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Nonlinear Attitude Control Design and Verification for a Safe Flight of a Small-Scale Unmanned Helicopter

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Abstract— **Autonomous small unmanned helicopter systems have been widely studied in the last decades. These systems are extremely agile due to their energy efficiency, overall costs and high levels of maneuverability compared to manned helicopters. This allows them to be used in urban environments for different applications such as search and rescue, aerial stunts for movie industry, fire fighting, surveillance, etc. Such applications require the control system to be robust and safe since a fault may lead to environmental damage and endangering human life. For reasons of the very high safety requirements, in this paper we propose a robust control design and also introduce formal verification of control for small-scale unmanned helicopters. The controller proposed is based on dynamic inversion control for a 3-DOF (degree-of-freedom) attitude dynamics while taking in to account the system modelling uncertainty with variable payloads and external disturbances of wind. An invariant set called control-enabled-set is defined for flight envelope, which represents the dynamical state vectors comprised of the attitude and rotation rates, for which the stable control of the craft is feasible with our control scheme. Then the controller is verified using formal methods represented by MetiTarski automated theorem prover to ensure controller stability and robustness. Our approach also paves the way to the possibility that the autopilot system monitors whether it is getting near the boundary of its flight envelope, in which case it can propose or plan and execute an emergency landing to a safe location.**

I. INTRODUCTION

Helicopters unmanned aerial vehicles (UAVs) are safety-critical systems which in some applications need to fly near buildings that required high maneuverability, accuracy, and fast response abilities. These systems are mostly autonomous which use in some critical applications such as surveillance, fire fighting, search and rescue, etc. Such applications require ensuring safe flight which is important as they are fly in urban areas hence human life may be at risk. Therefore, the controllers of these systems need to be formally verified then officially certified before flying by aviation authorities such as International Civil Aviation Organization (ICAO), Joint Authorities for Rulemaking on Unmanned Systems (JARUS), European Aviation Safety Agency (EASA) and member organisations such as the Civil Aviation Authority (CAA) in the UK.

Autonomous helicopter flight control has been widely studied in the last decades. Several controllers have considered the uncertainty and disturbances which are important aspects that affect aircraft stability and performance. However, maintaining attitude stability is still a major control problem due to the aerodynamic mechanism nonlinearity [1]. Previous works on helicopter UAVs control and verification have been reviewed and several are presented. Adaptive inverse dynamic control for an autonomous helicopter is proposed in [2] and [3]. In [4], attitude based model predictive control of an unmanned small helicopter is presented. Robust nonlinear control with considering wind disturbances is proposed in [5] and with H_∞ control in [6]. There are several projects working on verification of cyber-physical systems such as the European project: Integrated Tool Chain for Model-based Design of Cyber-Physical Systems (INTO-CPS) [7], Metamathematics for Systems Design (MMSD) [8], and several projects conducting in NASA Langley Research Center [9]. MetiTarski prover has been used in [10] to verify the stability of a flight controller using Nichols plots. Verifying the validity of Nichols plot analysis for two simple control systems using MetiTarski is proposed in [11]. Other works have been conducted by NASA Langley team on UAV verification in [12], [13] and [14].

In this paper, a robust nonlinear attitude controller is designed with considering the modelling uncertainty and external wind disturbances for an unmanned small helicopter system. The controller stability is demonstrated using Lyapunov direct method and an invariant set is defined with considering system constraints. The controller is then verified since the verification framework includes verifying that the control system is asymptotically stable and ensuring that the system states are within the defined control set using formal methods represented by MetiTarski [15] automated theorem prover (ATP). The aircraft control parameters are computed based on a VARIO Benzin Trainer helicopter [16].

Our motivation in this research is to work towards automating this verification framework and integrating MetiTarski with the autopilot system [17], since both are working on the same operating system (Linux), to perform an onboard real-time verification. The parameters required for the verification process can be passed from the autopilot to MetiTarski prover. The results produced by MetiTarski, proved or not proved, can be then passed back to the autopilot to check whether the aircraft is unstable or out of the designed constraints. Thereby, the autopilot could make the decision to cope with this, for example, avoiding any aggressive maneuvers or performing an emergency landing. By using this approach, the autopilot flight management system will be more trustworthy; i.e. if the vehicle becomes unstable or the controller specifications constraints are violated, the autopilot will either send warnings to the pilot, for semi-

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autonomous flight or do an emergency safe landing in case of full-autonomous flight. This verification method is general and can be applied to different kinds of autonomous UAVs that include autopilots such as multicopters and fixed wings crafts. To demonstrate our approach, we implemented the nonlinear dynamical model with the designed controller of the helicopter UAV in Simulink/Matlab and, as a first step towards the integration, we used simulation and MetiTarski to illustrate the possibility of implementing the framework with the autopilot system.

II. MATHEMATICAL MODEL

The small helicopter UAV is shown in Fig. 1. The helicopter aircraft is mainly controlled by four operating controls: the throttle T_M which determines the amount of thrust generated by the main motor P_M, the throttle T_R which determines the amount of side force produces by the tail motor P_R that required to rotate the aircraft (yaw), collective pitch for controlling the angles of main motor blades hence moving the aircraft up/down vertically, cyclic pitch for determining the flapping angles which are tilting the main rotor blades to move the aircraft forward/backward (pitch) or right/left (roll). Helicopter dynamics can be found with more details in [18], [19]. However, in this paper we will present only the attitude rotational control to illustrate our approach.

The helicopter three-dimensional attitude dynamics are represented in the body-fixed frame B by Euler-Lagrange rigid body rotational dynamics which described as follows

$$H(q)\ddot{q} + D(q,\dot{q})\dot{q} + w_d = \tau, \qquad (1)$$

where $q = [\psi(t)\ \theta(t)\ \phi(t)]^T \in \Re^3$ is the Euler angles vector with yaw, pitch and roll respectively. $\dot{q} \in \Re^3$ represents the Euler rates vector and $\ddot{q} \in \Re^3$ is the Euler acceleration vector. $w_d = [w_{d\psi}(t)\ w_{d\theta}(t)\ w_{d\phi}(t)]^T \in \Re^3$ is the external disturbances vector. $\tau = [\tau_\psi(t)\ \tau_\theta(t)\ \tau_\phi(t)]^T \in \Re^3$ is the torque vector. $D(q,\dot{q}) \in \Re^{3\times3}$ is the Coriolis matrix which the total matrix is shown in (equation 5.129, [20]). $H(q) \in \Re^{3\times3}$ is an invertible Jacobian symmetric positive-definite matrix

$$H(q) = \begin{bmatrix} J_x s_\theta^2 + J_y c_\theta^2 s_\phi^2 + J_z c_\theta^2 c_\phi^2 & J_y c_\theta s_\phi c_\phi - J_z c_\theta s_\phi c_\phi & -J_x s_\theta \\ J_y c_\theta s_\phi c_\phi - J_z c_\theta s_\phi c_\phi & J_y c_\phi^2 + J_z s_\phi^2 & 0 \\ -J_x s_\theta & 0 & J_x \end{bmatrix} \qquad (2)$$

where $J \in \Re^{3\times3}$ is the symmetric inertia matrix; s and c are *sin* and *cos* respectively. The relation between the Euler rates $\dot{q}$ and the vehicle angular velocities ω in B is represented as

$$\omega = \Lambda\ \dot{q}, \quad \begin{bmatrix} \omega_r \\ \omega_q \\ \omega_p \end{bmatrix} = \begin{bmatrix} -s\theta & 0 & 1 \\ c\theta s\phi & c\phi & 0 \\ c\theta c\phi & -s\phi & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}, \qquad (3)$$

where $\dot{q} = \Lambda^{-1}\omega$. The main motor P_M produces a vertical thrust T_M in Z_B axis and the tail motor P_R produces a lateral thrust T_R in Y_B axis. The total thrust vector of the main and tail motors are F_M and F_R respectively, where

$$F_M = \frac{|T_M|}{\sqrt{1 - s^2(a).s^2(b)}} \cdot \begin{bmatrix} -c(a).c(b) \\ c(a).s(b) \\ -s(a).c(b) \end{bmatrix}, \quad F_R = \begin{bmatrix} 0 \\ T_R \\ 0 \end{bmatrix}, \qquad (4)$$

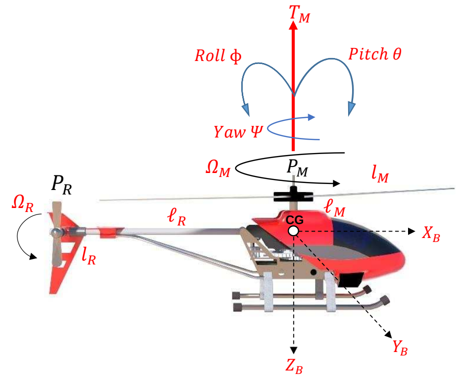

Fig. 1: Helicopter UAV configuration

since a and b are the longitudinal and lateral flapping angles respectively. The total torque vector τ in B-frame is

$$\begin{aligned} \tau &= \ell_M \times F_M + \ell_R \times F_R + E_M - E_R \\ &= \frac{|T_M|}{\sqrt{1 - s^2(a).s^2(b)}} \cdot \begin{bmatrix} \ell_M^y.s(a).c(b) + \ell_M^x.c(a).s(b) \\ \ell_M^x.c(a).c(b) - \ell_M^z.s(a).c(b) \\ -\ell_M^y.c(a).c(b) - \ell_M^z.c(a).s(b) \end{bmatrix} \\ &+ \begin{bmatrix} \ell_R^x T_R \\ 0 \\ -\ell_R^z T_R \end{bmatrix} + \begin{bmatrix} |\bar{E}_M| \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ |\bar{E}_R| \\ 0 \end{bmatrix}, \end{aligned} \qquad (5)$$

where the vectors ℓ_M and ℓ_R represent the length from the centre of mass of the helicopter to the hub of the main and tail motors respectively, E_M and E_R are vectors which represent the anti-torques that acting through the main and tail motors hubs which are generated from the aerodynamic drags on both motors. More details of the above derivations can be found in [20].

III. CONTROL SYSTEM DESIGN

The control system is designed based on the inverse dynamics control [21] with considering the system uncertainty and external disturbances. Controller stability is illustrated by Lyapunov second method. Considering the reference trajectory is q_{ref} and $\dot{q}_{ref}$, the error is defined

$$\hat{q} = q_{ref} - q \qquad (6)$$

$$\dot{\hat{q}} = \dot{q}_{ref} - \dot{q} \qquad (7)$$

$$\ddot{\hat{q}} = \ddot{q}_{ref} - \ddot{q}, \qquad (8)$$

where $\dot{q}_{ref}$ is computed such that

$$\dot{q}_{ref} = K_p\ q_{ref}, \qquad (9)$$

and $\ddot{q}_{ref}$ is the derivative of $\dot{q}_{ref}$. Considering the torque τ components are the control inputs, the nonlinear control law is chosen

$$\tau = \tilde{H}(q)u_c + u_a + \tilde{D}(q,\dot{q})\dot{q}, \qquad (10)$$

where $\tilde{H}(q)$ and $\tilde{D}(q,\dot{q})$ are the nominal matrices of $H(q)$ and $D(q,\dot{q})$ respectively. The control input u_c is chosen as

$$u_c = \ddot{q}_{ref} + K_d\dot{\hat{q}} + K_p\hat{q}, \qquad (11)$$

where $K_d = diag[K_{d1} \;\; K_{d2} \;\; K_{d3}] \in \Re^{3\times3}$ and $K_p = diag[K_{p1} \; K_{p2} \; K_{p3}] \in \Re^{3\times3}$ are positive-definite matrices. The auxiliary input u_a is dedicated to compensate the uncertainty and disturbances in (1) which will be chosen depending on the system stability. The following assumptions have been proposed to pursuit the robust control design:

1) As the helicopter actuators have limited rotational speed, the rotational Euler rates and acceleration can be upper bounded by positive constants $\alpha_1, \alpha_2 > 0$ such that

$$\|\dot{q}_{ref}\| \leq \alpha_1 \tag{12}$$

$$\|\ddot{q}_{ref}\| \leq \alpha_2. \tag{13}$$

2) The reference Euler angles are varying within limits such that

$$\|q_{ref}\| \leq \beta. \tag{14}$$

3) Due to the uncertainty in moments of the inertia matrix J, it is possible to set a lower and upper bound of the Jacobian matrix $H(q)$ such that

$$\|H^{-1}(q)\| \leq \gamma_1 \tag{15}$$

$$\|H^{-1}(q)\| \geq \gamma_2 \tag{16}$$

$$\|I - H^{-1}(q)\tilde{H}(q)\| \leq \gamma_3 \tag{17}$$

$$\|\tilde{H}(q)\| \leq \gamma_4, \tag{18}$$

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4 > 0$ and $I \in \Re^{3\times3}$ is an identity matrix.

4) From (12) and (13) where the Euler rates and acceleration are limited and using the assumptions in (15)-(18) for the inertia matrix uncertainty and setting $\hat{D}(q,\dot{q})\dot{q}$ as the difference between the actual $D(q,\dot{q})\dot{q}$ and nominal $\tilde{D}(q,\dot{q})\dot{q}$, the following constant bounds, $\lambda_1, \lambda_2 > 0$, are proposed

$$\|\tilde{D}(q,\dot{q})\dot{q}\| \leq \lambda_1 \tag{19}$$

$$\|\hat{D}(q,\dot{q})\dot{q}\| \leq \lambda_2. \tag{20}$$

5) The wind disturbance vector w_d is sufficiently smooth and an upper constant bound $\delta > 0$ is known such that

$$\|w_d\| \leq \delta, \tag{21}$$

where $\delta = \sup\|w(t)\|$; since $w(t)$ is the wind function that could violate the vehicle where its estimated superior value can be computed in practice.

The vehicle Euler acceleration is obtained from the dynamics in (1) and the control law in (10) as

$$\ddot{q} = H^{-1}(q)\tilde{H}(q)u_c + H^{-1}(q)[u_a + w_d + \hat{D}(q,\dot{q})\dot{q}], \tag{22}$$

and after few simplifications, we get

$$\begin{aligned} &\ddot{q} = H^{-1}(q)u_a + u_c - b, \\ \text{where} \quad &b = [I - H^{-1}(q)\tilde{H}(q)]u_c - H^{-1}(q)[w_d + \hat{D}(q,\dot{q})\dot{q}]. \end{aligned} \tag{23}$$

The error $\hat{q}$ in (8) becomes

$$\ddot{\hat{q}} = -K_p\hat{q} - K_d\dot{\hat{q}} - H^{-1}(q)u_a + b, \tag{24}$$

and in terms of the closed-loop dynamics, it can be written as

$$\begin{aligned} &\dot{\xi} = A\xi + \mathbb{I}[-H^{-1}(q)u_a + b], \\ \text{where} \quad &\xi = \begin{bmatrix}\hat{q} \\ \dot{\hat{q}}\end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix}, \quad \mathbb{I} = \begin{bmatrix} 0 \\ I \end{bmatrix}. \end{aligned} \tag{25}$$

Choosing the candidate Lyapunov function $L(\xi) > 0$ for $\forall \xi \neq 0$ as

$$L(\xi) = \xi^T M \xi, \tag{26}$$

where $L(0) = 0$ at the equilibrium point, $M \in \Re^{6\times6}$ and $Z \in \Re^{6\times6}$ are positive-definite matrices such that $-Z = A^T M + MA$. The rate of change of $L(\xi)$ with respect to the time is

$$\begin{aligned} \dot{L}(\xi) &= \dot{\xi}^T M \xi + \xi^T M \dot{\xi} \\ &= \xi^T [A^T M + MA]\xi + 2\xi^T M \mathbb{I}[-H^{-1}(q)u_a + b] \\ &= -\xi^T Z \xi + 2\eta^T[-H^{-1}(q)u_a + b], \end{aligned} \tag{27}$$

where $\dot{\xi}$ in (25) and $\eta = \mathbb{I}^T M \xi$. Equation (27) is needed to be strictly negative to ensure system stability. The first part of (27) is negative-definite while the second needs to be negative since it depends on the value of u_a. The auxiliary input u_a is defined as

$$u_a = \begin{cases} \nu(\xi,t)\|\eta\|^{-1}\eta & \|\eta\| \geq \rho \\ \nu(\xi,t)\rho^{-1}\eta & \|\eta\| < \rho \end{cases} \tag{28}$$

where $\nu(\xi,t)$ is a time varying scalar function to be defined later and ρ is a boundary that the error vary within. For $\|\eta\| \geq \rho$ and using (16), the second term $2\eta^T[-H^{-1}(q)u_a + b]$ in (27) is bounded such that

$$2\eta^T[-H^{-1}(q)u_a + b] \leq 2\|\eta\|[-\gamma_2\nu(\xi,t) + \|b\|]. \tag{29}$$

To ensure that (29) is negative hence stable control, $\nu(\xi,t)$ should be chosen such that the term $\gamma_2\nu(\xi,t)$ is semi-positive and greater than or equal to $\|b\|$. Thus, $\nu(\xi,t)$ is defined depending on the superior value of the vector b such that $\|b\| \leq \bar{b}$. From b in (23) and u_c in (11), we have

$$\begin{aligned} \|b\| \leq &\|I - H^{-1}(q)\tilde{H}(q)\|[\|\ddot{q}_{ref}\| + \|\mathbb{K}\|\|\xi\|] \\ &+ \|H^{-1}(q)\|[\|w_d\| + \|\hat{D}(q,\dot{q})\dot{q}\|], \end{aligned} \tag{30}$$

where $\mathbb{K} = [K_p \;\; K_d]^T \in \Re^{3\times6}$ and ξ in (25). Recalling the assumptions in (13)-(21), we get $\bar{b}$

$$\|b\| \leq \gamma_3[\alpha_2 + \|\mathbb{K}\|\|\xi\|] + \gamma_1[\delta + \lambda_2] := \bar{b}. \tag{31}$$

From (29) where the stability condition should be $\gamma_2\nu(\xi,t) \geq \bar{b}$ and using (31), the scalar function $\nu(\xi,t)$ is obtained

$$\nu(\xi,t) \geq \gamma_2^{-1}\bar{b} = \gamma_3\gamma_2^{-1}[\alpha_2 + \|\mathbb{K}\|\|\xi\|] + \gamma_1\gamma_2^{-1}[\delta + \lambda_2]. \tag{32}$$

Note that $\nu(\xi,t)$ is time dependent because it is relying on the error ξ which is vary with the time; since we set $\xi(t) \equiv \xi$ for reading clarity. Finally, the asymptotic stability is guaranteed since by substituting u_a in (28) (for $\|\eta\| \geq \rho$) in equation (27), we get

$$\dot{L}(\xi) = -\xi^T Z \xi + 2\eta^T[-H^{-1}(q)[\nu(\xi,t)\|\eta\|^{-1}\eta] + b] < 0, \tag{33}$$

and for $\|\eta\| < \rho$,

$$\dot{L}(\xi) = -\xi^T Z\xi + 2\eta^T[-H^{-1}(q)[\nu(\xi,t)\rho^{-1}\eta] + b] < 0. \quad (34)$$

The following section will illustrate determining a robust invariant set of the designed controller which will be used in the verification process to show that all the system trajectories will stay within this set; hence ensure controller stability and robustness.

IV. HANDLING OF CONSTRAINTS

This section defines the dynamical state vectors comprised of the attitude and rotation rates, for which the stable control of the craft is feasible with our control scheme with suitable chosen references of the guidance derivatives and under the constraints of the current state of attitude error and reference for the rotation rate and the current attitude itself. First, the state set is defined where a feasible control input exists under the *rpm* limitations of the motors of the helicopter. Then the state evolution within the set will be verified as illustrate in Section VI.

Definition 1: Let $x = [q\ \dot{q}]^T$, the helicopter rotational dynamics defined as

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \Lambda^{-1}\omega \\ H^{-1}(q)u_a + u_c - b \end{bmatrix}, \quad (35)$$

then a robust invariant set $\mathbf{S}(.) \subset \Re^6$ is called a *control enabled set*, if for any $x \in \mathbf{S}(.)$ at current time t_c there are continuous guidance functions $\dot{q}_{ref}$, $\ddot{q}_{ref}$ for any $t > t_c$, so that u_c in (11) is realisable by the motors of the vehicle under the constraints of the torque τ in (10) with considering the constraints of: first, the torque vector τ in (5) due to the limits of the thrusts, T_M and T_R and the flapping angles, a and b; second, the main motor angular velocity $0 < \Omega_M < \Omega_M^{max}$, and rear motor angular velocity $0 < \Omega_R < \Omega_R^{max}$; third, the assumptions in (12)-(21).

Control enabled set can be numerically computed for various values of their guidance parameters q_{ref} and $\dot{q}_{ref}$ with the constraints $\alpha_{1,2}$, $\gamma_{1,2,3,4}$, $\lambda_{1,2}$, $\kappa_{1,2}$, β and δ. Under the *rpm* and flapping angles constraints of (5), all possible vectors τ are in a convex set Ψ. The polytope Ψ is reduced due to the bounds of the constraints in (12)-(21) and transformed by feasible values of $\tilde{H}(q)$ and $\tilde{D}(q,\dot{q})\dot{q}$ to result in a polytope Ξ for the possible values of u_c. Then, for fixed $\dot{q}_{ref}$ and $\ddot{q}_{ref}$ the set of x for which the $u_a \in \Xi$ can be derived as by definition: $\hat{\dot{q}} = \dot{q}_{ref} - \dot{q}$.

Theorem 1: Assuming (33) and (34) are verified to be satisfied over a control enabled set $\mathbf{S}(.) \subset \Re^6$, then the state evolution of $x = [q\ \dot{q}]^T$ defined by $\ddot{q}$ in (23) remains in $\mathbf{S}(.)$ for any $\|\tilde{H}(q)\| \leq \gamma_4$, $\|\hat{D}(q,\dot{q})\dot{q}\| \leq \lambda_2$ and $\|w_d\| \leq \delta$, $t > t_c$, with the controller as defined by (10) with the constraints of τ in (5) and a suitable choice of adapted references $\dot{q}_{ref}$ and $\ddot{q}_{ref}$.

Proof: From the constraints of both motors $0 < \Omega_M < \Omega_M^{max}$ and $0 < \Omega_R < \Omega_R^{max}$ and (5), we can compute an upper limit of the torque τ such that

$$\|\tau\| \leq \tau_{max}. \quad (36)$$

Referring to the control law in (10), u_a in (28), and the assumptions in (12)-(21) and (36), we get

$$\begin{aligned} &\|\tau\| \leq \|\tilde{H}(q)\|\|u_c\| + \|u_a\| + \|\tilde{D}(q,\dot{q})\dot{q}\| \\ &\|\tilde{H}(q)\|\|u_c\| + \|u_a\| + \|\tilde{D}(q,\dot{q})\dot{q}\| \leq \tau_{max} \\ &\|u_c\| \leq [\tau_{max} - \|u_a\| - \|\tilde{D}(q,\dot{q})\dot{q}\|]/\|\tilde{H}(q)\| \\ &\qquad \leq [\tau_{max} - \nu(\xi,t) - \lambda_1]/\gamma_4, \end{aligned} \quad (37)$$

then taking (32), we have

$$-\nu(\xi,t) \leq -\gamma_3\gamma_2^{-1}[\alpha_2 + \|\mathbb{K}\|\|\xi\|] - \gamma_1\gamma_2^{-1}[\delta + \lambda_2]. \quad (38)$$

Recalling u_c definition in (11) and the assumption in (13),

$$\begin{aligned} -\|u_c\| &\geq -(\alpha_2 + \|K_d\|\|\hat{\dot{q}}\| + \|K_p\|\|\hat{q}\|) \\ &\geq -(\alpha_2 + \|\mathbb{K}\|\|\xi\|), \end{aligned} \quad (39)$$

then substituting (39) in (38), we get

$$-\nu(\xi,t) \leq -\gamma_3\gamma_2^{-1}\|u_c\| - \gamma_1\gamma_2^{-1}[\delta + \lambda_2]. \quad (40)$$

Substituting (40) in (37), the maximum control input $u_{c_{max}}$ is obtained

$$\begin{aligned} \|u_c\| &\leq [\tau_{max} - \gamma_3\gamma_2^{-1}\|u_c\| - \gamma_1\gamma_2^{-1}[\delta + \lambda_2] - \lambda_1]/\gamma_4 \\ &\leq [\tau_{max} - \gamma_1\gamma_2^{-1}[\delta + \lambda_2] - \lambda_1]/[\gamma_2^{-1}\gamma_3 + \gamma_4] := u_{c_{max}}. \end{aligned} \quad (41)$$

As the upper bound $u_{c_{max}}$ of the input u_c is now known, then from (11), (13) and (41) we have

$$\begin{aligned} &\|u_c\| \leq \|\ddot{q}_{ref}\| + \|K_d\|\|\hat{\dot{q}}\| + \|K_p\|\|\hat{q}\| \\ &\|\ddot{q}_{ref}\| + \|K_d\|\|\hat{\dot{q}}\| + \|K_p\|\|\hat{q}\| \leq u_{c_{max}} \\ &\alpha_2 + \kappa_1\|\hat{\dot{q}}\| + \kappa_2\|\hat{q}\| \leq u_{c_{max}}, \end{aligned} \quad (42)$$

where $\|K_d\| \leq \kappa_1$ and $\|K_p\| \leq \kappa_2$ with $\kappa_1, \kappa_2 > 0$. Recalling (6), (7), (12) and (14), we get

$$\kappa_2\|q\| + \kappa_1\|\dot{q}\| \leq u_{c_{max}} - \alpha_2 - \kappa_1\alpha_1 - \kappa_2\beta. \quad (43)$$

Finally, the control enabled set is obtained as

$$\mathbf{S}(.) = \{\forall [q\ \dot{q}]^T.\ \kappa_2\|q\| + \kappa_1\|\dot{q}\| \leq u_{c_{max}} - \alpha_2 - \kappa_1\alpha_1 - \kappa_2\beta\}. \quad (44)$$

■

The next section illustrates the application of the results in Simulink/Matlab.

V. SIMULATION

The nonlinear attitude dynamics in (1) and the designed controller are implemented in Simulink/Matlab for simulation and obtaining numerical parameters required for verification process; since the control inputs in τ and system states $q, \dot{q}$ with parameters are passed to the MetiTarski prover for verification. The simulation is based on a VARIO Benzin-Trainer unmanned helicopter. VARIO numerical parameters are shown in Table I(a). According to the maximum payload of the VARIO helicopter which is approximately $4kg$, the amount of variation of the inertia moments are computed. External disturbances are assumed to vary within a maximum 40% of the maximum torque. Table I(b) states the computed robust parameters of the controller. The controller results are shown in Fig. 2 since the tracking of reference Euler angles under disturbances are well performed by the controller. The next section illustrates the verification results.

TABLE I: Small helicopter UAV parameters and constraints

(a) Aircraft parameters

Parameter	***Value***
m	$7.5\ kg$
l_M	$1.8\ m$
l_R	$0.3\ m$
J_{xx}	$[0.02,\ 0.0307]\ k.gm^2$
J_{yy}	$[0.3,\ 0.46]\ k.gm^2$
J_{zz}	$[0.3,\ 0.46]\ k.gm^2$
ℓ^z_M	$-0.25\ m$
ℓ^x_R	$-0.75\ m$
ℓ^y_R	$-0.05\ m$
$\ell^x_M, \ell^y_M, \ell^z_R$	0
Ω^{max}_M	$132.9941\ rad/s$
Ω^{max}_R	$580.4989\ rad/s$
T^{max}_M	$135.7143\ N$
T^{max}_R	$2.4\ N$
$\bar{E}_M$	$0.02\lvert T_M\rvert$
$\bar{E}_R$	$0.02\lvert T_R\rvert$
a	$[-12°, 12°]$
b	$[-14°, 14°]$

(b) Control parameters

Parameter	***Value***
α_1	3.2657
α_2	2.1482
γ_1	60.8276
γ_2	39.0872
γ_3	0.5353
γ_4	0.46
λ_1	0.3337
λ_2	0.1161
β	3.4452
δ	4.2281
κ_1	0.135
κ_2	0.9
$u_{c_{max}}$	7.3385
τ_{max}	10.5703
K_{p1}	0.88
K_{p2}	0.8
K_{p3}	0.9
K_{d1}	0.0013
K_{d2}	0.12
K_{d3}	0.135
ρ	0.5

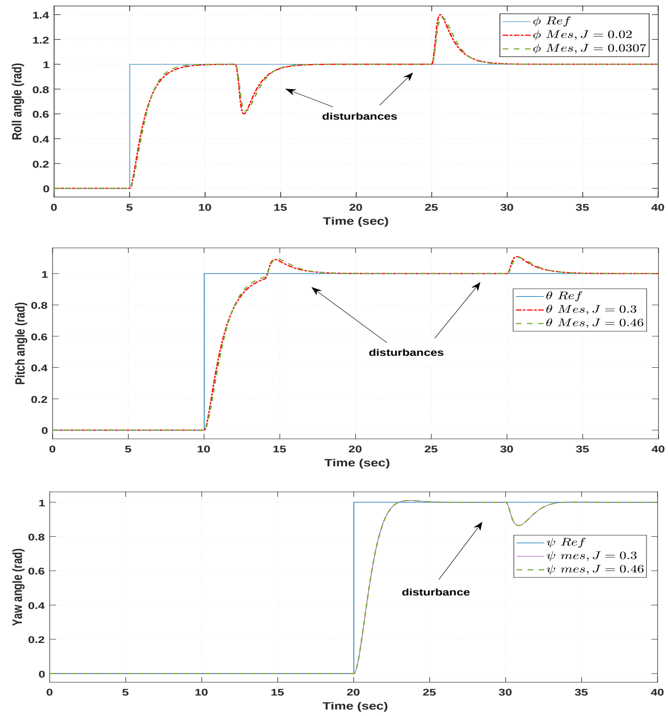

Fig. 2: Euler angles with disturbances

VI. CONTROL VERIFICATION

To ensure the validity of control scheme, the following verification objectives should be satisfied: 1) the controller produce torques which are with the maximum torques limits: $\|\tilde{H}(q)u_c + u_a + \tilde{D}(q,\dot{q})\dot{q}\| \leq \tau_{max}$; 2) the system is stable: $\dot{L}(\xi) < 0$ in (33) and (34); 3) all system states are vary and stay within the control enabled set: $\forall x = [q\ \ \dot{q}]^T.\ x \in \mathbf{S}(.)$ where $\mathbf{S}(.)$ is defined in (44).

Remark 1: If the translational control is designed then the limits of flapping angles a and b with thrusts T_M and T_R can be checked according to the produce control torques τ.

MetiTarski ATP is used as a verification tool to prove the above objectives. It is based on first-order logic (FOL) and designed to prove theorems on real numbers field with inequalities in particular for transcendental and some special functions such as *log*, *ln*, *exp*, *sin*, *cos*, *sqrt*. As the prover is limited to work on real scalar values, all vectors and matrices are simplified to scalar statements. Due to the space limit, we will illustrate examples of the proof while the complete code with proofs can be found in our web-repository[1]. However, the first objective is achieved by taking the torques produce from the controller in (10) and $\tau_{i_{max}}$ limits by (5) for each element (τ_i) then the objective inequality is formalized in FOL and proved as below: (note that this code is for $\tau_\psi \leq \tau_{\psi_{max}}$ only; see the web-repository for other codes)

```
fof(Torque_psi, conjecture, ![T_psi,TM,TR,A,B]:
%assumptions
(T_psi >= -0.0546 & T_psi <= 0.581 & TM <= 135.7143 & TR
<= 2.4 & A >= -0.2094 & A <= 0.2094 & B >= -0.2443 & B
<= 0.2443
%implies
=> T_psi <= (abs(TM)/sqrt(1-(sin(A)^2*sin(B)^2)))
   +(-0.75*TR)+(0.02*abs(TM)))).
```

[1] *https://github.com/uav-veri/helicopter*

where the notation fof refers to first-order logic, ! means $for\ all$, [ ] for variables, and the symbol $=>$ refers to *implies*. The second objective is achieved by formalizing equations (33), (34) and proving that they are strictly negative for all states under our control scheme. The following code illustrates a part of the stability implementation of (33).

```
fof(Stability_33, conjecture, ![V,Phi,Theta,Bb_1,Bb_2,Bb_3]
:?[Xi1,Xi2,Xi3,Xi4,Xi5,Xi6]:
%assumptions
(Xi1 !=0 & Xi1:(=0,1=) & Xi2 !=0 & Xi2:(=0,1=) & Xi3 !=0 &
Xi3:(=-0.0622,1=) & Xi4 !=0 & Xi4:(=-0.0897,0.88=) & Xi5
!=0 & Xi5:(=-0.0394,0.8=) & Xi6 !=0 & Xi6:(=-0.2419,0.9=)
& Phi:(=-1,1=) & Theta:(=-1,1=) & V:(=13.5797,13.6077=) &
Bb_1:(=-7.0404*10^(-18),7.5358*10^(-17)=) & Bb_2:(=-9.8665
*10^(-17), 1.0821*10^(-16)=) & Bb_3:(=-9.8665*10^(-17),
1.0821*10^(-16)=)
%implies
=> ...<0.
```

The code below shows formalising of the third objective which is proven by considering the upper and lower variation of the system states $q, \dot{q}$ is complying to the upper bound specified in (44).

```
fof(Helicopter_control_enabled_set, conjecture,
![Q1,Q2,Q3,Dot_Q1,Dot_Q2,Dot_Q3]:
% assumptions
(Q1 >= 0 & Q1 <= 1.0271 & Q2 >= 0 & Q2 <= 1 & Q3 >= 0
& Q3 <= 1 & Dot_Q1 >= -0.0735 & Dot_Q1 <= 0.6993 &
Dot_Q2 >= 0 & Dot_Q2 <= 0.5933 & Dot_Q3 >= -0.2798 &
Dot_Q3 <= 1.33
% implies
=> ((0.9*sqrt(Q1^2+Q2^2+Q3^2))+(0.135*sqrt(Dot_Q1^2+Dot_Q2
    ^2+Dot_Q3^2)) <= 1.6487))).
```

Our interaction approach between the simulation and the prover was useful since we resolved several unproved statements by retuning the parameters.

VII. DISCUSSION AND REMARKS

The proposed approach can be applied to different UAV systems as it is useful in two aspects: control design verification and onboard real-time validation. At the design stage, ensuring controller performance, robustness and stability are essential under physical limitations. This safety analysis cannot be achieved by simulation only as it relies on numerical computations as well as on co-simulation verification with symbolic computations. Regarding the control design, although many control schemes have been proposed in the literature, they are either taking into account uncertainty or disturbances but without considering both in a robust design, which are important factors together that affect control system performance. Therefore, we considered both factors in addition to taking into account dynamical actuator constraints based on practical parameters. For control verification, several attempts have been proposed to verify simple control systems such as in [22], [23], [24] and [25], and for hybrid verification systems as in [26]. These approaches have been developed based on interactive theorem provers, which need interaction with humans to complete proofs. Other approaches with the MetiTarski prover (mentioned in Sec. I) have been only used at the design stage. However, the remaining issue is how the autopilot knows whether the aircraft's dynamical envelope is violated by external forces such as gusts of wind. Therefore, we have proposed to integrate MetiTarski ATP with the autopilot system and to perform a real-time verification using our approach as presented. Based on our method, the autopilot can make decisions based on information from an onboard prover, which can send a warning to perform an emergency landing. This work is a first step towards this safety-integration, which will be useful to ensure a safe flight in the future.

VIII. CONCLUSION

A robust nonlinear attitude controller is presented for a small unmanned helicopter UAVs. Controller stability is demonstrated and verified using formal methods represented by MetiTarski ATP. The control system parameters constraints are computed and system states are verified to be vary within the defined invariant control set. A verification framework is proposed by merging the autopilot system with MetiTarski prover. The framework is demonstrated in simulation and MetiTarski outlined to illustrate its applicability. The framework is useful in particular when the vehicle is in a fully autonomous flight. If the controller performance is endangered by gusts of winds beyond its reaction abilities, then the autopilot could perform an emergency landing in a safe place.

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