Event-triggered adaptive control of minimum-phase systems

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Abstract: We study passification-based adaptive control of minimum-phase systems with sampled measurements. First, we remove the "relative degree one" assumption previously imposed in Selivanov et al. (2015). To achieve this, we introduce the shunting method that allows to obtain hyper-minimum-phase augmented system, which is further stabilized by a passification-based adaptive controller. Second, we introduce a switching event-triggering mechanism to reduce the number of transmitted measurements. The advantage of adaptive control and the benefit of the switching approach are demonstrated by an example of adaptive flight control.

1. INTRODUCTION

The passification method is a long-studied approach to adaptive control (Fradkov (1974, 1976); Barkana and Kaufman (1985); Kaufman et al. (1998); Iwai and Mizumoto (1992); Deng et al. (2001); Dolinar et al. (2000); Cho and Burton (2011); Amini and Javanbakht (2014)). Though passification-based adaptive controllers have rather simple structure, their practical implementation faces certain difficulties. First, disturbances, inherent in most systems, can cause infinite growth of the dynamically tuned controller gains. This issue has been overcome by introducing the so-called "σ-modification" (Narendra et al. (1971); Lindorff and Carroll (1973); Ioannou and Kokotovic (1984)). A similar modification is required for systems with measurement quantization (Selivanov et al. (2016)). An entirely different problem is the presence of input/output delays that unavoidably appear due to finite time of signals' processing and transmission. Just recently this problem has been resolved for minimum-phase systems with relative degree one, i.e., hyper-minimum-phase systems (Selivanov et al. (2015)). Since input/output delays can model control/measurement sampling (Fridman, 2014, Chapter 7), the results of Selivanov et al. (2015) are applicable to sampled-data systems.

In this paper, we remove the restrictive "relative degree one" assumption for sampled-data adaptive controller. To achieve this, we use the so-called shunting method (parallel feedforward compensator) in the form proposed in Fradkov (1994). This allows to obtain a hyper-minimum-phase augmented system, which is further stabilized by a passification-based adaptive controller.

One of the motivations for considering data sampling is networked control systems where signals cannot be continuously transmitted through communication medium. In such systems, it is beneficial to reduce the number of samplings (that is, the number of transmitted signals) to save computational and communicational resources. For this purpose we introduce the event-triggering mechanism (Åström and Bernhardsson (1999); Tabuada (2007); Heemels et al. (2012)), which basic idea is to send only those signals whose relative change is larger than a given threshold. We implement this idea using a switching approach recently proposed in Selivanov and Fridman (2016). We demonstrate its advantage over periodic sampling and periodic event-triggering (where event-triggering condition is checked periodically) by an example of adaptive flight control.

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Preliminaries

Lemma 1. (Wirtinger inequality, Hardy et al. (1952)). Let $f\colon [a,b] \to \mathbb{R}$ be an absolutely continuous function with a square integrable first derivative such that $f(a) = 0$ or $f(b) = 0$. Then

$$\int_a^b f^2(t)\,dt \le \frac{4(b-a)^2}{\pi^2}\int_a^b \left[\dot f(t)\right]^2\,dt.$$

Consider the system

$$\begin{aligned}\dot x &= Ax + Bu,\\ y &= Cx,\end{aligned}\qquad x\in\mathbb{R}^n,\ u\in\mathbb{R},\ y\in\mathbb{R}^l. \qquad (1)$$

For a given vector $g \in \mathbb{R}^l$ a scalar transfer function $g^TW(s) = g^TC(sI-A)^{-1}B$ is called *minimum-phase* if its numerator is stable. It is called *hyper-minimum-phase* (HMP) if its numerator is stable and has a positive leading coefficient, i.e., $g^TCB > 0$.

Lemma 2. (Passification lemma, Fradkov (1976, 2003)). For the existence of a matrix $P > 0$ such that $PB = C^Tg$ and

$$P[A - Bk_*g^TC] + [A - Bk_*g^TC]^TP < 0 \qquad (2)$$

with large enough $k_* \in \mathbb{R}$, it is necessary and sufficient that the rational function $g^TW(s) = g^TC(sI-A)^{-1}B$ is HMP.

If $g^TW(s) = g^TC(sI-A)^{-1}B$ is HMP then there exists k_* such that the input $u = -k_*g^Ty + v$ makes the system

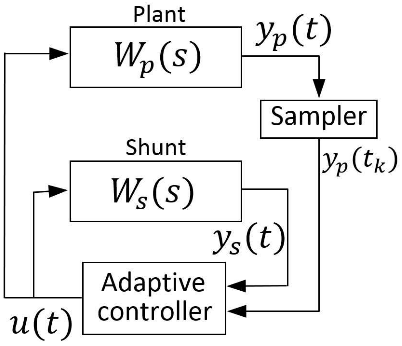

Fig. 1. Sampled-data adaptive controller with a shunt

(1) *strictly passive* from a new input v to the output $g^T y$, i.e., there exist functions $V(x) = x^T P x$, with $P > 0$, and $\varphi(x) \geq 0$, such that $\varphi(x) > 0$ for $x \neq 0$, satisfying

$$V(x(t)) \leq V(x(0)) + \int_0^t \left[y^T(s) g v(s) - \varphi(x(s)) \right] \, ds.$$

Lemma 3. (Fradkov (1994)). Let $g_p^T W_p(s) = g_p^T C_p (sI - A_p)^{-1} B_p$ be a minimum-phase transfer function with a relative degree $r > 1$ and a leading coefficient $g_p^T C_p A_p^{r-1} B_p > 0$. Let $P(s)$ and $Q(s)$ be Hurwitz polynomials of degrees $r-2$ and $r-1$ with positive coefficients. Then there exist a number $\kappa_0 > 0$ and a function $\lambda_0(\kappa) > 0$ such that $g_p^T W_p(s) + \kappa\lambda P(\lambda s)/Q(s)$ is HMP for any $\kappa > \kappa_0$, $0 < \lambda < \lambda_0(\kappa)$.

2. ADAPTIVE CONTROL WITH A SHUNT

Consider the linear system

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p, \\ y_p &= C_p x_p, \end{aligned} \qquad x_p \in \mathbb{R}^n, u_p \in \mathbb{R}, y_p \in \mathbb{R}^l \tag{3}$$

with a structured time-varying uncertainty

$$A_p \in \left\{ \sum_{i=1}^N \xi_i A_{p,i} \,\middle|\, 0 \leq \xi_i \leq 1, \sum_{i=1}^N \xi_i = 1 \right\}, \tag{4}$$

where $A_{p,i} \in \mathbb{R}^{n\times n}$ are some known matrices. The problem is to construct a controller that stabilizes the uncertain system (3) using only sampled measurements $y_p(t_k)$, where $0 = t_0 < t_1 < t_2 < \ldots$ are sampling instants.

Let there exist $g_p \in \mathbb{R}^l$ such that $g_p^T W_p(s) = g_p^T C_p (sI - A_p)^{-1} B_p$ is minimum-phase with a relative degree $r \geq 1$ and a leading coefficient $g_p^T C_p A_p^{r-1} B_p > 0$.

Remark 1. The transfer function $g_p^T W_p(s)$ depends on the uncertain matrix A_p from (4). To check that it is minimum-phase, one needs to check the stability of its numerator $b(s) = \det(sI - A_p)^{-1} g_p^T W_p(s)$ with uncertain coefficients. This can be done using Theorem 2 of Kharitonov (1979).

Choose some stable polynomials $P(s)$ and $Q(s)$ of degrees $r-2$ and $r-1$ with positive coefficients ($P(s) = 0$ if $r = 1$). Due to Lemma 3, there exist λ and κ such that $g_p^T W_p(s) + W_s(s)$ is HMP, where $W_s(s) = \kappa\lambda P(\lambda s)/Q(s)$. Consider a minimal realization of $W_s(s)$:

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + B_s u(t), \\ y_s(t) &= C_s x_s(t). \end{aligned} \tag{5}$$

Denoting $x = \mathrm{col}\{x_p, x_s\}$, $y = \mathrm{col}\{y_p, y_s\}$,

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_s \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ B_s \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0 \\ 0 & C_s \end{bmatrix}, \tag{6}$$

we obtain the augmented system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

which transfer function is

$$W(s) = \begin{bmatrix} W_p(s) \\ W_s(s) \end{bmatrix}.$$

Note that the uncertain matrix A belongs to (4) with $A_{p,i}$ replaced by $A_i = \mathrm{diag}\{A_{p,i}, A_s\}$. For $g = \mathrm{col}\{g_p, 1\}$, $g^T W(s) = g_p^T W_p(s) + W_s(s)$ is HMP. Uncertain HMP systems can be stabilized using passification-based adaptive controller (Andrievskii and Fradkov (2006)). Here we study the sampled-data form of such controller (see Fig. 1):

$$\begin{aligned} u(t) &= -k(t)[g_p^T y_p(t_k) + y_s(t)], \\ \dot{k}(t) &= \gamma [g_p^T y_p(t_k) + y_s(t)]^2, \end{aligned} \tag{7}$$

where $k \in \mathbb{R}$ is an adaptive coefficient and $\gamma > 0$.

3. EVENT-TRIGGERED SAMPLING

So far we did not specify how one should choose the sampling instants $0 = t_0 < t_1 < t_2 < \ldots$ A straight-forward approach is to take periodic sampling $t_k = kh$. However, in this case the measurements are sent even when the output fluctuation is small and does not significantly change the control signal. To avoid such "redundant" packets and reduce the amount of transmitted measurements, we consider the switching approach to event-triggered control proposed in Selivanov and Fridman (2016). Namely, we take $t_0 = 0$ and

$$\begin{aligned} t_{k+1} = \min\big\{ t \geq t_k + h \,\big|\, & [g_p^T (y_p(t_k) - y_p(t))]^2 \\ & \geq \sigma [g_p^T y_p(t)]^2 \big\}, \end{aligned} \tag{8}$$

where $h >$ and $\sigma > 0$ are the event-triggering parameters. The idea is that the sensor waits for at least h seconds after it sent the last measurement and sends the next one when its relative change is larger than σ. Under event-triggering sampling the signals are sent only when their change is significant enough. Moreover, the presence of the waiting time h guarantees that $\lim t_k = \infty$.

Theorem 1. For given event-triggering parameters $\sigma > 0$ and $h > 0$ let there exist a positive-definite matrix $P \in \mathbb{R}^{(n+r-1)\times(n+r-1)}$, positive scalars μ, w, and tuning parameter k_* such that [1]

$$PB = C^T g, \;\; \Phi_i|_{a=\pm M} < 0, \;\; \Psi_i|_{a=\pm M} < 0, \;\; i = 1, \ldots, N,$$

where Φ_i, Ψ_i are symmetric matrices combined of

$$\begin{aligned} \Phi_{11}^i &= P[A_i - Bk_* g^T C] + [A_i - Bk_* g^T C]^T P \\ &\quad + \mu\sigma \,\mathrm{diag}\{C_p^T g_p g_p^T C_p, 0_{r-1}\}, \\ \Phi_{12} &= \Psi_{12} = -PBk_* + C^T g a, \\ \Phi_{22} &= 2a - \mu, \\ \Psi_{11}^i &= P[A_i - Bk_* g^T C] + [A_i - Bk_* g^T C]^T P, \\ \Psi_{13}^i &= hw(A_i^T - C^T g B^T (k_* + a)) \begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times 1} \end{bmatrix}, \\ \Psi_{22} &= 2a - \pi^2 w/4, \end{aligned}$$

[1] MATLAB codes for solving the LMIs are available at `https://github.com/AntonSelivanov/IFAC17`

$$\Psi_{23} = -hwB^T(k_*+a)\begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times 1}\end{bmatrix},$$
$$\Psi_{33} = -w$$

with $A_i = \mathrm{diag}\{A_{p,i}, A_s\}$. Then, for the system (3) with initial conditions $\|x_p(0)\| < \delta$, the adaptive sampled-data controller (5), (7), (8) with

$$x_s(0) = 0, \quad |k(0)-k_*| < M, \quad \gamma < \frac{M^2-(k(0)-k_*)^2}{\lambda_{\max}(P)\delta^2}$$

guarantees $\lim_{t\to\infty} x_p(t) = 0$. Moreover, $\lim_{t\to\infty} x_s(t) = 0$ and $\lim_{t\to\infty} k(t) = \mathrm{const}$.

Proof. Consider the functional

$$V = \begin{cases} V_0 + V_\gamma + V_w, & t\in[t_k, t_k+h), \\ V_0 + V_\gamma, & t \in [t_k+h, t_{k+1}), \end{cases}$$

where

$$V_0 = x^T(t)Px(t),$$
$$V_\gamma = \gamma^{-1}(k(t)-k_*)^2,$$
$$V_w = h^2 w\int_{t_k}^{t}[g_p^T \dot y_p(s)]^2 ds - \frac{\pi^2 w}{4}\int_{t_k}^{t}[g_p^T(y_p(s)-y_p(t_k))]^2 ds.$$

The function $V_0 + V_\gamma$ is chosen following Fradkov (1974). The Wirtinger-based term V_w is taken from Liu and Fridman (2012). Due to Lemma 1, $V_w \geq 0$. Moreover, $V_w(t_k+h) \leq V_w(t_k+h-0)$, therefore, $V(t)$ does not increase at discontinuity points t_k+h. Since $V_w(t_k) = 0$, V is continuous at t_k.

Denote

$$a(t) = k(t) - k_*,$$
$$e(t) = g_p^T[y_p(t_k) - y_p(t)], \quad t\in[t_k, t_{k+1}).$$

Then the closed-loop system (3), (5), (7) can be presented in the form

$$\begin{aligned}\dot x &= Ax(t) - B(k(t)\pm k_*)(g_p^T[y_p(t_k)\pm y_p(t)] + y_s(t)) \\ &= Ax - B(k_*+a)(g^TCx+e).\end{aligned} \tag{9}$$

We have

$$\begin{aligned}\dot V_0 &= 2x^T(t)P\big[Ax - B(k_*+a)(g^TCx+e)\big] \\ &= 2x^TP[A-Bk_*g^TC]x - 2x^TPBk_*e \\ &\quad - 2(x^TPB \pm e)a(g^TCx+e) \\ &= 2x^TP[A-Bk_*g^TC]x - 2x^TPBk_*e \\ &\quad - 2a(g^TCx+e)^2 + 2ea(g^TCx+e),\end{aligned}$$

where we used $PB = C^Tg$ (note that x^TPB is a scalar). The penultimate term of $\dot V_0$ is canceled by

$$\dot V_\gamma = 2a[g_p^Ty_p(t_k) + y_s(t)]^2 = 2a[g^TCx+e]^2.$$

Thus,

$$\dot V_0 + \dot V_\gamma = 2x^TP[A-Bk_*g^TC]x - 2x^TPBk_*e + 2ea(g^TCx+e).$$

The remainder of the proof is divided into three parts. First, we show that $\dot V < -\varepsilon\left(\|x\|^2 + \|e\|^2\right)$ for small enough $\varepsilon > 0$ if $a(t) \in [-M, M]$. Then, we prove that $|a| < M$ is always true. Finally, we use the inequality $\dot V < -\varepsilon\left(\|x\|^2+\|e\|^2\right)$ to prove the statement of the theorem.

I. Proof that $|a| < M$ implies $\dot V < -\varepsilon(\|x\|^2 + \|e\|^2)$.

For $t \in [t_k, t_k+h)$ we have

$$\dot V_w = h^2 w\dot x^T \begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times n}\end{bmatrix}\begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times n}\end{bmatrix}^T \dot x - \frac{\pi^2 w}{4}e^2.$$

Therefore,

$$\dot V = \begin{bmatrix} x \\ e\end{bmatrix}^T \begin{bmatrix} \Psi_{11} & C^Tga - PBk_* \\ * & 2a - \pi^2 w/4\end{bmatrix}\begin{bmatrix} x \\ e\end{bmatrix} + h^2 w\dot x^T \begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times n}\end{bmatrix}\begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times n}\end{bmatrix}^T \dot x,$$

where $\Psi_{11} = P[A - Bk_*g^TC] + [A-Bk_*g^TC]^TP$. Substituting (9) for $\dot x$ and applying the Schur complement, we obtain that $\dot V < -\varepsilon(\|x\|^2 + \|e\|^2)$ for small enough ε if $\Psi < 0$, where Ψ is obtained from Ψ_i by replacing A_i with A. Since Ψ is affine in A and a, the conditions $\Psi_i|_{a=\pm M} < 0$ $(i = 1,\ldots,N)$ guarantee $\dot V < -\varepsilon(\|x\|^2 + \|e\|^2)$ for $t\in[t_k, t_k+h)$.

For $t \in [t_k+h, t_{k+1})$ the event-triggering condition (8) implies

$$\mu\sigma x^T \mathrm{diag}\{C_p^Tg_pg_p^TC_p, 0_{r-1}\}x - \mu e^2 \geq 0. \tag{10}$$

Then we have

$$\dot V + (10) = \begin{bmatrix} x \\ e\end{bmatrix}^T \Phi \begin{bmatrix} x \\ e\end{bmatrix},$$

where Φ is obtained from Φ_i by replacing A_i with A. Since Φ is affine in A and a, the conditions $\Phi_i|_{a=\pm M} < 0$ $(i = 1,\ldots,N)$ guarantee $\dot V < -\varepsilon(\|x\|^2 + \|e\|^2)$ for small enough ε and $t\in[t_k+h, t_{k+1})$.

II. Proof that $|a| < M$ for $t \geq 0$.

Note that $x_s(0)$, $k(0)$, and γ are chosen to ensure

$$V(0) \leq \lambda_{\max}(P)\delta^2 + \gamma^{-1}(k(0)-k_*)^2 < \gamma^{-1}M^2.$$

Let $V(t) \geq \gamma^{-1}M^2$ for some $t > 0$. Since $V(0) < \gamma^{-1}M^2$ and $V(t)$ decreases at its discontinuity points, there should exist t_* such that $V(t) < \gamma^{-1}M^2$ on $[0, t_*)$ and $V(t_*) = \gamma^{-1}M^2$. Then

$$|a(t)| = |k(t)-k_*| \leq \sqrt{\gamma V(t)} \leq M, \quad t\in[0,t_*].$$

As we have shown before, the latter implies $\dot V < -\varepsilon(\|x\|^2 + \|e\|^2)$ on $[0, t_*]$. But then $V(t_*) \leq V(0) < \gamma^{-1}M^2$ what contradicts to the definition of t_*. Thus, for $t\geq 0$ we have

$$V(t) < \gamma^{-1}M^2 \;\Rightarrow\; |a| < M \;\Rightarrow\; \dot V < -\varepsilon(\|x\|^2+\|e\|^2).$$

III. End of the proof.

Since V is positive and decreases, $\lim_{t\to\infty} V(t) < \infty$ and

$$\int_0^\infty \varepsilon\|x(s)\|^2\,ds + \int_0^\infty \varepsilon\|e(s)\|^2\,ds < -\int_0^\infty \dot V(s)\,ds = V(0) - \lim_{t\to\infty} V(t)$$

should be finite. That is, $x(\cdot), e(\cdot) \in L^2$. Boundedness of V implies boundedness of x and a and (10) implies boundedness of e. Therefore, $\dot x$ given by (9) is bounded and x is uniformly continuous. Then, from Barbalat's lemma (Khalil, 2002, Lemma 8.2), we have $\lim_{t\to\infty}\|x(t)\| = 0$. Moreover,

$$\begin{aligned}k(t) &= k(0) + \gamma\int_0^t [g^TCx(s) + e(s)]^2 ds \\ &\leq k(0) + \gamma\lambda_{\max}(C^Tgg^TC)\|x\|^2_{L^2} + \gamma\|e\|^2_{L^2}.\end{aligned}$$

Since $k(t) = k_* + a(t)$ monotonically increases and bounded, it converges to a constant value. □

Remark 2. The LMIs of Theorem 1 are always feasible for large k_*, small event-triggering parameters h, σ, and narrow uncertainty class (4). Indeed, since the system (3), (5) is HMP, Lemma 2 guarantees the existence of $P > 0$ such that $PB = C^T g$ and (2) is valid for large enough k_*. Then the Schur complement implies that the LMIs of Theorem 1 hold for $h = 0$, $\sigma = 0$, $A_i = A$ if μ and w are large enough. Then they hold for small h, σ and narrow enough uncertainty class (4).

Remark 3. The results of Theorem 1 are semi-global. That is, for any initial conditions $\|x_p(0)\|^2 < \delta$ with arbitrary large δ there always exists sufficiently small γ such that the adaptive controller (5), (7) stabilizes the system.

Remark 4. One can show that if the conditions of Theorem 1 are satisfied with $M = 0$ then the static feedback

$$u(t) = -k_*[g_p^T y_p(t_k) + y_s(t)]$$

stabilizes the system (3), (5). The advantage of adaptive control is that usually $\lim_{t\to\infty} k(t) < k_*$. That is, adaptive control allows to stabilize the system using a smaller controller gain (see Section 4).

Remark 5. For the sake of simplicity, we consider a *scalar* adaptive coefficient $k(t)$ in the adaptive controller (7). However, Theorem 1 can be extended to cope with a *vector* adaptive gain $\theta(t) = \mathrm{col}\{\theta_p(t), \theta_s(t)\}$, where $\theta_p \in \mathbb{R}^l$, $\theta_s \in \mathbb{R}$ and the adaptive controller is given by

$$\begin{aligned} u(t) &= -\theta_p^T(t) y_p(t_k) - \theta_s(t) y_s(t), \\ \dot{\theta}_p(t) &= \gamma y_p(t_k)[g_p^T y_p(t_k) + y_s(t)], \\ \dot{\theta}_s(t) &= \gamma y_s(t)[g_p^T y_p(t_k) + y_s(t)]. \end{aligned}$$

On the one hand, such extension brings more flexibility and allows to consider, e.g., adaptive PID controllers (see Andrievskii and Fradkov (2006)). On the other hand, one will have to check the LMIs of Theorem 1 for 2^{l+1} values of $a = \theta(t) - \theta_*$ (since each of $l + 1$ components will have two values $a_i = \pm M$), while for a scalar gain we have only two vertices $a = \pm M$.

Remark 6. Consider the periodic event-triggered sampling

$$\begin{aligned} t_{k+1} = \min\big\{t_k + ih \,\big|\, &[g_p^T(y_p(t_k) - y_p(t_k + ih))]^2 \\ &\geq \sigma[g_p^T y_p(t_k + ih)]^2,\ i \in \mathbb{N}\big\}. \end{aligned} \quad (11)$$

The stability conditions for the system (3), (5), (7), (11) can be obtained in a manner similar to Theorem 1 using the functional $V = V_0 + V_\gamma + V_w$ and representation $y(t_k) = y(t) + [y(t) - y(t_k + ih)] + [y(t_k) - y(t_k + ih)]$. In this case the matrices Φ_i and Ψ_i should be replaced by Ξ_i composed of the blocks [2]

$$\begin{aligned} \Xi_{11}^i &= P[A_i - Bk_* g^T C] + [A_i - Bk_* g^T C]^T P \\ &\quad + \mu\sigma\, \mathrm{diag}\{C_p^T g_p g_p^T C_p, 0_{r-1}\}, \\ \Xi_{12} &= \Xi_{13} = -PBk_* + C^T g a, \\ \Xi_{14}^i &= hw(A_i^T - C^T g B^T(k_* + a)) \begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times 1} \end{bmatrix}, \\ \Xi_{22} &= 2a - \pi^2 w/4, \\ \Xi_{23} &= 2a, \\ \Xi_{24} &= \Xi_{34} = -hwB^T(k_* + a) \begin{bmatrix} C_p^T g_p \\ 0_{(r-1)\times 1} \end{bmatrix}, \\ \Xi_{33} &= 2a - \mu, \quad \Xi_{44} = -w. \end{aligned}$$

[2] MATLAB codes for solving the LMIs are available at https://github.com/AntonSelivanov/IFAC17

The advantage of the sampling (11) is that it requires to check the event-triggering condition only at time instants $t_k + ih$ while (8) requires to check it continuously. However, the event-triggering with waiting time (8) may lead to a smaller amount of sent signals (see Section 4). A more detailed comparison of the event-triggering mechanisms (8) and (11) can be found in Selivanov and Fridman (2016).

4. EXAMPLE: ADAPTIVE FLIGHT CONTROL

The lateral motion of an aircraft considered as a rigid body can be described by (see Fradkov and Andrievsky (2011))

$$\begin{aligned} \dot{\beta}(t) &= a_1\beta(t) + r(t) + b_1\delta(t), \\ \dot{r}(t) &= a_2\beta(t) + a_3 r(t) + b_2\delta(t), \\ \dot{\psi}(t) &= r(t), \end{aligned} \quad (12)$$

where $\psi(t)$ and $r(t)$ are the yaw angle and the yaw rate, $\beta(t)$ is the sideslip angle, $\delta(t)$ is the rudder angle (the control signal), a_i and b_i are aircraft model parameters that depend on the flight conditions. Following Fradkov and Andrievsky (2011), we take $a_2 = 33$, $a_3 = -1.3$, $b_1 = 19/15$, $b_2 = 19$ and assume that $a_1 \in [-1.5, -0.7]$ is an uncertain parameter.

The first mode of the aircraft bending is modeled as

$$W_{bend}(s) = \frac{\Delta\psi(s)}{\delta(s)} = \frac{k_{bend}}{T_{bend}^2 s^2 + 2\xi_{bend} T_{bend} s + 1}, \quad (13)$$

where $k_{bend} = -1.5 \times 10^{-4}$ is the bending mode transition factor; $T_{bend} = \omega_{bend}^{-1}$ is the response time factor with $\omega_{bend} = 65$ s^{-1} being the first bending mode natural frequency; and $\xi_{bend} = 0.01$ is the damping ratio. The measured signal is given by

$$y(t) = \psi(t) + \Delta\psi(t). \quad (14)$$

The system (12)–(14) can be presented in the form (3) with

$$\left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array}\right] = \left[\begin{array}{ccccc|c} a_1 & 1 & 0 & 0 & 0 & b_1 \\ a_2 & a_3 & 0 & 0 & 0 & b_2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-2\xi_{bend}}{T_{bend}} & 1 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{bend}^2} & 0 & \frac{k_{bend}}{T_{bend}^2} \\ \hline 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right].$$

For $g_p = 1$ the transfer $g_p^T W_p(s) = g_p^T C_p(sI - A_p)^{-1} B_p$ is minimum-phase with the relative degree $r = 2$ and positive leading coefficient. As a shunt transfer function we take

$$W_s(s) = \frac{2}{s + 14}.$$

Then for $g = [1, 1]^T$ the function $g^T W(s) = W_p(s) + W_s(s)$ is HMP. In what follows, we study the adaptive controller (5), (7) for different choices of sampling instants t_k.

First, consider periodic sampling $t_k = kh$. Note that event-triggered sampling (8) coincides with periodic sampling if $\sigma = 0$. Therefore, we use Theorem 1 with $\sigma = 0$, $k_* = 30$, $M = 35$ to obtain the maximum periodic sampling $h = 0.0265$. This implies that $\lfloor \frac{10}{h} \rfloor + 1 = 378$ signals are transmitted within 10 seconds of system evolution.

Now consider the event-triggered sampling (8). For different values of σ we use Theorem 1 to find the maximum waiting time h. To obtain the average amount of sent signals, for each pair of (σ, h) we perform numerical

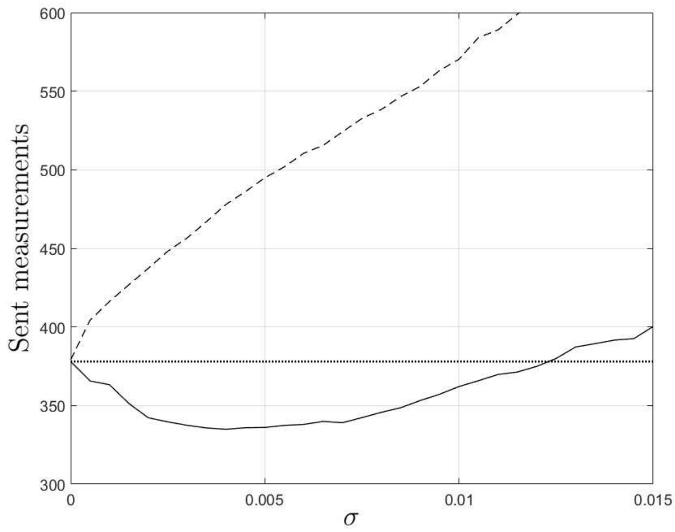

Fig. 2. Sent measurements with different σ for periodic sampling (dotted line), event-triggering (8) (solid line), periodic event-triggering (11) (dashed line).

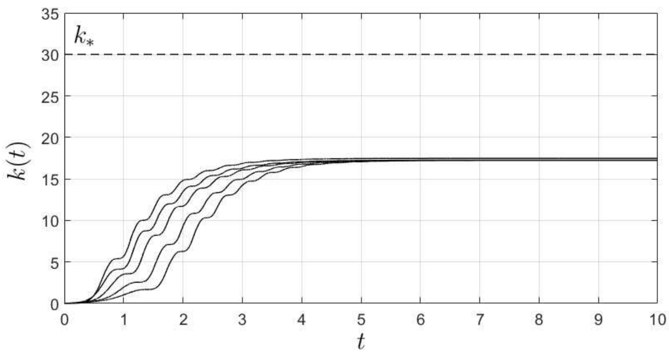

Fig. 3. Evolution of the controller gain $k(t)$ for 5 randomly chosen initial conditions and the value of $k_* = 30$.

simulations with 100 randomly chosen initial conditions $\|x_p(0)\| < 1$ and $a_1 = -0.75$, $x_s(0) = 0$, $k(0) = 0$. The results are depicted in Fig. 2 (solid line). The minimum corresponds to $\sigma = 4 \times 10^{-3}$, $h = 0.0261$ with 334.9 sent measurements, what is by more than 10% less than in the case of periodic sampling.

At the same time, as clearly seen from Fig. 2, periodic event-triggering (11) does not reduce the network workload. For instance, taking $\sigma = 4 \times 10^{-3}$ as above and using Remark 6, we find the maximum sampling $h = 0.0163$ and the average amount of sent signals 477.99. Therefore, the event-triggered control (5), (7), (8) reduces the network workload compared to both periodic sampling and periodic event-triggering (11).

In Fig. 3 one can see the evolution of $k(t)$ for 5 randomly chosen initial conditions $\|x_p(0)\| < 1$ under the event-triggered control (5), (7), (8). Clearly, the adaptive gains $k(t)$ converge to constants that are smaller than the value of the static gain $k_* = 30$ (see Remark 4). This happens because the static controller $u(t) = -k_*[g_p^T y_p(t_k) + y_s(t)]$ ensures stability of the system (3) for all $a_1 \in [-1.5, -0.7]$, while adaptive controller finds an appropriate static gain $\lim_{t\to\infty} k(t)$ for a particular value $a_1 = -0.75$.

5. CONCLUSION

We obtained the stability conditions in terms of linear matrix inequalities for event-triggered adaptive stabilization of minimum-phase systems with sampled measurements. The presence of measurement sampling allows to obtain only semi-global results because the deviation of the adaptive controller must be bounded (i.e. $|a| < M$ in Theorem 1). Possible extensions include consideration of time-varying network delays, system disturbances, and measurement noise.

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