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On the robustness of multidimensional counting poverty orderings

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Abstract
Counting poverty measures have gained prominence in the analysis of multidimensional poverty in recent decades. Poverty orderings based on these measures typically depend on methodological choices regarding individual poverty functions, poverty cut-offs, and dimensional weights whose impact on poverty rankings is often not well understood. In this paper we propose new dominance conditions that allow the analyst to evaluate the robustness of poverty comparisons to those choices. These conditions provide an approach to evaluating the sensitivity of poverty orderings superior to the common approach of considering a restricted and arbitrary set of indices, cut-offs, and weights. The new criteria apply to a broad class of counting poverty measures widely used in empirical analysis of poverty in developed and developing countries including the multidimensional headcount and the adjusted headcount ratios. We illustrate these methods with an application to time-trends in poverty in Australia and cross-regional poverty in Peru. Our results highlight the potentially large sensitivity of poverty orderings based on counting measures and the importance of evaluating the robustness of results when performing poverty comparisons across time and regions.

Keywords Multidimensional poverty · Counting measures · Dominance conditions

1 Introduction

Multidimensional counting poverty measures focusing on the number of dimensions in which individuals experience deprivation are widely used by academics and policymakers around the world. Since the contributions of Atkinson (2003), Chakravarty and D'Ambrosio (2006), and Alkire and Foster (2011), many governments and international institutions have

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adopted counting poverty measures in order to monitor poverty trends in developed and developing countries alike. These measures typically identify the poor using a weighted count of deprivations and a poverty cut-off representing the minimum level of deprivation required to be classified as poor.[1] Recent examples include World Bank (2016), the 'Multidimensional Poverty Index' used by UNDP in its Human Development Reports since UNDP (2010), and the measures of people at risk of poverty and social exclusion currently used by Eurostat to assess living conditions in Europe (Eurostat 2014). Moreover, the governments of Bhutan, Brazil, China, Colombia, El Salvador, Honduras, Malaysia, and Mexico, have already incorporated this type of measures into their set of national statistics. Meanwhile, other countries are expressing interest toward future adoption (Alkire et al. 2015).

Poverty evaluations based on those counting measures depend on a range of potentially arbitrary choices that are likely to influence poverty comparisons. These include the specific properties of the individual poverty function, the rule employed to identify the multidimensionally poor, and the weights assigned to each of the different dimensions or indicators. While the sensitivity of poverty estimates to these choices is generally acknowledged, the common approach in the literature proceeds by evaluating the sensitivity of poverty orderings considering a limited and usually arbitrarily chosen set of alternative indices, weights, and cut-offs (e.g., see Nusbaumer et al. 2012; Alkire and Santos 2014). Although easy to implement, this type of approach is inferior to the stochastic dominance approach commonly used in the income poverty literature, which reduces the problem of testing the robustness of alternative choices over a large, usually continuous domain, to a smaller set of finite distributional comparisons. Notwithstanding their widespread consideration in distributional analysis, including monetary poverty research, the use of dominance conditions for evaluating the robustness of counting poverty orderings to alternative methodological choices is still rare. However, the soaring popularity of counting poverty measures, together with their reliance on a range of potentially arbitrary methodological choices, justifies the development of testable conditions for gauging the robustness of poverty comparisons based on the counting approach.

This paper contributes to the existing literature by proposing new dominance criteria for multidimensional counting poverty measures. In contrast with previous results, our criteria allow a systematic evaluation of the robustness of poverty comparisons to all the methodological choices involved in the use of counting poverty measures. Thus, we derive conditions that are both necessary and sufficient to guarantee the robustness of multidimensional poverty orderings to the choice of the poverty index, the multidimensional poverty cut-off, and the vector of dimensional weights used to construct counting poverty scores. The new conditions are very intuitive and easy to test empirically as they involve the comparison of frequencies of people deprived in different sets of dimensions. For example, comparing the proportion of people deprived only in electricity in country A against their equally-deprived counterparts in country B, comparing the proportion deprived only in electricity and sanitation in A versus B, and so forth. Importantly, the new criteria apply to a broad class of counting poverty measures including the classes of measures proposed by Chakravarty and D'Ambrosio (2006), Alkire and Foster (2011), and Bossert et al. (2013) and combinations

[1] Counting poverty measures have a long tradition in the poverty literature. As cited in Atkinson (2003), early applications of counting measures include the works by Townsend (1979) for the United Kingdom, Erikson (1993) for Sweden, and Callan et al. (1999) for Ireland. Key recent methodological contributions in this literature also include Tsui (2002), Bourguignon and Chakravarty (2003), Duclos et al. (2006), Bossert et al. (2013), and Permanyer (2014).

thereof; in turn including the multidimensional headcount and the adjusted headcount ratio indices widely used in poverty research in developed and developing countries.

Existing proposals to analyse the robustness of poverty comparisons have succeeded in providing robustness tests based on changing a handful of key sets of parametric or functional choices while keeping the others constant. For example, in the case of counting poverty measures, Lasso de la Vega (2010) showed how to test for the robustness of comparisons to alternative poverty identification cut-offs and poverty functions, while keeping several other parameters constant (poverty lines and dimensional weights). More recently, Yalonetzky (2014) proposed a robustness test for ordinal variables, alternative functional forms, deprivation lines and weights, but only useful for extreme poverty identification approaches (union and intersection). Likewise, Permanyer and Hussain (2017) proposed a highly flexible robustness test based on first-order dominance conditions applied to multiple binary variables, but working exclusively under a union approach to poverty identification.

Our results build on the conditions proposed by Lasso de la Vega (2010) to identify unambiguous rankings for a particular class of poverty indices and poverty cut-offs. Our results extend hers in a number of ways. Firstly, while conditions in Lasso de la Vega (2010) apply *only to a particular vector of deprivation weights*, our new conditions guarantee the robustness of counting poverty orderings to changes in poverty indices and cut-offs *for any conceivable vector of dimensional weights*. Furthermore, we derive a set of useful necessary conditions that allow the analyst to rule out the robustness of poverty comparisons to changes in poverty functions, identification cut-offs, and dimensional weights. These conditions require only the comparison of the proportion of people deprived in each of the dimensions and the proportion of people deprived in all dimensions. For the statistical evaluation of these and the other conditions in the paper, we propose statistical tests based on the testing framework for pair-wise population comparisons proposed by Dardanoni and Forcina (1999) and Hasler (2007). To facilitate the application of the new results we have developed a Stata software package with documentation to guide practitioners on how to empirically implement the new dominance conditions. Taken together, we argue the new methods and software make a useful addition to the existing toolkit available to the research community for the study of multidimensional poverty.

We illustrate the new dominance conditions with an empirical evaluation of recent poverty trends in Australia and cross-regional poverty in Peru. Our results highlight the large sensitivity of poverty comparisons and the difficulty of establishing unambiguous orderings using counting poverty measures. For instance, our results for Australia show that, under standard parametric choices used in the literature, multidimensional poverty declined during the expansionary period between 2002 and 2006 and then started to increase in the aftermath of the Global Financial Crisis, so that by 2010 poverty levels were higher than in 2006 but still remained below those observed in 2002. However, the new robustness conditions enable us to conclude that, while multidimensional poverty in Australia unambiguously declined between 2002 and 2006, the 2002-2010 and 2006-2010 comparisons are not robust and the ordering of those years depends on the particular choice of poverty index, poverty cut-offs, and dimensional weights.

The evaluation of poverty levels across the 25 provinces in Peru also illustrates the lack of robustness of poverty orderings based on counting measures. In fact, results from the dominance analysis reveal that only 3 out of the 300 pairwise regional poverty comparisons appear to be robust to changes in poverty indices, cut-offs, and dimensional weights. We argue that these results call for caution when monitoring poverty using counting measures like those currently used by institutions like UNDP and Eurostat. They also highlight the importance of systematically evaluating the robustness of poverty orderings based on those measures. The new conditions and methods proposed in this paper go some way in providing the analytical tools required for that task.

The rest of the paper proceeds as follows. The next section presents the measurement framework and the class of counting poverty measures considered in the analysis. Key poverty statistics and some notation relevant for the derivation of the dominance results are also discussed in this section. The third section discusses the existing dominance conditions and develops the new dominance results for counting poverty measures. The fourth section briefly explains the statistical tests to evaluate the new conditions. The fifth section provides the empirical illustrations on multidimensional poverty in Australia and Peru. Finally, the paper closes with some conclusions.

2 The Counting Approach to Poverty Measurement

2.1 Measurement Framework

We consider a population with $N \geq 2$ individuals and $D > 1$ indicators of well-being. Let X be a matrix of attainments where the typical element x_{nd} denotes the level of attainment by individual n in dimension d. If $x_{nd} < z_d$, where z_d is a deprivation line for dimension d from a D-dimensional vector of deprivation lines, Z, then we say that individual n is deprived in indicator d.

Let $y_{nd} = \mathbb{I}(x_{nd} < z_d)$ where $\mathbb{I}$ is the indicator function that takes value 1 if the argument in parenthesis is true, and 0 otherwise. Let Y denote the $N \times D$ deprivation matrix whose typical element y_{nd} informs whether individual n is deprived in dimension d or not. There are different ways of defining the elements of matrix Y ranging from simple binary comparisons to complex logical operations. For example, y_{nd} could be a binary indicator of access to electricity where $y_{nd} = 0$ could denote access and $y_{nd} = 1$ would mean lack of access. On the other extreme, y_{nd} could also be a complex binary indicator taking the value of 1 whenever a set of conditions are fulfilled. For example, we could say that $y_{nd} = 1$ if at least one construction material (e.g. for floor, walls, roof, etc.) is of substandard quality, otherwise $y_{nd} = 0$ (i.e. a type of union approach for y_{nd}). But we could also say that $y_{nd} = 1$ if every adult in the family is illiterate, otherwise $y_{nd} = 0$ (i.e. a type of intersection approach for y_{nd}). Unlike the proposals by Yalonetzky (2014) and Permanyer and Hussain (2017), the dominance conditions proposed in this paper do not apply directly to the joint distribution of the variables whose logical combinations lead to matrix Y. Our conditions build from Y once the rules used to construct the matrix of deprivations are set. Therefore a change in those rules would require implementing our proposed tests (or any other tests taking the construction of Y for granted) again.[2]

[2] Another potential complication in the construction of matrices X and Y is that some attainments or deprivations may not always be observable, either directly or indirectly. This will often depend on the choice and definition of the well-being indicator, as well as the degree of complexity of the decision rules used to define deprivations based on several indicators. For example, if a cohabiting couple is surveyed too early into their partnership before they have children, one will be unable to report the health issues affecting the children. Likewise, if one or two household heads are surveyed too late into their lives, one will be unable to retrieve information about the education of children in the household if their children do not live with them anymore already, unless the heads are asked retrospective questions about their offspring and their memory does not fail them. This is a challenge common to the literature, e.g. it would affect indices like the UNDP's MPI (see Alkire and Santos 2014, table 1).

In order to account for the breadth of deprivations, most counting measures rely on individual deprivation scores defined as a weighted count of deprivations. Let $W = (w_1, w_2, ..., w_D)$ denote the vector of dimensional weights such that $w_d > 0 \wedge \sum_{d=1}^{D} w_d = 1$. Let $C = Y \times W^T$ denote the vector of deprivation scores where W^T is the transposed vector of dimensional weights and let Θ denote the set of all admissible vectors of deprivation counts. The nth element of vector C is the deprivation score for individual n which is given by

$$c_n(W) = \sum_{d=1}^{D} y_{nd} w_d,$$

i.e., the weighted sum of the deprivations experienced by the individual. There is only one vector of possible values of c_n for each particular choice of deprivation lines and weights. Moreover it is easy to show that the maximum number of possible values is given by: $\sum_{i=0}^{D} \binom{D}{i} = 2^D$. The vector of possible values is defined as: $V = (v_1, v_2, ..., v_l)$, where $\max l = 2^D$, $v_i < v_{i+j}$, $v_1 = 0$ and $v_l = 1$.[3]

Following Alkire and Foster (2011) we characterise the set of multidimensionally poor with an identification rule $\rho_k(c_n)$ that equals 1 when the individual is poor and 0 otherwise. The indicator function ρ_k compares individuals' c_n with a multidimensional cut-off $k \in [0, 1] \subset \mathbb{R}_+$ so that any person n is deemed poor if and only if $c_n \geq k$. As shown in Lasso de la Vega (2010), the function ρ_k is the only identification rule that satisfies the property of poverty consistency which requires $\rho_k(c_{n'}) = 1$ whenever $\rho_k(c_n) = 1$ and $c_n \leq c_{n'}$.

Let $P(C(Y, W), k)$ denote a poverty counting measure depending on the vector of deprivation scores, C, and the cut-off k used for the identification rule $\rho_k(c_n)$. This notation explicitly recognises the role of the vector of deprivations, Y, and the vector of weights, W, in determining overall poverty through their influence on the vector of deprivation scores, C. However, in those parts of the paper where the dependency of C on Y and W is not worth stating explicitly, we refer to the poverty measure by the simpler notation $P(C, k)$ for ease of exposition.

Following Foster and Shorrocks (1991) and Lasso de la Vega (2010), we consider a broad class of social poverty measures satisfying standard axioms in the literature on poverty measurement including:

Axiom 1 *Symmetry (SYM): For all* $k \in (0, 1]$ *and* $C \in \Theta$, $P(\dot{C}, k) = P(C, k)$ *if* $\dot{C} = \Pi C$, *where* Π *is some* $N \times N$ *permutation matrix.*

SYM requires that the poverty measure is not affected by the exchange of deprivation scores across individuals, ensuring that the measure does not put more emphasis on some individuals than others. The following axiom ensures that poverty evaluations are invariant to replications of the population, allowing the comparison of populations of different sizes:

Axiom 2 *Population-replication invariance (PRI): For all* $k \in (0, 1]$ *and* $C \in \Theta$, $P(C, k) = P(C_R, k)$ *if* $C_R = (C, C, ..., C)$ *is any replication of the vector of deprivation scores* C.

[3] As shown by Permanyer (2014, table 1), alternative forms for the deprivation score are possible when variables are cardinal and not partitioned or dichotomised. However this diversity significantly contracts when we work with binary deprivation indicators, as is the case in the counting framework.

The focus axiom is a common axiom in the poverty literature which requires poverty measures to be unaffected by improvements in the well-being of the non-poor. We say that $\dot{C}$ is obtained from C by a simple *deprivation score decrease among the non-poor* if $0 \leq \dot{c}_i < c_i$ for some individual i with $c_i < k$, while $\dot{c}_j = c_j$ for every other $j \neq i$. Note this type of transformation simply reduces the level of deprivation experienced by a non-poor person, where the decline in the deprivation score could be the result of a reduction in the number of deprivations experienced by the individual, a decline in the weight of the dimensions in which the person is deprived, or any combination of the two. The following axiom states the focus property in our multidimensional setting:

Axiom 3 *Poverty focus (FOC): For all* $k \in (0, 1]$ *and* $C \in \Theta$*,* $P(\dot{C}, k) = P(C, k)$ *if* $\dot{C}$ *is obtained from C by a simple deprivation score decrease among the non-poor.*

The focus axiom thus ensures that poverty remains unchanged when the poverty score of a non-poor person decreases.[4]

We say that $\dot{C}$ is obtained from C by a simple *deprivation score decrease among the poor* if $0 \leq \dot{c}_i < c_i$ for some person i with $c_i \geq k$ while $\dot{c}_j = c_j$ for every other $j \neq i$. The following axiom is a restatement of the dimensional monotonicity axiom by Lasso de la Vega (2010) establishing how the poverty measure should change when a poor person experiences a decrease in the deprivation score.

Axiom 4 *Monotonicity (MON): For all* $k \in (0, 1]$ *and* $C \in \Theta$*,* $P(\dot{C}, k) < P(C, k)$ *if* $\dot{C}$ *is obtained from C by a simple deprivation score decrease among the poor.*

The monotonicity axiom thus ensures that poverty decreases whenever a poor person experiences a reduction in the deprivation score. Note the axiom does not exclude the possibility that the poor person is lifted out of poverty as a consequence of the deprivation decrease.[5] The following axiom is a restatement of the distribution sensitivity axiom by Lasso de la Vega (2010) requiring the poverty measure to be sensitive to the distribution of deprivation scores among the poor:

Axiom 5 *Distribution sensitivity (DS): Let* $\dot{C}$ *and* $\ddot{C}$ *denote the deprivation score vectors obtained from C by a decrease of size h in the deprivation scores of two poor individuals i and j, respectively. Then, for all* $k \in (0, 1]$ *and* $h > 0$*,* $P(C, k) - P(\dot{C}, k) > P(C, k) - P(\ddot{C}, k)$ *if* $c_i > c_j \geq k$ *and both* $\dot{C}$ *and* $\ddot{C}$ *belong to the set of admissible vectors of deprivation counts* Θ*.*

Note that axiom DS essentially prioritises the reduction of deprivation scores among those with higher initial deprivation scores, i.e. the poorest among the poor.

[4] Alkire and Foster (2011) also introduce a deprivation focus axiom whereby the social poverty measure should not be affected by an improvement in a wellbeing indicator whose value was already above the deprivation line, whether the beneficiary person is poor or not. In our context this axiom is not relevant because we work only with either binary or dichotomised variables, therefore further improvements above the deprivation line are not considered.

[5] Alternatively, Alkire and Foster (2011) proposed a monotonicity axiom requiring a poverty measure to decrease whenever a poor person ceases to be deprived in one indicator. Our monotonicity axiom, following Lasso de la Vega (2010), is more general in that it allows for decreases in deprivation score due to either deprivation reductions or changes in the weighting vector.

Finally, we denote by $\mathbb{P}_1$ the class of poverty counting measures P satisfying FOC, MON, SYM and PRI. And let $\mathbb{P}_2 \subset \mathbb{P}_1$ denote the class of social poverty measures satisfying DS in addition to those five axioms. In this paper we propose dominance conditions for these two classes of poverty measures.

2.2 Useful Poverty Statistics

The following poverty statistics are important for the derivation of the dominance conditions. The multidimensional poverty headcount is widely used in poverty analysis based on counting measures and is given by:

$$H(C, k) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(c_n \geq k). \tag{1}$$

The measure $H(C, k)$ provides the proportion of people whose poverty score c_n is at least as high as the multidimensional poverty cut-off k. This is a crude measure of poverty that fails to satisfy the monotonicity axiom as it does not take into account the depth of poverty. However, as shown in Lasso de la Vega (2010), even if $H(C, k)$ does not belong to the classes $\mathbb{P}_1$ and $\mathbb{P}_2$ of poverty measures, the orderings based on the $H(C, k)$ statistic for all k are useful to identify unambiguous rankings within the class $\mathbb{P}_1$.

We also use the adjusted headcount ratio proposed by Alkire and Foster (2011) which can be expressed as:

$$M(C, k) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(c_n \geq k) c_n. \tag{2}$$

The statistic $M(C, k)$ is defined as the censored population average score, in which the censorship trait stems from setting the scores of non-poor people to zero, in order to fulfil the focus axiom. In contrast with $H(C, k)$, $M(C, k)$ takes into account the breadth of deprivation to characterise the overall level of poverty. The measure $M(C, k)$ fails to satisfy the Distribution Sensitivity axiom and therefore does not belong in the class $\mathbb{P}_2$. However, as we discuss below, unambiguous orderings with respect to $M(C, k)$ for all k imply robust orderings within the class $\mathbb{P}_2$.

To derive the new dominance conditions it is also useful to consider the *uncensored deprivation headcount*, which measures the proportion of people deprived in dimension d irrespective of their deprivation in other dimensions:

$$U_d(Y) = \frac{1}{N} \sum_{n=1}^{N} y_{nd}. \tag{3}$$

3 Dominance Conditions for Counting Measures

In this section we present the new dominance conditions to assess the robustness of counting poverty orderings within the classes of poverty measures $\mathbb{P}_1$ and $\mathbb{P}_2$. These conditions build on the dominance results derived by Lasso de la Vega (2010). Let $P(C(Y_A, W), k)$ and $P(C(Y_B, W), k)$ refer to the social poverty indices of populations A and B with deprivation matrices Y_A and Y_B , respectively. Let $H(C(Y_A, W), k)$ and $H(C(Y_B, W), k)$ refer to their multidimensional headcounts. The following result sets out the conditions for unambiguous poverty orderings within the class $\mathbb{P}_1$:

Condition 1 *$P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all P in $\mathbb{P}_1$ and any identification cut-off $k \in (0, 1]$, if and only if $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ for all $k \in (0, 1]$.*

Proof See Lasso de la Vega (2010, proposition 3, p. 164) which relates $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ for all $k \in (0, 1]$ to comparisons of poverty measures in $\mathbb{P}_1$. □

Condition (1) states that poverty comparisons of A and B are robust to the choice of the poverty function satisfying FOC, MON, SYM, and PRI only when the ordering of headcount measures is the same for every value of k.

Now let $M(C(Y_A, W), k)$ and $M(C(Y_B, W), k)$ refer to the adjusted headcount ratio of populations A and B, respectively. The following result establishes the conditions for unambiguous poverty rankings within the class $\mathbb{P}_2$:

Condition 2 *$P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all P in $\mathbb{P}_2$ and any identification cut-off $k \in (0, 1]$, if and only if $M(C(Y_A, W), k) \leq M(C(Y_B, W), k)$ for all $k \in (0, 1]$.*

Proof See Lasso de la Vega (2010, proposition 6, p. 167) which relates $M(C(Y_A, W), k) \leq M(C(Y_B, W), k)$ for all $k \in (0, 1]$ to comparisons of poverty measures in $\mathbb{P}_2$. □

Thus, when the adjusted headcount ratio in population A is never higher than in B for every value of k then we can claim that poverty in A is never higher than in B for any poverty measure in $\mathbb{P}_1$ satisfying DS (i.e. $\mathbb{P}_2$). The following remark links condition (1) to (2):

Remark 1 If $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ for all $k \in (0, 1]$ then $M(C(Y_A, W), k) \leq M(C(Y_B, W), k)$ for all $k \in (0, 1]$.

Proof From condition (1) we know that $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ for all $k \in (0, 1]$ implies $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all P in $\mathbb{P}_1$ and any identification cut-off $k \in (0, 1]$. Additionally, M is a member of the class $\mathbb{P}_1$. Therefore $M(C(Y_A, W), k) \leq M(C(Y_B, W), k)$ for all $k \in (0, 1]$. □

Remark (1) states that the existence of dominance within the class $\mathbb{P}_1$ implies dominance within the class $\mathbb{P}_2$, which is not surprising given that $\mathbb{P}_2 \subset \mathbb{P}_1$. Following Lasso de la Vega (2010, Proposition 7), conditions (1) and (2) can also be restricted to apply only to a subset of relevant k values, ruling out the lowest ones below a minimum k_{min}. In order to proceed this way, we construct censored deprivation scores such that: $c_n = 0$ whenever $c_n < k_{min}$. Then conditions (1) and (2) apply only to those $P(C, k)$ for which $k_{min} \leq k \leq 1$.

The conditions presented above allow us to assess the sensitivity of poverty orderings to the choice of the social poverty measure. However, *these conditions hold only for a particular choice of dimensional weights*. With alternative selection of weights the conditions would need to be evaluated again as the values of the poverty statistics H and M depend on the specific values of the multidimensional poverty cut-off and weights.

We propose new dominance conditions to examine the robustness of poverty orderings to the choice of weighting schemes. First, we present the necessary and sufficient conditions whose fulfilment guarantees, *separately*, the robustness of conditions (1) and (2) to *any possible choice* of dimensional weights. Then, we present a sufficient condition whose fulfilment guarantees the robustness of condition (1), as well as the robustness of condition (Condition (2)) by implication, to any possible choice of dimensional weights. Finally, we present a set of conditions whose fulfilment is necessary (but insufficient) to guarantee the robustness of poverty orderings to changes in the poverty index, identification cut-off, and dimensional weights. The advantage of both the exclusively sufficient and the exclusively necessary conditions resides in their easier implementation for testing purposes vis-a-vis the jointly necessary and sufficient conditions. Before presenting the new dominance results, the next subsection introduces additional notation necessary for the derivation of the new conditions.

3.1 Additional Notation and Useful Poverty Statistics

We denote by $S(D) = \{\{1\}, \{2\}, ..., \{D\}, \{1, 2\}, ..., \{1, 2, ..., D\}\}$ the power set with all possible combinations of D welfare dimensions (each denoted by a natural number from 1 to D) *excluding the empty set*. The number of elements in $S(D)$ is equal to $2^D - 1$. Let O_s denote the population subgroup deprived *only* in the subset of dimensions $s \in S(D)$ and let $c^s(W)$ denote the poverty score for those deprived in the dimensions in s when the weighting vector is W (whenever there is no room for ambiguity, we use c^s below for ease of exposition). Thus, for instance, for $D = 3$, the sets O_1, $O_{2,3}$, and $O_{1,2,3}$ include, respectively, the persons deprived only in dimension 1, those deprived in dimensions 2 and 3 but not in dimension 1, and those deprived in the three dimensions. Clearly, $O_s \bigcap O_{s'} = \emptyset$ for all $s \neq s'$.

For each subset s we define the *subset headcount ratio*, H_s, as the proportion of people who are deprived *only* in the subset of dimensions $s \in S(D)$. For any $s \in S(D)$, the measure H_s is equal to:

$$H_s = \frac{|O_s|}{N}, \tag{4}$$

where $|O_s|$ is the number of people deprived exclusively in dimensions $s \in S(D)$. For any weighting vector W, all individuals in O_s share the same score denoted by $c^s(W)$. Note that when the whole set of dimensions is considered, $H_{1,2,...,D}$ is the proportion of people who are deprived in each and every possible dimension, also known as the multidimensional intersection headcount index (Duclos and Tiberti 2016).

Now we make a crucial distinction between the *potential* sets of the multidimensionally poor and the *actual* set of the multidimensionally poor. The former includes all possible groups of people that would be deemed poor under all potential combinations of W and k. The latter refers to the unique set of people that ends up being identified as poor once we choose and fix a particular combination of W and k. That is, the actual poverty set is an element of the set of potential poverty sets. Any combination of W and k selects only one actual poverty set of the poor among all the potential sets (although, of course, different combinations of W and k could select the same actual poverty set).

We denote by Γ the set containing all *potential* poverty sets consistent with any given identification rule. Any potential poverty set $\gamma \in \Gamma$ can be expressed as the union of groups O_s. Meanwhile $\gamma(W, k) \in \Gamma$ stands for the unique *actual* poverty set (among the potential ones) once W and k are chosen and fixed. That is, $\gamma(W, k)$ is the set of people for whom $c_n(W) \geq k$.

For instance, in the case of $D = 2$, the set of potential poverty sets is given by $\Gamma = \{(O_{1,2}), (O_{1,2} \bigcup O_1), (O_{1,2} \bigcup O_2), (O_{1,2} \bigcup O_1 \bigcup O_2)\}$, where the first and last elements in this set correspond to the cases where the group identified as multidimensionally poor includes those deprived in all the dimensions and those deprived in any dimension, respectively. Note that because we are only considering identification rules satisfying the property of poverty consistency, any $\gamma \in \Gamma$ must always include the group of those deprived in all dimensions $(O_{1,2,\ldots,D})$. Then, for instance, $\gamma(W, 1) = (O_{1,2})$ for all W is the actual poverty set corresponding to the intersection approach to poverty identification. Likewise, $\gamma(W, k) = (O_{1,2} \bigcup O_1)$ for all W and k such that: $w_2 < k \leq w_1$; and so forth. That is, the actual poverty set depends on the threshold k and the vector of dimensional weights W which determines the score $c^s(W)$ of the different groups O_s.

We define $\Pi(\gamma)$ as the proportion of the population belonging in γ, which can be expressed as follows:

$$\Pi(\gamma) = \frac{1}{N} \sum_{O_s \subset \gamma} |O_s| = \sum_{O_s \subset \gamma} H_s, \tag{5}$$

For any $\gamma \in \Gamma$, the headcount ratio $\Pi(\gamma)$ can be expressed as the sum of the subset headcount ratios of the sets O_s included in γ.

For any $\gamma \in \Gamma$, let $\gamma_d \subset \gamma$ denote the subset of the multidimensionally poor in γ who are also deprived in dimension d. For instance, for $D = 2$, the sets γ_1 and γ_2 associated to $\gamma = (O_{1,2} \bigcup O_1 \bigcup O_2)$ are given by $\gamma_1 = (O_{1,2} \bigcup O_1)$ and $\gamma_2 = (O_{1,2} \bigcup O_2)$. For $\gamma = (O_{1,2} \bigcup O_1)$ the sets are $\gamma_1 = (O_{1,2} \bigcup O_1)$ and $\gamma_2 = (O_{1,2})$. For any $\gamma \in \Gamma$ it is easy to show that $\gamma = \bigcup_{d=1}^{D} \gamma_d$. Let Γ_d denote the set of all γ_d that can be part of a multidimensional poverty set γ. In the case of $D = 2$, the sets Γ_1 and Γ_2 have only two elements and are given by $\Gamma_1 = \{(O_{1,2}), (O_{1,2} \bigcup O_1)\}$ and $\Gamma_2 = \{(O_{1,2}), (O_{1,2} \bigcup O_2)\}$. The proportion of the population belonging in γ_d is given by the following expression:

$$\Pi(\gamma_d) = \frac{1}{N} \sum_{O_s \subset \gamma_d} |O_s| = \sum_{O_s \subset \gamma_d} H_s, \tag{6}$$

It is important to note that for any number of dimensions D: $\sum_{d=1}^{D} dim(\Gamma_d) \leq dim(\Gamma)$. The sets Γ and Γ_d will play a key role in the new dominance conditions and they will be discussed in detail in the next subsection.

3.2 Necessary and Sufficient Conditions

Let $\Pi^A(\gamma)$ and $\Pi^B(\gamma)$ denote the proportions of the population belonging in the set γ in A and B, respectively. The following condition is both necessary and sufficient to guarantee unambiguous poverty orderings within the class of measures $\mathbb{P}_1$:

Condition 3 *The following three statements are equivalent:*

1. $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ *for all* $P \in \mathbb{P}_1$, *all poverty thresholds* $k \in (0, 1]$ *and all weighting vectors,* W.
2. $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ *for all poverty thresholds* $k \in (0, 1]$ *and all weighting vectors,* W.
3. $\Pi^A(\gamma) \leq \Pi^B(\gamma)$ *for all* $\gamma \in \Gamma$.

Proof The equivalence between statements (1) and (2) follows immediately from condition (1). To complete the proof we show the equivalence between statements 2 and 3.

To prove the sufficiency of the condition, note that the multidimensional headcount ratio H can be expressed in terms of the number of people in the groups O_s who are deemed poor for a particular combination of W and k, as per expression 7:

$$H(C(Y,W),k) = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n(W)\geq k) = \frac{1}{N}\sum_{s\in S(D)}\mathbb{I}(c^s(W)\geq k)|O_s|$$
$$= \sum_{O_s\subset\gamma(W,k)} H_s = \Pi(\gamma(W,k)). \quad (7)$$

Now note that, for any given D, there is only a finite number of *potential* poverty sets, γ, because these sets are unions of $2^D - 1$ disjoint sets O_s. Hence the number of γ sets only depends on D; i.e. it is independent of W or k. By contrast, the *actual* observed poverty set, $\gamma(W,k)$, depends on W and k. Therefore, by Eq. 7 we know that even though W and k are continuous variables, $H(C(Y,W),k)$ can only take a finite number of values (for a given Y) from a vector of the proportions of the population in each potential poverty set γ.

Then, by definition, if $H(C(Y_A,W),k) \leq H(C(Y_B,W),k)$ for all poverty thresholds $k \in (0,1]$ and all weighting vectors, W, then the proportion of people in all potential poverty sets is never higher in A versus B. This is exactly what is meant by the expression $\Pi^A(\gamma) \leq \Pi^B(\gamma)$ for all $\gamma \in \Gamma$.

To prove the necessity, let's assume there is a weighting vector W' and poverty threshold k' such that: $H(C(Y_A,W'),k') > H(C(Y_B,W'),k')$. Then, by Eq. 7 we would get: $\Pi^A(\gamma(W',k')) = H(C(Y_A,W'),k') > H(C(Y_B,W'),k') = \Pi^B(\gamma(W',k'))$. □

Condition (3) implies that dominance within the class $\mathbb{P}_1$ can be evaluated by comparing the headcounts ratios of all potential poverty sets, $\Pi(\gamma)$. Essentially, condition (3) transforms the problem of assessing the robustness of poverty comparisons to infinite alternatives (since W and k are continuous and the class $\mathbb{P}_1$ is not finite) into a much simpler evaluation of a finite number of headcount ratio comparisons based on $\Pi(\gamma)$.

The following result establishes the necessary and sufficient conditions for unambiguous poverty orderings within the class $\mathbb{P}_2$:

Condition 4 *The following three statements are equivalent:*

1. $P(C(Y_A,W),k) \leq P(C(Y_B,W),k)$ *for all* $P \in \mathbb{P}_2$, *all poverty thresholds* $k \in (0,1]$ *and all weighting vectors,* W.
2. $M(C(Y_A,W),k) \leq M(C(Y_B,W),k)$ *for all poverty thresholds* $k \in (0,1]$ *and all weighting vectors,* W.
3. $\Pi^A(\gamma_d) \leq \Pi^B(\gamma_d)$ *for all* $\gamma_d \in \Gamma_d$ *and* $d = 1, ..., D$.

Proof The equivalence between statements (1) and (2) follows immediately from condition (2). We complete the proof by demonstrating the equivalence between statements (2) and (3). For this latter purpose, note first that, for a given combination of weights and multidimensional cut-off, the adjusted headcount ratio, M, can be expressed as the weighted sum of the relative frequencies of the sets γ_d, i.e. $\Pi(\gamma_d)$, included in the set of multidimensionally poor γ associated to that particular combination of k and W:

Let $\gamma_d(W, k)$, i.e. the set of people for whom $c_n(W) \geq k$ and $y_{nd} = 1$, be a subset of $\gamma(W, k)$. Then:

$$M(C(Y, W), k) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n(W) \geq k)c_n(W) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n(W) \geq k)\sum_{d=1}^{D} w_d y_{nd}$$
$$= \sum_{d=1}^{D} w_d \Pi(\gamma_d(W, k)), \tag{8}$$

where $\gamma(W, k) = \bigcup_{d=1}^{D} \gamma_d(W, k)$. Based on Eq. 8, the difference in adjusted headcount ratios can then be expressed as:

$$M(C(Y_A, W), k) - M(C(Y_B, W), k) = \sum_{d=1}^{D} w_d[\Pi^A(\gamma_d(W, k)) - \Pi^B(\gamma_d(W, k))]. \tag{9}$$

Now, just as in the case of γ, the sets γ_d stem from unions of sets O_s, of which there are only $2^D - 1$. Therefore the number of sets γ_d is also finite, dependent on D and independent of W and k; whereas the *actual* observed sets, $\gamma_d(W, k)$ for $d = 1, 2, ..., D$, depend on W and k.

Therefore, if $\Pi^A(\gamma_d) - \Pi^B(\gamma_d) \leq 0$ for every $\gamma_d \in \Gamma_d$ and all $\Gamma_1, \Gamma_2, ..., \Gamma_D$, then, by Eq. 9, it must be the case that $[M(C(Y_A, W), k) - M(C(Y_B, W), k)] \leq 0$ for all weights, W, and poverty thresholds, k, which proves the sufficiency part of the equivalence.

Now assume that, for some γ_d, $[\Pi^A(\gamma_d) - \Pi^B(\gamma_d)] > 0$. Then we could find a combination of W and k (e.g. $k = 0$ with $w_d \to 1$), such that $[M(C(Y_A, W), k) - M(C(Y_B, W), k)] > 0$. But this contradicts statement (2). Therefore it must be true that $\Pi^A(\gamma_d)$ is not greater than $\Pi^B(\gamma_d)$ for all $\gamma_d \in \Gamma_d$ and $d = 1, ..., D$. □

The following remark establishes the link between conditions (3) and (4):

Remark 2 If $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ for all $k \in (0, 1]$ and any vector of weights, W, then $M(C(Y_A, W), k) \leq M(C(Y_B, W), k)$ for all $k \in (0, 1]$ and any weighting vector, W.

Proof This remark is an extension of remark (1) to any possible vector of weights. From condition (3) we know that $H(C(Y_A, W), k) \leq H(C(Y_B, W), k)$ for all $k \in (0, 1]$ and any vector of weights, W, implies that $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all $P \in \mathbb{P}_1$, for any weighting vector, W, and poverty threshold $k \in (0, 1]$. Additionally, M is a member of the class $\mathbb{P}_1$. Therefore $M(C(Y_A, W), k) \leq M(C(Y_B, W), k)$ for all $k \in (0, 1]$ and any weighting vector, W. □

Remark 2 implies the existence of dominance for the class of poverty measures $\mathbb{P}_2 \subset \mathbb{P}_1$ whenever there exists dominance within the class $\mathbb{P}_1$ of poverty measures.

3.3 General Sufficient Conditions

Conditions (3) and (4) provide a simple way to ascertain the existence of dominance in poverty comparisons based on counting measures. However, testing those conditions may require comparing a large number of statistics. In fact, as we show in the next section, the number of elements in the sets Γ and Γ_d increases exponentially with the number of dimensions involved in the poverty comparisons. With that concern in mind, we derive a set of useful conditions which are much easier to implement in practice, especially when D is relatively large, as they require a much smaller number of statistics. Firstly, we derive a sufficient condition whose fulfillment guarantees a robust pairwise poverty ordering for any poverty measures in the most general classes $\mathbb{P}_1$ and $\mathbb{P}_2$, as well as, the measures H and M. Secondly, in the next subsection, we introduce two necessary conditions whose violation implies that no unambiguous poverty ordering can be established when comparing two populations.

Let H_s^A and H_s^B denote the subset headcounts for $s \in S(D)$ in populations A and B, respectively. The sufficient condition is the following:

Condition 5 *If $H_s^A \leq H_s^B$ for all $s \in S(D)$, then $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all $P \in \mathbb{P}_1$, all poverty thresholds $k \in (0, 1]$ and all weighting vectors W.*

Proof From equation (5) we know that, for any $\gamma \in \Gamma$, the measure $\Pi(\gamma)$ can be expressed as a sum of subset headcounts. Therefore if all the subset headcounts of A are never higher than those of B, then the value of $\Pi^A(\gamma)$ will never be higher than $\Pi^B(\gamma)$ for any $\gamma \in \Gamma$. From condition (3) this implies that $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all $P \in \mathbb{P}_1$, any poverty cut-off, k, and weighting vector, W. □

Note that this sufficient condition applies to the M index and the class $\mathbb{P}_2$, both included in $\mathbb{P}_1$. It also applies to the index H. This is because, by conditions (3), dominance within the class $\mathbb{P}_1$ implies dominance within the class $\mathbb{P}_2$, as well as the poverty index H. Being just sufficient, a violation of condition (5) does not rule out poverty dominance of A over B. However, as shown in the necessary condition (6) below, if condition (5) is violated because $H^A_{(1,2,\ldots,D)} > H^B_{(1,2,\ldots,D)}$, then we can actually conclude that A does not dominate B. Hence a combination of condition (5) and the necessary conditions of the next subsection, can go a long way in ascertaining pairwise poverty dominance (or lack thereof) when D is large.

3.4 General Necessary Conditions

We derive two useful necessary conditions which are easy to implement, as they require one and D statistics, respectively. The first of these necessary conditions is the following:

Condition 6 *If $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all $P \in \mathbb{P}_1$, all poverty thresholds, k, and weighting vectors, W, then: $H^A_{(1,2,\ldots,D)} \leq H^B_{(1,2,\ldots,D)}$.*

Proof From condition (3) we know that, when $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all W and k, then $\Pi^A(\gamma) \leq \Pi^B(\gamma)$ for all $\gamma \in \Gamma$, which implies that $\Pi^A(O_{1,2,\ldots,D}) = H^A_{(1,2,\ldots,D)} \leq H^B_{(1,2,\ldots,D)} = \Pi^B(O_{1,2,\ldots,D})$. □

Condition (6) states that whenever multidimensional poverty in population A is lower than in population B for every possible weighting vector, W, and identification cut-off, k, then it must be the case that the percentage of people deprived in every dimension in A (i.e. following an intersection approach to poverty identification) cannot be higher than the percentage of people from B in the same situation. This is a simple but powerful condition: it basically means that we can rule out the possibility of dominance between two populations by simply comparing the percentage of people deprived in all dimensions in each population. Note that this condition applies to multidimensional headcount index, H, as well as the measures in the class $\mathbb{P}_1$ (including the indices in the class $\mathbb{P}_2$ and the index M).

The second necessary condition is:

Condition 7 *If $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all $P \in \mathbb{P}_1$, all poverty thresholds, k, and weighting vectors, W, then: $U_d^A \leq U_d^B$ for all $d \in \{1, 2, ..., D\}$.*

Proof Note that for all $d \in \{1, 2, ..., D\}$, it is easy to show that the set including all those individuals deprived in dimension d is one of the potential poverty sets γ included in Γ.

Let Q_1 be the set of people deprived in dimension 1. Then U_1 is the proportion of people in Q_1. Also, by definition, $Q_1 = (O_1 \bigcup O_{1,2} \bigcup O_{1,3} \bigcup ... \bigcup O_{1,...,D})$, which means that $Q_1 \in \Gamma$, i.e. Q_1 is a potential poverty set (for example, if we choose W and k such that: $\sum_{d=2}^{D} w_d < k < w_1$ then Q_1 will be the actual observed poverty set). Likewise, all the Q_d for $d = 1, 2, ..., D$ are potential poverty sets (i.e. elements of the set Γ).

Then, from condition (3) we know that, when $P(C(Y_A, W), k) \leq P(C(Y_B, W), k)$ for all W and k, then $\Pi^A(\gamma) \leq \Pi^B(\gamma)$ for all $\gamma \in \Gamma$, which implies $U_d^A \leq U_d^B$ for all $d \in \{1, 2, ..., D\}$. □

Condition (7) states that if poverty in population A is unambiguously lower than in B then it must be the case that all the uncensored deprivation headcount ratios in A cannot be higher than their respective counterparts from B. This is, again, a simple but powerful condition: without comparing the relative frequencies of all elements in the sets Γ and Γ_d, if there exists just one variable d for which $U_d^A > U_d^B$, then we can rule out the possibility that A unambiguously dominates B according to any poverty measure in $\mathbb{P}_1$ (including those in $\mathbb{P}_2$ and M) and the multidimensional headcount H.

4 Application of the New Dominance Conditions

The application of the dominance conditions derived in the previous section requires two things from the analyst: the set of poverty statistics (M, H, H_s, Γ, Γ_d, U_d) relevant to each condition; and a testing framework to evaluate whether differences in those statistics between populations are statistically significant. In this section we address these two issues.

4.1 Number of Statistics Needed to Evaluate Dominance

The evaluation of the dominance conditions requires the computation and comparison of a number of statistics which varies with the number of welfare dimensions. Table 1 shows the number of statistics involved in conditions 1-7 for values of D from 2 to 5. In general, conditions (1) and (2) involve a small number of statistics vis-a-vis the conditions applicable to the case of variable weights, i.e. (3) and (4). This is not surprising as the first two conditions permit to assert poverty dominance within the classes $\mathbb{P}_1$ and $\mathbb{P}_2$ only for *a given vector of dimensional weights*. Thus, for any vector of weights, conditions (1) and (2) require the comparison of the indices $H(C(Y, W), k)$ and $M(C(Y, W), k)$ for all relevant values of the threshold k. These values depend on the specific vector of weights and it is easy to show that the number of relevant values is never greater than $\sum_{i=0}^{D} \binom{D}{i} = 2^D$.

Table 1 Number of statistics involved in each dominance condition

	Statistic	$D=2$	$D=3$	$D=4$	$D=5$
Fixed weights					
Necessary & sufficient					
Condition 1	$H(C,k)$	4	8	16	32
Condition 2	$M(C,k)$	4	8	16	32
Variable weights					
Necessary & sufficient					
Condition 3	$\Pi(\gamma)$	4	18	166	7,579
Condition 4	$\Pi(\gamma_d)$	3	10	63	690
Sufficient					
Condition 5	H_s	3	7	15	31
Necessary					
Condition 6	$H_{(1,2,\ldots,D)}$	1	1	1	1
Condition 7	U_d	2	3	4	5

The necessary and sufficient conditions (3) and (4) are the most demanding of all conditions since they require the comparison of all the sets γ and γ_d belonging to the sets Γ and Γ_d, respectively. While derivation of these sets is trivial when D is small, it gets more complex as the number of dimensions increases. This is because the number of elements in Γ and Γ_d grows fast with D as the combinations of groups O_s that can make up the set of the multidimensionally poor rise exponentially with the number of dimensions.

In order to derive the sets Γ and Γ_d we developed two search algorithms that identify the combinations of O_s that can form any plausible poverty set γ and their dimensional components γ_d.[6] The key to the identification of the potential poverty sets in these algorithms is the consistency property of the poverty identification function ρ_k (Lasso de la Vega 2010). This property requires that, for any two sets O_s and $O_{s'}$ with $c^s \leq c^{s'}$, if the set O_s belongs in a given poverty set γ then that must also be the case of the set $O_{s'}$. Thus, for instance, if a poverty set γ includes the group O_1 comprising those deprived only in dimension 1, then it must also include all those sets O_s with larger $c^{s'}$ involving combinations of deprivation in dimension 1 and any other dimensions. For instance, in the case of $D=3$, if O_1 belongs to any set γ then that must also be the case of the groups $O_{1,2}$, $O_{1,3}$, and $O_{1,2,3}$, including those deprived in dimensions 1 and 2; 1 and 3; and 1, 2, and 3; respectively.

As Table 1 shows, the number of potential poverty sets grows more than exponentially for conditions (3) and (4); with the number of dimensions jumping from 18 when $D=3$ to 7,579 when $D=5$, in the case of condition (3). Although smaller, the number of sets γ_d required to evaluate condition (4) also grows significantly fast, with D reaching 690 when $D=5$.

The sufficient condition (5) involves the comparison of the subset headcounts H_s for all combinations of dimensions s in the power set $S(D)$. The number of elements in this set, excluding the empty set, is equal to 2^D-1 which gives the number of statistics to be compared. Finally the necessary conditions (6) and (7) are the easiest to evaluate as they require, respectively, the comparison of the percentage of people deprived in all dimensions,

[6] The algorithms *gamma* and *gammad* are coded in Stata version 14.0 and are included as part of the Stata package *Domcount* specifically developed to empirically implement the new dominance conditions. The package is available at https://drive.google.com/file/d/0B4MaiGQpsjKqeUlvQWJhUWpJbk0/view.

and the uncensored deprivation headcounts U_d reporting the proportion of people deprived in each of the dimensions.

In practice, the evaluation of dominance conditions (3), (4), and (5) may require the comparison of fewer statistics than those listed in Table 1. This occurs whenever any of the sets involved in a given dominance condition (e.g., γ, γ_d, O_s) are empty in the pair of compared distributions simultaneously.[7] As it happens, in this paper's empirical illustrations none of the sets was empty which means that all statistics had to be compared.

4.2 A General Testing Framework

To test the dominance conditions we propose an intersection-union, multiple comparison test which is convenient for its simplicity, generally low size, and decent power for pair-wise population comparisons (Dardanoni and Forcina 1999; Hasler 2007). Evaluating each of the dominance conditions requires computing and comparing $R \geq 1$ sample statistics in the forms of sample means — e.g. the indices H and M for all relevant values of k in conditions(1) and (2) or the probabilities $\Pi(\gamma)$ in condition (3) —which are asymptotically standard-normally distributed (i.e. the assumptions of the central limit theorem hold).

Let $z(r) = \frac{X^A(r)-X^B(r)}{SE[X^A(r)-X^B(r)]}$, where $X^A(r)$ is a sample mean for A (e.g. the index M for a given value of k) and $SE[X^A(r) - X^B(r)]$ is the standard error of the difference $X^A(r) - X^B(r)$. We propose the following null and alternative hypotheses:

$$Ho : z(r) = 0 \text{ for all } r = 1, 2, ..., R$$
$$Ha : z(r) < 0 \text{ for all } r = 1, 2, ..., R$$

When testing these hypotheses we reject the null in favour of the alternative if $\max\{z(1), z(2), ..., z(R)\} < z_\alpha < 0$, where z_α is a left-tail critical value, and α is both the size of a single-comparison test as well as the overall level of significance of the multiple-comparison test. It is not difficult to show that, generally, the overall size of the test will be lower than α. Given the nature of the conditions, if we reject the null in favour of the alternative hypothesis then A dominates B in the sense of being deemed less poor for a broad class of poverty measurement choices (which depends on the condition in question).

The formula of the specific z-statistics varies across conditions as different conditions look at different aspects of the distribution of deprivations. Below we present the statistics used for each condition.

4.2.1 Test of conditions 1 and 2

For condition (1) we use z-statistics of the form:

$$z(k) = \frac{H^A(C,k) - H^B(C,k)}{\sqrt{\frac{\sigma^2_{H^A(C,k)}}{N^A} + \frac{\sigma^2_{H^B(C,k)}}{N^B}}}, \tag{10}$$

where the expression for the variance is:

$$\sigma^2_{H(C,k)} = H(C,k)[1 - H(C,k)]. \tag{11}$$

[7] We thank an anonymous referee for pointing this out.

For condition (2) we use the same statistic but replacing $H(C, k)$ with $M(C, k)$, and noting that the variance in this case is given by:

$$\sigma^2_{M(C,k)} = \frac{1}{N}\sum_{n=1}^{N}[c_n]^2\mathbb{I}(c_n \geq k) - [M(C, k)]^2 \tag{12}$$

4.2.2 Test of conditions 3 and 4

These conditions require the comparison of the measure of the sets $\gamma \in \Gamma$ and $\gamma_d \in \Gamma_d$. To this purpose, for condition (3) we consider statistics of the form:

$$z(\gamma) = \frac{\Pi^A(\gamma) - \Pi^B(\gamma)}{\sqrt{\frac{\sigma^2_{\Pi^A(\gamma)}}{N^A} + \frac{\sigma^2_{\Pi^B(\gamma)}}{N^B}}}, \tag{13}$$

where $\Pi(\gamma)$ is given by expression (5) and the variances is:

$$\sigma^2_{\Pi(\gamma)} = \Pi(\gamma)[1 - \Pi(\gamma)]. \tag{14}$$

For condition (4) the formulae are the same but simply replacing $\Pi(\gamma)$ with $\Pi(\gamma_d)$.

4.2.3 Test of condition 5

This condition compares the subset headcounts H_s for all combinations of dimensions s in the power set $S(D)$. We use the following statistic:

$$z(s) = \frac{H^A_s - H^B_s}{\sqrt{\frac{\sigma^2_{H^A_s}}{N^A} + \frac{\sigma^2_{H^B_s}}{N^B}}}, \tag{15}$$

where H_s is given by Eq. (4) and the variance:

$$\sigma^2_{H_s} = H_s[1 - H_s]. \tag{16}$$

4.2.4 Test of condition 6 and 7

For the necessary condition (7) we use z-statistics of the form:

$$z_d = \frac{U^A_d - U^B_d}{\sqrt{\frac{\sigma^2_{U^A_d}}{N^A} + \frac{\sigma^2_{U^B_d}}{N^B}}}, \tag{17}$$

where the variance is given by the expression:

$$\sigma^2_{U_d} = U_d[1 - U_d]. \tag{18}$$

These formulae can also be used for condition (6) but noting that evaluating this condition requires only the comparison of the percentage of people deprived in all possible dimensions which is given by $H_{1,2,\ldots,D}$.

5 Empirical illustrations

5.1 Poverty in Australia in the 2000s

We use the new dominance results to evaluate the robustness of poverty trends in Australia over the first decade of the 21st century. This was a period of strong income growth in which Australia outperformed most developed countries. This was particularly true during the period 2001-2007, where incomes grew at an average rate above 3 per cent largely driven by the mining boom and favourable trends in commodity prices. Although to a lesser extend than the US and European countries, Australia's economic performance was also affected by the Global Financial Crisis (GFC) as reflected in the rapid increase in unemployment between April 2008 and June 2009 (from 4.1 to 5.7 per cent). This negative shock, together with the declining mining boom, led to slower income and employment growth in the period 2008-2010 relative to the pre-GFC years.

We evaluate poverty trends in Australia using data from the Household Income and Labour Dynamics in Australia (HILDA) survey. This is a nationally representative survey initiated in 2001, which collects detailed socio-economic information from more than 7,000 households and their members every year. For the illustration we consider three indicators of economic disadvantage: a binary income poverty indicator equal to 1 if the household's annual disposable income is below 60 per cent of the median equivalent income; an asset-poverty indicator which is equal to 1 when the household lacks enough assets to sustain its members above the income poverty line for three months; and a measure of financial hardship equal to 1 whenever the household reports that at least three of the following circumstances occurred along the financial year: could not pay electricity, gas or telephone bills on time; could not pay the mortgage or rent on time; pawned or sold something; went without meals; was unable to heat the home; asked for financial help from family, friends, or community organizations. For the income and wealth poverty indicators, the income and wealth variables were adjusted by household size using the OECD modified equivalence scale that assigns a value of 1 to the first adult, 0.5 to subsequent adults in the household, and 0.3 to every member under the age of 15. The unit of analysis for poverty comparisons is the individual and each individual is assigned the value of the poverty indicators computed at the household level.

Table 2 shows the prevalence of the poverty indicators for the years 2002, 2006, and 2010. The levels of economic deprivation declined substantially during the years of strong economic growth that preceded the GFC. Income and wealth poverty rates fell, respectively, about 2 and 1.4 percentage points from 2002 to 2006. The income and wealth gains led to a decline in the proportion of people experiencing financial hardship which, by 2006, was more than 1.7 percentage points below that in 2002. By contrast, economic disadvantage increased in the years following the GFC. By 2010 the income and wealth poverty rates were above those of 2006 but still below the levels observed at the start of the decade.

Figure 1 shows the multidimensional headcount $H(k)$ (horizontal axis) and the adjusted headcount ratio $M(k)$ (vertical axis) for the years 2002, 2006, and 2010 assuming equal

Table 2 Poverty indicators in Australia(%)

Year	Income	Wealth	Financial hardship
2002	18.47	8.36	6.65
2006	16.55	6.97	4.93
2010	18.21	7.39	5.89

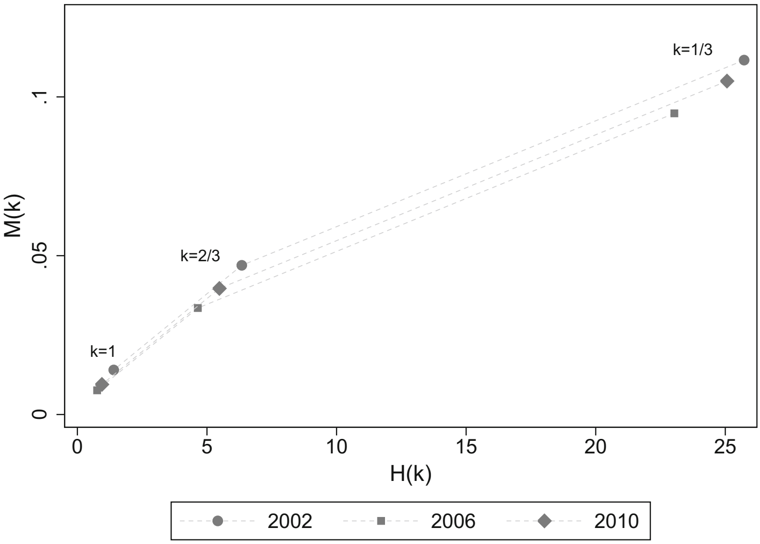

Fig. 1 M(k) and H(k) indices (equal weights)

weights for the three dimensions. Estimates of the indices are displayed for each relevant values of the poverty threshold k (1, 2/3, and 1/3, from the origin outward). A point in the graph thus represents the vector (H, M) for a year and poverty cut-off, such that, for a given value of k, points located further away from the origin indicate higher levels of multidimensional poverty. Inspection of the figure reveals a substantial decline in poverty in the years preceding the GFC. Our estimates of M and H for 2002 are larger than those for 2006 for any relevant value of k. This positive trend was partially reversed in the years following the GFC. Indeed, poverty estimates for 2010 are greater or equal than those in 2006 for any poverty cut-off. Despite this change, poverty levels by 2010 were still lower than those at the start of the decade.

To evaluate whether the poverty orderings based on the $H(k)$ and $M(k)$ indices for the case of equal weights coincide with those of any measure in the classes $\mathbb{P}_1$ and $\mathbb{P}_2$ we apply the dominance conditions (1) and (2). Tables 3 and 4 present the statistics required to test each of those conditions.[8] For this and all subsequent tests, we present the results for all pairwise comparisons such that the statistic in each cell serves to test whether the year in the column dominates (i.e., has less poverty than) the year in the row. For conditions (1) and (2), the statistics in the tables correspond to the maximum value of the $z(k)$ statistics ($k = 1, 2/3, 1/3$) relevant for each pairwise comparison.[9] When comparing 2002 with 2006, we find statistical evidence to reject the hypothesis of equal poverty in favour of the alternative whereby poverty declined between the two years. Thus, under the assumption of

[8]These statistics, as well as those used to test the other conditions, were computed using the Stata program *robust* included in the Stata package *Domcount* available at https://drive.google.com/file/d/0B4MaiGQpsjKqeUlvQWJhUWpJbk0/view.

[9]Note that the statistics in the column for 2006 are the same for conditions (1) and (2). This is because, for the statistics based on both the M and H measures, the maximum difference between 2006 and the other two years occurs for $k = 1$, and we know that $H(1) = M(1)$.

Table 3 Test of condition 1 (maximum statistics). $Ho: H(C(Y_A, W), k) = H(C(Y_B, W), k)$ for all k versus $Ha: H(C(Y_A, W), k) < H(C(Y_B, W), k)$ for all k

B \ A	2002	2006	2010
2002	0.00	−4.70	−1.14
2006	5.64	0.00	3.54
2010	3.19	−1.49	0.00

equal weights, using standard significance levels we can conclude that poverty in 2006 was lower than in 2002 for any poverty index in the class $\mathbb{P}_1$. Based on our estimates, we cannot unambiguously assert that poverty levels in 2010 were different to those in 2006. However, the results for 2010 and 2002 show that the level of poverty in 2010 was still below that in 2002, although this results holds only for the class $\mathbb{P}_2$ of poverty measures as we fail to reject the null hypothesis for condition (1).

These dominance results apply only to the case of equal weights. Nothing a priori ensures that they will hold under different weighting schemes. Can we unambiguously claim that poverty in 2006, or 2010, was lower than in 2002 regardless of the choice of dimensional weights? In order to answer this question we now turn to the new poverty dominance conditions.

We start the analysis looking at the necessary conditions as these allow us to rule out the existence of dominance by checking only a limited number of conditions. Table 5 shows the statistics to test the necessary condition (7) which involves the comparison of the uncensored deprivation headcount U_d of the different dimensions. The value reported in each cell corresponds to the maximum value of the z_d statistics ($d = 1, 2, 3$) relevant for each pairwise comparison. A sufficiently large negative value of the statistic is taken as evidence against the null hypothesis and the failure to reject this hypothesis means that we can rule out the existence of dominance between the compared years. Interestingly, our results rule out the existence of dominance for all pairwise comparisons except that between 2006 and 2002. Thus, we cannot establish any unanimous ranking for any of the poverty comparisons involving 2002 versus 2010 and 2006 versus 2010. *This result illustrates the sensitivity of multidimensional poverty orderings based on counting poverty measures to the choice of dimensional weights and poverty cut-off.*

In order to evaluate whether poverty in 2006 was unambiguously lower than in 2002 we use the necessary and sufficient conditions. Table 6 shows the statistics required to test the sufficient condition (5). This condition involves the comparison of the subset headcounts and the rejection of the null hypothesis implies that the sufficient condition for dominance is satisfied. Using standard levels of significance, we find no statistically significant evidence to reject the null in any pairwise comparison. In particular, the value of the statistic for the comparison of 2006 against 2002 is 0.07, which implies that the dominance of 2006 over 2002 cannot be unambiguously asserted using the sufficient condition. However, this result does not rule out the possibility of dominance, since condition (5) is sufficient but not necessary.

Table 4 Test of condition 2 (maximum statistics). $Ho: M(C(Y_A, W), k) = M(C(Y_B, W), k)$ for all k versus $Ha: M(C(Y_A, W), k) < M(C(Y_B, W), k)$ for all k

B \ A	2002	2006	2010
2002	0.00	−4.70	−2.41
2006	6.30	0.00	3.86
2010	3.19	−1.49	0.00

Table 5 Test of necessary condition 7 (maximum statistics). $Ho : U_d^A = U_d^B$ for all $d \in [1, 2, ..., D]$ versus $Ha : U_d^A < U_d^B$ for all $d \in [1, 2, ..., D]$

B \ A	2002	2006	2010
2002	0.00	−3.82	−0.50
2006	5.60	0.00	3.26
2010	2.69	−1.22	0.00

Table 7 shows the statistics to evaluate the necessary and sufficient condition (3). Evaluating this condition requires the comparison of the measure of all poverty sets $\gamma \in \Gamma$. Interestingly, the result for the comparison of 2006 and 2002 suggests there is enough evidence to reject the null and therefore to assert that poverty by 2006 was unambiguously lower than in 2002 for any poverty measure in $\mathbb{P}_1$ (including those in $\mathbb{P}_2$ and the index M) and the measure H, and any choice of dimensional weights and poverty cut-off.

5.2 Inter-provincial Poverty Comparisons in Peru

For our second illustration we consider multidimensional poverty comparisons across the 25 provinces of Peru, known as departments. According to a wide dashboard of development indicators, the coastal departments (Tacna, Moquegua, Arequipa, Ica, Lima, Callao, Ancash, La Libertad, Lambayeque, Piura, and Tumbes) feature lower deprivation headcounts vis-a-vis highland departments (Puno, Cusco, Apurimac, Ayacucho, Huancavelica, Junin, Pasco, Huanuco, and Cajamarca) and rainforest departments (Madre de Dios, Ucayali, Loreto, San Martin, and Amazonas). Particularly, rainforest and southern highland departments (Puno, Cusco, Apurimac, Ayacucho, and Huancavelica) are known for having the most acute levels of material and non-material deprivation.

For the empirical exercise we use data from the 2013 Peruvian National Household Surveys (ENAHO). Our multidimensional poverty measure relies on four dimensions, and on the household as the unit of analysis. Firstly, household education, comprising two indicators: (1) school delay, which is equal to one if there is a household member in school age who is delayed by at least one year, and (2) incomplete adult primary, which is equal to one if the household head or his/her partner has not completed primary education. The household is considered deprived in education if any of these indicators takes the value of one.

The second dimension considers two indicators on infrastructure dwelling conditions: (i) overcrowding, which takes the value of one if the ratio of the number of household members to the number of rooms in the house is larger than three; and (ii) inadequate construction materials, which takes the value of one if the walls are made of straw or other (almost certainly inferior) material, if the walls are made of stone and mud or wood combined with soil floor, or if the house was constructed at an improvised location inadequate for human inhabitation. The household is deprived in living conditions if any of the above indicators takes the value of one.

The third dimension is access to services. The household is deemed deprived in this dimension if any of the following indicators takes the value of one: (i) lack of electricity for

Table 6 Test of sufficient condition 5 (maximum statistics). $Ho : H_s^A = H_s^B$ for all $s \in S(D)$ versus $Ha : H_s^A < H_s^B$ for all $s \in S(D)$

B \ A	2002	2006	2010
2002	0.00	0.07	1.05
2006	4.70	0.00	3.33
2010	3.19	0.29	0.00

Table 7 Test of necessary and sufficient condition 3 (maximum statistics). $Ho : \Pi^A(\gamma) = \Pi^B(\gamma)$ for all $\gamma \in \Gamma$ versus $Ha : \Pi^A(\gamma) < \Pi^B(\gamma)$ for all $\gamma \in \Gamma$

B \ A	2002	2006	2010
2002	0.00	−3.45	−0.50
2006	6.79	0.00	4.27
2010	3.78	−0.50	0.00

lighting, (ii) lack of access to piped water, (iii) lack of access to sewerage or septic tank, and (iv) lack of access to a telephone landline. The fourth dimension is household vulnerability to dependency burdens. The household is deprived or vulnerable if household members who are younger than 14 or older than 64 are three times or more as numerous as those members who are between 14 and 64 years old (i.e. in working age).

We start the analysis looking at the ranking of regions assuming equal weights for the four dimensions of poverty. Figure 2 shows the M and H measures for the Peruvian departments for the relevant values of the poverty threshold k. Interestingly, there appear to

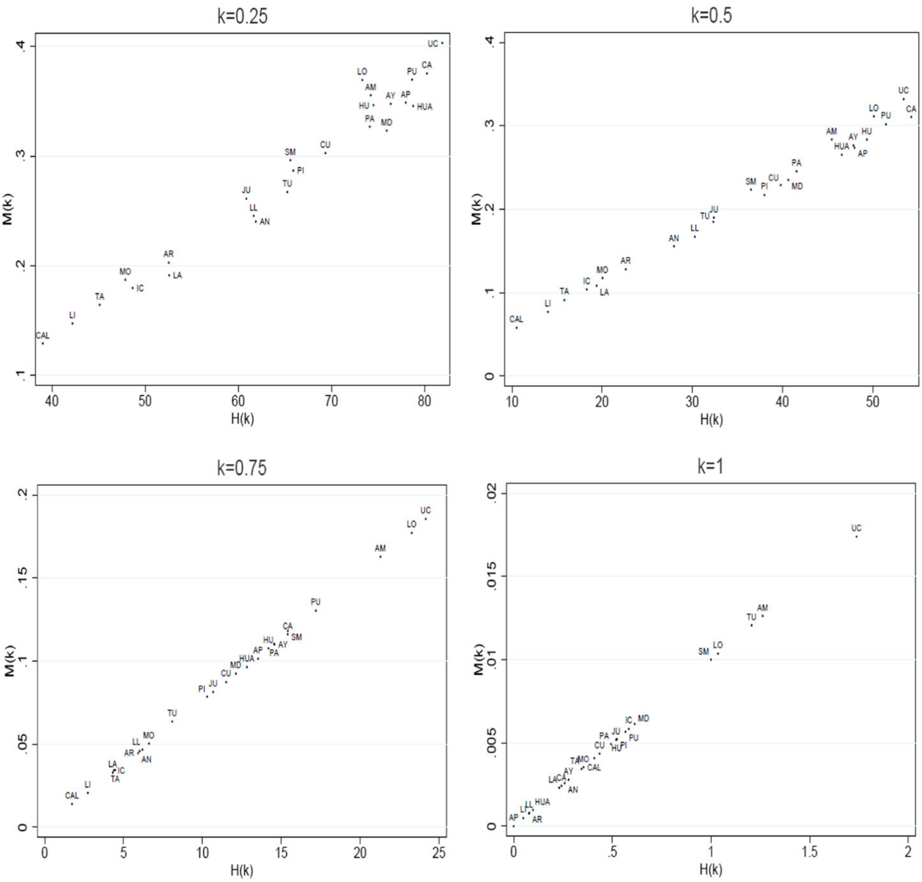

Fig. 2 M(k) and H(k) indices (equal weights). Note: AM=Amazonas, AN=Ancash, AP=Apurimac, AR= Arequipa, AY=Ayacucho, CA=Cajamarca, CAL=Callao, CU=Cusco, HUA=Huancavelica, HU=Hunuco, IC=Ica, JU= Junan, LL=La Libertad, LA=Lambayeque, LI=Lima, LO=Loreto, MD= Madred de Dios, MO= Moquegua, PA=Pasco, PI=Piura, PU=Puno, SM=San Martin, TA=Tacna, TU=Tumbes, UC=Ucayali

be three regional "poverty clusters". The panels feature on their bottom left a cluster with low values of H and M, made exclusively of coastal departments (Callao, Lima, Tacna, Moquegua, Arequipa, Ica, La Libertad). Meanwhile, on the top right, we see a cluster with high values of H and M made mainly of rainforest (Ucayali, Loreto, Madre de Dios, Amazonas) and highland (Cajamarca, Huanuco, Huancavelica, Pasco, Ayacucho, Puno) departments. The other departments appear with intermediate values of H and M. Naturally the size of the bottom left cluster increases with the values of k.

We evaluate the robustness of poverty comparisons between Peruvian departments using the new dominance results. In particular, we apply the dominance conditions (1) and (2) to evaluate whether the poverty orderings based on the $H(k)$ and $M(k)$ indices for the case of equal weights coincide with those of any measure in the classes $\mathbb{P}_1$ and $\mathbb{P}_2$. Furthermore,

Table 8 Poverty dominance between Peruvian departments, 2013

	Equal weights				Variable weights			
	Condition 1		Condition 2		Condition 3		Condition 4	
	DS	DD	DS	DD	DS	DD	DS	DD
Amazonas	0	12	0	13	0	0	0	0
Ancash	4	0	5	0	0	0	0	0
Apurimac	1	0	3	0	0	0	0	0
Arequipa	11	0	11	0	0	0	0	0
Ayacucho	1	0	2	0	0	0	0	0
Cajamarca	0	0	0	0	0	0	0	0
Callao	5	0	5	0	0	0	0	0
Cusco	3	3	3	3	0	0	0	0
Huancavelica	1	0	3	0	0	0	0	0
Huanuco	1	3	1	3	0	2	0	2
Ica	2	1	2	1	0	0	0	0
Junin	2	2	2	2	0	1	0	1
La Libertad	9	0	9	0	1	0	1	0
Lambayeque	5	0	5	0	0	0	0	0
Lima	13	0	13	0	2	0	2	0
Loreto	0	9	0	12	0	0	0	0
Madre de Dios	1	3	1	3	0	0	0	0
Moquegua	4	0	4	0	0	0	0	0
Pasco	1	1	2	1	0	0	0	0
Piura	2	3	2	3	0	0	0	0
Puno	0	3	1	5	0	0	0	0
San Martin	0	8	0	8	0	0	0	0
Tacna	5	0	5	0	0	0	0	0
Tumbes	0	5	0	6	0	0	0	0
Ucayali	0	18	0	19	0	0	0	0
Total	71	71	79	79	3	3	3	3

Note: DS and DD indicate, for each condition, the number of comparisons (out of 24) in which each department dominates and is dominated, respectively we assess the robustness of poverty orderings within those classes to changes in the dimensional weights using the necessary and sufficient conditions (3) and (4). Table 8 presents the dominance results. For each of the 25 departments, and for conditions (1) to (4), we report the number of comparisons (out of a maximum of 24 per department) in which each department dominates, and the number in which each department is dominated.[10]

Several results from Table 8 are noteworthy. Firstly, the probability of finding unambiguous rankings of regions is remarkably low, even when dimensional weights are held constant and equal across dimensions. Thus, only 71 of the 300 pairwise regional comparisons are robust to changes in the individual poverty function and poverty cut-off when weights are set equal across dimensions. This number slightly increases up to 79 when the narrower class of poverty functions $\mathbb{P}_2$ is considered instead of $\mathbb{P}_1$. Inspection of the table reveals that rainforest regions are in general the most dominated (i.e. have more poverty) of all regions. The Amazonas and Ucayali departments appear to be the poorest ones as they are dominated by 13 and 19 departments, respectively. In contrast, coastal regions such as Arequipa and Lima are the most dominant (i.e. have less poverty) with the latter dominating up to 13 regions. Interestingly, the number of unambiguous poverty comparisons significantly drops when weights are allowed to vary. In fact, only 3 of the 300 pairwise poverty comparisons are found to be robust. This means that the vast majority of regions cannot be unambiguously ranked in terms of multidimensional poverty as their comparison will depend on the particular combination of poverty index, cut-off, and dimensional weights.

6 Concluding remarks

In this paper we sought to derive new dominance conditions to evaluate the sensitivity of poverty orderings based on counting measures. Widely used to monitor poverty trends in developed and developing countries, counting measures depend on a range of methodological choices including dimensional weights, poverty functions, and poverty cut-offs. As the empirical evidence presented in this paper clearly shows, poverty comparisons based on counting measures are highly sensitive to those choices which calls for new methods to systematically evaluate the robustness of those comparisons.

Poverty analyses based on counting measures typically test the robustness of counting poverty comparisons by recalculating the indices with alternative choices of parameters (e.g. dimensional weights). In principle, we do not see any inconvenient with that approach, as long as those alternative parameters are in themselves meaningful to the researcher. The problem, we argue, is to use such a limited and arbitrary approach to make general robustness statements regarding poverty comparisons, i.e. in relation to both tested and untested sets of parameters. In that sense, we state that our dominance approach is superior: it enables us to examine robustness over a vast domain of alternative parameter values and choices of functional forms, relying on a finite and tractable battery of testable conditions.

Building on the results in Lasso de la Vega (2010), we propose fundamental conditions whose fulfilment is both necessary and sufficient to ensure that poverty comparisons are robust to changes in individual poverty functions, dimensional weights and poverty cut-off. However, since these conditions may be cumbersome to implement when the number of variables is large,[11] we also derived two useful conditions whose fulfillment is necessary,

[10] The statistics used to test those conditions are not reported here for the sake of space but are available upon request.

[11] Even though we have also rendered a ready-to-use algorithm available to Stata users.

but insufficient, for robust first- and second-order poverty comparisons. While these conditions are insufficient, they are fewer in number and much easier to compute. Furthermore, when they are not met we can immediately rule out the robustness of poverty orderings. We also provided a useful sufficient condition whose fulfillment guarantees the existence of poverty dominance for a broad class of poverty measures and any choice of dimensional weights and poverty cut-off. Though this condition is not necessary (hence its violation would not preclude the existence of a dominance relationship), it also bears the advantage of a much easier implementation vis-a-vis the set of jointly necessary and sufficient conditions.

Moreover, above and beyond the conditions derived in this paper, it is also possible to derive sets of necessary and sufficient conditions which guarantee robust poverty comparisons for a restricted *subset of weights*, as well as for broader sets of weights (e.g. admitting zero values). Likewise further useful necessary conditions are derivable if we opt to restrict the set of admissible weighting vectors, or the domain of k cut-offs, or both jointly. Some examples are available upon request. The development of general methods for the derivation of conditions whose fulfillment guarantees *partial robustness*, i.e. full robustness only to combinations of subsets of parameters (e.g. joint restrictions on weights and cut-offs, etc.) is left for future research.

Finally, the empirical application to Australia and Peru illustrated the usefulness of the new robustness conditions. In the Australian case, we learned that the rapid expansion in the early 2000s led a to significant reduction in poverty. Based on our dominance results, we can claim that poverty in Australia by 2006 was unambiguously lower than in 2002 for a broad range of poverty measures and this comparison is robust to the choice of dimensional weights and poverty cut-off. By contrast, the apparent trend of poverty increasing from 2006 to 2010 but leading to overall lower poverty levels compared to 2002, did not prove robust to every conceivable combination of the aforementioned parameters. Meanwhile, in the Peruvian case, we learned that only a handful of cross-province comparisons pass the full-robustness test when we allow dimensional weights and poverty cut-offs to vary freely. In other words, the seemingly robust results showcasing rainforest regions as poorer than most of the rest of the country and Lima and Arequipa less poor than most of the rest, crucially depended on a particular choice of dimensional weights. Hence, the poverty reduction experience in Australia between 2002 and 2006, together with the cross-province comparisons in Peru in 2013, provide two extreme empirical examples of the relative degrees of robustness that one could, in theory, expect when performing poverty comparisons with multidimensional counting measures.

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