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Improvements on “Secure multi-party quantum summation based on quantum Fourier transform”

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1 **Abstract** Recently, a quantum multi-party summation protocol based on the
2 quantum Fourier transform has been proposed [Quantum Inf Process 17: 129,
3 2018]. The protocol claims to be secure against both outside and participant
4 attacks. However, a closer look reveals that the player in charge of generating
5 the required multi-partite entangled states can launch two kinds of attacks to
6 learn about other parties’ private integer strings without being caught. In this
7 paper, we present these attacks, and propose countermeasures to make the
8 protocol secure again. The improved protocol not only can resist these attacks
9 but also remove the need for the quantum Fourier transform and encoding
10 quantum operations by participants.

11 **Keywords** Quantum summation · Quantum Fourier transform · Participant
12 attacks

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1 Introduction

Since the advent of quantum key distribution (QKD) by Bennett and Brassard [1], a variety of other secure quantum communication protocols has emerged. This includes novel techniques for quantum secure direct communication [2], quantum secret sharing [3], secure multi-party quantum computation [4, 5, 6, 7, 8], remote state preparation [9, 10], which will be part of the growing number of applications that can be run on quantum networks [11, 12]. Of particular interest is the secure multi-party quantum summation [13, 14, 15, 16], as a fundamental primitive for secure multi-party quantum computation [4, 5, 6, 7, 8], whose goals are the protection of participants' input privacy and the guarantee of the correctness of the summation result. Quantum summation has potential applications in quantum anonymous ranking [17], quantum private comparison [18], and quantum anonymous voting [19]. Ji et al. [20] have discussed how to implement these applications by using quantum summation protocols. A considerable number of quantum summation protocols have thus been proposed by using various quantum states, including single photons in two degrees of freedom [21], Bell states [22, 23], Greenberger-Horne-Zeilinger (GHZ) states [24], genuinely maximally entangled six-qubit states [25], and multi-partite high-dimensional entangled states [16, 20, 26, 27, 28].

Lately, Yang et al. [28] have proposed a quantum summation protocol, which we refer to by YY2018's protocol, hereafter, based on the quantum Fourier transform. It has been claimed that YY2018's protocol is secure against outside and participant attacks. In this work, we show that, although P_1 , who prepares the d -level n -component entangled states, cannot collude with other dishonest participants, she can launch attacks to learn about other participants' private integer strings without being detected. We present two kinds of such attacks and propose an improved protocol that renders these attacks invalid. In addition, our new protocol removes the requirement for the quantum Fourier transform and encoding quantum operations to be performed by participants [28].

Note that Shi et al. [26] also proposed a quantum summation protocol earlier based on the quantum Fourier transform. However, Shi et al.'s and YY2018's protocols are quite different. Shi et al. protocol adopts the circle-type transmission mode while YY2018's protocol adopts the tree-type transmission mode [28]. In addition, Shi et al. employ controlled NOT (CNOT) gates and oracle operators, which are not required in Yang et al.'s work. In Shi et al.'s protocol I, as the authors mention, even though the initiators private input (P_1 's private input) is unconditionally secure against other dishonest parties, the dishonest parties P_{k-1} and P_{k+1} ($k \neq 1$) can collaborate to steal P_k 's private input. To overcome this weakness, they design their improved protocol III in their paper, which is secure against the collusive attacks from any less than $n - 1$ dishonest parties.

The rest of this paper is organized as follows. In Section 2, we review YY2018's protocol and give a brief analysis of its security. In Section 3, we present in detail the two attacks that can be attempted by the first player, P_1 .

In Section 4, we discuss how to detect P_1 's attacks and propose an improved protocol, followed by its correctness and security analysis. Conclusions are given in Section 5.

2 The YY2018's protocol

Let us begin with a review of YY2018's protocol [28]. In their protocol, P_1 is assumed not to conspire with other participants and the quantum channels are assumed to be ideal without any noise.

In a group of n players, suppose player P_i , $i = 1, 2, \dots, n$ and $n > 2$, has a private integer string K_i of length N in the following form.

$$K_i = (k_i^1, k_i^2, \dots, k_i^N), \quad (1)$$

where $k_i^t \in \{0, 1, \dots, d-1\}$, for $t = 1, 2, \dots, N$. P_1, P_2, \dots, P_n intend to jointly compute the summation of their private integer strings, K , given by

$$\begin{aligned} K &= K_1 \oplus K_2 \oplus \dots \oplus K_n \\ &= (k_1^1 \oplus k_2^1 \oplus \dots \oplus k_n^1, k_1^2 \oplus k_2^2 \oplus \dots \oplus k_n^2, \dots, k_1^N \oplus k_2^N \oplus \dots \oplus k_n^N), \end{aligned} \quad (2)$$

where \oplus denotes addition modulo d .

YY2018's protocol can be described as follows.

(Step 1) P_1 prepares N d -level n -component entangled states, each of which is in the state $\frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 |r\rangle_2 \dots |r\rangle_n$, and then arranges them into an ordered sequence

$$\left[\frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1^1 |r\rangle_2^1 \dots |r\rangle_n^1, \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1^2 |r\rangle_2^2 \dots |r\rangle_n^2, \dots, \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1^N |r\rangle_2^N \dots |r\rangle_n^N \right], \quad (3)$$

where the superscripts $1, 2, \dots, N$ present the order of d -level n -component entangled states in the sequence. Later, P_1 picks up the i -th, $i = 1, 2, \dots, n$, component from each state to construct n sequences labelled as:

$$\begin{aligned} S_1 &= (p_1^1, p_1^2, \dots, p_1^N), \\ S_2 &= (p_2^1, p_2^2, \dots, p_2^N), \\ &\vdots \\ S_n &= (p_n^1, p_n^2, \dots, p_n^N), \end{aligned} \quad (4)$$

where p_i^t , $t = 1, 2, \dots, N$, denotes the i -th component of the t -th entangled state. Next, P_1 prepares $n-1$ groups of decoy photons and each photon is randomly chosen from $\{|r\rangle\}_{r=0}^{d-1}$, i.e., the computational basis, or $\{F|r\rangle\}_{r=0}^{d-1}$, i.e., the Fourier basis, where F is the d -th order discrete quantum Fourier transform. P_1 randomly selects one group of the decoy photons and randomly inserts them into S_j , $j = 2, 3, \dots, n$, to form a new sequence S'_j . At last, P_1 keeps S_1 in her hand and sends S'_j to P_j .

- (Step 2) In this step, P_1 uses the decoy photons to check if the transmission of S'_j , $j = 2, 3, \dots, n$, is secure with P_j . If the error rate is higher than a predetermined threshold, they will terminate the protocol; otherwise, they will proceed to the next step.
- (Step 3) P_j , $j = 2, 3, \dots, n$, discards the decoy photons in S'_j and obtain S_j . P_j , $j = 1, 2, \dots, n$, encodes her private integer string K_j on the components in S_j by performing $U_{k_j^t} F$, $t = 1, 2, \dots, N$, on component p_j^t , where $U_{k_j^t} = \sum_{u=0}^{d-1} |u \oplus k_j^t\rangle\langle u|$ and \oplus represents addition modulo d . The new sequence of S_j after encoding is denoted by ES_j . P_1 also encodes her private integer string K_1 on the components in S_1 by performing $U_{k_1^t} F$ on component p_1^t . The new sequence of S_1 after encoded is denoted by ES_1 .
- (Step 4) After all parties have finished encoding their private integer strings, each of them measures all components in their hand in the computational basis $\{|r\rangle\}_{r=0}^{d-1}$ and obtains the corresponding measurement results as follows

$$\begin{aligned}
M_1 &= (m_1^1, m_1^2, \dots, m_1^N), \\
M_2 &= (m_2^1, m_2^2, \dots, m_2^N), \\
&\vdots \\
M_n &= (m_n^1, m_n^2, \dots, m_n^N),
\end{aligned} \tag{5}$$

where m_i^t , with $i = 1, 2, \dots, n$ and $t = 1, 2, \dots, N$, is the measurement result of component p_i^t after encoding. P_i , $i = 2, 3, \dots, n$, then announces M_i to P_1 . Finally, P_1 obtains the summation of all parties' private integer strings by computing $M_1 \oplus M_2 \oplus \dots \oplus M_n$, which she can then announce.

A quantum summation protocol should be secure against outside as well as participant attacks. In the YY2018's protocol, resilience against outside attacks is achieved by using decoy photons [29,30], which ensure that the quantum transmission between P_1 and other parties is secure. We find, however, that YY2018's protocol is not necessarily secure against participant attacks. In particular, P_1 has a chance to carry out effective attacks to achieve other participants' private inputs. Furthermore, we can see that all parties use $U_k F$ to encode their private integer strings. This is equivalent to the measurement in the Fourier basis and the addition modulo d of their measurement results and private integer strings. As a result, the quantum operation $U_k F$ can be replaced with the measurement in the Fourier basis and the simple addition modulo d operation. We describe these points in the following sections.

3 P_1 's attacks

In this section, we show how P_1 can initiate attacks to obtain other parties' private integer strings without being detected. As we mentioned in Section 2, YY2018's protocol in its current form is only able to detect the outside attacks, which makes P_1 's attacks possible due to her advantage over other parties. P_1 is able to perform different quantum operations on the states she has prepared,

although she cannot conspire with other parties. Here, we describe two kinds of such attacks that enables P_1 to steal other parties' private integer strings.

3.1 Attack 1

- (1) P_1 prepares N d -level n -component entangled states, with each of them being in the state $\frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 |r\rangle_2 \dots |r\rangle_n$, and arranges them into an ordered sequence as in equation (3). She then measures each component in the basis $\{|r\rangle\}_{r=0}^{d-1}$. The collapsed states after measurement become

$$[(|r^1\rangle_1, |r^1\rangle_2, \dots, |r^1\rangle_n), (|r^2\rangle_1, |r^2\rangle_2, \dots, |r^2\rangle_n), \dots, (|r^N\rangle_1, |r^N\rangle_2, \dots, |r^N\rangle_n)], \quad (6)$$

where $|r^t\rangle_i$, with $r^t \in \{0, 1, \dots, d-1\}$, $t = 1, 2, \dots, N$, and $i = 1, 2, \dots, n$, denotes the collapsed state of the i -th component in the t -th d -level n -component entangled state after measurement. After that, P_1 performs F^\dagger on each component in the sequence, which results in the following states

$$[(F^\dagger|r^1\rangle_1, F^\dagger|r^1\rangle_2, \dots, F^\dagger|r^1\rangle_n), (F^\dagger|r^2\rangle_1, F^\dagger|r^2\rangle_2, \dots, F^\dagger|r^2\rangle_n), \dots, (F^\dagger|r^N\rangle_1, F^\dagger|r^N\rangle_2, \dots, F^\dagger|r^N\rangle_n)], \quad (7)$$

where F is the quantum Fourier transform and $F^\dagger F = F F^\dagger = I$.

Later, P_1 rearranges the n sequences in the following way:

$$\begin{aligned} S_1 &= (F^\dagger|r^1\rangle_1, F^\dagger|r^2\rangle_1, \dots, F^\dagger|r^N\rangle_1), \\ S_2 &= (F^\dagger|r^1\rangle_2, F^\dagger|r^2\rangle_2, \dots, F^\dagger|r^N\rangle_2), \\ &\vdots \\ S_n &= (F^\dagger|r^1\rangle_n, F^\dagger|r^2\rangle_n, \dots, F^\dagger|r^N\rangle_n). \end{aligned} \quad (8)$$

P_1 then sends S_j , $j = 2, 3, \dots, n$, to P_j using the decoy-photon technique.

- (2) After confirming that S_j , $j = 2, 3, \dots, n$, has been securely received by P_j , P_j performs $U_{k_j^t} F$ on component $F^\dagger|r^t\rangle_j$, $t = 1, 2, \dots, N$, as in (Step 3) of the protocol. The corresponding encoded state becomes

$$(U_{k_j^t} F) F^\dagger|r^t\rangle_j = U_{k_j^t} F F^\dagger|r^t\rangle_j = U_{k_j^t}|r^t\rangle_j = |k_j^t \oplus r^t\rangle_j. \quad (9)$$

Next, P_j measures all components in the basis $\{|r\rangle\}_{r=0}^{d-1}$, and her measurement results will be

$$M_j = (m_j^1, m_j^2, \dots, m_j^N), \quad (10)$$

where $m_j^t = r^t \oplus k_j^t$. P_j then announces M_j to P_1 in (Step 4) of the protocol. Clearly, P_1 can easily extract k_j^t from m_j^t because she knows r^t exactly. P_1 can therefore obtain other parties' private integer strings without being caught.

3.2 Attack 2

In Attack 2, we consider, for simplicity, how P_1 can steal one integer of one party's (say P_j 's, $j \neq 1$) private integer string.

- (1) P_1 prepares one d -level n -component entangled state, in the state $\frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 |r\rangle_2 \dots |r\rangle_n$, and then performs the quantum Fourier transform on each of the last $n - 1$ components. The resulting state is given by

$$\frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 F|r\rangle_2 \dots F|r\rangle_n. \quad (11)$$

She then sends the first component of the above state to P_j using the decoy photon technique and holds the remaining $n - 1$ components in her hand.

- (2) After P_j performs $U_{k_j} F$ (assume that one integer of her private integer string is k_j) on the received components, the state turns into

$$\begin{aligned} & \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} U_{k_j} F|r\rangle_1 F|r\rangle_2 \dots F|r\rangle_n \\ &= d^{-\frac{n-1}{2}} \sum_{l_1 \oplus l_2 \oplus \dots \oplus l_n = 0} |l_1 \oplus k_j\rangle \otimes |l_2\rangle \otimes \dots \otimes |l_n\rangle, \end{aligned} \quad (12)$$

where \oplus denotes addition modulo d .

Once P_j measures her components and P_1 measures the remaining $(n - 1)$ ones in the basis $\{|r\rangle\}_{r=0}^{d-1}$, the measurement result will be

$$|l_1 \oplus k_j\rangle \otimes |l_2\rangle \otimes \dots \otimes |l_n\rangle. \quad (13)$$

Next, P_j announces $|l_1 \oplus k_j\rangle$ to P_1 . In the end, P_1 can achieve k_j by computing

$$\begin{aligned} & l_1 \oplus k_j \oplus l_2 \oplus \dots \oplus l_n \\ &= k_j \oplus l_1 \oplus l_2 \oplus \dots \oplus l_n = k_j. \end{aligned} \quad (14)$$

Because the original n sequences are constructed by P_1 , she could also adopt a similar method for obtaining other parties' private integer strings without being caught.

Note that, in Attack 2, stealing one integer of each party's private integer string costs one d -level n -component entangled state. In order to take all other parties' private strings, $N(n - 1)$ such entangled states are required, which is more expensive compared with Attack 1 where only N such entangled states are needed. Similarly, P_1 can employ other entangled states, such as d -level Bell states and d -level GHZ states, to get other parties' private integer strings.

4 The improved protocol

In the previous section, we showed how P_1 could succeed in obtaining other parties' private integer strings without being caught. This is mainly because of the lack of a detection mechanism, in YY2018's protocol, to check if the N states shared among n parties are genuinely d -level n -component entangled states in the form of $\frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 |r\rangle_2 \dots |r\rangle_n$. One solution would then be to add a detection step to the protocol before other parties encode their private integer strings on the components sent by P_1 .

Before presenting our method for detecting a dishonest P_1 , let us explore some of the properties of the main states used in YY2018's protocol. Let $r \in \{0, 1, \dots, d-1\}$ and $d \geq 2$. The set $\{|r\rangle\}_{r=0}^{d-1}$ forms a d -level computational basis. The d -th order quantum Fourier transform is defined as

$$F|r\rangle = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \zeta^{lr} |l\rangle, \quad (15)$$

where $\zeta = e^{2\pi i/d}$ and the set $\{F|r\rangle\}_{r=0}^{d-1}$ is a d -level Fourier basis. Expanding each $|r\rangle$, $r = 0, 1, \dots, d-1$, in the Fourier basis gives

$$|r\rangle = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \zeta^{-lr} F|l\rangle. \quad (16)$$

The d -level n -component entangled state, $|\omega\rangle_{12\dots n} = \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 |r\rangle_2 \dots |r\rangle_n$, can then be rewritten as

$$\begin{aligned} & |\omega\rangle_{12\dots n} \\ &= \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} |r\rangle_1 |r\rangle_2 \dots |r\rangle_n \\ &= \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} \left(\frac{1}{\sqrt{d}} \sum_{l_1=0}^{d-1} \zeta^{-l_1 r} F|l_1\rangle_1 \right) \left(\frac{1}{\sqrt{d}} \sum_{l_2=0}^{d-1} \zeta^{-l_2 r} F|l_2\rangle_2 \right) \dots \left(\frac{1}{\sqrt{d}} \sum_{l_n=0}^{d-1} \zeta^{-l_n r} F|l_n\rangle_n \right) \\ &= \left(\frac{1}{\sqrt{d}} \right)^{n+1} \sum_{r=0}^{d-1} \sum_{l_1, l_2, \dots, l_n} \zeta^{-r(l_1+l_2+\dots+l_n)} F|l_1\rangle_1 F|l_2\rangle_2 \dots F|l_n\rangle_n \\ &= \left(\frac{1}{\sqrt{d}} \right)^{n-1} \sum_{l_1 \oplus l_2 \oplus \dots \oplus l_n = 0} F|l_1\rangle_1 F|l_2\rangle_2 \dots F|l_n\rangle_n. \end{aligned} \quad (17)$$

The above state has an interesting property. If each component of $|\omega\rangle_{12\dots n}$ is measured in the d -level computational basis, then all the measurement results would be the same as, i.e., all subsystems will be in state $|r\rangle$, for a single $r \in \{0, 1, \dots, d-1\}$. If each component of $|\omega\rangle_{12\dots n}$ is measured in the d -level Fourier basis, however, then the outcome would be in the form $F|l_1\rangle_1 F|l_2\rangle_2 \dots F|l_n\rangle_n$, where $l_1 \oplus l_2 \oplus \dots \oplus l_n = 0$. We can, therefore, employ this property of $|\omega\rangle_{12\dots n}$ to check if it is shared properly among n parties.

We are now ready to explain our improved protocol, whose assumptions are the same as that of YY2018's protocol. The improved protocol is described in the following.

- (S 1) P_1 prepares $N+q$ d -level n -component entangled states, with q being a security parameter, each of which is in the state $|\omega\rangle_{12\dots n}$. She then chooses the i -th, $i = 1, 2, \dots, n$, component from each state to construct the following n sequences:

$$\begin{aligned} S_1 &= \left(p_1^1, p_1^2, \dots, p_1^{N+q} \right), \\ S_2 &= \left(p_2^1, p_2^2, \dots, p_2^{N+q} \right), \\ &\vdots \\ S_n &= \left(p_n^1, p_n^2, \dots, p_n^{N+q} \right), \end{aligned} \quad (18)$$

where p_i^t , $i = 1, 2, \dots, n$ and $t = 1, 2, \dots, N+q$, represents the i -th component of the t -th entangled state $|\omega\rangle_{12\dots n}$. Afterward, P_1 randomly inserts decoy photons into S_j , $j = 2, 3, \dots, n$, to form a new sequence, denoted by S'_j . Finally, P_1 sends S'_j to P_j and keeps S_1 in her hand.

- (S 2) Similarly to the (Step 2) of YY2018's protocol, once P_j ($j = 2, 3, \dots, n$) confirms that she has received S'_j , P_j and P_1 utilize the decoy photons to check if the transmission of S'_j is secure. If the error rate is greater than a predetermined threshold, they will terminate the protocol; otherwise, they will proceed to the next step.
- (S 3) In this step, P_j , $j = 2, 3, \dots, n$, will check if entangled states $|\omega\rangle_{12\dots n}$ are genuinely shared among n parties. By removing the decoy photons from S'_j , P_j obtains S_j . Next, P_2, P_3, \dots, P_n agree to randomly choose q entangled states $|\omega\rangle'_{12\dots n}$ from S_1, S_2, \dots, S_n . For each of the chosen states, they randomly ask P_1 to measure her component in the computational basis or in the Fourier basis and to announce her measurement result. Other parties also measure their corresponding components in the same basis as P_1 did. The measurement results, according to equation (17), are correlated if all components come from the same entangled state $|\omega\rangle_{12\dots n}$ and are measured in the same basis. They compute the error rate relying on these measurement results. If the error rate is higher than a predetermined threshold, they will terminate the protocol; otherwise, they will proceed to the next step.
- (S 4) By removing q entangled states used for detection, all parties share, with a high probability, N genuine d -level n -component entangled states $|\omega\rangle_{12\dots n}$. All parties measure all the components in their hand in the Fourier basis and get certain measurement results. By mapping $F|r\rangle$, as the output measurement state, to the integer r , for $r = 0, 1, \dots, d-1$, P_j ($j = 1, 2, \dots, n$) will obtain an integer string $L_j = (l_j^1, l_j^2, \dots, l_j^N)$, with $l_j^t \in \{0, 1, \dots, d-1\}$ and $t = 1, 2, \dots, N$. After that, P_j , $j = 2, 3, \dots, n$, computes $M_j = K_j \oplus L_j = (k_j^1 \oplus l_j^1, k_j^2 \oplus l_j^2, \dots, k_j^N \oplus l_j^N)$, and sends it to P_1 . P_1 also computes $M_1 = K_1 \oplus L_1$. Because $l_1^t \oplus l_2^t \oplus \dots \oplus l_n^t = 0$, in accordance with equation (17), P_1 can eventually achieve the summation of parties' private

integer strings by the following computation

$$\begin{aligned}
& M_1 \oplus M_2 \oplus \dots \oplus M_n \\
&= \left(\bigoplus_{j=1}^n k_j^1 \oplus l_j^1, \bigoplus_{j=1}^n k_j^2 \oplus l_j^2, \dots, \bigoplus_{j=1}^n k_j^N \oplus l_j^N \right) \\
&= \left(\bigoplus_{j=1}^n k_j^1 \oplus \bigoplus_{j=1}^n l_j^1, \bigoplus_{j=1}^n k_j^2 \oplus \bigoplus_{j=1}^n l_j^2, \dots, \bigoplus_{j=1}^n k_j^N \oplus \bigoplus_{j=1}^n l_j^N \right) \\
&= \left(\bigoplus_{j=1}^n k_j^1, \bigoplus_{j=1}^n k_j^2, \dots, \bigoplus_{j=1}^n k_j^N \right) \\
&= K_1 \oplus K_2 \oplus \dots \oplus K_n = K,
\end{aligned} \tag{19}$$

where $\bigoplus_{i=1}^n x_i = x_1 \oplus x_2 \oplus \dots \oplus x_n$. P_1 then announces the result.

4.1 Analysis

Let us move on to the analyses of our improved protocol's correctness and security.

We first analyze the correctness. Note that we also utilize d -level n -component entangled states $|\omega\rangle_{12\dots n}$ to design the improved protocol and that the state $F|r\rangle$ corresponds to the integer r , for $r = 0, 1, \dots, d - 1$. For each entangled state used for the summation of all parties' private integer strings, P_j , $j = 1, 2, \dots, n$, measures her component in the Fourier basis; if P_j obtains the measurement result $F|l_j^t\rangle$, $t = 1, 2, \dots, N$, she will get the corresponding integer l_j^t that has the relation $l_1^t \oplus l_2^t \oplus \dots \oplus l_n^t = 0$ according to equation (17). Thus, after receiving $M_j = K_j \oplus L_j = (k_j^1 \oplus l_j^1, k_j^2 \oplus l_j^2, \dots, k_j^N \oplus l_j^N)$, $j = 2, 3, \dots, n$, sent by P_j , P_1 can eventually obtain the correct result, as shown in equation (19).

For the security, because we also employ decoy photon technique to detect outside attacks, the improved protocol is still secure against all kinds of outside attacks. It also remains secure against attacks from $n - 2$ dishonest parties (excluding P_1 because she cannot conspire with any dishonest party). Suppose they want to steal P_1 's and P_i 's private integer strings, they have to learn about the exact values of l_1^t and l_i^t , $t = 1, 2, \dots, N$, that are used to encrypt P_1 's and P_i 's private integer strings, respectively. This, however, is impossible, because l_1^t and l_i^t are only known to P_1 and P_i , respectively. Consequently, the $(n - 2)$ -party collusive attack would not be effective in the improved protocol.

We focus now on P_1 's attacks. As we can see from equation (17), if the genuine entangled state $|\omega\rangle_{12\dots n}$ is shared in the same way as proposed in the improved protocol, P_1 is unable to attain other parties' private integer strings since she does not know the exact values of l_i^t , $i = 2, 3, \dots, n$ and $t = 1, 2, \dots, N$. She will try to construct special quantum states to steal other

parties' secret, as presented in Section 3. However, if P_1 launches such attacks, she will be caught in Step (S 3) of the improved protocol, as the correlations will be destroyed, according to equation (17).

Suppose that P_1 makes the similar Attack 1 to obtain some parties' private integer strings, she may prepare fake states as

$$(F|r\rangle_1, F|r\rangle_2, \dots, F|r\rangle_n), \quad (20)$$

where $r \in \{0, 1, \dots, d-1\}$. In order to pass the detection with high probability in Step (S 3) of the improved protocol, P_1 has to prepare the fake states as few as possible.

Let us consider the case where P_1 only wishes to attain one integer of each party's integer string. She can then prepare only one fake quantum state $(F|r\rangle_1, F|r\rangle_2, \dots, F|r\rangle_n)$ and $N + q - 1$ genuine entangled states $|\omega\rangle_{12\dots n}$. In this case, P_1 has the maximum possibility of passing the detection.

The probability of P_1 not being detected by our proposed mechanism can be calculated as follows. Note that the fake quantum state should not be chosen for detection so that P_1 can succeed in obtaining one integer of each party's integer string. Since P_2, P_3, \dots, P_n randomly choose q entangled states for detection, the probability that P_1 passes the detection and gets the private integer is

$$\binom{N+q-1}{q} / \binom{N+q}{q} = \frac{N}{N+q}, \quad (21)$$

which will approach to 0 when q is sufficiently large.

If the fake quantum state is chosen for detection, P_1 could still pass the detection with probability $(\frac{1}{2} + \frac{1}{2d})$. However, she cannot obtain any information about other parties' private integer strings, as she shares $|\omega\rangle_{12\dots n}$ among all the parties and the measurement results of other parties are unknown to her. The analysis of the attacks based on fake entangled states (Attack 2) is similar. These two kinds of attacks thus become ineffective in the improved protocol.

In fact, once the genuine entangled state $|\omega\rangle_{12\dots n}$ is shared among n parties, it would be impossible for any group of less than $n-1$ parties (including P_1) to obtain the remaining two parties' private inputs as their private keys cannot be exactly obtained. Note that in the improved protocol, there is no need for parties to encode their private integer strings. That has been replaced with the measurement in the Fourier basis and classical addition modulo d .

We would like to point out that two months after submission of this manuscript, a similar work, by Gu and Hwang, appeared on arXiv [32]. Let us compare Gu and Hwang's work with our work in terms of the attacks, the detection methods, and the improved protocols. In terms of attacks, Gu and Hwang's only attack is almost the same as our Attack 1. The only difference is in Step 1. If P_1 in our protocol can also initially prepare n single d -level states, with each of which from the basis $\{F|0\rangle, F|1\rangle, \dots, F|d-1\rangle\}$, instead of an n -component entangled state, then these two attacks are the same. In our case, we also present another attack, Attack 2, based on entangled states,

which is not discussed in Gu and Hwang's work. To solve the security problem, Gu and Hwang's work and our work both use the same method to check if the initial state $|\omega\rangle_{1,2,\dots,n}$ is correctly shared among participants. For the improved protocols, Gu and Hwang's and Yang et al.'s are the same except for the step added to detect P_1 's dishonesty. In our work, we changed YY2018's protocol a bit more. In our improved protocol, in addition to the new step for P_1 's dishonesty detection, we also remove the need for the quantum Fourier transform and encoding quantum operations by participants. Instead, noting that the operation $U_k F$ to encode private integer strings is equivalent to the measurements in the Fourier basis followed by addition modulo d of the measurement results, we offer a simpler technique to implement the protocol.

5 Conclusions

We showed that the protocol in [28] could not prevent P_1 , who prepares d -level n -component entangled states, from obtaining other parties' private integer strings. We analyzed in detail P_1 's two kinds of attacks and proposed an improved protocol that rectified such security threats. In addition, our improved protocol did not require quantum operations to encode parties' private integer strings.

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