a}_t^1(\phi^1) \ \mathbf{a}_t^2(\phi^2) \ ... \ \mathbf{a}_t^K(\phi^K)\right]$. The equivalent channel with analog combiner and analog beamformer applied is

$$\begin{aligned}\mathbf{h}_{\mathrm{eq}_k} &= \sqrt{N_t N_r}\alpha^k {\mathbf{a}_t^k}^H(\phi^k)\left[\mathbf{a}_t^1(\phi^1) \ ... \ \mathbf{a}_t^K(\phi^K)\right] \\ &\approx \sqrt{N_t N_r}\alpha^k \mathbf{e}_k^H,\end{aligned} \tag{21}$$

where $\mathbf{e}_k$ is a K-length column vector with 1 at the k^{th} position and zero elsewhere. Further, we can write

$$\mathbf{h}_{\mathrm{eq}_k}^H \mathbf{h}_{\mathrm{eq}_k} = N_t N_r \mathbf{A}_k, \tag{22}$$

where $\mathbf{A}_k$ is a $K \times K$ matrix in which ${\mathbf{A}_k}^{(k,k)} = \left|\alpha^k\right|^2$ while all other entries are zero. Also,

$$\tilde{\mathbf{H}}_{\mathrm{eq}_k}^H \tilde{\mathbf{H}}_{\mathrm{eq}_k} + \frac{1}{\gamma_k} \mathbf{F}_{\mathrm{R}}^H \mathbf{F}_{\mathrm{R}} \approx N_t N_r \tilde{\mathbf{D}}_k + \frac{1}{\gamma_k} \mathbf{I}_{N_t}$$
$$= diag\{d_1, d_2, ..., d_K\}, \quad (23)$$

where $\tilde{\mathbf{D}}_k = diag\{\tilde{d}_1, \tilde{d}_2, ..., \tilde{d}_K\}$ with $\tilde{d}_m = |\alpha^m|^2, m = 1, 2, ..., K$ and $m \neq k$, and $\tilde{d}_k = 0$. And, $d_m = N_t N_r |\alpha^m|^2 + \frac{1}{\gamma_k}, m = 1, 2, ..., K$ and $m \neq k$, and $d_k = \frac{1}{\gamma_k}$. Thus, the digital part of the k^{th} hybrid precoder is given by

$$\begin{aligned} \mathbf{f}_{\mathrm{D}_k} &\propto \text{max eigenvector}\left(N_t N_r \mathbf{A}_k, diag\{d_1, d_2, ..., d_K\}\right) \\ &\propto \text{max eigenvector}(diag\left\{\frac{1}{d_1}, \frac{1}{d_2}, ..., \frac{1}{d_K}\right\} * N_t N_r \mathbf{A}_k) \\ &\propto \text{max eigenvector}\left(N_t N_r \mathbf{A}_k \gamma_k\right) \\ &\propto \mathbf{e}_k. \end{aligned} \quad (24)$$

Normalizing the obtained $\mathbf{f}_{\mathrm{D}_k}$,

$$\mathbf{f}_{\mathrm{D}_k} = \frac{\mathbf{e}_k}{|\mathbf{F}_{\mathrm{R}} \mathbf{e}_k|} = \frac{\mathbf{e}_k}{|\mathbf{f}_{\mathrm{R}_k}|} = \mathbf{e}_k. \quad (25)$$

The hybrid precoder for the k^{th} MS is $\mathbf{f}_k = \mathbf{F}_{\mathrm{R}} \mathbf{e}_k = \mathbf{f}_{\mathrm{R}_k}$. Hence, it implies that in single path channel when $N_t \to \infty$, no SLNR precoding is required and analog beamsteering is sufficient. The achievable rate of k^{th} user is given by

$$R_k = \log_2\left(1 + \gamma_k N_t N_r \left|\alpha^k\right|^2\right). \quad (26)$$

VI. SIMULATION RESULTS

Similar to [9], we consider mmWave channel with $L = 3$. We employ UPA at both BS and MS. The azimuth AoAs/AoDs are uniformly distributed in the range $[0, 2\pi]$, and the elevation AoAs/AoDs are uniformly distributed in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ while the complex path gains, $\alpha_\ell^k \sim \mathcal{CN}(0, 1)$. The noise variance per receive antenna is assumed to be same for all users, *i.e.*, $\sigma_1^2 = \sigma_2^2 = ... = \sigma_K^2 = \sigma^2$. The SNR used in the plots is defined as SNR $= \frac{P}{\sigma^2}$. The spectral efficiency plotted in the Fig. 2, 4, 5 refers to the average achievable rate per user, *i.e.*, $\frac{1}{K}\mathbb{E}\left(\sum_{k=1}^K R_k\right)$. The sum spectral efficiency plotted in Fig. 3 refers to the average total achievable rate, *i.e.*, $\mathbb{E}\left(\sum_{k=1}^K R_k\right)$. Unless it is a varying parameter, we have considered 8×8 BS UPA, 4×4 MS UPA, $K = 8$ and SNR $= 0$ dB in all the figures.

We compare the performance of the proposed hybrid SLNR-based precoder (which we will refer to as *Hy-SLNR precoder* henceforth) with hybrid ZF precoder [9], no interference case (a hypothetical single user case which is used as a comparison benchmark), fully digital or unconstrained SLNR precoder and spatially sparse precoder [3] extended to maximize SLNR in MU-MIMO environment. We have considered analog-only combining at the MS in all the precoding schemes which ensures a fair comparison. We assume perfect channel knowledge at MS and perfect effective channel knowledge at the BS. The

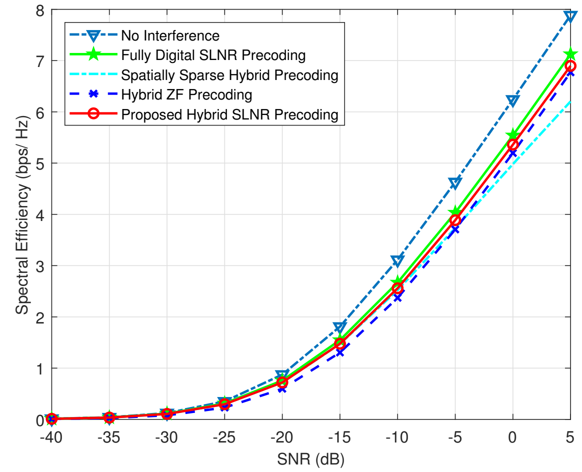

Fig. 2: Achievable spectral efficiency in different precoding schemes.

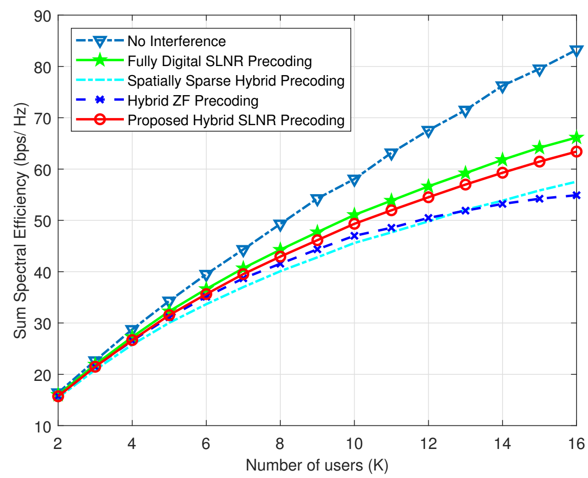

Fig. 3: Achievable sum rates as a function of number of users.

beamsteering codebooks at the MS and the BS are assumed to be AoA/AoD codebooks, *i.e.*, $\mathcal{W}_k$ consists of array response vectors of the actual AoA's at the k^{th} MS and $\mathcal{F}$ consists of the array response vectors of the actual AoD's.

We compare the spectral efficiency as a function of SNR of the several schemes in Fig. 2. Hy-SLNR precoder clearly offers the achievable rate, quite close to fully digital precoder which establishes that the Hy-SLNR precoder is capable of curbing the interference due to other users. Hy-SLNR precoder accomplishes a better performance than the hybrid ZF precoder.

Fig. 3 depicts the effect of number of users on achievable sum-rates. Hy-SLNR precoder closely follows the sum-rate performance of the fully digital precoder, with its sum-rate increasing with K, while the sum-rate performance of hybrid ZF precoder satiates at high values of K. The hybrid ZF precoder is equivalent to the ZF precoder developed for a

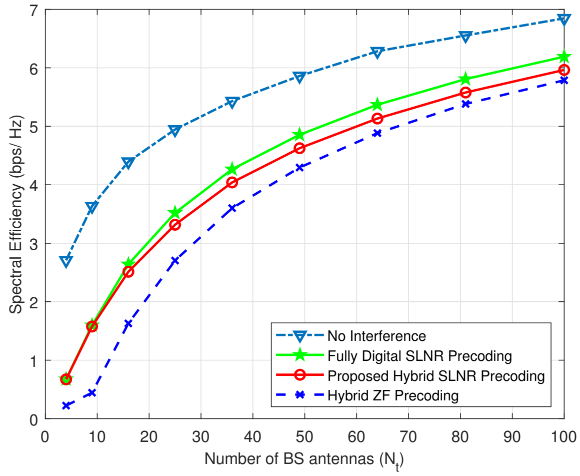

Fig. 4: Achievable spectral efficiency as a function of number of BS antennas.

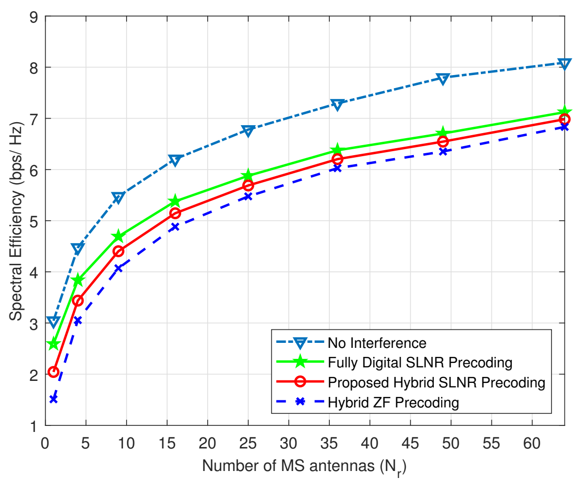

Fig. 5: Achievable spectral efficiency as a function of number of MS antennas.

system with K transmit antennas and K MS's equipped with single antenna. In such a system, the sum-rate performance of the ZF precoder does not grow at higher number of users [11], [14]. The spatially sparse hybrid precoder tries to follow the fully digital precoder but falls quite short of both digital SLNR precoder and hybrid SLNR precoder.

Fig. 4 and Fig. 5 portray the effect of N_t and N_r on achievable rates. With the growing number of N_t and N_r, the achievable spectral efficiency of Hy-SLNR precoder increases expectedly as there is a rise in array gain with the increase in N_t and N_r. In Fig.5, unlike at higher values of N_t in Fig. 4, Hy-SLNR precoder does not narrow down the gap between its achievable rate and the no-interference rate at higher values of N_r. This can be attributed to the fact that the analog-only combiner employed at the MS is unable to combine the signals received at different antennas to maximize the received power.

VII. CONCLUSION

In this paper, we have proposed a hybrid SLNR-based precoder for downlink mmWave multi-user massive MIMO systems. In the presented method, analog-only combiner is designed at MS first and then hybrid precoder is constructed at BS to maximize SLNR of the effective channel. The proposed scheme, while using a small number of RF chains, still produces spectral efficiency almost on a par with the fully-digital precoder and superior to those of hybrid ZF precoder and spatially sparse precoder. In addition, the proposed hybrid SLNR precoder, unlike hybrid ZF precoder, doesn't require the equivalent channel matrix to be well-conditioned. We will extend our precoding scheme to support multiple data streams in our future works.

REFERENCES

[1] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up mimo: Opportunities and challenges with very large arrays," *IEEE Signal Processing Magazine*, vol. 30, pp. 40–60, Jan 2013.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive mimo for next generation wireless systems," *IEEE Communications Magazine*, vol. 52, pp. 186–195, February 2014.

[3] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, "Spatially sparse precoding in millimeter wave mimo systems," *IEEE Transactions on Wireless Communications*, vol. 13, pp. 1499–1513, March 2014.

[4] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski, "Five disruptive technology directions for 5g," *IEEE Communications Magazine*, vol. 52, pp. 74–80, February 2014.

[5] A. L. Swindlehurst, E. Ayanoglu, P. Heydari, and F. Capolino, "Millimeter-wave massive mimo: the next wireless revolution?," *IEEE Communications Magazine*, vol. 52, pp. 56–62, Sep. 2014.

[6] O. E. Ayach, R. W. Heath, S. Abu-Surra, S. Rajagopal, and Z. Pi, "Low complexity precoding for large millimeter wave mimo systems," in *2012 IEEE International Conference on Communications (ICC)*, pp. 3724–3729, June 2012.

[7] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Communications Magazine*, vol. 49, pp. 101–107, June 2011.

[8] A. Alkhateeb, O. E. Ayach, G. Leus, and R. W. Heath, "Hybrid precoding for millimeter wave cellular systems with partial channel knowledge," in *2013 Information Theory and Applications Workshop (ITA)*, pp. 1–5, Feb 2013.

[9] A. Alkhateeb, G. Leus, and R. W. Heath, "Limited feedback hybrid precoding for multi-user millimeter wave systems," *IEEE Transactions on Wireless Communications*, vol. 14, pp. 6481–6494, Nov 2015.

[10] W. Ni and X. Dong, "Hybrid block diagonalization for massive multiuser mimo systems," *IEEE Transactions on Communications*, vol. 64, pp. 201–211, Jan 2016.

[11] D. H. N. Nguyen, L. B. Le, T. Le-Ngoc, and R. W. Heath, "Hybrid mmse precoding and combining designs for mmwave multiuser systems," *IEEE Access*, vol. 5, pp. 19167–19181, 2017.

[12] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user mimo channels," *IEEE Transactions on Wireless Communications*, vol. 6, pp. 1711–1721, May 2007.

[13] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual sinr constraints," *IEEE Transactions on Vehicular Technology*, vol. 53, pp. 18–28, Jan 2004.

[14] C. Peel, B. Hochwald, and A. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—part i: Channel inversion and regularization," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 195–202, 2005.