A digital design process for shell structures

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Abstract

Over the last few decades, the design of freeform structures has undergone a radical change: powerful computational tools within parametric environment associated with digital fabrication techniques are pushing the boundaries of architecture towards bold solutions. The present work proposes a digital workflow for a shell in compression. The design process starts with the form-finding phase, which generates a hanging model. Through the interoperability of digital tools within parametric environment, optimization of the shape and structural analysis were carried out in order to investigate its behavior. The resulting surface is subject to tessellation, planarization of its cells that take into account fabrication constrains, and the 3D generation of panels composing the thickness of the structure. In order to accomplish an easier assembly process a hypothesis of a puzzle-like connection system was developed.

The whole process provides a guidance for the design of freeform shell by the creation of a "customized" digital workflow implemented by digital fabrication techniques for the realization phase.

Keywords: fabrication, hexagonal tessellation, parametric modelling, planarization, shell design, structural analysis.

1. Introduction

By definition, a shell structure is a system represented by a curved surface, in which one dimension is smaller than the other two. Its structural efficiency is mainly due to its geometrical features and its capacity to carry membrane stresses throughout the structure "passively" [1]. In the contemporary scenario, architects and engineers are promoting a radical change in the techniques used for the design of shell structures. Shell design has a very long tradition: the great masters of the past paved the way of obtaining efficient structures, providing powerful techniques. This remarkable legacy combined with the translation of these techniques in innovative digital tools is allowing an increased knowledge and interest in their use even from designers that never used such techniques before. The investigation of innovative methods may give a strong contribution in the architecture scenario as well as encouraging new architects to follow this direction, providing more efficient solutions.

This work presents a two-stage optimization process: in the first stage, the form-finding is used to obtain a compressed shell defined on the basis of given boundary conditions; tessellation and planarization are applied to "optimize" the shape through the definition of panels that can be easily fabricated and joint together. In the following sections, the process is discussed starting with the development of shell having a rectangular plan.

2. Design process

This paper addresses the design process applied on a shell working mainly in compression. The process phases include: form finding and planarization process, a preliminary structural analysis and the elaboration of a connection system. The brief required for this case is a rectangular plane (13.3 x 9.1 m) (the dimensions are not integers since they take into account of the hexagonal discretization provided at the beginning of the process). The geometrical parameters include 4 point supports and 4 openings. This approach takes into consideration geometrical parameters together with structural and fabrication constraints that will be justified in the next sections. The process is implemented by parametric modelling, relying on Grasshopper® functions and its plugins. Thanks to their interoperability, it is possible to work on the whole design process in a unique working field, with a greater control of the outcomes regarding each aspect of the process. Working simultaneously from different perspectives gives the possibility to fulfill a wider range of requirements by acting on the parameters involved in the design.

Grasshopper® is a visual editor for algorithms. One of the main peculiarities of this tool is the possibility to create a custom work field by adding plug-ins for specific tasks, enriching its potentialities. Most of this plug-ins are open source like Grasshopper® itself enabling to work with a high degree of freedom in a wide range of applications, such as Finite Elements analysis (FEA), optimization, environmental analysis, mesh subdivision and rationalization, among others.

2.1. Form-finding-hexagonal subdivision and planarization

Form-finding is the earliest phase in the design process. It allows the definition of shape taking into account geometrical constraints and allows to develop shapes which are efficient for a specific set of forces [2]. Historically, graphic statics and hanging chains were used to define geometries and nowadays those methodologies have been translated in digital tools [3].

Such project makes use of an intuitive technique for form finding, which is Particle-Spring Systems (PSS). PSS was originally used for animation and was based on dynamic simulations and nowadays it represents an efficient methodology for the generation of 3D geometries [4]. In a general context, the starting surface is subject to a discretization process composed of particles and springs: the particles have a position and velocity and forces represented by vectors are applied to them, while the springs are the connection between the particles, and the interaction between them is governed by Hooke's law [5].

The hanging model is generated based on:

- Initial geometry, which represents the input for the generation of the 3D shape;
- Load cases, in this case gravity load is considered;
- Supports that represent anchor points.

This methodology is applied in the digital workflow through Kangaroo2 plug-in [6].

Starting from a rectangular plan and after defining the point supports the geometry is defined through a process carried out with Kangaroo2. The subdivision is a constraint applied in the process of form-finding to control deformation due to the planarization. Therefore, a subdivision composed of hexagonal cells is produced before to run the simulation. Fig. 1 shows the initial subdivision and the starting geometry. The form- finding is performed and at this stage the geometry is not planar yet, hence a further refinement which allows to planarise the cells, incorporated in this Grasshopper definition, is carried out. Fig. 2 shows the non-planar geometry, which is the result of the simulation.

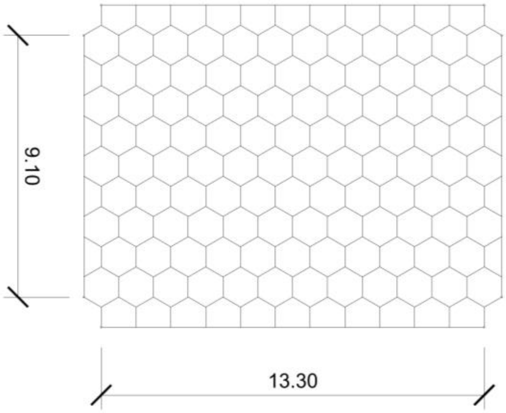

Figure 1: Starting plan with the hexagonal subdivision

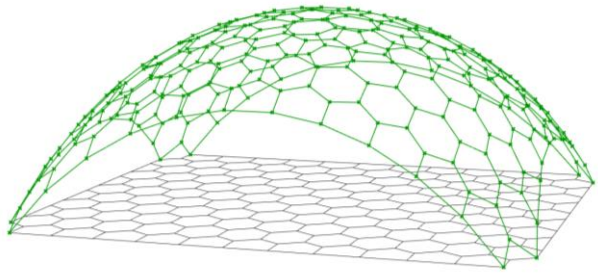

Figure 2: 3D shape after form-finding before planarization

Shell structures are double curvature surfaces and the evaluation of the curvature is an important parameter in the planarization process. This work deals with a shell with positive curvature. Positive curvatures allow the planarization to achieve good results, while in the case of negative curvature where the panels are concave some issues such as severe geometrical deformations and self-intersections could occur. When form-finding is generated and the planarization is an operation applied afterwards, then some deformations can occur in critical areas of the shell, such as at the supports. With the proposed approach, these deformations can be avoided although the resulting shape is affected. This is motivated by the presence of more constraints during the form-finding process that, indeed, controls the planarization in a more rigid way.

Therefore, the planarization process is incorporated in the form-finding process providing control during the simulation and avoiding severe deformations of the cells. By constraining vertices of a polygon in the same plane, the design process provides more optimized shapes that translate into a more efficient fabrication and potential low manufacturing costs.

This project refers to a specific work done by Muller [7], which addresses the thematic of conformal hexagons. A hexagon $(z_0, \dots, z_5)$ is called *conformal* if both cr$(z_0; z_1; z_2; z_3)$ = -1/2 and cr$(z_0; z_5; z_4; z_3)$ = -1/2

Cr is cross-ratio and for 4 complex numbers $(z_0; z_1; z_2; z_3)$ it is given by:

$$cr = \frac{(z_0 - z_1)(z_2 - z_3)}{(z_1 - z_2)(z_3 - z_0)} \quad (1)$$

As Muller stated, in a conformal hexagon both quadrilaterals $z_0; z_1; z_2; z_3$ and $z_0; z_5; z_4; z_3$ are circular since their cross-ratios are real and all vertices of a circular polygon are contained in a sphere or plane (Fig. 3).

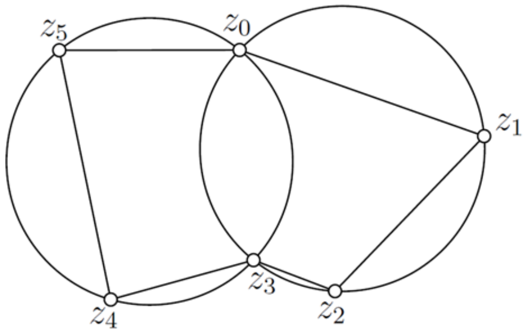

Figure 3: Conformal hexagon in two circles [Muller, 2011]

The simulation with Kangaroo2 allows to translate these properties by constraining the vertices of the hexagons in circles by using a specific component called “CoCircular”. This constraint allows to control the transformations of the cells during the form-finding process and the planarization, whose successful outcome depends on another component from Kangaroo2, which is “CoPlanar”. It is the principal force acting in order to run planarization allowing to “pull” a collection of point within their best fit plane. A best-fit plane is found generally by minimizing the sum of quadratic distances (perpendicular to the plane) between the plane and points. Fig. 4 shows the planar geometry proving how the use of the initial subdivision as constraint minimized the deformation of the planar cells.

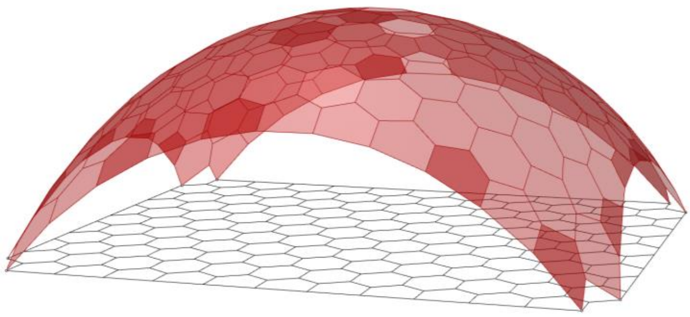

Figure 4: 3D shape after planarization

2.2. Preliminary structural analysis

The preliminary structural analysis has been performed by FEA within the Grasshopper® platform [8, 9]. A continuous surface was considered as input geometry, to perform a simplified preliminary analysis. A further structural analysis involving the discretized elements will be carried out in the future work.

The macromodel was analysed by taking into account the following conditions:

- Four point supports;
- Material properties are related to a material working mainly in compression as Table 1 displays; Material properties refer to Concrete C30/37 since such values represent a good approximation of a structure working mainly in compression. The values are taken from Eurocode 2 [10].
- A defined cross section;
- A load condition, to investigate the structural behavior in a series of context (Table 2).

Table 1: Material properties used for FEA [Eurocode]

Density	1900 kg/m3
Elastic Modulus	3300 N/mm^2
Compressive Strength Tensile strength	30 N/mm^2 2.9 N/mm^2
Shear Modulus (G)	1375 N/mm^2
Poison’s ratio (V)	0.2

Table 2: Load combination

Dead load + wind load	5.7 + 0.8 (lateral wind) + 0.5 (up-lift wind)

The load combination (Table 2) consisted of dead loading acting with a horizontal wind load applied on the windward face of the structure with a vertical uplift wind load to create the maximum tensile stresses in the structure.

In order to contain tension stresses generated throughout the structure a thickness of 20 cm was considered appropriate. The results are showed in Table 3 where maximum compression stress, maximum tension stress and displacement proved a good insight of the shell’s behavior. Fig. 5 and Fig. 6 show the results of the finite element analysis in terms of stress distribution and deformation.

At this stage, the results were considered acceptable according to material properties.

Table 3: Results extracted from Karamba

Max comp. stress N/mm^2	0.25
Max tensile stress N/mm^2	0.25
Max Displacement (mm)	0.65

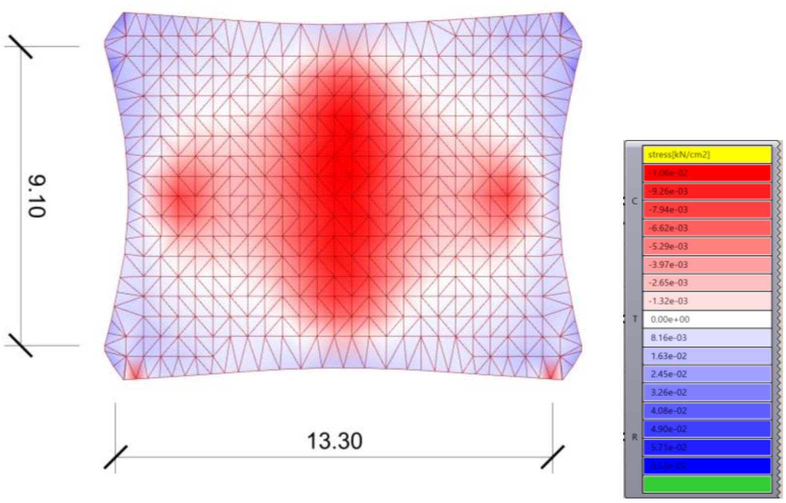

Figure 5: Stress distribution (blue-tension, red-compression)

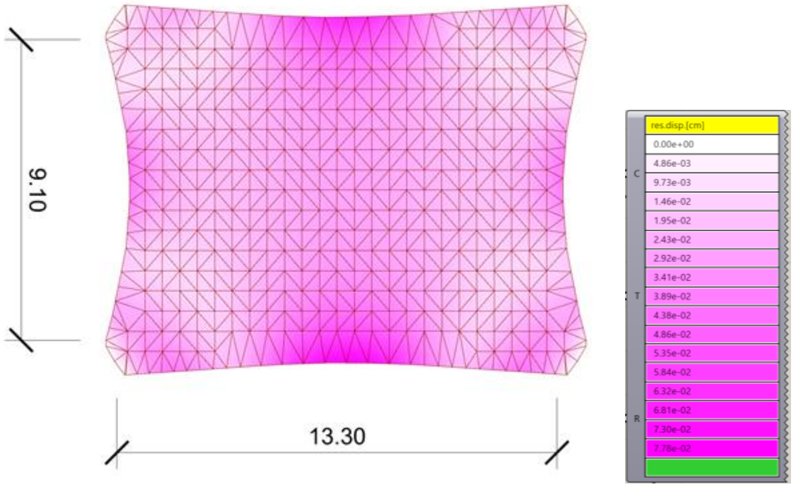

Figure 6: Displacement values

2.3. 3D generation

The 3D generation of the panels is a crucial part of the design process. It is based on the extrusion of the intradoses and the generation of the side faces. Research by Rippmann and Block [11, 12], have given important assumptions for designing shell structure. Such assumptions have provided guidelines for the next steps in this work, especially regarding structural requirements, explained as follows:

- Suitable thickness to assure safety of the structure;
- In order to avoid sliding the extrusion must follow the normal vectors of the surface.

Starting from these conditions a geometrical process has been carried out in order to generate 3D hexagonal elements. The first step was to retrieve normal vectors belonging to the continuous surface that represents the shell geometry. Fig. 7 depicts the normal vectors of the surface providing the direction of the extrusion of the hexagonal panels.

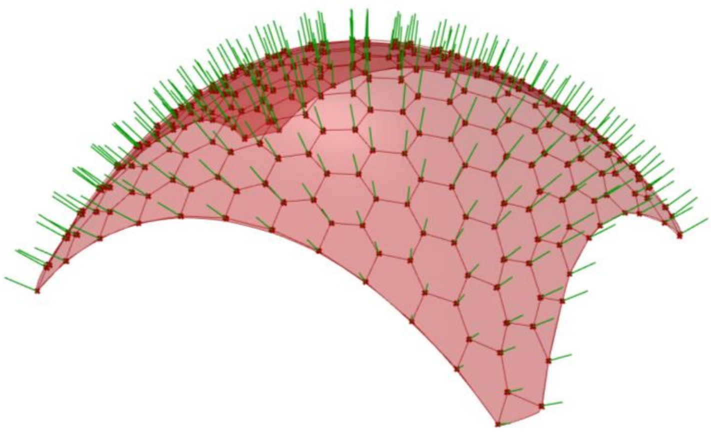

Figure 7: Normal vectors generation on the surface

The extradoses are generated, guaranteeing parallelism between corresponding faces. It is important to check the planarity of the external faces since it is not ensured for all the faces, this means that in case of non-planar extradoses a further refinement is necessary to planarise the faces. Finally, the side contacts faces are generated by a loft operation according to the shell thickness, thus common sides are provided between the 3D panels. The whole process for a piece of the shell is described in Fig. 8.

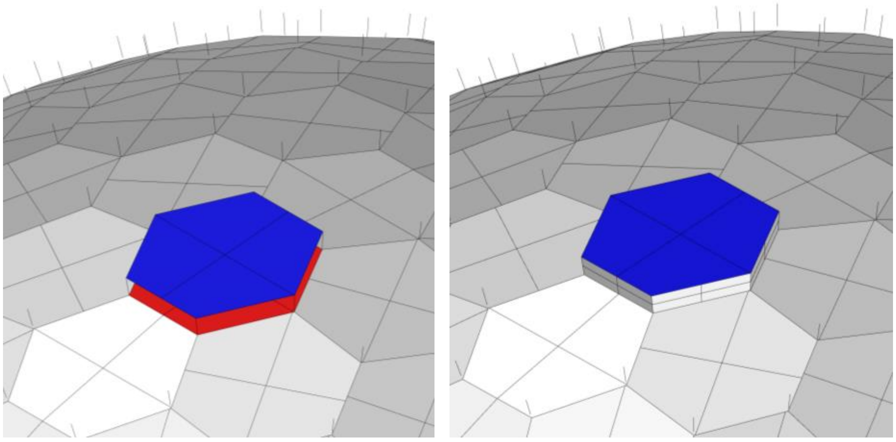

Figure 8: Generation of the extrados (on the left) and contact faces (on the right)

2.5. Connection system

A parametric definition has been developed to create connection for each panel. Although shell structures do not require connection system to ensure stability according to the assumptions described in the previous section and due to the compression stresses, an interlocking system has been elaborated mainly finalized to the realization phase. Starting from a typical puzzle layout, the system has been refined by generating a circular tapered section in order to assure interlocking during assembly test. The circular sections have been extruded by loft operation and successively Boolean operations have been carried out for addition or subtraction of the elements to the panels. The planarity was an essential requirement in order to retrieve semi-circle sections on the top and bottom part of the hexagonal panels, since they are contained in the same planes of the extrados and intrados faces. Fig. 9 summarizes the process including the application to a portion of the shell.

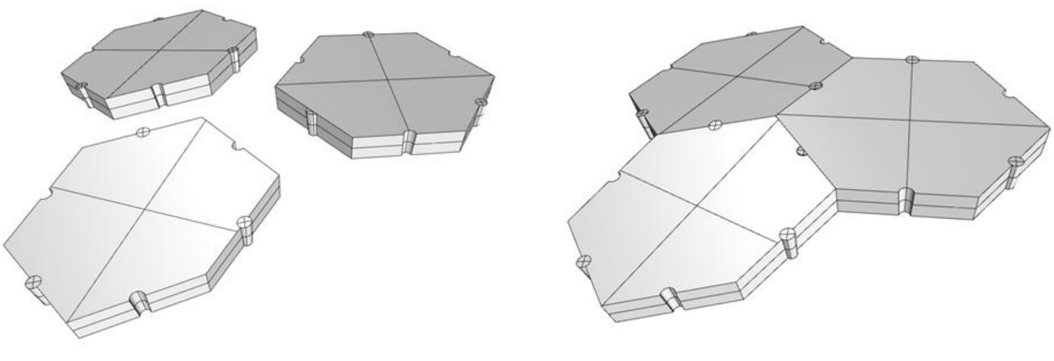

Figure 9: Connection system of the hexagonal panels

3. Conclusion

A digital process applied on a shell structure has been presented, generated within parametric modelling, through Grasshopper® functionalities. The aim of this work was to present a process for the design of a shell, which enables to optimize the realization phase by carrying out planarization and designing a connection system, which was designed for simplifying the final assembling. Firstly, a form-finding technique was used to generate a form-found shell and it was based on a preliminary hexagonal subdivision, and then the planarization process was embedded within this simulation to have a better control of potential geometrical deformations. Its structural efficiency was evaluated through Finite Element Analysis with particular focus on the material properties, concerning a material working mainly in compression. Finally, a tessellation process consisting of 3D generation of the hexagonal panels and the elaboration of a puzzle-like connection system was developed meeting specific structural and fabrication requirements.

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