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Developing a locally adaptive spatial multi-level logistic model to analyse ecological effects on health using individual census records

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Abstract

Geographical variable distributions often exhibit both macro-scale geographic smoothness and micro-scale discontinuities or local step changes. Nonetheless, accounting for both effects in a unified statistical model is challenging, especially when the data under study involves a multi-scale structure and non-Gaussian response variables. This study develops a locally adaptive spatial multi-level logistic model to examine binomial response variables, which integrates an innovative locally adaptive spatial econometric model with a multi-level model. It takes into account global spatial auto-correlation, local step changes, and vertical dependence effects arising from the multi-scale data structure. Another appealing feature is that the spatial correlation structure, implied by a spatial weights matrix, are *learned* along with other model parameters via an iterative estimation algorithm, rather than being presumed to be invariant. Bayesian Markov chain Monte Carlo (MCMC) samplers are derived to implement this new spatial multi-level logistic model. A data augmentation approach, drawing upon recently devised Pólya-Gamma distributions, is adopted to reduce computational burdens of calculating binomial likelihoods with a *logit* link function. The validity of the developed model is evaluated by a set of simulation experiments, before being applied to analyse self-rated health for the elderly in Shijiazhuang, the capital city of Hebei province, China. Model estimation results highlight a nuanced geography of self-rated health, and identify a range of individual- and area-level correlates of health for the elderly.

Key Words: *Spatial auto-correlation*, *local spatial modelling*, *spatial econometrics*, *multi-level models*, *geography of health*

Introduction

Distributions of many geographical variables over space exhibit clear global patterns, most often, spatial clusters—attributes in nearby areas tend to be similar, implying positive spatial dependency or auto-correlation. Accordingly, statistical modelling of such variables requires a careful treatment of spatial correlation, leading to a wide use of spatial statistics and econometrics models in social and environmental science research (Anselin 1988; Griffith 2003; Haining 2003; Banerjee, Carlin, and Gelfand 2014). Meanwhile, locally abrupt changes are also often observed in the distributions of geographical variables, i.e., moving from one side of a geographic border to the other is however associated with contrasting attribute values (e.g. Mitchell and Lee 2014). Local step changes might reflect distinct socio-economic processes in the effect of interest, even for areas of close geographical proximity. Therefore, ideally we would want to capture the co-existing global auto-correlation and local step change effects in a unified statistical methodology when modelling spatial data. A further complication rises when the data under investigation has a multiple-scale structure such as individuals nesting into census geographies or cities into regions. This multi-scale data structure tends to induce group dependency effects or vertical spatial dependency effects (Dong and Harris 2015). If they were ignored, model parameter estimates and the associated statistical inferences would be adversely affected (Raudenbush and Bryk 2002; Goldstein 2010). A simultaneous treatment of global spatial auto-correlation, local step changes and multi-scale data structure in a unified spatial statistical model poses methodological challenge. Developing such a statistical model is the primary objective of this study.

Capturing global spatial auto-correlation and local step changes

In a single-scale spatial modeling context, an innovative proposal has been put forward by Lee and Mitchell (2013), in which spatial auto-correlation is conceptualised as a global process but adjusted locally in the presence of step changes. This is achieved by learning and updating the correlation structure of spatial units (i.e. the spatial weights matrix W) through data, rather than assuming it to be invariant and exogenous to the outcome variable under examination. Local step changes are inferred by comparing the empirical distributions of spatially dependent random effects (i.e. model residuals) of bordering areas. Should a step change be identified between two neighbouring areas, the (conditional) correlation between them is set to be zero by disconnecting them in W. The rationale is that if significant differences were detected (net of covariate effects) when crossing the border of two areas, it would make little sense to impose a global spatial smoothing mechanism between them. The approach is termed as a locally adaptive spatial modelling approach (Lee and Mitchell 2013, 2014; Dean et al. 2019).

A small number of localised spatial statistical models explicitly treat entries of W as unknown random quantities, which are modelled via a logistic regression model (Lu et al. 2007; Ma, Carlin, and Banerjee 2010). However, issues of over-parameterisation and poor identification of individual entries of W in such approaches have been recognised (Lee and Mitchell 2013). Rushworth, Lee, and Sarran (2017) estimate the vector of adjacency elements of W (on the logit scale) by using a Gaussian Markov Random Field (GMRF) prior. In their proposal, the connection structure of areas is regarded as a graph where areas are represented by vertices and an edge linking two vertices is presented if these two areas are adjacent. As a GMRF prior is assumed for the adjacency structure of *edges* and the number of edges are usually much larger than that of areas, the implementation of such approach is computationally expensive and can be impossible in the presence of large spatial dataset. This study, therefore, adopts the proposal in Lee and Mitchell (2013) to simultaneously model spatial auto-correlation and local step change effects. We refer to Lee, Rushworth, and Sahu (2014) for a thorough review on localised spatial structure estimation in a single-scale spatial modelling context.

Multi-level modelling and spatial auto-correlation

Multi-level modeling has been well-recognised as a rigorous statistical modelling framework to deal with data with a multiple-scale structure and geographically clustered survey or census data in particular (Raudenbush and Bryk 2002; Goldstein 2010). It simultaneously models the outcome variable of interest at different scales or levels, thus with great potentials to address the scale effects such as the individualistic and ecological fallacies (Jones 1991; Subramanian, Jones and Duncan 2003; Subramanian et al. 2008). Nonetheless, what standard multi-level models capture is a vertical dependence effect arising from the group membership structure of data, while a horizontal dependence effect among units due to geographical proximity (i.e. spatial auto-correlation) tends to be left unmodelled.

Scholarship on an integrated multi-level and spatial auto-correlation modelling approach is nascent but expanding. The key idea is to conceptualise the higher-level (or area-level) random effects as spatially dependent by using a simultaneous auto-regressive model (SAR, Anselin 1988; Haining 2003) or a conditional auto-regressive model (CAR, Banerjee et al. 2014; Congdon 2014). The former is seen in Smith and LeSage (2004), Savitz and Raudenbush (2009), Lacombe, Holloway and Shaughnessy (2014), and Dong et al. (2018, 2019). The latter-type extension has been proposed in Arcaya et al. (2012), Ma et al. (2017), Ma, Chen, and Dong (2018), and in Dong et al. (2016) where regression coefficients are further allowed to be spatially varied. In the so-called hierarchical spatial auto-regressive models proposed in Dong and Harris (2015), SAR models are integrated into each level of a geographically hierarchical data. The usefulness of a spatially explicit multi-level model in dealing with multi-scale geographical data has been assessed via Monte Carlo simulation and empirical studies (Dong et al. 2015; Owen, Harris, and Jones 2016; Bivand et al. 2017). However, these spatial extensions on the multi-level models only consider *global* spatial auto-correlation, thus ignoring the possibility of local step change or boundary effects in the distributions of geographical variables. A recent model proposed by Dong, Wolf, Alexiou, and Arribas-Bel (2019) deals with global spatial auto-correlation and local step changes in a multi-level modelling context, but is designed only for Gaussian response variables. Moreover, their approach to calibrate a spatial weights matrix W is based on the distributions of estimated outcomes at a higher (or more aggregated) spatial scale rather than the distributions of differences in genuine areal effects (discussed below). It thus risks the potential conflation of covariate effects and areal effects when estimating the spatial correlation structure of units.

Innovation of this study

This study develops a new class of multi-level model to investigate geographically hierarchical *binomial* data where individuals nest into geographical units. It is termed as *a locally adaptive spatial multi-level logistic model* and differs from previous spatial extensions on multi-level models in a few important aspects. Foremost, it integrates a locally adaptive spatial auto-correlation model with a multi-level logistic model, thus being able to capture both global spatial auto-correlation and potential local step change effects. This can lead to a more realistic modelling of spatial effects at the ecological scale. Secondly, it separates effects of covariates at different levels on an outcome variable, so the interpretations of regression coefficients are intuitive. This resonates with the idea that different processes might be operating at different scales, and that outcomes at different scales could be affected by different sets of predictor variables. With an adaptive SAR model, rather than a CAR model used in Lee and Mitchell (2013), specified for the areal level *latent* outcomes, spatial spillover or feedback effects are allowed (discussed below). Thirdly, it permits the links between individuals and geographical contexts to be learned through data in the sense that the model identifies a set of areas (through estimates on W), by which each individual is affected. This, to some extent, alleviates the uncertain geographic context problem (Kwan 2012) by moving beyond the restrictive assumption that individuals are only affected by areas where they live. Lastly, we demonstrate that the Mundlak correction (Mundlak 1978) can be easily incorporated into the proposed model to deal with potential correlations between individual-level covariates and unobservable areal level random effects.

The locally adaptive spatial multi-level logistic model is implemented by using an iterative algorithm following Lee and Mitchell (2013). Overall, it cycles between estimating model parameters via a Bayesian *global* spatial multi-level logistic model, and updating W based on estimated areal level random effects (net of covariate effects to avoid conflation), until a convergence criterion is met (detailed below). Bayesian MCMC samplers have been derived to implement the *global* spatial multi-level logistic model, which constitutes the core component of the overall algorithm. To reduce computational cost, we derive MCMC samplers by exploiting a new class of Pólya-Gamma distribution specifically devised to deal with binomial likelihoods with a *logit* link function (Polson, Scott, and Windle 2013).

The methodology is applied to explore the social and spatial disparity of self-rated health for the elderly in Shijiazhuang, the capital city of Hebei province in China, using a unique individual census record data. These census records are further linked to the finest-resolution census geographical units publicly available in China, for which a range of social, environmental and industrial development variables are extracted. With the linked dataset, we aim to understand the individual- and area-level correlates of self-rated health for the elderly in the study area.

The remainder of this paper is structured as follows. Section 2 describes our methodological development and model estimation strategy. In Section 3, we conduct a simulation study to assess the validity of the developed methodology. We then describe the data and variables used in our empirical study in Section 4, and model estimation results are presented in Section 5. The final section concludes with a brief summary of our findings and a discussion of future work.

Methodological development

A standard non-spatial multi-level logistic model

Consider a two-level data where surveyed (or census) individuals (Level-1 units) nest into J non-overlapping areal units (Level-2 units) that constitutes a study region $D = \{A_1,\ldots,A_J\}$. There are n_j individuals in A_j. A standard random intercept multi-level logistic model is expressed as (e.g. Goldstein 2010),

$$Y_{ij} \sim \text{Binomial}\,(1, p_{ij});\quad i = 1,2,\ldots,n_j;\ j = 1,2,\ldots,J$$

$$\log\frac{p_{ij}}{1-p_{ij}} = \eta_{ij} = \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \varsigma_j$$

$$\varsigma_j = \boldsymbol{z}'_j\boldsymbol{\gamma} + \mu_j;\qquad \mu_j \sim N(0, \sigma^2) \tag{1}$$

where i and j are individual and areal (sub-district in this study) indicators. p_{ij} is the probability of success, e.g., the probability of the ith individual living in the jth sub-district reporting good health status, which is related to a set of predictors via a *logit* link function. Individual outcome Y_{ij} then follows a Binomial distribution with probability of p_{ij}. The logit link function is chosen over the cumulative Normal distribution function because of its intuitive and straightforward interpretation of covariate effects in terms of odds ratios. ς_j measures the effect of spatial unit j on individuals located within it or the average outcome of area j on the *logit* scale, with mean-centred individual-level covariates $\boldsymbol{x}$ (Raudenbush and Bryk 2002).

At the areal level, $\boldsymbol{\varsigma} = [\varsigma_1,..., \varsigma_J]$ is a linear model of the areal level covariates (z) and a vector of independent area-level random residuals $\boldsymbol{\mu} = [\mu_1,\ldots, \mu_J]$. Elements of $\boldsymbol{\mu}$ are assumed to be independent, each of which follows a Normal distribution, $N(0, \sigma^2)$. $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of fixed regression coefficients of $\boldsymbol{x}$ and $\boldsymbol{z}$, respectively. We note that cross-level interaction terms between individual- and area-level covariates can also be added into the individual-level equation.

Developing a global spatial multi-level logistic model

The independence assumption imposed on $\boldsymbol{\varsigma}$ has been questioned in a spatial context on the basis that the areal effects are likely to be correlated because of geographical proximity (e.g. Dong and Harris 2015; Bivand et al. 2017). Spatial auto-correlation or dependence is a critical issue to deal with when modelling geographical data (Ver Hoef et al. 2018). In the multi-level logistic modelling context, Ma et al. (2017) capture spatial auto-correlation in $\boldsymbol{\varsigma}$ by specifying a CAR model for $\boldsymbol{\mu}$,

$$\mu_j \mid \boldsymbol{\mu}_{-j}, W,\rho,\tau^2 \sim N\left(\frac{\rho\sum_{j\sim l}\mu_l}{1-\rho+\rho w_{j+}},\frac{1}{\tau^2(1-\rho+\rho w_{j+})}\right) \tag{2}$$

where $\boldsymbol{\mu}_{-j} = [\mu_1,...,\mu_{j-1},\mu_{j+1},...,\mu_J]$ includes random effects other than area j. The neighbourhood structure or spatial weights matrix is presented by W, with elements defined on the basis of geographical contiguity: $w_{jl} = 1$ if areas j and k share a border (denoted by $j \sim l$) and 0 otherwise. w_{j+} records the total number of geographical neighbours of area j. The scalars τ^2 and ρ are precision and spatial correlation parameters, respectively. Equation (2) is a specific CAR model, developed by Leroux, Lei, and Breslow (1999) and widely used in the spatial statistics literature (e.g. Lee 2011). The whole set of full conditionals for all J areas form a unique GMRF, $\boldsymbol{\mu} \sim \text{MVN}(\mathbf{0}, \Omega_{\text{CAR}})$ with a J by J precision matrix $\Omega_{\text{CAR}} = \tau^2 (L_W - W)$ where $L_W = \text{diag}(1 - \rho + \rho \boldsymbol{w}_+)$.

Alternatively, a SAR model, common in the spatial econometrics literature, can be used to capture potential spatial dependence in $\boldsymbol{\varsigma}$ (but has not been employed to our knowledge in the multi-level logistic modelling context), which is specified as (e.g. Anselin 1988; Haining 2003),

$$\boldsymbol{\mu} = \rho W \boldsymbol{\mu} + \boldsymbol{\varepsilon}; \quad \boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{I}_J \sigma^2) \tag{3}$$

In SAR, the spatial weights matrix W is usually row-normalised so that the maximum possible value of ρ is constrained to less than one. The distribution of $\boldsymbol{\mu}$ is also a multivariate Normal distribution, $\text{MVN}(\mathbf{0}, \Omega_{\text{SAR}})$ with a precision matrix $\Omega_{\text{SAR}} = (\sigma^2)^{-1}(I_J - \rho W)^{\text{T}}(I_J - \rho W)$.[1]

Substituting the formulation of $\boldsymbol{\mu}$ in Equation 2 or 3 for the independently distributed $\boldsymbol{\mu}$ in Equation 1 gives a baseline *global* spatial multi-level logistic model. However, an unfavourable feature of both types of spatial multi-level model specification is the absence of substantive spatial spillover effect arising from changes in the areal level covariate effects ($\boldsymbol{z}'\boldsymbol{\gamma}$). In other words, the effect of changes in a covariate z_p at area j will only affect the outcome of area j (ς_j), and subsequently, the outcomes of individuals belonging to this area; it cannot be passed on to surrounding areas. This is due to the fact that spatial dependence is included in the areal level residual term ($\boldsymbol{\mu}$) so the partial derivatives of the areal level outcome variable $\boldsymbol{\varsigma}$ with respect to z_p is simply the corresponding regression coefficient γ_p.

To model spatial dependence in a substantive way that allows for spillovers and feedbacks among Level-2 areal units, we formulate Level-2 outcomes $\boldsymbol{\varsigma}$ as a SAR model (written as succinct matrix-vector form),

$$\boldsymbol{\eta} = \boldsymbol{x}\boldsymbol{\beta} + \Delta\boldsymbol{\varsigma}$$

$$\boldsymbol{\varsigma} = \rho W \boldsymbol{\varsigma} + \boldsymbol{z\gamma} + \boldsymbol{\mu}, \quad (4)$$

where the linear predictor vector $\boldsymbol{\eta}$ is of dimension N × 1 with N = $\sum n_j$; Δ is a random effect design matrix of dimension N × J, linking Level-2 outcomes or effects to individual outcomes; $\boldsymbol{x}$ and $\boldsymbol{z}$ are individual- and area-level covariate design matrices; elements of $\boldsymbol{\mu}$ are assumed to be independent, each of which follows a Normal distribution, $N(0, \sigma^2)$. Under Equation 4, the effect of a Level-2 (or sub-district level) predictor variable (e.g. $\boldsymbol{z}_p$) on $\boldsymbol{\varsigma}$ will not be equal to estimated regression coefficient of this variables (γ_p) because of spatial spillover effects. This is a result well-established in the SAR or spatial econometric modelling literature (LeSage and Pace 2009; Elhorst 2010). Thus, we interpret Level-2 covariate effects in terms of direct, indirect and total impacts following the parameter interpretation convention in the spatial econometrics literature (LeSage and Pace 2009).

Developing a locally adaptive spatial multi-level logistic model

The key issue of the global spatial multi-level logistic model (Equation 4) is the global conceptualisation of spatial correlation (or auto-correlation), ignoring potential step change or boundary effects in distributions of spatial outcomes often observed in real-world data. In the locally adaptive spatial multi-level model, the spatial weights matrix W is calibrated based on estimates of differences in the paired areal effects $\boldsymbol{\varsigma}$ (net of the covariate effects $\boldsymbol{z\gamma}$) so that the spatial auto-correlation structure among areas is learned through data. The model is formulated as,

$$Y \sim \text{Binomial}\,(1, \boldsymbol{p})$$

$$\log \frac{\boldsymbol{p}}{\boldsymbol{1-p}} = \boldsymbol{\eta} = \boldsymbol{x\beta} + \Delta\boldsymbol{\varsigma}$$

$$\boldsymbol{\varsigma} = \rho \tilde{W} \boldsymbol{\varsigma} + \boldsymbol{z\gamma} + \boldsymbol{\mu}, \quad (5)$$

in which $\tilde{W}$ is the final estimate of W. In the model, whilst the parameter ρ controls the strength of global spatial correlation in the areal random effects ($\boldsymbol{\varsigma}$), $\tilde{w}_{lk}$ specifically defines whether sub-districts l and k are correlated or not (conditionally or at least in terms of the first-order correlation structure). In the case that w_{lk} is estimated as 0, *converted from 1 in W*, a boundary or step-change between (l, k) is detected. Comparing W with $\tilde{W}$ reveals the locations of boundaries in the areal random effect surface and the shapes of clusters of high and low values.

On model specification, we finally note that an implicit assumption underlying Equation (5) is the *independence* between individual-level covariates $\boldsymbol{x}$ and Level-2 residuals $\boldsymbol{\mu}$ (or Level-2 outcomes $\boldsymbol{\varsigma}$). The presence of correlation between $\boldsymbol{x}$ and $\boldsymbol{\mu}$, a standard endogeneity issue, can lead to inconsistent estimates of $\boldsymbol{\beta}$. The well-studied Mundlak correction approach (Mundlak 1978) can be adopted to deal with this issue. Re-writing an individual-level covariate $x_k = \bar{x} + \tilde{x}_k$ where $\bar{x}$ is the area-wise (or group-wise) mean of x_k and, $\tilde{x}_k$ the remaining within-area part, the source of correlation between x_k and $\boldsymbol{\mu}$ is the possible relation between the $\bar{x}$ and $\boldsymbol{\mu}$ because $cov(\tilde{x}_k, \boldsymbol{\mu}) \equiv 0$.[2] Therefore, an auxiliary regression for $\boldsymbol{\mu}$ can be expressed as: $\boldsymbol{\mu} = \bar{\boldsymbol{x}}\phi + \boldsymbol{\nu}$ where $\boldsymbol{\nu}$ is assumed to be independent of $\boldsymbol{x}$. Plugging this expression into Equation 5 yields a locally adaptive spatial multi-level logistic model that would further deal with potential dependence between $\boldsymbol{x}$ and $\boldsymbol{\mu}$. This offers an advantage for applied researcher who are interested in identifying causal effects of area-level covariates on individual outcomes (Bell and Jones 2015).

Before discussing estimation algorithms for the proposed locally adaptive spatial multi-level logistic model, we briefly summarise some key new features of the methodology. First, specifying the Level-2 equation as a SAR model allows for an investigation of substantive spatial dependency effects, i.e., feedbacks and spillovers between areas arising from changes in area-level covariate effects. Second, it conceptualises that an individual's outcome is affected not only by the immediate neighbourhoods—the areas where she/he lives, but also by surrounding neighbourhoods. In doing so, the correlation or dependency between individuals (Level-1 units) is permitted to move beyond Level-2 areal boundaries (or group-membership structures). Lastly, with a more realistic geographical correlation structure revealed by $\tilde{W}$, the latent Level-2 outcomes ($\boldsymbol{\varsigma}$) and the associated uncertainty measures can be more reliably estimated in the locally adaptive spatial multi-level model than in its counterpart global model. In relation to this, uncertainties of estimates on $\boldsymbol{\varsigma}$ can also be propagated to the estimates of Level-2 covariate effects.

An iterative estimation algorithm is devised to implement the locally adaptive spatial multi-level logistic model. A similar algorithm was proposed by Lee and Mitchell (2013) to estimate a locally adaptive spatial CAR model, applied to *single-level* spatial data. Here we prove its usefulness to deal with multi-scale data. In the locally adaptive spatial multi-level logistic model, model parameters were divided into two sets: $\boldsymbol{\theta} = [\boldsymbol{\beta}, \boldsymbol{\gamma}, \rho, \sigma^2, \boldsymbol{\varsigma}]$ and binary entries of W. Only entries of 1s in W (areas sharing common borders) are to be estimated, with entries of 0s being fixed. The iterative algorithm cycles between estimating $\boldsymbol{\theta}$ given W, $f(\boldsymbol{\theta} \mid W, Y, \boldsymbol{x}, z)$, and a deterministic updating of W given $\boldsymbol{\theta}$, $f(W \mid \boldsymbol{\theta}, Y, \boldsymbol{x}, z)$, until a convergence criterion is met. $f(\boldsymbol{\theta} \mid W, Y, \boldsymbol{x}, z)$ represents posterior distributions of parameters $\boldsymbol{\theta}$ from a global spatial multi-level logistic model. Details on the derivation of MCMC algorithms for $f(\boldsymbol{\theta} \mid W, Y, \boldsymbol{x}, \boldsymbol{z})$ are provided in the *Appendix*. A deterministic method is used to update W based on the posterior distributions of Level-2 model residuals $\boldsymbol{\mu}$, obtained by $(\boldsymbol{I} - \hat{\rho}W)\hat{\boldsymbol{\varsigma}} - \boldsymbol{z}'\hat{\boldsymbol{\gamma}}$. For geographically adjacent areas l and k, w_{lk} is set to 0 if the 95% credible intervals of μ_l and μ_k do not overlap, and kept to 1 otherwise. The pseudo-code of the iterative algorithm is presented in Figure 1. The model implementation algorithm is coded by using the open-source R language. The R codes and a demonstration are made publicly available on the Open Science Framework platform (https://osf.io/6pzcm/?view_only=83315448dabf42dda4c600602174c9ed). Statistical inferences on parameters are based on two MCMC chains, each of which consists of

10,000 iterations with a burn-in period of 5,000 in the following analyses. Convergence of samplers is checked by both visual inspection of trace plots of parameters and the Brooks-Gelman-Rubin scale reduction statistics (Brooks and Gelman 1998; Gelman et al. 2014).

[Figure 1 about here]

Simulation study

This section presents a small-scale Monte Carlo simulation study to assess the validity and performance of the locally adaptive spatial multi-level logistic model and its global counterparts. In the two global models that are included, spatial auto-correlation is represented by either a SAR model (as in Equation 4) or a CAR model in Ma et al. (2017). The Level-2 units are census geographical units (sub-districts) in Shijiazhuang, the capital city of Hebei Province, China (the study area in our empirical analysis). In total, there are 276 sub-districts, and for each area we randomly generate a number of individuals (ranging from 5 to 100) to mimic sample size distributions often observed in real-world data sets. This leads to a hierarchical data structure with 5,773 individuals nesting into 276 sub-districts. The linear predictor ($\boldsymbol{\eta}$ in Equation 5) includes an intercept term and a single covariate at each scale. The regression coefficients of covariates at each level are fixed to 1 and −1. The covariates are drawn randomly and independently from a standard Normal distribution. The variance of Level-2 residuals is set to 0.2 while the spatial auto-correlation parameter is set to 0.9, reflecting relatively strong spatial correlations in the Level-2 random effects. The Level-2 equation (Equation 5) is used to generate spatially dependent random effects. The boundary location template is depicted in Figure 2, which delineates the city into four hypothetical clusters (in grey colour) and one main (or comparison) area (in white colour).

Two sets of simulation experiments are conducted. The first evaluates the performance of the three models in retrieving true regression coefficient parameters under scenarios of with and without boundaries in the true data generating process. In the scenario where the hypothetical boundary locations are presented in Figure 2, the true spatial weights matrix implied by the map is used to generate areal level random effects **ς**. Individual outcomes are then generated by using a Binomial distribution. In the scenario where boundaries are absent, contiguity-based W is used to generate Level-2 random effects. The second set of simulation experiments are designed to test the performance of the locally adaptive spatial multi-level model in terms of retrieving the hypothetical boundary locations under the above two scenarios. Following Lee and Mitchell (2013), in the second set of simulation study we only include two intercept terms in the data generating process and add a value of one to the spatial random effects of grey areas (Figure 2) to reflect local step changes.

Under each scenario, 100 data sets are generated, and results from the first sets of experiments are presented in Table 1. The bias and root-mean-squared error (RMSE) of regression coefficients of covariates are presented as percentages of their true values. Under the scenario where boundaries are present, biases of the Level-2 covariate coefficient estimates are higher in the two global spatial multi-level models than in the locally adaptive model. This is expected as the latter model is the true data generating process. It seems that, with respect to estimate bias, the global spatial multi-level CAR model is relatively less sensitive to the issue of local step change effects than the SAR model. However, the RMSE of estimates on γ is the highest in the spatial multi-level CAR model among the three models. The biases of Level-1 covariate coefficient (β) estimates are small (less than 2%) in all three models and the RMSEs are also comparable. Under the scenario where boundaries are absent, i.e., the true data generating process is a global spatial multi-level SAR model (Equation 4), biases of estimates on γ are all small (less than 2%) and comparable between the locally adaptive and global spatial multi-level logistic models. Similarly, with respect to the biases of Level-1 covariate effects, they are again small and comparable across the three models. These results show the validity of the locally adaptive spatial multi-level model in retrieving true covariate effects.

The second set of experiments evaluate whether the locally adaptive spatial multi-level model can correctly identify hypothetical boundary locations (dotted lines in Figure 2). Again, 100 data sets are generated under the two scenarios. Following Lee and Mitchell (2013), two summary statistics are calculated: sensitivity which measures the percentage of true boundaries identified by the proposed model; and specificity which measures the percentage of non-boundaries correctly identified by the model. In the first scenario, the sensitivity and specificity are 96.1% and 97.9% respectively, indicating that the proposal model can accurately estimate locations of boundaries and non-boundaries. The specificity is 98.1% in the second scenario, suggesting again that the model does not tend to falsely identify boundaries when they do not exist.

[Figure 2 about here]

[Table 1 about here]

Data and Variables

The empirical study primarily draws upon a unique individual census data, containing about 130,000 records of individuals aged above 60 in Shijiazhuang, the capital city of Hebei province, China. Hebei province surrounds Beijing and is the main component of the Beijing-Tianjin-Hebei urban cluster, the largest urbanised region in North China. It is also one of the most heavily polluted regions in China partly because of the sitting of a large number of heavy industries such as mining, cement and steel industries. Moreover, due to its immediate geographic proximity to Beijing, Hebei province has been gradually undertaking polluting heavy industries transferred from Beijing. The population of Hebei province was about 71.9

million, of which 10.2 million lived in Shijiazhuang according to the Sixth population census in 2010 (National Bureau of Statistics of China, NBSC 2010). The elderly accounted for about 13% of the total population in Hebei province (NBSC 2010). The individual census records consist of approximately 10% of randomly selected households in the capital city. These household members were required to fill in a long census form that records individual socio-demographic and economic characteristics. In addition, the elderly people were further required to report their self-rated health status. We therefore select all elderly samples to investigate the social and spatial disparity of self-rated health. This leads to a data set of 130,051 elderly samples out of more than one million individual census records in Shijiazhuang. To prevent potential residential mobility effects, we focus on the long-term local resident samples—the elderly whose household registrations are at the current sub-districts where they reside and who never lived in other areas for more than six months. Given our key interest in the areal level covariate effects on health outcomes, it is important to control for potential residential selection effect, i.e. healthy people self-sort themselves into areas with observable and unobservable characteristics that promote good health (e.g. Chen, Chen, and Landry 2013).

The final elderly samples are located in 276 sub-districts (or *Jiedao*), which are the basic census spatial units in China with geographical boundary data publicly available. The average population of sub-districts is about 38,000 with a relatively large standard deviation of about 26,000. The relative location of Shijiazhuang in Hebei province as well as the sub-district boundaries in the city are depicted in Figure 3.

[Figure 3 about here]

The outcome variable is the self-rated health status for the elderly. It is quantified on a 4-point Likert scale from 1 (very bad) to 4 (very good). About half of the elderly reported very good health status, followed by more than a third reporting good health status and 16% of the elderly reporting poor or very poor health statuses (Table 2). To facilitate model implementation and allows for comparability with prior health studies focusing on the elderly in China (e.g. Feng et al. 2012; Chen, Chen, and Landry 2013; Ma et al. 2017), self-rated health status is recoded into a binary variable: 1 for good and very good health, and 0 otherwise.

Through aggregating individual records to sub-districts, the spatial distribution of self-rated health is illustrated in Figure 3, with the breaking points being quintiles of the variable. It shows an overall northwest-southeast divide with the southeast of the city showing better health outcomes than the northwest on average. A relatively strong positive spatial auto-correlation in the sub-district level health outcome is found, as indicated by a statistically significant Moran's *I* of 0.592 with a *p*-value <0.001.

The independent variables are measured at the individual and sub-district scales, respectively. The individual-level covariates include: individual socio-demographics such as age, gender, educational achievement and marital status; poverty; settlement type; and physical living environment. Poverty is measured by whether a person lives on minimum living allowance or unemployment insurance. Minimum living allowance provides residents with the basic safety net under China's social security system (Besharov and Baehler 2013). It is mean-tested, and only offers help according to local minimum living standards. The settlement type is divided into urban and rural categories to test potential urban-rural divide in health outcomes. This information is extracted from an individual's household registration status, either being agricultural *hukou* (*nong-ye hukou*, usually living in rural areas) or non-agricultural *hukou* (*ju-min hukou*, usually living in urban areas).[3] The physical living environment of the elderly includes living space per capita, the presence of tap water, and shower facilities. Physical living environments of households are often excluded in previous studies on the elderly's health in China (Feng et al. 2012), but are deemed important to health.

At the sub-district scale, we focus on the impacts of environmental pollution, poverty concentration, industrial development, and climatic conditions on health. Real-time air pollution data from monitoring stations are recorded by the Ministry of Environmental Protection of China, but they are only available from 2013, thus not compatible with the time frame of our individual census data. Instead, we use model-based estimates on ground annual concentrations of $PM_{2.5}$, provided by the Atmospheric Composition Analysis Group (van Donkelaar *et al.* 2016, public available at http://fizz.phys.dal.ca/~atmos/martin/?page_id=140). The key features of this dataset are its relatively long temporal span (from 1998 to 2015), fine spatial resolution (about 1.1 × 1.1 km) and global coverage. It allows for an aggregation of $PM_{2.5}$ concentrations to sub-districts with a standard GIS areal weighting approach (Lloyd et al. 2017), and for the calculation of cumulative air pollution concentrations from 2000 to 2010 for each sub-district. This is a more reasonable proxy variable for pollution exposures of individuals residing in an area than a snapshot transient pollution concentration measure. Poverty concentration is measured by the proportion of individuals living on minimum living allowance or unemployment insurance in a sub-district. Moreover, drawing upon the industrial unit census data in 2010 published by NBSC, we geocoded each industrial economic unit in Hebei province, aggregated revenues of each unit to the sub-district scale, and calculated the proportions of revenue for each sub-district by industry type. The aim is to explore industrial development and structural impacts on individual self-rated health. Lastly, climatic factors and land vegetation conditions are further included in our health model. Climate factors include changes in maximum and minimum daily temperatures from 2000 to 2010. Land vegetation condition is measured by the Normalised Difference Vegetation Index (NDVI), a popular indicator quantifying the greenness and amount of vegetation of an area (Curran 1980). Both temperature and NDVI data are collected from the Resource and Environmental Data Cloud Platform, Chinese Academy of Sciences (http://www.resdc.cn) and mapped to sub-districts in the study area (Lloyd et al. 2017). Summaries on variables are provided in Table 2. In the analyses that follow, continuous predictor variables are standardised by using the approach in Gelman (2008) to facilitate comparisons of covariate effects. The method subtracts a variable from its mean and further divides it by two standard deviations so that a one unit change in the variable means a change of ± one standard deviation from the mean.

[Table 2 about here]

Results

Level-2 covariate effects on health

Both the global and locally adaptive spatial multi-level logistic models developed above are applied to explore the social and spatial health disparity in the study area. Deviance information criterion (DIC, Spiegelhalter et al. 2002), the common model fit index in Bayesian inference that penalises model complexity, is used for model comparison with smaller value of DIC indicating better model fit. Moving from a global model to a locally adaptive one is associated with a significant increase in model fit, as indicated by a substantive decrease in DIC values (about 37.5). We also implemented a locally adaptive model with the Mundlak correction terms, however, none of the correction terms are statistically significant at the 95% credible interval and the increase in model fit is only marginal (the decrease in DIC is about 1.14). This suggests that the correlations between individual-level covariates and the sub-district level residuals are negligible in this particular study. As such, we rely on estimation results from the locally adaptive model without the Mundlak correction terms to interpret health inequality in Shijiazhuang (Table 3). With the posterior samples of regression coefficients and the spatial auto-regressive parameter, the impacts of sub-district level covariates and the associated 95% credible intervals are also calculated and reported in Table 4.

[Tables 3 and 4 about here]

The direct, indirect and total impacts of poverty concentration are all statistically significantly associated with the self-rated health of the elderly, net of the individual covariate effects. The direct impact of a one unit increase in poverty concentration (i.e. a change from one standard deviation below the mean to one standard deviation above the mean) is associated with a 15.7% decrease in the odds of reporting good health while the total impact is a 39.9% decrease in the odds (Table 4), *ceteris paribus*. The difference between the direct and total impacts of poverty concentration is attributable to the spillover or feedback effects among sub-districts. In terms of industrial development effects on health inequality, individuals living in sub-districts with higher levels of concentration of mining industry are associated with lower odds of reporting good health. Manufacturing and electricity industry concentrations are, however, not statistically significantly associated with health status of the elderly in the study area. Whilst different industrial sector impacts on health are clear, the results highlight the relatively severe detrimental effects of mining-related economic activities on the health status of the elderly. This detrimental effect might come from the adverse in-situ and diffusive pollution impacts on environments (e.g. water resources, air quality and soils) of mining processing activities in China (e.g. Zhang et al. 2012).

Air pollution is found to be negatively associated with self-rated health, but this association is not statistically significant. It might be attributable to two reasons. The first is related to the poor public awareness and concern of air pollution and its hazardous health effects, a situation that does not change until 2013 when long spells of toxic haze blanketed most inland areas of China (Ma et al. 2017). The health survey was conducted as a component of the 2010 population census so that the lack of public awareness might contribute to the insignificant association between air pollution and self-rated health. Secondly, the measure of pollution exposure might entail measurement errors, for instance it does not consider seasonal changes in air pollution and individual daily mobility (Kwan 2012; Park and Kwan 2017). Thus, the insignificant impacts of air pollution upon health found in this study needs to be interpreted with caution.

With respect to climate change factors, changes in the maximum daily temperature during 2000 and 2010 are found to be statistically significantly associated with health. A one unit increase in maximum temperature (i.e. about 0.6 ^{o}C) appears to be associated with a direct impact of 31.3% decrease in the odds of reporting good health (Table 4), *ceteris paribus*. Taking into account the spillover effects between sub-districts, the total impact of maximum temperature change amounts to about a 67.4% decrease in the odds of reporting good health status. Increasing empirical evidences of the adverse effects on mental health imposed by climate change (primarily increases in temperature) have been documented at the global and national scales (Obradovich et al. 2018). Our results suggest that climate change effects on health could also manifest at a relatively local scale. The impacts of changes in minimum daily temperature and land vegetation conditions during the same period on self-rated health is, however, not statistically significant. To check the robustness of the sub-district level covariate effects on health, we also estimated a model with additional spatial lag terms of these covariates, formed by using the original spatial weights matrix. However, none of the regression coefficients of spatial lag terms are found statistically significant at the 95% credible level. DICs of the two models with and without lagged predictor variables differ only marginally (full estimation results are available upon request).

Level-1 covariate effects on health

All Level-1 covariates are statistically significantly associated with self-rated health of the elderly. Age is related to self-rated health in a nonlinear way, indicated by the statistically significant coefficients of the linear and quadratic age variables, holding other variables constant. Females tend to be associated with lower probabilities of reporting good health than males. The elderly people with compulsory educational qualification are less likely to report good health status than those with secondary education or with degree, *ceteris paribus*. Family structure also makes a difference—the elderly who live alone (being single, divorced or widowed) tend to report poor health status than those living with spouse, everything else equal.

Poverty is found to be a very significant correlate of self-rated health for the elderly. Being in poverty is associated with a 71.7% decrease in the odds of reporting good health, *ceteris paribus*. This highlights the substantive detrimental impact of poverty on health for the elderly and suggests that tackling the elderly poverty seems to be an effective policy tool to promote health among the elderly in Shijiazhuang. Despite being a capital city, the level of economic development of Shijiazhuang substantively lags behind other prosperous mega-cities such as Beijing and Shanghai. With respect to the urban-rural divide in health, the elderly with agricultural *hukou* tend to report poorer health status than those with non-agricultural hukou do, net of the poverty effect. The urban-rural health divide might reflects the strong disparity in healthcare provision and its quality between urban and rural areas (e.g. Feng et al. 2012). A favorable everyday living environment, one with spacious living space, tap water and shower facility, tends to promote good health for the elderly, everything else equal.

Geographies of health at the sub-district scale

The estimated sub-district level health outcomes $\boldsymbol{\varsigma}$ (e^{ς}, to be precise) are mapped in Figure 4 with breaking points being the quintiles of the variable. Estimated boundary locations are superimposed on the distribution of $\boldsymbol{\varsigma}$. The strength of global spatial autocorrelation of $\boldsymbol{\varsigma}$ is about 0.725, implying a relatively smooth pattern of health outcomes. The variance of $\boldsymbol{\varsigma}$ is about 0.122, accounting for about 3.7% of the total variation of health at the log-odds scale. Of 753 borders that geographically separate 326 sub-districts, about 16.3% of them are found to be boundaries or local step changes. Figure 4 reveals a complex geography of health in the study area: visible local step changes scattering on a globally smoothing surface. Another feature is that most of the identified boundaries are open—one area's health outcome significantly differs from the health outcomes of *a subset of* its geographical neighbours (Lee and Mitchell 2013). Open boundaries consider potential directionality in the spatial inequality of health by allowing for the possibility that one area could differ from some of its neighbours in certain directions but blends into others in other directions (Figure 4). This differs from geographies of health that would be identified by conventional clustering methods (e.g. the *k*-means algorithm).

[Figure 4 about here]

Conclusion and future work

A suitable treatment of spatial auto-correlation is a long-standing challenge in the spatial analysis and modelling literature. The difficulty partly arises from the potential co-existence of global spatial auto-correlation and local step changes, and partly from the multi-scale structure of spatial data. We addresses this challenge by developing a Bayesian locally adaptive spatial multi-level logistic modelling approach. It integrates an adaptive spatial econometric model, in which both global spatial smoothness and local step change effects are captured, into a multi-level logistic model. Bayesian MCMC samplers are derived to implement the global spatial multi-level logistic model, which constitutes the core component of the devised iterative estimation algorithm for implementing the locally adaptive model.

Computational burden of calculating Binomial likelihoods with a *logit* link function is reduced by the use of a data augmentation approach, drawing upon the recently proposed Pólya-Gamma distribution (Polson, Scott, and Windle 2013).

Results from our Monte Carlo simulation experiments show the validity of the locally adaptive spatial multi-level model in retrieving regression coefficients and locations of boundaries under the hypothetic scenario when both global spatial auto-correlation and local step change effects are present. It also produces low chances of false positive identification of boundaries when the true data generating process only entails a global spatial auto-correlation effect. Nonetheless, in the presence of a localised spatial dependence structure the *global* spatial multi-level models tend to produce moderately biased and imprecise estimates on regression coefficients of Level-2 covariates. These results together suggest that the proposed methodology can be a useful complement to the existing spatial analytics tools.

The empirical study provides insights into the individual- and area-level correlates of self-rated health for the elderly in the study area. The geography of self-rated health, net of individual-level covariate effects, shows a complex pattern including both large-scale smoothness and local step changes. A range of correlates of self-rated health are identified. At the sub-district scale, the concentrations of poverty, mining economic activity and climate change are adversely associated with self-rated health for the elderly. At the individual scale, poverty seems to be the most important correlate of self-rated health for the elderly. Age, education, family structure, settlement types (urban versus rural), and physical living environments are all found to be significant correlates of self-rated health for the elderly. However, we note that given the cross-sectional nature of our data, effects of covariates on self-rated health estimated from the model should not be interpreted as causal effects.

Although the identification of health boundaries is interesting and meaningful on its own, an important avenue for future research is to explore potential mechanisms of boundary formation. This could include spatial differences in social welfare systems, quality of healthcare provision, and physical environments. Another important future extension to the model developed here is a full stochastic estimation of W (Rushworth, Lee, and Sarran 2017) so that the uncertainties associated with the estimated spatial correlation structures W could be measured. However, the practical issue to solve is the vastly increased computational burden. We shall explore the possibility of applying various machine learning algorithms (e.g. stochastic gradient descent) to implement the complex and computationally intensive locally adaptive spatial multi-level models.

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Notes

[1] Although SAR and CAR models have been used almost in parallel in different fields (e.g. spatial econometrics and geographical analysis in general versus spatial statistics), they are closely linked, and detailed descriptions on the similarities between the two models are provided in Assunção and Krainski (2009).

[2] $\tilde{x}_k$ is an area-wise mean-centered variable so the expectation of the product between $\tilde{x}_k$ and $\boldsymbol{\mu}$ $\mathrm{E}[\tilde{x}_k\boldsymbol{\mu}] = 0$. Originally, the Mundlak correction is proposed in the panel data modelling context and deals with potential dependence between time-variant predictors and individual random effects. However, this approach is readily applicable to general multi-level models (Raudenbush and Bryk 2002; Bell and Jones 2015) and spatial panel econometrics models (Debarsy 2012).

[3] The urban-rural divide here is more of institutional than physical landscape separation of the population, although the vast majority of people with agricultural hukou live in rural areas. This *hukou* system, implemented in 1958, has supported and strengthened a rural-urban dual structure in China which results in unequal distribution of resources (e.g. health services and facilities), and thus large gaps in terms of health outcomes in rural and urban areas (e.g. Chan 2009).

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Figure Captions

Figure 1. The pseudo-code of the iterative algorithm to implement a locally adaptive spatial multi-level logistic model.

Figure 2. Locations of hypothetical boundaries in the sub-district level random effects.

Figure 3. The study area of the capital city of Hebei province and the spatial distribution of self-rated health status, measured by percentages of good and very good health status reported by the elderly in 2010.

Figure 4. Estimated boundary locations of self-rated health in the study area, superimposed by the sub-district level random effects presented at the odds scale.

Appendix

In this appendix, we derive the Bayesian MCMC algorithm to implement the global spatial multi-level logistic model (Equation 4). At its heart, Bayesian estimation is based on the joint posterior distribution of all model parameters, which is the product of data likelihood, denoted by f(Y|.), and prior densities specified for model parameters, denoted by p(.) as below,

$$f(\boldsymbol{\beta},\boldsymbol{\gamma},\rho,\sigma^2,\boldsymbol{\varsigma}\,|\,Y,W) \propto f(Y\,|\,\boldsymbol{\beta},\boldsymbol{\gamma},\rho,\sigma^2,\boldsymbol{\varsigma},W)p(\boldsymbol{\varsigma}\,|\,\rho,\sigma^2,\boldsymbol{\gamma})p(\rho)p(\sigma^2)p(\boldsymbol{\beta})p(\boldsymbol{\gamma}). \quad \text{(A1)}$$

The prior distributions for ($\boldsymbol{\beta}$, γ, ρ, σ^2) are assumed to be independent. To be specific, $p(\boldsymbol{\beta})$ and $p(\boldsymbol{\gamma})$ are both set to a multivariate Normal distribution with mean M_0 and variance matrix T_0, MVN (M_0, T_0). An uniform distribution over (-1, 1) is assigned to ρ, allowing for the possibility of a negative spatial auto-correlation (Griffith 1980). Inverse gamma distribution (IG) is used for the variance parameter σ^2: $p(\sigma^2) \sim \text{IG}(a_0, b_0)$ with a_0 and b_0 being the shape and scale parameters. The above prior distributions are commonly used in the Bayesian multi-level and spatial econometrics literature (e.g. LeSage and Pace 2009; Gelman et al. 2014).

The likelihood function for the model is,

$$l(\boldsymbol{\beta},\rho,\sigma^2,\boldsymbol{\varsigma}) = \prod p_{ij}^{y_{ij}}(1-p_{ij})^{1-y_{ij}} = \prod \frac{\{\exp\,(x'_{ij}\boldsymbol{\beta}+\varsigma_j)\}^{y_{ij}}}{1+\exp\,(x'_{ij}\boldsymbol{\beta}+\mu_j)} \quad \text{(A2)}$$

The posterior distribution of regression coefficients $\boldsymbol{\beta}$ is not a standard density function in a logistic model, usually leading to the use of a Metropolis-Hastings (M-H) sampling method to draw inferences parameters (Gelman et al. 2014). Based on their devised Pólya-Gamma distribution, Polson, Scott, and Windle (2013) proposed a computationally effective data-augmentation strategy to conduct posterior inferences on regression coefficients in logistic models. This innovation is important and useful as it enables Gibbs samplers to be derived for posterior distributions of other model parameters. *Theorem 1* in their study established the critical link between a logistic likelihood function (Equation A2) and a Pólya-Gamma distribution. The likelihood contribution of an individual (i, j), $l_{ij}(.)$, is expressed as (Polson, Scott, and Windle 2013, p. 1342),

$$l_{ij}(\boldsymbol{\beta}, \rho, \sigma^2, \boldsymbol{\varsigma}) = \frac{\{\exp(x'_{ij}\boldsymbol{\beta} + \varsigma_j)\}^{y_{ij}}}{1 + \exp(x'_{ij}\boldsymbol{\beta} + \varsigma_j)}$$

$$\propto \exp\{\kappa_{ij}(x'_{ij}\boldsymbol{\beta} + \varsigma_j)\} \int_0^{+\infty} \exp\{-\omega_{ij}(x'_{ij}\boldsymbol{\beta} + \varsigma_j)^2/2\} p(\omega_{ij} \mid 1,0), \tag{A3}$$

where $\kappa_{ij} = y_{ij} - 0.5$, and $p(\omega_{ij}|1,0)$ is the density of a Pólya-Gamma random variable $PG(1,0)$. With Equation A3 it is readily seen that conditioning on the Pólya-Gamma latent variable ω_{ij}, $l_{ij}(.|\omega_{ij})$ is proportional to

$$l_{ij}(\boldsymbol{\beta}, \rho, \sigma^2, \boldsymbol{\varsigma}|\omega_{ij}) \propto \exp\{\kappa_{ij}(x'_{ij}\boldsymbol{\beta} + \varsigma_j) - \omega_{ij}(x'_{ij}\boldsymbol{\beta} + \varsigma_j)^2/2\}. \tag{A4}$$

This simplifies the overall likelihood function, conditioning on a vector $\boldsymbol{\omega}$, to

$$l(\boldsymbol{\beta}, \rho, \sigma^2, \boldsymbol{\varsigma}|\boldsymbol{\omega}) \propto \exp\{-0.5(\boldsymbol{\xi} - \boldsymbol{x}'\boldsymbol{\beta} - \Delta\boldsymbol{\varsigma})'\Omega(\boldsymbol{\xi} - \boldsymbol{x}'\boldsymbol{\delta} - \Delta\boldsymbol{\mu})\} \tag{A5}$$

where $\boldsymbol{\xi} = \boldsymbol{\kappa} / \boldsymbol{\omega}$ and Ω is a diagonal matrix with entries of $\boldsymbol{\omega}$. Δ is the random effect design matrix as described above. The vector $\boldsymbol{\xi}$ now serves as a working response variable and is used to derive Gibbs samplers for other model parameters.

Combining Equation A5 and prior distributions gives the joint posterior distribution for model parameters. The full conditional posterior distribution for regression coefficients of Level-1 covariates $\boldsymbol{x}$, $f(\boldsymbol{\beta} \mid Y, W, \rho, \sigma^2, \boldsymbol{\varsigma}, \boldsymbol{\omega})$ is a multivariate Normal distribution, MVN (M_β, Σ_β) with

$$M_{\boldsymbol{\beta}} = \Sigma_{\boldsymbol{\beta}}\left[\boldsymbol{x}'\Omega(\boldsymbol{\xi} - \Delta\boldsymbol{\varsigma}) + \mathrm{T}_0^{-1}\mathrm{M}_0\right];\;\; \Sigma_{\boldsymbol{\beta}} = \left(\boldsymbol{x}'\Omega\boldsymbol{x} + \mathrm{T}_0^{-1}\right)^{-1} \quad \text{(A6)}$$

Based on Equation 4 and using the Jacobian method that transforms the spatially dependent vector $\boldsymbol{\varsigma}$ to an independent vector (e.g. Anselin 1988), the prior distribution of $\boldsymbol{\varsigma}$ is,

$$p(\boldsymbol{\varsigma}\,|\rho,\sigma^2,\boldsymbol{\gamma}) = |\mathrm{B}|\left(2\pi\sigma^2\right)^{-\frac{J}{2}}\exp\left\{-0.5\sigma^{-2}(B\boldsymbol{\varsigma} - \boldsymbol{z\gamma})'(B\boldsymbol{\varsigma} - \boldsymbol{z\gamma})\right\} \quad \text{(A7)}$$

where $\mathrm{B} = I_J - \rho\mathrm{W}$ and $|\mathrm{B}|$ is the absolute value of the determinate of B. Combining Equations A5 and A7 gives the full conditional distribution of $\boldsymbol{\varsigma}$, which is a multivariate Normal distribution, MVN (M_{ς}, Σ_{ς}) with

$$M_{\boldsymbol{\varsigma}} = \Sigma_{\boldsymbol{\varsigma}}\left[\Delta'\Omega(\boldsymbol{\xi} - \boldsymbol{x}'\boldsymbol{\beta}) + B'\boldsymbol{z\gamma}/\sigma^2\right];\;\; \Sigma_{\boldsymbol{\varsigma}} = \left(\mathrm{B}'\mathrm{B}/\sigma^2 + \Delta'\Omega\Delta\right)^{-1}. \quad \text{(A8)}$$

The posterior distribution of $\boldsymbol{\gamma}$ (regression coefficients of Level-2 covariates) is a multivariate Normal distribution, MVN (M_{γ}, Σ_{γ})) with

$$M_{\boldsymbol{\gamma}} = \Sigma_{\boldsymbol{\delta}}\left[\boldsymbol{z}'\mathrm{B}\boldsymbol{\varsigma}/\sigma^2 + \mathrm{T}_0^{-1}\mathrm{M}_0\right];\;\; \Sigma_{\boldsymbol{\gamma}} = \left(\boldsymbol{z}'\boldsymbol{z}/\sigma^2 + \mathrm{T}_0^{-1}\right)^{-1}. \quad \text{(A9)}$$

The posterior distribution for σ^2 is an Inversed Gamma distribution IG(a_1, b_1) where

$$a_1 = J/2 + a_0;\;\; b_1 = (B\boldsymbol{\varsigma} - \boldsymbol{z\gamma})'(B\boldsymbol{\varsigma} - \boldsymbol{z\gamma})/2 + b_0. \quad \text{(A10)}$$

The conditional posterior distribution for the spatial autoregressive parameter ρ is

$$f(\rho\,|\,.) = |I_J - \rho W|\,\exp\{-0.5\sigma^{-2}(B\boldsymbol{\varsigma} - \boldsymbol{z\gamma})'(B\boldsymbol{\varsigma} - \boldsymbol{z\gamma})\} \quad \text{(A11)}$$

which is not a commonly-recognised probability density function, thus a Gibbs sampler is not directly applicable (Gelman et al. 2014). Following Smith and LeSage (2004) and Dong and Harris (2015), an inversion sampling approach is employed to update ρ. In short, two steps are involved. In the first step, the log-posterior density function of ρ, log $f(\rho)$, is evaluated empirically based on the updated values of ($\boldsymbol{\gamma}^{(k)}$, $\sigma^{2(k)}$, $\boldsymbol{\varsigma}^{(k)}$, $\boldsymbol{\omega}^{(k)}$) in the kth MCMC iteration. log $f(\rho)$ is evaluated as,

$$f(\rho\,|\,.) = \log|I_J - \rho W| - (e_0^{(k)} - \rho e_d^{(k)})'((e_0^{(k)} - \rho e_d^{(k)})/2\sigma^{2(k)} + C \quad \text{(A12)}$$

$$e_0^{(k)} = \left(I_J - \mathbf{z}(\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}\right);\, e_d^{(k)} = W\boldsymbol{\varsigma}^{(k)}\left(I_J - \mathbf{z}(\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}\right). \quad \text{(A13)}$$

C is a constant. e_0 and e_d are two vectors of residuals when regressing $\boldsymbol{\varsigma}^{(k)}$ and $W\boldsymbol{\varsigma}^{(k)}$ on the areal level covariates $\boldsymbol{z}$. In the second step, we numerically integrate log$f(\rho)$ on ρ over the range of (-1, 1) and draw $\rho^{(k)}$ from its empirical cumulative distribution.

In the last step, we update the Pólya-Gamma latent variable $\boldsymbol{\omega}$ to calculate our working response variable ξ. As proved by Polson, Scott, and Windle (2013), the posterior distribution of $\boldsymbol{\omega}$, $f(\boldsymbol{\omega}|Y, W, \boldsymbol{\beta}, \rho, \sigma^2, \boldsymbol{\varsigma})$ is also a Pólya-Gamma distribution, $PG(1, \boldsymbol{x}'\boldsymbol{\beta} + \Delta\boldsymbol{\varsigma})$.

Table 1. Summary of Monte Carlo simulation results.

Metrics	Model	Scenario 1: *With boundary*	Scenario 2: *Without boundary*
% Bias (γ)	Locally adaptive spatial multi-level	4.94	1.40
	Global spatial multi-level SAR	9.01	1.46
	Global spatial multi-level CAR	5.94	1.78
% RMSE (γ)	Locally adaptive spatial multi-level	6.42	5.89
	Global spatial multi-level SAR	6.38	5.94
	Global spatial multi-level CAR	7.11	6.07
% Bias (β)	Locally adaptive spatial multi-level	1.63	1.17
	Global spatial multi-level SAR	1.32	1.19
	Global spatial multi-level CAR	1.32	1.46
% RMSE (β)	Locally adaptive spatial multi-level	3.96	4.07
	Global spatial multi-level SAR	3.99	3.99
	Global spatial multi-level CAR	4.01	4.09
% Bias (ρ)	Locally adaptive spatial multi-level	0.55	-1.48
	Global spatial multi-level SAR	-2.28	-1.44

Table 2. Descriptive summaries on variables used in the study.

Variables	Definitions	Mean/proportions (Std.dev)
Self-rated health	4 = Very good	47.3%
	3 = Good	36.7%
	2 = Bad	12.8%
	1 = Very Bad	3.2%
Individual-level variables		
Age	Continuous age variable	69 (7.4)
Gender	Males as base category	53.4%
Education	Compulsory education	88.5%
	Secondary education	6.7%
	College and above	4.7%
Family structure	Single	2.1%
	With spouse	71.1%
	Divorced	0.5%
	Widowed	26.3%
Poverty	Living on minimum living allowance or unemployment insurance	1.7%
Rural household	Agricultural *hukou* holders	68.4%
Living environment	Living area per capita	40.6 (29.2)
	With tap water	31.2%
	With shower facilities	48.8%
Sub-district level variables		
Poverty concentration	Proportions of people receiving minimum living allowance or unemployment insurance	0.5% (0.4%)
Cumulative air pollution	Cumulative air concentrations during 2000 to 2010	780.2 (274.2)
Mining industry proportion	Proportion of mining industry revenue	8.4% (26.1%)
Manufacturing industry proportion	Proportion of manufacturing industry revenue	51.7% (48.1%)
Electricity industry proportion	Proportion of electricity industry revenue	3.1% (13.8%)
Land vegetation condition change	Changes of land vegetation conditions from 2000 to 2010 measured by NDVI	-0.047 (0.041)
Maximum daily temperature change	Changes of maximum daily temperature from 2000 to 2010 (unit: °C)	2.793 (0.307)
Minimum daily temperature change	Changes of minimum daily temperature from 2000 to 2010 (unit: °C)	-6.739 (1.033)

Table 3. Estimation results from the locally adaptive spatial multi-level model.

Variables	*Posterior distributions of regression coefficients*		
	Median	*2.5%*	*97.5%*
Individual-level covariates			
Age	0.225*	0.215	0.236
Age squared	1.301*	1.231	1.373
Gender	0.881*	0.85	0.915
Compulsory education	0.797*	0.722	0.874
College or above	1.182*	1.002	1.385
Single	0.887*	0.791	0.985
Divorced	0.833*	0.706	0.964
Widowed	0.806*	0.775	0.839
Poverty	0.283*	0.257	0.314
Rural household	0.675*	0.615	0.743
Living area per capita	1.107*	1.069	1.147
With tap water	1.302*	1.211	1.387
With shower facilities	1.221*	1.171	1.275
Sub-district level covariates			
Cumulative air pollution	0.988	0.941	1.008
Poverty concentration	0.868*	0.780	0.958
Electricity industry proportion	1.007	0.926	1.103
Manufacturing industry proportion	1.041	0.936	1.153
Mining industry proportion	0.848*	0.769	0.938
Land vegetation condition change	0.666	0.223	1.941
Maximum daily temperature change	0.734*	0.596	0.907
Minimum daily temperature change	0.952	0.890	1.024
ρ	0.725*	0.614	0.828
Marginal variance of ($\boldsymbol{\varsigma}$)	0.122	0.089	0.172
Number of individuals	129,809		
Number of sub-districts	276		
DIC	88630		

Note: the symbol "*" presents a 95% significance level and a regression coefficient is statistically significant if its 95% credible interval does not contain zero. DIC indicates the deviance information criterion. Odds ratios (the exponentials of estimated regression coefficients) are reported in the table.

Table 4. Estimation results of the direct, indirect and total impacts of sub-district level covariates.

Variables	*Direct* impacts			*Indirect* impacts			*Total* impacts		
	Median	*2.5%*	*97.5%*	*Median*	*2.5%*	*97.5%*	*Median*	*2.5%*	*97.5%*
Poverty concentration	0.843	0.731	0.950	0.715	0.205	0.924	0.601	0.159	0.875
Mining industry proportion	0.818	0.717	0.926	0.665	0.189	0.892	0.545	0.145	0.824
Maximum daily temperature change	0.687	0.519	0.894	0.481	0.034	0.834	0.326	0.017	0.730

Note: Odds ratios are reported in the table. Reported estimates on the impacts of the sub-district level variables in the table are all statistically significant at the 95% credible interval. The direct, indirect and total impacts of individual-level covariates are all equal to their regression coefficients so that they are omitted in the table.

Input: Outcome variable Y, Level-1 and Level-2 covariates $\boldsymbol{x}$ and z, original contiguity-based spatial weights matrix $W^{(0)}$.
Output: Final estimates on W and $\boldsymbol{\theta}$, $\widetilde{W}$ and $\widetilde{\boldsymbol{\theta}}$.
Initialization:
Implement a standard multi-level logistic model with $\{Y, \boldsymbol{x}, \boldsymbol{z}\}$
    Obtain Level-2 random effects or residuals $\boldsymbol{\mu}^{(0)}$
    Update $W^{(0)}$ to $W^{(1)}$
    FOR each pair of bordered areas $(l \sim k)$ **DO**
    IF the 95% credible intervals of $u_l^{(0)}$ and $u_k^{(0)}$ do not overlap
        Set $w_{lk}^{(1)} = 0$
    ELSE
        Set $w_{lk}^{(1)} = 1$
    ENDIF
Store each updated W in a set $\Phi = \{W^{(0)}, W^{(1)}, \ldots\}$
WHILE $W^{(r)} \neq W^{(r-1)}$ and $W^{(r)} \notin \{W^{(1)}, \ldots, W^{(r-1)}\}$ **DO**
    Implement a *global spatial multi-level logistic model* with $W^{(r)}$
    Obtain Level-2 random effects or residuals $\boldsymbol{\mu}^{(r)}$
    Update $W^{(r)}$ to $W^{(r+1)}$
    FOR each pair of bordered areas $(l \sim k)$ **DO**
    IF the 95% credible intervals of $u_l^{(r)}$ and $u_k^{(r)}$ do not overlap
        Set $w_{lk}^{(r+1)} = 0$
    ELSE
        Set $w_{lk}^{(r+1)} = 1$
    ENDIF
ENDWHILE
Obtain the final estimate of W, $\widetilde{W}$ based on the convergence condition above
    IF $W^{(R)} = W^{(R-1)}$
        $\widetilde{W} = W^{(R)}$
    ELSE
        Find index s ($s < R$) such that $W^{(s)} = W^{(R)}$, and from $\{W^{(s)}, \ldots, W^{(R)}\}$, find $W^{(R^*)}$ that produces the smallest Moran's I values for Level-2 residuals, and assign $W^{(R^*)}$ to $\widetilde{W}$
    ENDIF
Implement the locally adaptive spatial multi-level logistic model with $\widetilde{W}$
Return $\widetilde{W}$ and $\widetilde{\boldsymbol{\theta}}$

Figure 1. The pseudo-code of the iterative algorithm to implement a locally adaptive spatial multi-level logistic model.

127x173mm (300 x 300 DPI)

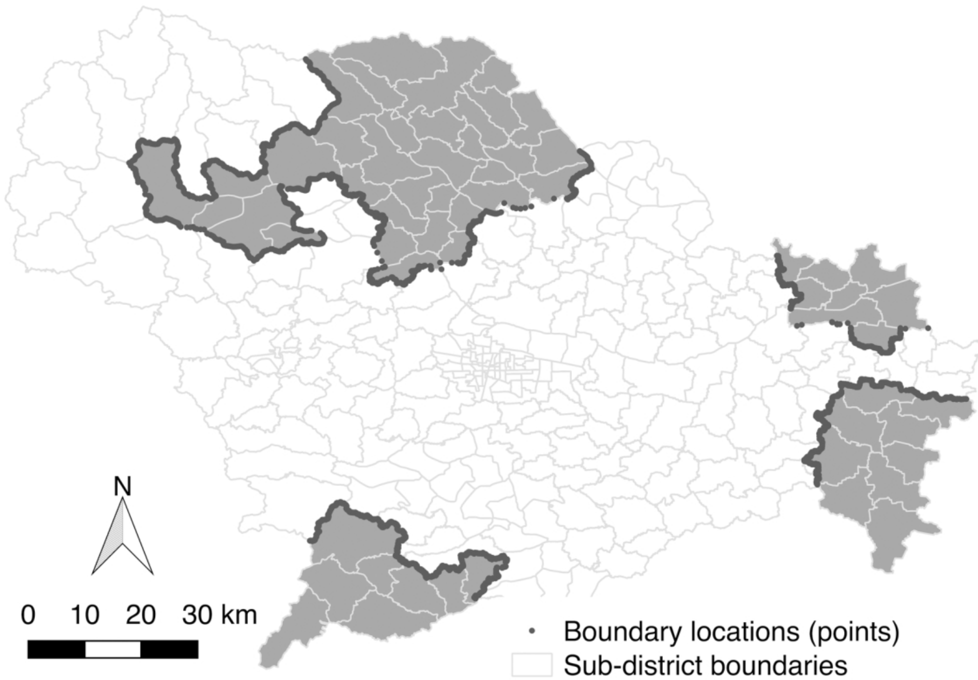

Figure 2. Locations of hypothetical boundaries in the sub-district level random effects.

148x102mm (300 x 300 DPI)

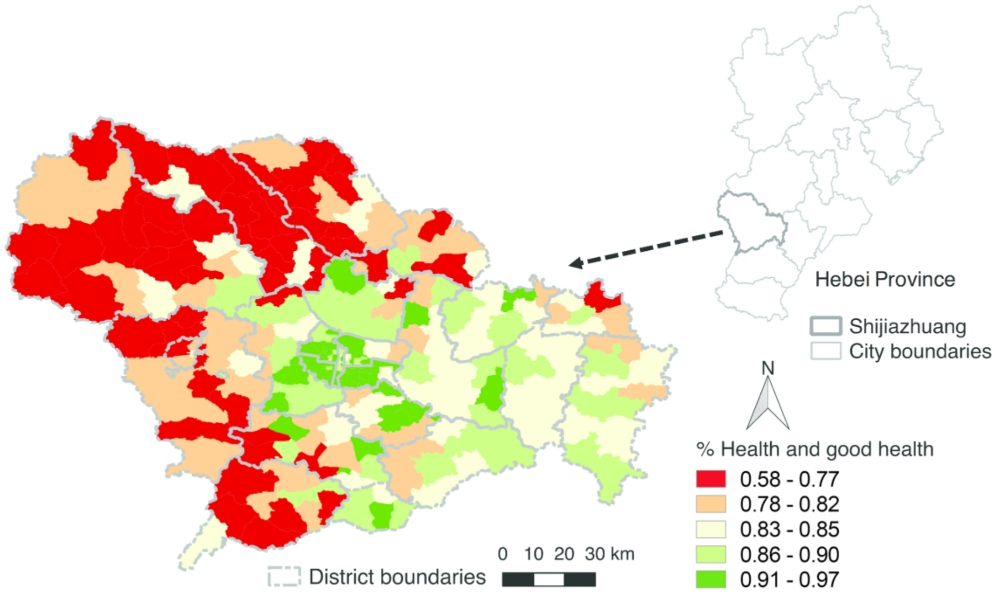

Figure 3. The study area of the capital city of Hebei province and the spatial distribution of self-rated health status, measured by percentages of good and very good health status reported by the elderly in 2010.

177x108mm (300 x 300 DPI)

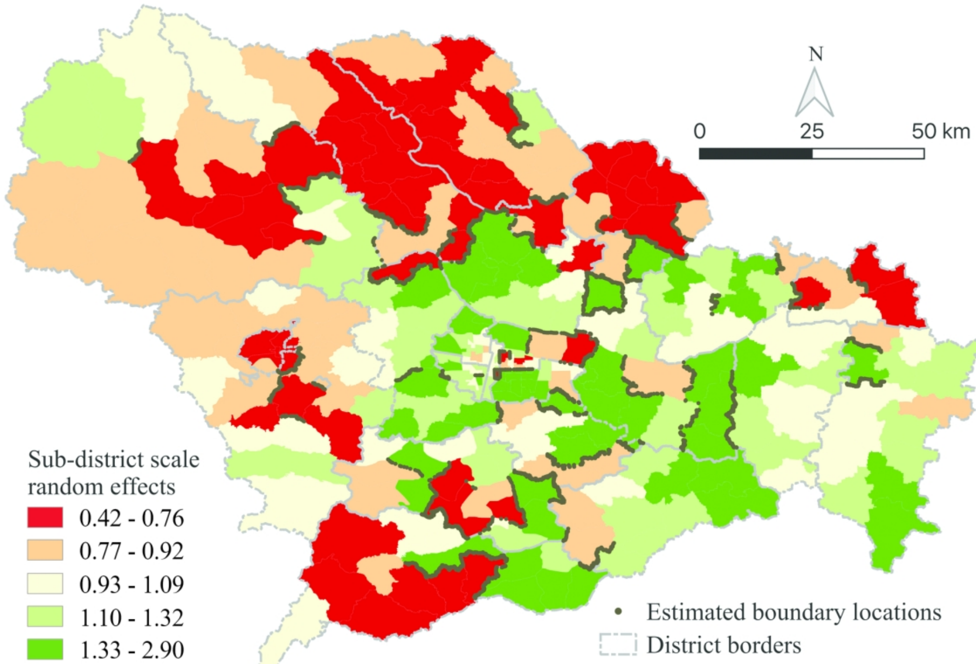

Figure 4. Estimated boundary locations of self-rated health in the study area, superimposed by the sub-district level random effects presented at the odds scale.

168x114mm (300 x 300 DPI)