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Distributed Combining Techniques for Distributed Detection in Fading Wireless Sensor Networks

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***Abstract*—We investigate distributed combining techniques for distributed detection in wireless sensor networks (WSNs) over Rayleigh fading multiple access channel (MAC). The MAC also suffers from with path loss and additive noise. The WSN is modelled as a Poisson point process (PPP). Two distributed transmit combining techniques are proposed to mitigate fading; distributed equal gain transmit combining (ddEGTC) and distributed maximum ratio transmit combining (dMRTC). The performance of the previous methods is analysed using stochastic geometry tools, where the mean and variance of the detector's test statistic are found thus enabling the fitting of the received signal distribution by a log-normal distribution. Surprisingly, simulation results show a that ddEGTC outperforms dMRTC.**

***Index Terms*—Wireless sensor networks, distributed detection, Rayleigh fading, multiple access channel, distributed transmit combining, stochastic geometry.**

I. Introduction

Wireless sensor networks (WSNs) are becoming a mainstream technology constituting the backbone of several emerging technologies, such as the internet of things (IoT) [1]. However, several WSNs aspects remain a fertile research ground, one of which the application of WSNs in distributed detection [2]. Having geographical distributed battery-powered sensor nodes (SNs) connected via wireless channels to a fusion center (FC) adds extra dimensions to the distributed detection problem. But wireless channel imperfections and SN power failure lead to having a random number of SNs involved in the distributed detection operation. In this case, it is unfeasible to assigns dedicated communication channels to all the SNs. One solution is to use a multiple access channel (MAC) in which the channel is shared among all the SNs.

Distributed detection over MAC was investigated in [3], in which the MAC is only affected by noise. The performance of distirbuted detection under Rayleigh fading MAC was investigated in [4] fortype-based communication. A further extension to the case of non-coherent fading channels was investigated in [5]. The authors in [6] then presented distributed detection in random WSNs, modelled by a Poisson point process (PPP) [7]. Distributed detection in random clustered WSNs over an ideal MAC was discussed in [8] and for a noisy MAC [9].

In this paper, we take a broader approach in which we investigate the case of a MAC suffering from Rayleigh fading, path loss, and additive noise. To mitigate fading, distributed transmit diversity combining is used. Fortunately, by virtue of the MAC, the received signal is aggregated at the FC facilitating the use of transmit diversity techniques. We propose two schemes; distributed equal gain transmit combining (dEGTC) and distributed maximum ratio transit combining (dMRTC) [10]. The PPP model for the WSN on the other hand, enables us to leverage stochastic geometry tools to find the mean and variance of the received signal at the FC. This, in turn, permits approximation of the received signal's distribution by a log-normal distribution, which enables finding the detector's probability of detection and probability of false alarm.

The paper is organized as follows. Section II formally introduces the system model. In Section III, distributed detection is discussed in addition to transmit diversity combining schemes and the statistics of the detector. Simulation results and discussion are provided in Section IV. Finally, the paper is concluded with Section V.

II. System Model

Consider a WSN deployed randomly in a sensing field $\mathcal{F} \in \mathbb{R}^2$. The number of SNs assumed to be random due to power or communication failures. Such a network is elegantly modelled by a PPP $\Phi = \{\mathbf{X}_i\}$ with mean λ, where $\mathbf{X}_i \in \mathcal{F}$ is the location of the ith SN. All the SNs report to a FC located at $\mathbf{x}_0 = (0, 0)$, without loss of generality, over flat fading channels. The channel between the ith SN and FC is $H_i = |H_i|e^{j\theta_i}$ where $|H_i|$'s are assumed to be i.i.d. Rayleigh random variables (RVs) with parameter σ_H^2 and θ_i's are i.i.d. uniform RVs in the interval $[0, 2\pi]$. The SNs estimate the channels with the aid of a pilot signal sent by the FC in the network initialization stage. Note, however, that the channels are known to the SNs but not to the FC.

The WSN is tasked with the detection of any intruders entering the sensing field. Such an intruder has a power of P_t located at $\mathbf{x}_t \in \mathcal{F}$. Each SN samples the sensing field thus acquiring the signal [11]:

$$\mathcal{H}_0 : S_i = V_i \tag{1}$$

$$\mathcal{H}_1 : S_i = \frac{\sqrt{P_t}}{\|\mathbf{X}_i - \mathbf{x}_t\|^{\eta}} + V_i \tag{2}$$

where $\mathcal{H}_0, \mathcal{H}_1, \eta$, and V_i are the null hypothesis, alternative hypothesis, sensing path-loss exponent, and the sensing noise respectively. Where the latter is normally distributed with variance σ_s^2, i.e., $V_i \sim \mathcal{N}(0, \sigma_s^2)$ and sensing SNR here is defined as $\text{SNR}_s = P_t/\sigma_s^2$. The ith SN local decision, $I(\mathbf{X}_i)$, is positive (1) if $S_i \geq \gamma$, where γ is the local threshold. It is negative (0) if $S_i < \gamma$. Consequently, the local detection probability is

$$\begin{aligned} P_d(\mathbf{x}, \mathbf{x}_t) &= \mathbb{P}\{I(\mathbf{X}_i) = 1; \mathcal{H}_1\} \\ &= Q\left(\frac{\gamma}{\sigma_s} - \frac{\sqrt{P_t}}{\sigma_s \|\mathbf{x} - \mathbf{x}_t\|^{\eta}}\right) \end{aligned} \tag{3}$$

whereas the local false alarm probability is

$$P_{fa} = \mathbb{P}\{I(\mathbf{X}_i) = 1; \mathcal{H}_0\} = Q\left(\frac{\gamma}{\sigma_s}\right). \tag{4}$$

The SNs use on-off-keying (OOK) to send their decisions to the FC over a shared MAC. Moreover, the SNs employ transmit diversity schemes via pre-multiplying the transmitted signal with G_i. The received signal is:

$$Z = Y + W \tag{5}$$

$$Y = \sum_{\mathbf{X}_i \in \Phi} \frac{\sqrt{P_{tx}} H_i G_i}{\|\mathbf{X}_i - \mathbf{x}_0\|^{\frac{\alpha}{2}}} I(\mathbf{X}_i) \tag{6}$$

where P_{tx} is the SN transmission power, α is communication path-loss exponent, and W is the MAC's circular AWGN with zero mean with variance σ_c^2.

III. Distributed Detection over Rayleigh Multiple Access Channel

A. Distributed Transmit Diversity Combining

Transmit combining schemes can be realized in a distributed manner by virtue of the shared MAC, since all the transmitted signals are combined at the FC as shown in (6). Distributed maximum ratio transmit combining (dMRTC) is implemented if $H_i = G_i^*$, whereas the distributed equal gain transmit combining (dEGTC) is implemented if $H_i = e^{-j\theta_i}$. In order to represent all cases, define $f(G_i) = H_i G_i$, which is $|H_i|^2$ in the case of dMRTC and H_i in the case of dEGTC.

B. Statistics of the Received Signal

The noiseless received signal Y in (6) is actually a random sum over the point process of detecting SNs. Unfortunately, its distribution does not have a closed-form. Nonetheless, the mean and variance of Y can be found via stochastic geometry tools. Firstly, the mean is given below as

$$\begin{aligned} \mu_j &= \mathbb{E}[Y; \mathcal{H}_j] = \mathbb{E}\left[\sqrt{P_{tx}} \sum_{\mathbf{X}_i \in \Phi} \frac{f(G_i)}{\|\mathbf{X}_i\|^{\frac{\alpha}{2}}} I(\mathbf{X}_i); \mathcal{H}_j\right] \\ &= \sqrt{P_{tx}} \mathbb{E}[f(G)] \mathbb{E}_{\Phi}\left[\sum_{\mathbf{X}_i \in \Phi} \frac{1}{\|\mathbf{X}_i\|^{\frac{\alpha}{2}}} I(\mathbf{X}_i); \mathcal{H}_j\right] \end{aligned} \tag{7}$$

where $\mathbb{E}_{\Phi}[\cdot]$ is the expectation with respect to Φ and $j = 0, 1$ denotes the $\mathcal{H}_0$ and $\mathcal{H}_1$ hypotheses. The mean can be further simplified as given in the following proposition.

Proposition 1: The mean of Y defined in (6) is given by

$$\mu_j = \begin{cases} \lambda\sqrt{P_{tx}} \mathbb{E}[f(G)] I_{\mu_0} & , j = 0 \\ \lambda\sqrt{P_{tx}} \mathbb{E}[f(G)] I_{\mu_1} & , j = 1 \end{cases}. \tag{8}$$

where

$$I_{\mu_0} = \int_{\mathcal{F}} \|\mathbf{x}\|^{-\frac{\alpha}{2}} P_{fa} d\mathbf{x} \tag{9}$$

$$I_{\mu_1} = \int_{\mathcal{F}} \|\mathbf{x}\|^{-\frac{\alpha}{2}} P_d(\mathbf{x}, \mathbf{x}_t) d\mathbf{x}. \tag{10}$$

Proof: Realizing that the local detection is actually is a thinning of the PPP, then Campbell's theory [7] can be applied to find the average of the expectation in (7) yielding the result in (8). ■

The variance, on the other hand, is not as straightforward. The following proposition provides the variance.

Proposition 2: The variance of Y defined in (6) is given by

$$\begin{aligned} \sigma_j^2 &= \text{var}(Y; \mathcal{H}_j) \\ &= \begin{cases} \lambda P_{tx} \mathbb{E}\left[f^2(G)\right] I_{\sigma_0^2} & , j = 0 \\ \lambda P_{tx} \mathbb{E}\left[f^2(G)\right] I_{\sigma_1^2} & , j = 1 \end{cases}. \end{aligned} \tag{11}$$

where

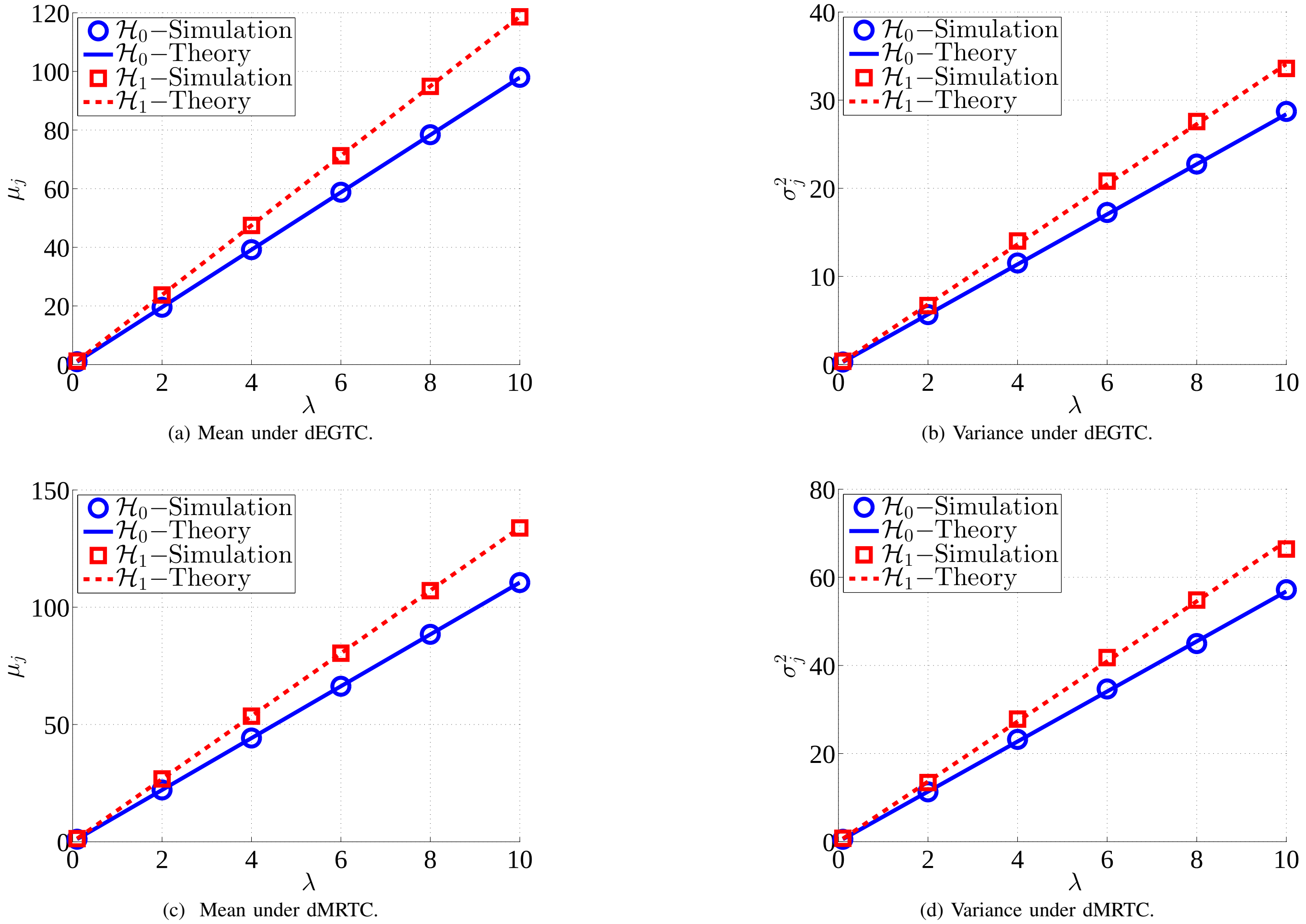
$$I_{\sigma_0^2} = \int_{\mathcal{F}} \|\mathbf{x}\|^{-\alpha} P_{fa} d\mathbf{x} \tag{12}$$

$$I_{\sigma_1^2} = \int_{\mathcal{F}} \|\mathbf{x}\|^{-\alpha} P_d(\mathbf{x}, \mathbf{x}_t) d\mathbf{x} \tag{13}$$

Proof: See Appendix A. ■

Having found the mean and variance, it is possible to approximate the distribution of Y via the traditional moment matching method. However, the choice of the approximating distribution is not straightforward. One should note that Y's distribution is skewed due to being based on Rayleigh random variables and rare event detection. So the lognormal distribution is an appropriate candidate due to its flexible shape. The approximate distribution is given by

$$f_Y(y) = \frac{1}{y\sigma_{a,j}\sqrt{2\pi}} \exp\left(-\frac{(\log y - \mu_{a,j})^2}{2\sigma_{a,j}^2}\right) \tag{14}$$



(a) Mean under dEGTC.

(b) Variance under dEGTC.

(c) Mean under dMRTC.

(d) Variance under dMRTC.

Fig. 1: Mean (μ_j) and variance (σ_j^2) of Y in (6) for dEGTC and dMRTC, with $P_{tx} = 10$.

where the arithmetic mean and variance are

$$\mu_{a,j} = \log\left(\frac{\mu_j^2}{\sqrt{\sigma_j^2+\mu_j^2}}\right) \quad (15)$$

$$\sigma_{a,j}^2 = \log\left(1+\frac{\sigma_j^2}{\mu_j^2}\right). \quad (16)$$

C. Distributed Detection

The FC reaches its global decision on the target's presence by comparing the received signal with a global detection threshold, Γ. One way to suppress the MAC channel noise is to increase the received signal SNR at the FC. If the SNR is chosen appropriately under $\mathcal{H}_0$ then the Z's distribution is guaranteed to be lognormal under both hypotheses ((1) and (2)). The SNR at the FC under $\mathcal{H}_0$ is

$$\mathrm{SNR}_c = \frac{\lambda P_{tx}}{\sigma_c^2}\mathbb{E}\left[f^2(G)\right] I_{\sigma_0^2}. \quad (17)$$

Hence, the SNR can be arbitrarily large by appropriately choosing λ and P_{tx}. Now assuming negligible AWGN and equipped with the lognormal distribution in (14) the global detection performance can be readily found. The global probability of false alarm is

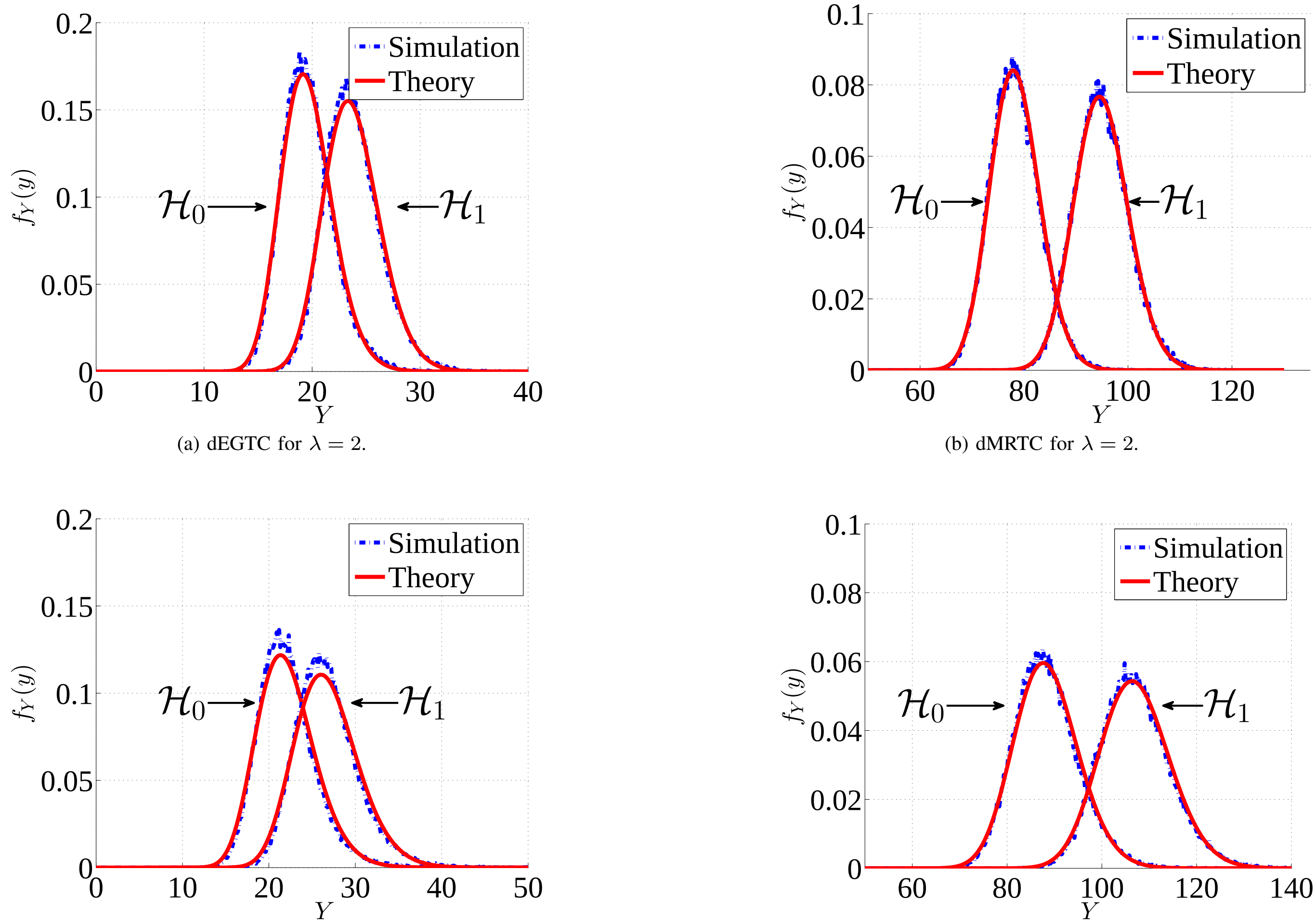
$$P_{FA} = \mathbb{P}(Z > \Gamma; \mathcal{H}_0) = Q\left(\frac{\log\Gamma - \mu_{a,0}}{\sigma_{a,0}}\right) \quad (18)$$

Consequently, given P_{FA} the global detection threshold can be found as $\log\Gamma = \sigma_{a,0}Q^{-1}(P_{FA}) + \mu_{a,0}$. On the other hand, the global probability of detection is

$$P_D = \mathbb{P}(Z > \Gamma; \mathcal{H}_1) = Q\left(\frac{\log\Gamma - \mu_{a,1}}{\sigma_{a,1}}\right) = Q\left(\frac{\mu_{a,0} - \mu_{a,1} + \sigma_{a,0}Q^{-1}(P_{FA})}{\sigma_{a,1}}\right). \quad (19)$$

Unfortunately, (19) does not provide an insight into the performance of the detector due to the complications in (15) and (16) w.r.t λ and P_{tx}. Therefore, we choose to investigate the deflection coefficient [12] in terms of the means and variances given by propositions 1 and 2, which is

$$d^2 = \frac{(\mu_1 - \mu_0)^2}{\sigma_1^2} = \lambda g_{tc}\frac{(I_{\mu_1} - I_{\mu_0})^2}{I_{\sigma_1^2}}. \quad (20)$$



(a) dEGTC for $\lambda = 2$. (b) dMRTC for $\lambda = 2$. (c) dEGTC for $\lambda = 10$. (d) dMRTC for $\lambda = 10$.

Fig. 2: Empirical distribution of Y in (6) and the theoretical Lognormal approximation under $\mathcal{H}_0$ and $\mathcal{H}_1$ at $P_{tx} = 10$ and $\lambda = 10$.

where $g_{tc} = \mathbb{E}^2\left[f(G)\right]/\mathbb{E}\left[f^2(G)\right]$ is the transmit combining gain. Interestingly, d^2 depends on λ, local detection through I_{μ_1}, and the transmit diversity. But does not depend on the transmission power. Note that it can be shown that for dMRTC $g_{tc} = 1/2$ whereas for dEGTC $g_{tc} = \pi/4$, so predict that dEGTC has better performance compared to dMRTC.

IV. Results and Discussion

We simulate a WSN in a field of 100×100. The intruder is arbitrarily located at $\mathbf{x}_t = (20, 20)$ with power of $P = 10$ and $\eta = 1$. The sensing SNR (SNR_s) is 5 dB. The local probability of false alarm (P_{fa}) is 0.01. The path loss exponent for the communication channel is $\alpha = 2$ whereas the channel gains are distributed as iid Rayleigh RV with parameter of $\sigma_G^2 = 1/\sqrt{2}$. The communication SNR is defined as $\text{SNR}_{c,0} = P_{tx}/\sigma_c^2$ where $\sigma_c^2 = 0.01$. The WSN is simulated for 10^5 Monte Carlo iterations.

Fig. 1 shows an almost perfect match between the simulated and theoretical mean and variance for both dMRTC and dEGTC under $\mathcal{H}_0$ and $\mathcal{H}_1$ for different values of λ. This shows the verifies the analytic expressions for mean and variance provided by propositions 1 and 2.

Fig. 2 shows how the lognormal distribution provides an excellent approximation for the distribution of pure MAC received signal, Y. The excellent match is due to shape properties of the lognormal distribution that can assume different skewness values, in contrast for the Gaussian distribution for example, which is always symmetrical.

Fig. 3 illustrates the theoretical and simulated ROC graphs for the dMRTC and dEGTC. A close resemblance is also observed here. Interestingly, the dEGTC performs better than the dMRTC as predicted by the deflection coefficient in (20). This trend is also observed in Fig. 4 where the probability one detection is achieved when λ is increased.

Fig. 5 shows the performance under ideal and Gaussian MACs with respect to the communication SNR. Interestingly, the ideal MAC is independent of the transmission power, as also predicted by d^2. It is also evident that the ideal MAC is the upper bound for the Gaussian MAC.

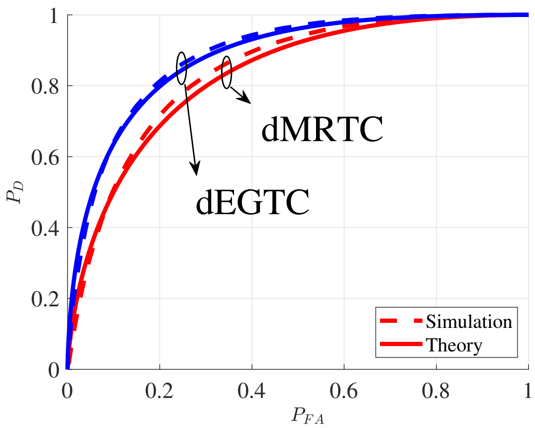


Fig. 3: ROC for $\lambda = 2$ and $P_{tx} = 10$.

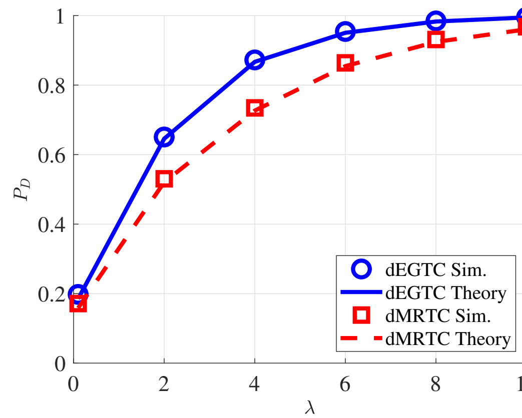


Fig. 4: P_D versus λ at $P_{tx} = 10$.

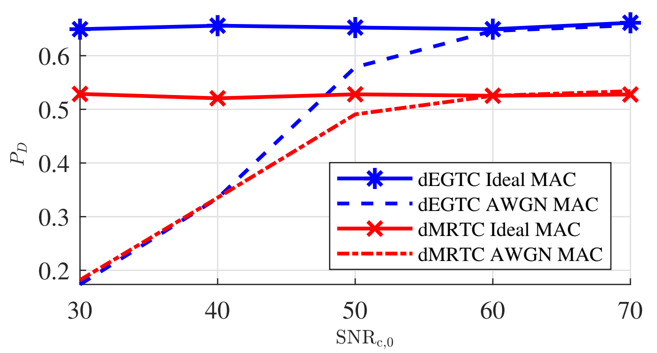


Fig. 5: P_D versus SNR_c for dEGTC and dMRTC under ideal MAC and Gaussian MAC plotted at $\lambda = 2$.

V. Conclusions

We have investigated distributed detection in WSNs over a shared MAC suffering from Rayleigh fading and additive noise. To mitigate the effect of the fading channel, distributed transmit combining methods are used, in particular dMRTC and dEGTC. To quantify the performance of both schemes, stochastic geometry tools were used to find the statistics of the detector's test statistics, which were, in turn, used to fit the distribution with a log-normal distribution. Interestingly, it has been shown that the dEGTC is better than the dMRTC in terms of the detector's performance.

Appendix
Proof of Proposition 2

Using the total variance identity the variances are

$$\begin{aligned}\sigma_j^2 &= \text{var}_\Phi\left(\mathbb{E}_G\left[\sqrt{P_{tx}}\sum_{\mathbf{X}_i\in\Phi}\frac{f(G_i)}{\|\mathbf{X}_i\|^{\frac{\alpha}{2}}}I(\mathbf{X}_i)\middle|\Phi;\mathcal{H}_j\right]\right)\\ &+ \mathbb{E}_\Phi\left[\text{var}_G\left(\sqrt{P_{tx}}\sum_{\mathbf{X}_i\in\Phi}\frac{f(G_i)}{\|\mathbf{X}_i\|^{\frac{\alpha}{2}}}I(\mathbf{X}_i)\right)\middle|\Phi;\mathcal{H}_j\right]\end{aligned} \quad (21)$$

where $\text{var}_\Phi(\cdot)$ is the variance w.r.t the PPP, Φ. Next,

$$\begin{aligned}\sigma_j^2 &= P_{tx}\mathbb{E}^2[f(G)]\,\text{var}_\Phi\left(\sum_{\mathbf{X}_i\in\Phi}\frac{1}{\|\mathbf{X}_i\|^{\frac{\alpha}{2}}}I(\mathbf{X}_i);\mathcal{H}_j\right)\\ &+ P_{tx}\text{var}(f(G))\,\mathbb{E}_\Phi\left[\sum_{\mathbf{X}_i\in\Phi}\frac{1}{\|\mathbf{X}_i\|^{\alpha}}I(\mathbf{X}_i);\mathcal{H}_j\right]\end{aligned} \quad (22)$$

From Campbell's theorem, for a given $f(\mathbf{x})$ we can write $\text{var}\left(\sum_{\mathbf{X}_i\in\Phi} f(\mathbf{x})\right) = \lambda\int f^2(x)d\mathbf{x} = \mathbb{E}\left[\sum_{\mathbf{X}_i\in\Phi} f^2(\mathbf{x})\right]$. Hence, by using the variance identity we get

$$\sigma_j^2 = \mathbb{E}\left[f^2(G)\right]P_{tx}\mathbb{E}_\Phi\left[\sum_{\mathbf{X}_i\in\Phi}\frac{1}{\|\mathbf{X}_i\|^{\alpha}}I(\mathbf{X}_i);\mathcal{H}_j\right]. \quad (23)$$

Finally, applying Campbell's theorem yields (11).

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