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Simulation-based robust optimization of limited-stop bus service with vehicle overtaking and dynamics: A response surface methodology

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Abstract

We propose a robust optimization model for limited-stop bus service with vehicle overtaking and demand dynamics. Time-dependent stochastic travel time is also considered. The objective is to minimize the total cost (user cost and operation cost) at the planning phase given a target reliability imperative. We further propose a simulation-based optimization framework incorporating response surface methodology to solve the problem efficiently. A real-world application result shows that vehicle overtaking and demand dynamics have significant impacts on the performance of limited-stop service and that the stop patterns are quite distinct when overtaking and demand dynamics are considered.

Keywords: Public transport; Bus stop-skipping; Overtaking; Dynamics; Response surface methodology

1. Introduction

Transit operation in peak periods is characterized by spatially heterogeneous demand and scarcity of resources, which can be efficiently addressed by proper transit line alignment and fleet allocation (Yu et al., 2012a; Chen et al., 2018; Wu et al., 2019; Li et al., 2019). Limited-stop bus service (or a bus stop-skipping scheme) is one of the proven fleet allocation strategies to handle the unbalanced demand, which allows some vehicles to visit only a fixed subset of stops (Yu et al., 2012b; Liu et al., 2013). By skipping a set of stops, the cycle time is reduced and thus the commercial speed is improved. Limited-stop service is favourable to both users and operators. For users, its use improves the level of service, particularly for those passengers with long travel distance, whilst it enables operators to meet the demand in a cost-effective manner.

In spite of its advantages, the employment of limited-stop service might worsen headway irregularity and induce bus overtaking. There are two primary reasons for this. First, trips with different stop patterns lead to speed distinctions and overtaking even without exogenous stochastic factors. It is expected that an express trip is likely to catch up with a full-length regular trip, and a leapfrogging phenomenon frequently results. Second, those passengers who are skipped must wait for the next bus, such that the next bus will accommodate more accumulated demand, which further causes imbalanced dwell times and irregular arrivals at stops. As a result, compared to the full-length regular service, buses are more likely to overtake in the provision of limited-stop services.

Previous research on the stop-skipping problem presented simplified models without consideration of overtaking and dynamics, which may lead to inaccurate estimation of system performance and irrational schedule design. Furthermore, due to the presence of high uncertainties in transit operation, the tight schedules are usually susceptible to disruptions. Unfortunately, to date, there is no literature assessing schedule robustness against uncertainties in the design of limited-stop service. Alternately, the performance of transit operation passenger is closely related to demand dynamics (Sun et al., 2014), whereas a deterministic uniform distribution is commonly used to model the passenger arrival process in the literature, underlying that the demand variability is low. However, urban transit demand usually exhibits high heterogeneity in both time and space, particularly in the high-density populated cities with large bus passenger demands (Yu et al., 2011). From their empirical and simulation analysis, Liu and colleagues (Liu and Siha, 2007; Sorratini et al., 2008; Fonzone et al., 2015) have shown that variable demand distribution, both spatially and temporally, is the factor that affects bus reliability the most. Traditional passenger demand data, which are usually aggregated into one hour or even the entire planning period, are too coarse to describe the passenger behaviour. The rapid development of information (i.e., the Internet of Things) enables the acquisition of real-time high-resolution data on passengers and vehicles. This facilitates transit operators to leverage on the potential flexibility embodied in limited-stop service in catering more efficiently to the prevailing passenger demand variations.

To fill in these gaps, our paper sets out to address the stop-skipping problem considering these realistic factors. To this end, we first develop an enhanced bus propagation model under given stop patterns in which vehicle overtaking and capacity constraints are explicitly taken into account, while accommodating a given passenger OD matrix. Second, we develop a robust optimization model subject to a risk threshold in the context of time-dependent stochastic travel time. To improve the computational efficiency, we further propose a simulation-based optimization framework incorporating response surface methodology to cope with the problem. The models are tested via a real-world bus route in Guangzhou, China. We also examine the Pareto frontier for robust solutions under different uncertainty levels. Our findings show that the stop patterns are quite distinctive when overtaking and demand dynamics are considered. We thus suggest that it is imperative to consider vehicle overtaking and dynamics in the design of robust limited-stop bus service.

The remainder of this study consists of five sections. Following the literature introduction, Section 3 describes the mathematical models. In Section 4, solution methods are provided. Section 5 verifies the effectiveness of the models through the data of a real-world bus route. Finally, Section 6 provides the conclusions and future works.

2. Literature review and main contributions

There are extensive studies on bus operational strategies for public transport management. The main objective of deploying bus operational strategies is to correspond and scale to passenger demand by providing different fleet supplies along the route. Generally, they can be divided into three types of categories: short-turn, deadheading, skip-stop and zonal service. Short-turn strategies are to allow some trips to serve only a route segment to increase frequencies for regions with greater demand while reducing frequent stops at the less demanded area, which is useful in matching the transit supply with the demand profile in different segments of a linear corridor (Furth, 1987). Deadheading is another form of limited-stop service heading empty from one location to the other one in order to start up another trip as soon as possible in the most demanded direction. This strategy can be added to a subset of a predefined itinerary to realize certain trip connections (Ceder and Stern, 1981; Furth, 1985), while in most scenarios it is used as a corrective control method at the operational stage. For example, Eberlein et al. (1998) investigated a dynamic deadheading problem, whereby a vehicle runs empty from a terminal to a designated stop in case of disruptions. Yu et al. (2012) presented a partway deadheading strategy with short-term future reliability assessment. Stop-skipping or express service visits only a subset of stops along the route. Similar to the stop-skipping operation, zonal service visits all stops within specific zones while skipping other stops on the route. Furth (1986) proposed a zonal express service where buses only pick up passengers in their zones and then run express to the CBD. This work is extended by Larrain et al. (2015) by visiting all stops in a route's initial and final segments, skipping all stops in between.

Stop-skipping service can be undertaken at both the planning level and operational level, whereas the objectives and mechanisms at different levels are distinct. At the operational level, such a strategy is often employed in tandem with other tactics, such as real-time holding control to improve transit operation reliability. The main objective is to adapt to the changes in operating conditions by dynamically adjusting the number and locations of the skipped stops. In this sense, the stop-skipping control aims to catch up with the schedule, which resembles speed adjustment and schedule recovery. Unlike the stop-skipping control implemented at the operation level, the predetermined skipping stops cannot be changed in response to the varying operating conditions at the planning level, which may lead to bus overtaking and the leapfrogging effect. Leapfrogging is a special variant of bus bunching, in which two vehicles cannot separate from each other over multiple stops, and one bus overtakes another and is overtaken again by the same bus (Wu et al., 2017). Sun and Hickman (2005) investigated real-time stop-skipping policy in response to service disruptions. The results showed that the travel time variability could have a great impact on the performance of the policy. Milla et al. (2012) integrated the stop-skipping and holding control as a means of anti-bunching to maintain regular headways between consecutive buses. By utilizing real-time headway prediction, Moreira-Matias et al. (2016) developed an online approach with both stop-skipping and holding methods to eliminate bus bunching. Hadas and Ceder (2010) used stop-skipping, in tandem with holding and short-turning in the simulation of real-time transfer synchronization. Nesheli and Ceder (2014) refined the simulation framework by introducing a skip-segment strategy apart from skipping a stop.

At the planning level, there exist a number of studies focusing on the determination of headways and limited-stop routes for a bus corridor. Leiva et al. (2010) investigated a limited-stop service design problem by optimizing the service headways to minimize the total social cost. Chen et al. (2015) formulated a model to simultaneously optimize the headways for both full-length and limited-stop services and the subset of skipping stops. Freyss et al. (2013) presented a skip-stop operation mode for rail transit using a continuous approximation approach. Larrain and Muñoz (2016) investigated the factors influencing the benefits from limited-stop services. Soto et al. (2017) presented a bi-level model of limited-stop service design considering transfer activities and passenger behaviour. Wang et al. (2018) developed an optimization model for combined limited-stop and full-length service in the context of a common line. In practice, there exist large amounts of uncertainties in vehicle travel time due to traffic congestion, bad weather, etc. (Yao et al., 2014; Wu et al., 2016; Wu et al., 2018). To increase the operational accuracy, a handful of works have considered stochastic travel time in the design of limited-stop service at the planning level. Liu et al. (2013) presents a model which optimizes the stop-skipping decision of each trip along the route with random travel time. Later, this model was extended by Chen et al. (2015) to explicitly consider the capacity constraints and crowding effect. However, their models aim for pure cost-efficiency in terms of minimal planned costs while neglecting the effect of vehicle overtaking.

Another significant simplification in existing studies is the deterministic uniform arrival rate of passengers. More specifically, the arrival rate is assumed to remain constant in a relatively long period (generally one hour). However, in reality, passenger demand variations are commonly observed over time during transit operation. Such temporal demand heterogeneity may exert an influence on the optimal fleet assignment along the bus route. Incorporating the system dynamics, however, has created a series of challenges in both modelling formulations and algorithmic efficiency. The main challenges lie in the fact that there exist high uncertainties in the presence of limited-stop service (see Section 4), which requires conducting a larger number of simulation replications to mitigate the influence of random simulation errors. Thus, the objectives will become computationally expensive-to-evaluate as a result of the increased simulation noise and integral formulations for dynamic demand. According to our preliminary experiments, the algorithmic efficiency is quite low when using the conventional methods, such as enumeration and random search algorithms. The computational time for searching for an optimized solution could be many hours or even several days depending on the scale of the bus route and the number of simulation runs. Therefore, enumeration or random search algorithms requiring intensive objective evaluations are not suitable to address the problem.

Surrogate models aim to fit the response surface featuring the relationships between the decision variables and simulation outputs. Through a set of initial input data and corresponding objective outputs, the response surface can be fitted as an approximation of the expensive-to-evaluate objective (Chen et al., 2014; Chen et al., 2016; Zhang et al., 2017; Zheng et al., 2019). Commonly used response surface methodologies include quadratic polynomial regression (Yalçınkaya, 2009), the Kriging model, and support vector machine (Chen et al., 2014). Based on the tuned response surface, the optimal decision variables can be easily obtained. Surrogate-based optimization has been applied to transportation systems. For example, Chen et al. (2016) investigated the congestion pricing problem for freeway mitigation using a surrogate-based optimization approach. Xiong et al. (2017) investigated optimal travel information provision strategies via the integration of agent-based and surrogate modelling. For this sake, response surface methodology could be a proper tool to address the issues of an expensive-to-evaluate objective as presented in our study.

Distinctly from previous research, in this paper, we develop a more generalized optimization model for the stop-skipping problem explicitly considering vehicle overtaking and dynamics. As poor service reliability indicates a drop in demand and possible loss of route operating franchise, the target service reliability imperative is also included in the service performance. The objective is to minimize the expected total cost, including both user cost and operation cost, while offering an appropriate level of reliable service.

The main contributions of this study are summarized as follows: (1) we explicitly consider vehicle overtaking and dynamic demand in the design of limited-stop bus service, which is more general and realistic; (2) we investigate schedule robustness against operation uncertainties in addition to cost-efficient solutions, which has significant implications in the realm of operational design; (3) to reduce the problem complexity, we propose a simulation-based optimization framework by constructing the response surface with only a few evaluations of objective functions. In this manner, it is possible to obtain high-quality solutions for the expensive-to-evaluate problem in a reasonable amount of time. In particular, we propose an effective coding method to handle the binary decision variables adapting to the response surface methodology; (4) we compare the performance of the stop-skipping scheme to those without overtaking and dynamics under various operational settings and demonstrate that the stop patterns are quite distinct when overtaking and dynamics are considered. *As far as we are aware, this is the first time the response surface methodology has been applied in the design of robust limited-stop bus service with explicit consideration of vehicle overtaking and dynamics.*

3. Model specification

3.1 Problem description

Suppose a number of buses are planned to operate on a bus route given departure headway in a specified planning period. Similar to Liu et al. (2013) and Chen et al. (2015), the stop-skipping problem aims to determine whether a designated bus stop should be skipped by each vehicle at the planning level. The information of the specific stopping scheme of each vehicle can be delivered to the passengers by the information board at the bus stops and various graphical and verbal indications on the windscreen. Fig. 1 shows a simple illustration of limited-stop service, where 6 bus trips are planned to operate in a period. This paper seeks to develop an improved stop-skipping optimization model, specifically by incorporating bus bunching (and overtaking) and dynamics, and investigates an efficient solution method for the problem. To this end, we firstly propose a capacity-constraint bus movement model with overtaking and demand dynamics under given stop-skipping scheme, which is an extension of the bus-bunching model proposed by Wu et al. (2017). Based on the bus movement model, we develop a cost-minimization model featuring the target reliability imperative. Then, we propose a solution method based on a response surface methodology. To demonstrate the benefit, the enhanced model is compared with the respective versions without overtaking and dynamics under different operational settings.

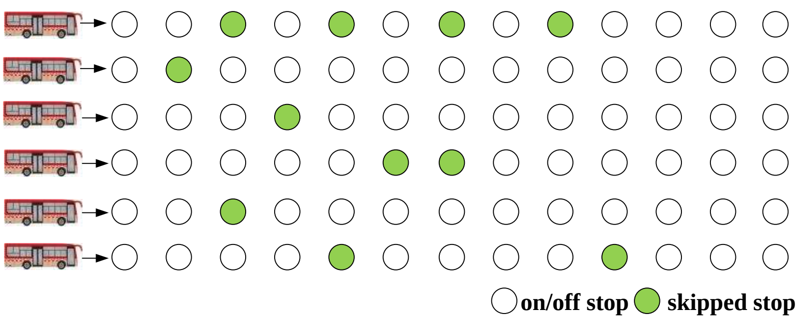


Fig. 1 Illustration of limited-stop service

3.2 Assumptions and notation

The following assumptions are adopted and presented as follows:

(A1) Vehicle overtaking is allowed, as opposed to no allowance for bus bunching and overtaking in previous works (i.e., Liu et al., 2013; Chen et al., 2015). This assumption is to make sure that the leapfrogging phenomenon can be reproduced.

(A2) Passenger OD demand is known, and the passenger arrival rate varies as a function of time, as opposed to deterministic uniform arrival in the literature.

(A3) The link travel time is stochastic and follows a time-dependent general distribution with a positive lower bound.

(A4) Passengers take the first direct trip to their destinations. Transfers between different trips are not allowed. The validity of this assumption depends on the cost of a transfer. Leiva et al (2010) reported that transfers would not occur in the limited stop services with high transfer penalties, while in a single bus line the transfer penalties between different trips are usually high.

The following notations in Table 1 are adopted in this paper.

Table 1 Primary notations used in this article

Symbols	Definition	Unit
Indices and Sets		
$i,\ k$	The subscript of a bus	—
j	The subscript of a bus stop	—
K	The set of buses of a bus route	—
Model parameters		
C	The vehicle capacity	pax
N	The number of bus stops on the bus line	—
$\lambda_{j,r}(t)$	Passenger arrival rate to stop j heading to stop r at time t	pax/min
$\lambda_j(t)$	Total passenger arrival rate at stop j at time t, $\lambda_j(t) = \sum_{r=j+1}^{N} \lambda_{j,r}(t)$	pax/min
$r_j(t)$	Travel time between stops j and $j+1$ given the vehicle leaves stop j at time t	min
ρ_j	The alighting proportion at stop j	%
α	The average alighting time per passenger	min
b	The average boarding rate	pax/min
δ	Total deceleration and acceleration time for a bus at a stop	min
τ	Minimum safety interval	min
Auxiliary variables		
$B_{i,j}$	The total number of passengers waiting for bus i at stop j during inter-departure headway	pax
$\bar{B}_{i,j}$	The actual number of boarding passengers for bus i at stop j during inter-departure headway	pax
$B_{i,jr}$	The total number of passengers waiting for bus i at stop j heading to the destination stop r during inter-departure headway	pax
$\bar{B}_{i,jr}$	The actual number of boarding passengers for bus i at stop j heading to the destination stop r during inter-departure headway	pax

$A_{i,j}$	Alighting demand for bus i at stop j	pax
$D'_{i,j}$	Boarding time for all waiting passengers of bus i at stop j	min
$D_{i,j}$	Actual boarding time of bus i at stop j with a capacity constraint	min
$\bar{D}_{i,j}$	Dwell time of bus i at stop j	min
$l_{i,j}$	The number of leftover passengers of bus i when departing from stop j	pax
$L_{i,j}$	The number of on-board passengers of bus i between stops j and $j+1$	pax
$a_{i,j}$	Arrival time of bus i at stop j	min
$d_{i,j}$	Departure time of bus i from stop j	min
$h_{i,j}$	Inter-departure headway between bus i and preceding bus at stop j	min
Decision variables		
$y_{i,j}$	Binary variable to indicate whether bus i serves stop j, i.e., if stop j is skipped, $y_{i,j}=0$; otherwise, $y_{i,j}=1$.	—

3.3 Bus movement model for limited-stop service

Fig. 2 provides an illustration of the operation of the stop-skipping scheme with two adjacent buses. A bus overtaking event corresponds to two intersecting space-time trajectories. Naturally, the commercial speeds are differential among bus trips with distinct stopping plans. As a result, even without stochastic factors (e.g., random travel time, exogenous delay), bus overtaking can arise as a result of the speed differences. The existence of stochastic factors further worsens overtaking conditions. In other words, the speed distinction induced by the use of the stop-skipping scheme is another source of perturbation triggering bunching and overtaking in addition to stochastic factors. Therefore, compared to the full-length regular service, bus overtaking may occur more frequently in the presence of limited-stop service.

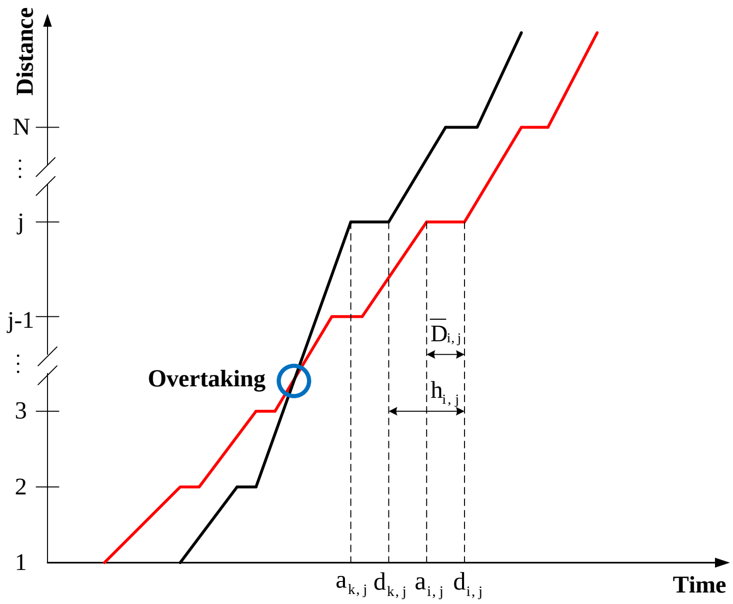


Fig.2 The space-time diagram of limited-stop service with overtaking

Now we develop a bus movement model given stopping plans for a set of buses. Generally, a bus movement model is composed of the following components: the vehicle departure events, dwell times at stops and link travel times. The arrival time at the current stop equals the departure time from the last visited stop plus the stochastic link travel time (between the last visited stop and the current stop).

$$a_{i,j} = d_{i,j-1} + r_{j-1}(d_{i,j-1}) + \frac{\delta}{2} y_{i,j-1} + \frac{\delta}{2} y_{i,j} \tag{1}$$

where $\frac{\delta}{2} y_{i,j-1}$ is the acceleration time at stop $j-1$ and $\frac{\delta}{2} y_{i,j}$ is the deceleration time at stop j. $r_{j-1}(d_{i,j-1})$ stands for the time-dependent stochastic link travel time depending on the departure time from a stop.

The vehicle departure time from a designated stop can be obtained as the addition of the arrival time and dwell time, where the dwell time is related to the time-headway between buses and the capacity constraint.

$$d_{i,j} = a_{i,j} + \overline{D}_{i,j} \tag{2}$$

Let the buses be indexed by their dispatching sequence from the terminal, i.e., $i = \{1,2,\cdots,M\}$. The headway between buses is defined as the departure time interval between consecutive buses. Because bus order may change over time, the ranking order of the subject bus adjacent to bus i is not necessarily bus $i-1$. The formulation of inter-departure headway is expressed as follows:

$$h_{i,j} = d_{i,j} - d_{k,j} \tag{3}$$

where $d_{k,j}$ is the departure time of the bus immediately before bus i departs, and it can be derived as:

$$k = arg_{k \in K} max\{d_{k,j} | d_{k,j} < d_{i,j}\} \tag{4}$$

where K is the set of buses of a bus route. Note that bus k changes from time to time.

Because the passenger arrival rate is assumed to be time-dependent, the number of arriving passengers at stop j is determined by integrating the time-dependent passenger arrival function $\lambda_{j,r}(t)$ during the inter-departure headway. The time-dependent passenger arrival function can be inferred by mining GPS and smart card data (Gordon et al., 2013). Given the capacity constraint, the actual boarding demand cannot exceed the residual available space in a bus. As a result, when the boarding demand is sufficiently high, a number of passengers may be prevented from boarding and become what we term 'leftover passengers'. The total number of waiting passengers is the summation of the numbers of arriving passengers and leftover passengers from the preceding bus.

$$B_{i,j} = \sum_{r=j+1}^{N} y_{i,r} \int_{d_{k,j}}^{d_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,j} \tag{5}$$

The actual number of boarding passengers should not exceed the remaining capacity, yielding

$$\overline{B}_{i,j} = y_{i,j} \cdot min\{B_{i,j}, C - L_{i,j-1} + A_{i,j}\}$$

(6)

The number of leftover passengers at a stop depends on the corresponding stop-skip decision. When the designated stop is served, the number of leftover passengers equals the difference between passengers waiting and those who board. Alternately, when the designated stop is skipped, it is simply equal to the number of waiting passengers.

$$l_{i,j} = (1 - y_{i,j}) \cdot B_{i,j} + y_{i,j} \cdot max\{0, L_{i,j-1} + B_{i,j} - A_{i,j} - C\} \tag{7}$$

The number of on-board passengers of bus i when it departs from stop j is related to the boarding and alighting demand at stop j, and the vehicle load of bus i from the previous stop $j-1$.

$$L_{i,j} = L_{i,j-1} + \bar{B}_{i,j} - A_{i,j} \tag{8}$$

Similarly, the total number of waiting passengers heading to destination stop r is not only the number of OD-specific arriving passengers but also passengers who are left from the preceding bus due to bus capacity constraints.

$$B_{i,jr} = \int_{d_{k,j}}^{d_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,jr} \tag{9}$$

Without loss of generality, it is assumed that the waiting passengers have equal access to each available in-vehicle space; then, the boarding probability equals the ratio between the actual number of boarding passengers $\bar{B}_{i,j}$ and that of total waiting passengers $B_{i,j}$. Therefore, the actual number of boarding passengers heading to destination stop r is

$$\bar{B}_{i,jr} = \frac{\bar{B}_{i,j}}{B_{i,j}} \cdot B_{i,jr} = \frac{\bar{B}_{i,j}}{B_{i,j}} \left[\int_{d_{k,j}}^{d_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,jr}\right]$$

(10)

Accordingly, the number of leftover passengers heading to the destination stop is:

$$l_{i,jr} = \left(1 - \frac{\bar{B}_{i,j}}{B_{i,j}}\right) \int_{d_{k,j}}^{d_{i,j}} \lambda_{j,r}(t)\, dt \tag{11}$$

The total number of passengers alighting from bus i at stop j is related to the boarding demand from previous stops.

$$A_{i,j} = y_{i,j} \sum_{r=1}^{j-1} \bar{B}_{i,rj}\, y_{i,r} \tag{12}$$

Assuming each passenger consumes an average time to complete the entire boarding process, then the boarding time at the stop is given as follows, given that the bus can hold all of the waiting passengers at the stop.

$$D'_{i,j} = y_{i,j} \frac{1}{b}\left[\sum_{r=j+1}^{N} y_{i,r} \int_{d_{k,j}}^{d_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,j}\right] = y_{i,j} \frac{1}{b}\left[\sum_{r=j+1}^{N} y_{i,r} \int_{d_{k,j}}^{a_{i,j}+D'_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,j}\right]$$

$$= y_{i,j} \frac{1}{b}\left\{\sum_{r=j+1}^{N} y_{i,r} \left[\int_{d_{k,j}}^{a_{i,j}} \lambda_{j,r}(t)\, dt + \int_{a_{i,j}}^{a_{i,j}+D'_{i,j}} \lambda_{j,r}(t)\right] dt + l_{k,j}\right\} \tag{13}$$

To reduce the computational complexity, we approximate the dwell time by assuming uniform arrivals during the boarding process. This approximation is reasonable since the boarding time is relatively short and the corresponding demand variability is low.

$$D'_{i,j} \approx y_{i,j} \frac{1}{b}\left\{\sum_{r=j+1}^{N} y_{i,r} \left[\int_{d_{k,j}}^{a_{i,j}} \lambda_{j,r}(t)\, dt + \lambda_{j,r}(a_{i,j}) \frac{D'_{i,j}}{2}\right] + l_{k,j}\right\} \tag{14}$$

Merging the term of boarding time $D'_{i,j}$ on both sides in Eq. (14) yields the closed-form expression of boarding time without a capacity constraint as follows:

$$D'_{i,j} = y_{i,j} \frac{2}{2b - \sum_{r=j+1}^{N} y_{i,r} \lambda_{j,r}(a_{i,j})} \left[\sum_{r=j+1}^{N} y_{i,r} \int_{d_{k,j}}^{a_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,j} \right] \tag{15}$$

Accordingly, the actual boarding time with a capacity constraint takes the following form:

$$D_{i,j} = y_{i,j} min \left\{ \frac{2}{2b - \sum_{r=j+1}^{N} y_{i,r} \lambda_{j,r}(a_{i,j})} \left[\sum_{r=j+1}^{N} y_{i,r} \int_{d_{k,j}}^{a_{i,j}} \lambda_{j,r}(t)\, dt + l_{k,j} \right], \frac{1}{b} \left(C - L_{i,j-1} + A_{i,j} \right) \right\} \tag{16}$$

Because the boarding and alighting process usually occur simultaneously during the stop service, the actual dwell time at a stop should be the larger of boarding and alighting times.

$$\overline{D}_{i,j} = max\{D_{i,j}, \alpha A_{i,j}\} \tag{17}$$

Remark 1: The calculation of the boarding demand depends on the headways at stops. Liu et al. (2013) and Chen et al. (2015) considered the headways as the interval between the arrival time of the current vehicle and the departure time of the preceding vehicle. Although such an assumption could simplify the modelling of bus movement, it may lead to an underestimation of the actual demand since the boarding demand during the dwell time is missing. To better reflect reality, in this paper, we consider the inter-departure headway (Eq. (3)) in the calculation of boarding demand.

3.4 Optimization model

The purpose of this paper is to address the stop-skipping problem, where different trips serving a bus route may have different stopping plans. Limited-stop service contributes to reducing the bus dwell time and thus increasing the commercial speed, such that the passengers' in-vehicle travel time cost and operation cost are reduced. Alternately, the use of such a strategy may also increase the passenger waiting time, especially for those whose origin or destination stop is passed. Therefore, it is essential to develop an optimization model to make a trade-off between the two sides and achieve a minimum total cost from the system perspective. More importantly, to achieve solutions that are less vulnerable to disruptions, we enhance the solution approach for pure cost-efficient schedules by additionally incorporating uncertainty tolerance.

Because there exists capacity binding in the boarding process, the total passenger waiting time should include the waiting time for overall awaiting passengers (the newly arriving passengers and leftover passengers) between the inter-departure headway, and the additional waiting time for leftover passengers. The average waiting time for the overall awaiting passengers between the inter-departure headway is $\frac{1}{2} h_{i,j}$, then the corresponding total waiting time can be approximated by:

$$Z_w^1 = \frac{1}{2} \sum_i \sum_j B_{i,j} h_{i,j} \tag{18}$$

The additional waiting time for a leftover passenger is $h_{i,j}$. Therefore, the additional waiting time for leftover passengers is a product of the leftover demand and the following inter-departure headway.

$$Z_w^2 = \sum_i \sum_j l_{i,j} h_{i,j} \tag{19}$$

Consequently, the total waiting time is the summation of these two parts.

$$Z_w = Z_w^1 + Z_w^2 = \frac{1}{2}\sum_i \sum_j B_{i,j} h_{i,j} + \sum_i \sum_j l_{i,j} h_{i,j} \tag{20}$$

It is worth mentioning that, in average, the total waiting time for a leftover passenger is $\frac{1}{2}h_{k,j} + h_{i,j}$.

The total in-vehicle time for passengers consists of two components: link travel times and dwell times. They are calculated as the product of the total number of on-board passengers on specified links and the link travel times/dwell times. Therefore, summing up the total link travel times and dwell times yields the total in-vehicle time, which is expressed as follows:

$$Z_v = \sum_i \sum_j \{[r_j(d_{i,j}) + \delta y_{i,j}]L_{i,j} + (\bar{D}_{i,j} y_{i,j})L_{i,j}\} \tag{21}$$

Similarly, the total operating time takes the following form:

$$Z_o = \sum_i \sum_j [r_j(d_{i,j}) + (\bar{D}_{i,j} + \delta) y_{i,j}] \tag{22}$$

Then, the system's total cost is the weighted sum of waiting time, in-vehicle travel time and riding time.

$$Z = \varphi_1 Z_w + \varphi_2 Z_v + \varphi_3 Z_o \tag{23}$$

$$= \varphi_1 \sum_i \sum_j (\frac{1}{2}\sum_i \sum_j B_{i,j} h_{i,j} + l_{i,j} h_{i,j}) + \varphi_2 \sum_i \sum_j \{[r_j(d_{i,j}) + \delta y_{i,j}]L_{i,j} + (\bar{D}_{i,j} y_{i,j})L_{i,j}\}$$

$$+\varphi_3 \sum_i \sum_j [r_j(d_{i,j}) + (\bar{D}_{i,j} + \delta) y_{i,j}]$$

where φ_1, φ_2, and φ_3 are the weighting values for each term.

The objective of limited-stop service design is to explore the best decision variables $y_{i,j}$ that minimize the expected total cost while offering an appropriate level of reliable service. The existence of stochastic travel times implies that the objective function would be variable, particularly in the provision of vehicle overtaking (as explained in Section 4.1). Therefore, to ensure the schedule robustness, the standard deviation of the objective should not exceed a threshold. As a result, the bus stop-skipping problem could be formulated as integer nonlinear programming models:

$$min\, E(Z) \tag{24}$$

subject to

$$Std(Z) \leq T \tag{25}$$

where Eq. (25) specifies that the standard deviation of total cost should not be larger than a critical value T.

In addition, considering the practical circumstances, some other optional rules/constraints can be further combined to the limited-stop service design. For example, because each bus trip of the same bus line will have different stops, to avoid long waiting times for passengers, it is reasonable to assume that bus i is not allowed to skip two adjacent bus stops simultaneously. Then, the following constraint can be added:

$$1 \leq y_{i,j} + y_{i,j+1} \leq 2 \tag{26}$$

3.5 Model properties

Proposition 1. The size of the solution space increases exponentially with the number of trips and bus stops along the route.

Proof. Let us assume that the number of stops along the route is N, and that the number of bus trips is M. Because the decision variable is a binary variable, the number of combinations that need to be evaluated for one trip is 2^N. If one needs to determine the optimal stop-skipping scheme for all trips, then the number of combinations is $2^{N \cdot M}$. The size of the solution space is still within its original order of magnitude in spite of the optional constraint Eq. (26).

Q.E.D.

Proposition 2. The model in Eqs.(24) and (25) is equivalent to the following robust optimization model (with decision variables $y_{i,j}$ and θ):

$$\min_{y_{i,j}} \max_{\theta} E(Z) + \theta[Std(Z) - T] \tag{27}$$

s.t.

$$\theta \geq 0$$

Proof. A former proof of this proposition can be found in Zhang and Xu (2017).

Q.E.D.

Proposition 3. The decision variable θ increases as the risk threshold T decreases.

Proof. Eq. (27) is equivalent to $\max_{\theta} \min_{y_{i,j}} E(Z) + \theta[Std(Z) - T]$. Because the term θT is not related to the stop decision y_{ij}, we can focus on the term $E(Z) + \theta Std(Z)$. Solving the minimization problem on this term yields an optimal solution $y^*(\theta)$ satisfying $E(Z^*) + \theta Std(Z^*) := r(\theta)$. Then, Eq. (27) can be transformed into:

$$\max_{\theta \geq 0} F(\theta, T) \tag{28}$$

where $F(\theta, T) := r(\theta) - \theta T$

Let θ_1 be the optimal value that maximizes $F(\theta, T)$ given the risk threshold T_1, and let θ_2 be the optimal value that maximizes $F(\theta, T)$ given the risk threshold T_2. Because the objective function can be solved to optimality given θ_1 and T_1 (or given θ_2 and T_2), then the objective function will become sub-optimal given θ_2 and T_1 (or given θ_1 and T_2). That is,

$$F(\theta_1, T_1) \geq F(\theta_2, T_1) \quad (29)$$

Then, we have that

$$r(\theta_1) - \theta_1 T_1 \geq r(\theta_2) - \theta_2 T_1 \quad (30)$$

$$= r(\theta_2) - \theta_2 T_2 + \theta_2 T_2 - \theta_2 T_1$$

Similarly, the following inequality holds:

$$F(\theta_2, T_2) \geq F(\theta_1, T_2) \quad (31)$$

$$r(\theta_2) - \theta_2 T_2 \geq r(\theta_1) - \theta_1 T_2 \quad (32)$$

Adding the term $\theta_2 T_2 - \theta_2 T_1$ on both sides of Eq. (32) yields:

$$r(\theta_2) - \theta_2 T_2 + \theta_2 T_2 - \theta_2 T_1 \geq r(\theta_1) - \theta_1 T_2 + \theta_2 T_2 - \theta_2 T_1 \quad (33)$$

The left-hand side of Eq. (33) is equivalent to

$$r(\theta_2) - \theta_2 T_1 = F(\theta_2, T_1)$$

Combing Eqs. (30) with (33), we have that

$$r(\theta_1) - \theta_1 T_1 \geq r(\theta_1) - \theta_1 T_2 + \theta_2 T_2 - \theta_2 T_1 \quad (34)$$

Eliminating the term $r(\theta_1)$ on both the left-hand side and right-hand side yields:

$$(\theta_1 - \theta_2)(T_1 - T_2) \leq 0 \quad (35)$$

This means that θ increases when the risk threshold T decreases.

Q.E.D.

Proposition 4. The total costs decrease as the risk threshold T increases.

Proof. According to Proposition 3, θ increases as the risk threshold T decreases. In other words, θ decreases as the risk threshold T increases. Therefore, for the objective function $E(Z) + \theta[Std(Z) - T]$ (i.e., total cost), the term $\theta[Std(Z) - T]$ will be reduced as T decreases given $E(Z)$ and $Std(Z)$. As a result, the value of the objective function will be reduced as T increases.

Q.E.D.

Proposition 4 indicates that a higher risk level could lead to smaller cost (Zhang and Xu, 2017). In other words, to minimize the costs, we should have to suffer from a certain level of risk. Hence, there is a trade-off between cost minimization and service robustness, as will be experimentally validated in Section 5.

4. Solution methods

4.1 Framework

Distinctly from previous research (i.e., Liu et al., 2013; Chen et al., 2015), the effect of vehicle overtaking and dynamic demand is explicitly considered in the design of the limited-stop service. In our preliminary experiment, we found that the objective functions are quite expensive-to-evaluate. The reason mainly lies in two aspects: First, there exist large amounts of uncertainty that induce random simulation outputs given the same input, such as random travel time and dynamic passenger demand. In addition, the implementation of the stop-skipping scheme leads to the speed difference across bus trips, which further worsens bus overtaking effect. As a result, compared to the full-length regular service, more random seeds may exist in the simulation. This requires conducting a larger number of simulation replications to mitigate the influence of the random simulation errors so as to generate less biased results. Second, to capture the demand dynamics, the calculation of the objective function in the form of an integration operation is generally time-consuming. Because the size of the solution space increases exponentially with the number of trips and bus stops (Proposition 1), it is almost impossible to obtain the optimized results for large-scale scenarios in a reasonable time using conventional methods such as enumeration and random search algorithms.

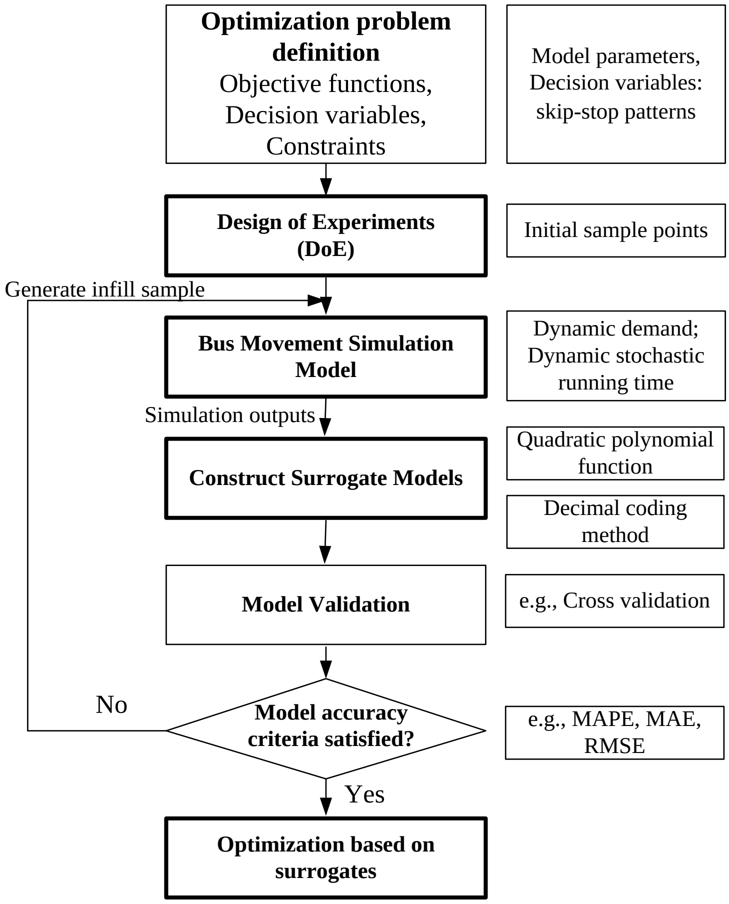


Fig.3 Simulation-based optimization framework

To address these issues, we propose a simulation-based optimization framework incorporating response surface methodology (RSM). The principle is that, with an initial input dataset and corresponding output of the objective function, the RSM can be fitted as a surrogate or analytical approximation of the expensive-to-evaluate objective. Based on the exploration and exploitation of the computationally efficient surrogate, the optimal decision variables could be obtained in a reasonable amount of time, enabling the model's applicability in large-scale scenarios. The main steps are as follows:

- Step 1: Given the objective function Eq. (24) evaluated by simulation outputs, an initial number of stop patterns $\boldsymbol{x}$ are generated using a design of experiment method. For details of the design of experiment method, readers can refer to Chen et al. (2014).
- Step 2: Simulate the bus propagation model using the initial set of stop patterns $\boldsymbol{x}$ and obtain the corresponding expected values of the objective function. The expected value is calculated as the mean value of objective function $\hat{f}_1(x)$ and standard deviation of objective function $\hat{f}_2(x)$ obtained by a number of simulation runs S.

$$\hat{f}_1(x) = \frac{1}{S}\sum_{i=1}^{S} Z_i \tag{36}$$

$$\hat{f}_2(x) = \frac{1}{S}\sum_{i=1}^{S} \left(Z_i - \hat{f}_1(x)\right) \tag{37}$$

- Step 3: Develop the input-output relationships using quadratic polynomial function models (see Section 4.2) to construct the response surface for both the expected value $\hat{f}_1(x)$ and the respective standard deviation $\hat{f}_2(x)$. If the response surface does not satisfy the validation criteria, go to Step 2 to generate infill points and run simulations.
- Step 4: Solve the following constraint optimization problem under various risk threshold values T to give the Pareto frontier (Kleijnen, 2008; Zhang and Xu, 2017).

$$\min \hat{f}_1(x) \tag{38}$$

$$\text{s.t. } \hat{f}_2(x) \leq T \tag{39}$$

Fig.3 illustrates the solution framework adopted in this paper.

4.2 Response surface methodology (RSM)

The first step of RSM is to explore an analytical approximation for the relationship between response and input decision variables. In this paper, the quadratic polynomial function, which is the most commonly used RSM, is applied to fit the response surface for both the expected value $\hat{f}_1(x)$ and the respective standard deviation $\hat{f}_2(x)$. The quadratic polynomial function takes the following form:

$$\hat{f}(\mathbf{x}) = \beta_0 + \sum_{i=1}^{m} \beta_i x_i + \sum_{i<j}\sum_j \beta_{ij} x_i x_j + \sum_{i=1}^{m} \beta_{ii} x_i^2,\ \mathrm{x} \in \mathbb{R}^m \tag{40}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_m]^T$ is a m-dimensional point to be predicted. $\hat{f}(\mathbf{x})$ is the estimation of the real objective function $f(\mathbf{x})$. β_0 is the intercept. β_i is the linear coefficient. β_{ij} is the coefficient of interaction terms. β_{ii} is the quadratic coefficient.

Because the simulation-based objective function (with integration operation) is replaced by the analytical RSM function, the optimal solution can be obtained much more efficiently. The fitted quadratic model is non-convex and contains many integer decision variables, so it is difficult to find an exact method to solve this model in polynomial time, which is known as a NP-hard (non-deterministic polynomial-time hard) problem. In view of the NP-hardness of the model, a variety of evolutionary algorithms, such as genetic algorithm, can be adopted to solve the model.

4.3 Coding method for decision variables

In the optimization model (Section 3.4), the decision variable $y_{i,j}$ is defined as a binary variable to indicate the stopping decision at a designated stop for a bus trip. Because the number of input variables depends on the number of bus stops along the route, the number of parameters to be estimated will be dramatically large if the quadratic polynomial function is adopted as the surrogate, which is rather difficult since there is only a limited number of input-output data from the outcomes of expensive simulations.

To address this issue, we propose an effective decimal coding method to transform the binary stop-skipping decision of a bus along the route into a decimal value. To illustrate, we consider a bus route consisting of 16 intermediate bus stops (as shown in our case study). Let x_i be a decimal value representing the stop pattern of a trip; then, it can be represented by a combination of binary variables $y_{i,j}$. For instance, the stop pattern of a vehicle 1111100101100110 can be transformed into a decimal value 63846. In other words, the decision variables can be replaced by a number of decimal values instead of binary variables. In this manner, the number of transformed decision variables equals the number of trips while being independent of the number of stops. Such an approach yields far fewer input variables and facilitates fitting the regression model. Additionally, this approach scales well when the scale of the bus route expands.

Now we discuss the representation of the constraints given the new coding method. Naturally, the range of x_i should be between 0 and 1111111111111111. Therefore, the constraint should be satisfied, i.e., $0 \leq x_i \leq 65535$. For the optional constraint that two adjacent bus stops are not allowed to be skipped simultaneously (Eq. (26)), i.e., $1 \leq y_{i,j} + y_{i,j+1} \leq 2$, we have that

$$x_i + x_{i+1} \geq 1 \times (2^0 + 2^1 + \cdots + 2^{15}) = 65535 \text{ and} \quad (41)$$

$$x_i + x_{i+1} \leq 2 \times (2^0 + 2^1 + \cdots + 2^{15}) = 131070$$

Therefore, Eq. (26) can be replaced by the following constraint:

$$65535 \leq x_i + x_{i+1} \leq 131070 \quad (42)$$

4.4 Trajectories evolution simulation

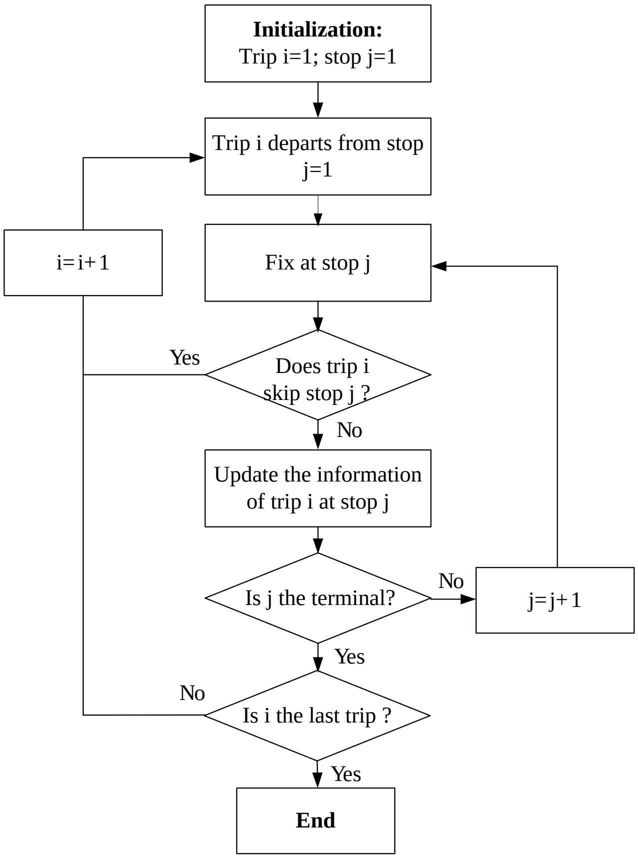


Fig. 4 Simulation framework of bus stop-skipping operation without overtaking

In this section, simulation frameworks are developed to calculate the bus trajectories under given stopping plans and overtaking policies. Bus trajectories are composed of the departure time, travel time and dwell time. The calculation of passenger flow and resultant dwell time depends on the inter-departure headways. In the case without overtaking, the information of the front vehicle has been available before the calculation for the subject vehicle, and thus the bus trajectories can be simply calculated trip-by-trip. In other words, the trajectory of a vehicle is calculated only after the calculation for the preceding vehicle. Fig. 4 shows the simulation framework of bus stop-skipping operation without overtaking.

For the case with overtaking, however, the calculation method is more complex. The allowance for overtaking poses challenges for simulation since the bus order may change from stop to stop. In other words, the bus order at the subsequent stops may differ from that at the terminal, as opposed to the case without overtaking. Because the calculation of passenger flow depends on the inter-departure headway, it requires sorting the arrival times of all trips at a designated stop in chronological order to update the trip order. Therefore, unlike the trip-by-trip manner in the case without overtaking, the vehicle trajectories should be computed stop-by-stop. Hence, we here propose a new simulation framework of bus stop-skipping operation with overtaking. Because the bus order will not change after trip sorting, this simulation framework can also be adapted to the case without overtaking. Algorithm 1 provides the general simulation framework of bus stop-skipping operation with overtaking.

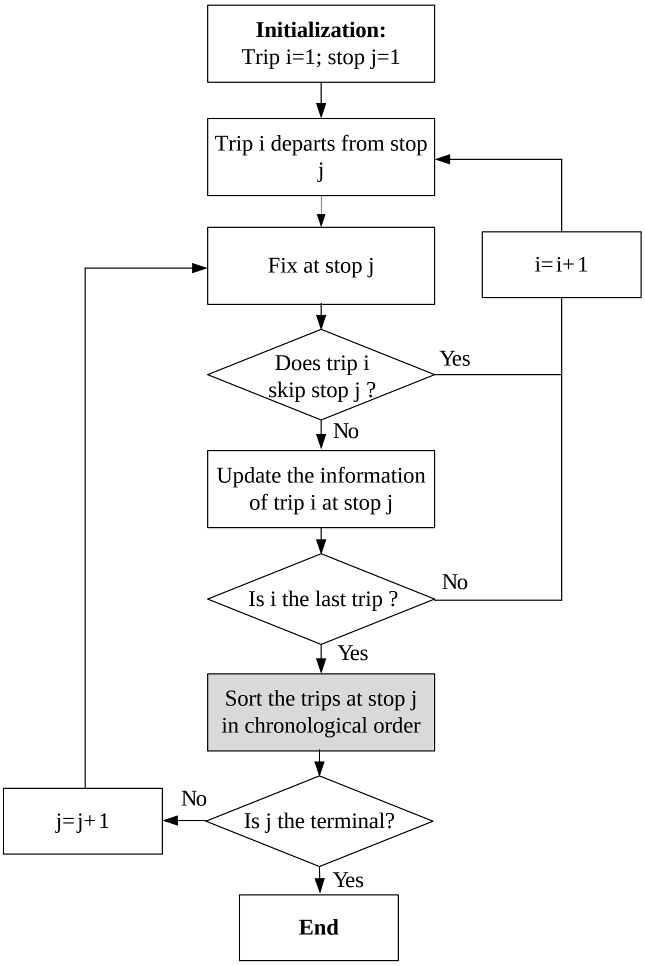


Fig.5 Simulation framework of bus stop-skipping operation with overtaking

Algorithm 1. The algorithm of trajectories evolution with overtaking

Initialization: Set input parameters and the number of instances

for each instance

Generate the dispatching time for all bus trips from the terminal

Draw the stochastic link travel times from a given distribution

Generate the trajectories for all trips in the planning period

for each stop

Sort the trips in order of arrival times at the stop

Calculate the boarding time according to *Algorithm 2*

Calculate the departure time of all buses at the stop

Prior to providing a detailed description of the algorithm, we introduce some additional notations.

Let $AT(j)$ denote the set of sorted arrival time at stop j, i.e., $AT(j) = [\vec{a}_{1,j}, \vec{a}_{2,j}, \cdots, \vec{a}_{n,j}, \cdots]$, where $\vec{a}_{n,j}$ denotes the arrival time of n^{th} bus satisfying $\vec{a}_{n-1,j} \le \vec{a}_{n,j}$. To achieve the mapping between the information to those ordered ones, the position of bus i in $AT(j)$ is drawn and denoted as $\mathcal{L}(i,j)$. Algorithm 2 describes how to calculate the passenger demand and dwell time at a stop for the case with overtaking.

Algorithm 2. Calculating the passenger flows and dwell time at stop j for the overtaking case

Step 1: Obtain the information of the ordered buses

for n^{th} bus

Calculate the boarding time of n^{th} bus at the current stop with Eq. (16)

Calculate the passenger demand and dwell time of n^{th} bus with Eqs. (1)-(12) and Eq. (17)

Obtain the departure time of n^{th} bus

The above results are put into a matrix I, in which n^{th} row corresponds to the passenger demand of n^{th} bus.

Step 2: Obtain the information of the original buses from those of ordered ones

for bus i

Draw the passenger demand and the trajectory (arrival time, dwell time, and departure time) of bus i at the stop by mapping to the $\mathcal{L}(i,j)^{\text{th}}$ row in matrix I

To examine the impact of overtaking operation, we also present the case without overtaking as a benchmark. Unlike the case with overtaking, for the case without overtaking, the ranking order of the subject bus that is adjacent to bus i should be bus $i-1$. In other words, the bus order at the subsequent stops remains the same as that at the terminal. Therefore, to avoid the bunching and overtaking phenomenon, the inter-departure headway is required to be higher than a constant parameter, that is,

$$d_{i,j} \ge d_{i-1,j} + \tau \quad (43)$$

5. Numerical example

5.1 Data settings

We implement the enhanced stop-skipping model for a busy route on bus number 113 in Guangzhou city of China with 18 stops (16 intermediate stops). The map is shown in Fig. 6. The route length is approximately 13 km. The fleet are of the same vehicle size with a capacity of 100 pax per vehicle. The boarding demand data are provided by Guangzhou Trolleybus. Since limited-stop service is usually employed at peak periods, in the example we investigate the schedule design during the entire morning peak hours (8:00-10:00 am). Although all trips depart in the first hour, the last few trips may reach the terminal in the next hour due to the long duration of vehicle tours. Therefore, the historical data in the next hour is also needed for modelling. To this end, we use the data of three consecutive hours (8:00-11:00 am) in one direction of this route (from Nantian Road Terminal to Tangan Road Terminal). The link travel time data are collected from on-board GPS vehicle tracker, where the statistics of link travel times are calculated. Fig. 7 shows the percentages of average passenger arrivals at stops, from which the uneven demand can be seen. Some stops attract more commuters possibly because of geographical advantages or other reasons. This results in spatially heterogeneous demand along the route, which may be addressed by operational strategies such as limit-stop service.

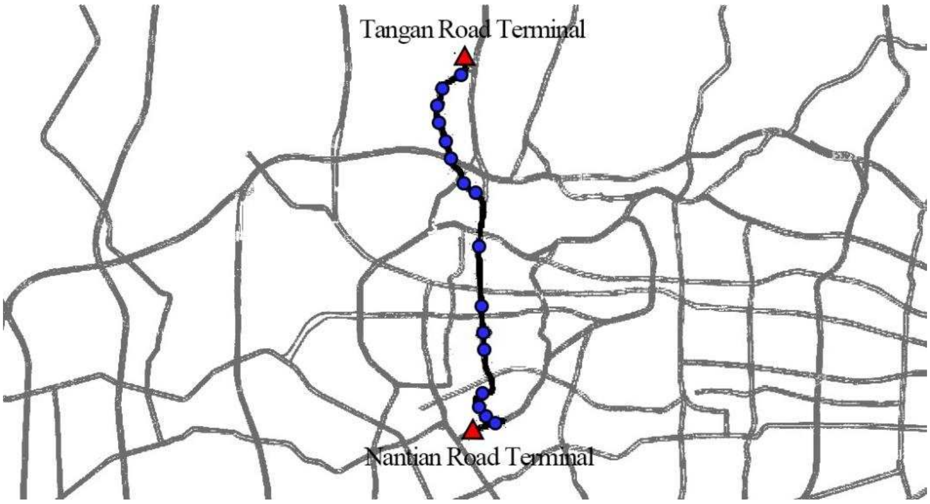


Fig. 6. A map of Bus Route Number 113

The bus OD matrix can be directly obtained by an automatic fare collection (AFC) system that can provide user information for both boarding and alighting locations. For the entry-only smart card data, a variety of methods in the literature could be used to forecast the destinations of smart-card users (Jung and Sohn, 2017). Because the smart card data is entry-only in Guangzhou, in this study, we estimate the bus OD matrix using the alighting probabilities estimation method following Liu et al. (2013) and Chen et al. (2015), that is, passengers boarding at a designated stop are assumed to evenly alight at the remaining stops. In this way, the passenger arrival rate heading to the downstream stops $\lambda_{j,r}(t)$ can be obtained given the stop-level arrival rate $\lambda_j(t)$. Subsequently, the passenger flow and alighting distributions along the route can be derived. The values of model parameters are provided in Table 2. The departure time of the first trip is 8:00. The departure headway is taken as 5 min, and the number of bus trips is taken as 6.

Fig. 8 presents the time-dependent passenger arrival rate at different stops for two successive periods (8:00-9:00 am and 9:00-10:00 am). We can see that the boarding demand varies considerably over time, particularly for those stops at the first half of the route. This indicates that the assumption of uniform arrival rate in the existing research may lead to inaccurate predictions of vehicle loads and the resultant system performance.

Because the travel time is naturally larger than a minimum positive value, the link travel time is assumed to be random following a time-dependent truncated normal distribution with a positive lower bound. Fig. 9 shows the mean and standard deviation of link travel times in the period of interests. As we can see, the mean travel times change slightly, while the standard deviations are quite different across some stops (e.g., stop 3 and stop 15).

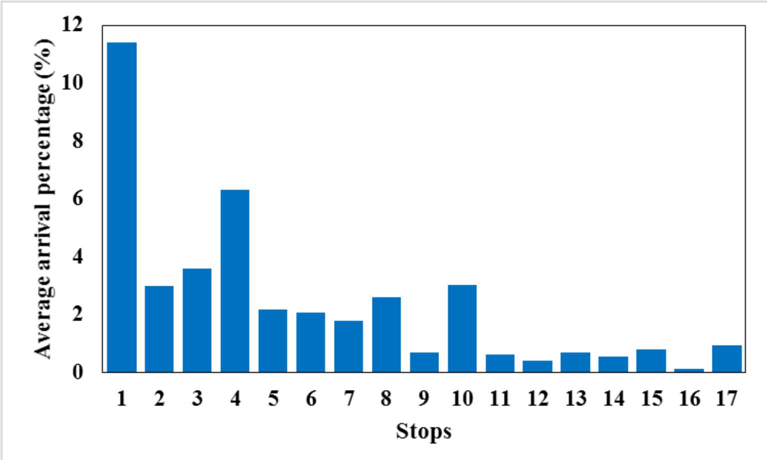
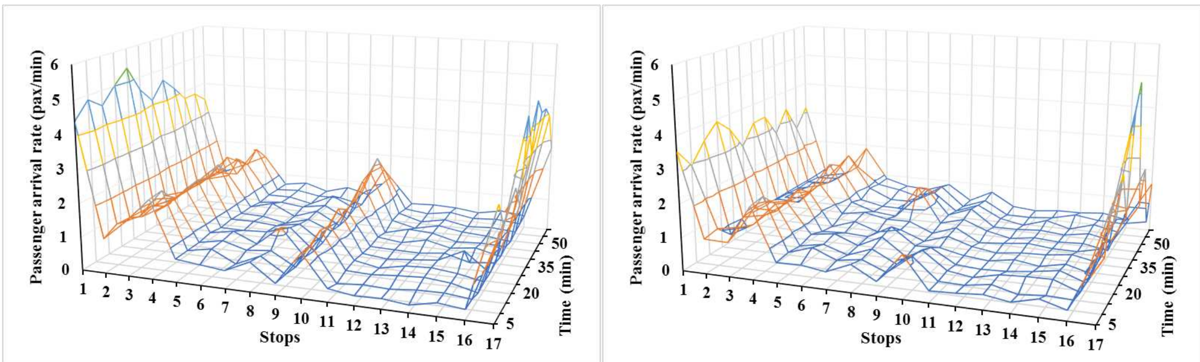


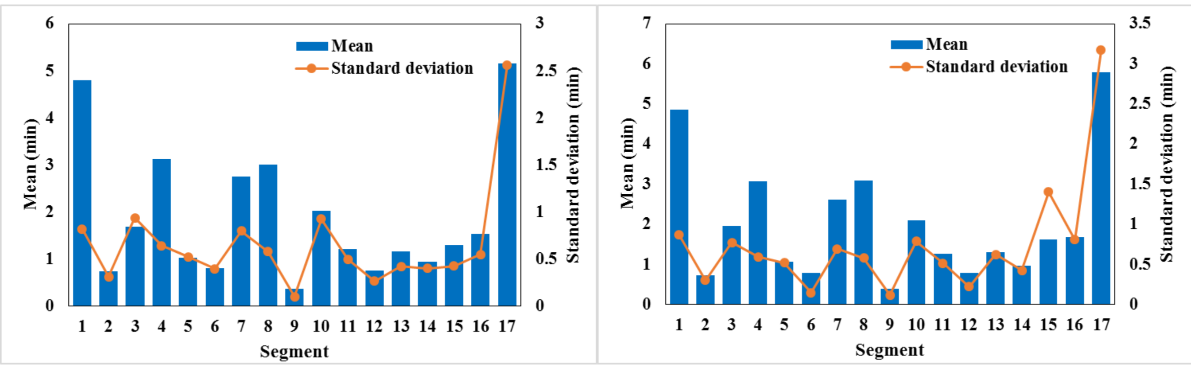
Fig. 7. Percentages of average passenger arrivals at stops



(a) 8:00-9:00

(b) 9:00-10:00

Fig. 8. Passenger arrival rate over time at different bus stops



(a) 8:00-9:00 am

(b) 9:00-10:00 am

Fig. 9. Mean and standard deviation of link travel times

Table 2 Default values of model parameters

Notation	Unit	Value	Description
C	pax/veh	100	The vehicle capacity
-	-	6	The number of trips
-	-	18	The number of bus stops
S	-	100	The number of simulation runs for each sample
-	min	5	Departure headway
φ_1	\$/h	15	The value of passenger waiting time
φ_2	\$/h	10	The value of passenger in-vehicle time
φ_3	\$/h	150	The value of bus operating time
α	s	2	The average alighting time per passenger
b	pax/min	15	Boarding rate
δ	min	0.3	Deceleration and acceleration time for a bus at a stop
τ	min	0.3	Minimal headway for the case without overtaking

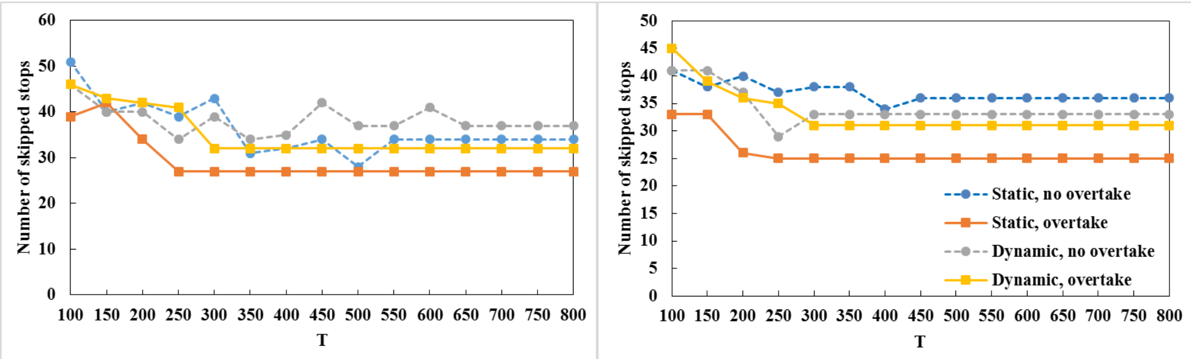
5.2 Results and discussion

5.2.1 Impact of dynamic demand and overtaking

To examine the impact of overtaking and dynamic demand on the bus stop-skipping scheme, in this section the results of the enhanced model (i.e., with overtaking and dynamic demand) are compared to the base case without either overtaking or dynamic demand. With regard to the base case without dynamic demand (or namely, static demand), the hourly arrival rate of passengers is assumed to be constant, which is calculated as the total number of arriving passengers within one hour divided by the time interval (60 min if the unit for arrival rate is pax per minute). The assumption of uniform arrival rate is common practice in the literature (e.g., Liu et al., 2013; Chen et al., 2015; Wu et al., 2017).

Because in our example the number of stops along the route is $N = 16$, and the number of bus trips is $M = 6$, then the number of combinations of stop-skipping scheme is $2^{N \cdot M} = 2^{96}$ (Proposition 1). The programs are implemented in Matlab 2014a on an Intel(R) Core(TM) i5-5200U CPU @ 2.20 GHz with 8GB RAM PC. The evaluation for each sample lasts for approximately 70 seconds. Hence, it is nearly impossible to obtain the optimal results in a reasonable time using enumeration. Using only an initial sample and corresponding values of objective function, the surrogate models can be used as an approximation of the expensive-to-evaluate objective.

Based on the simulation results, we found that the surrogate models statistically pass the goodness-of-fit tests. The fit performance and the fitted quadratic polynomial models in 8:00-9:00 am are illustrated in Appendix A and Appendix B. According to the fitted surrogate models, we obtain the optimal stopping plans that minimize the expected total cost. Keeping the variance of total cost being lower than a given threshold T, we solve the problem Eq. (38) via varying the threshold to explore a set of solutions to estimate the Pareto frontier. The positive infinity threshold corresponds to the pure cost-efficient solution.



(a) 8:00-9:00 am (b) 9:00-10:00 am

Fig. 10. Number of skipped stops under different risk thresholds for different schemes

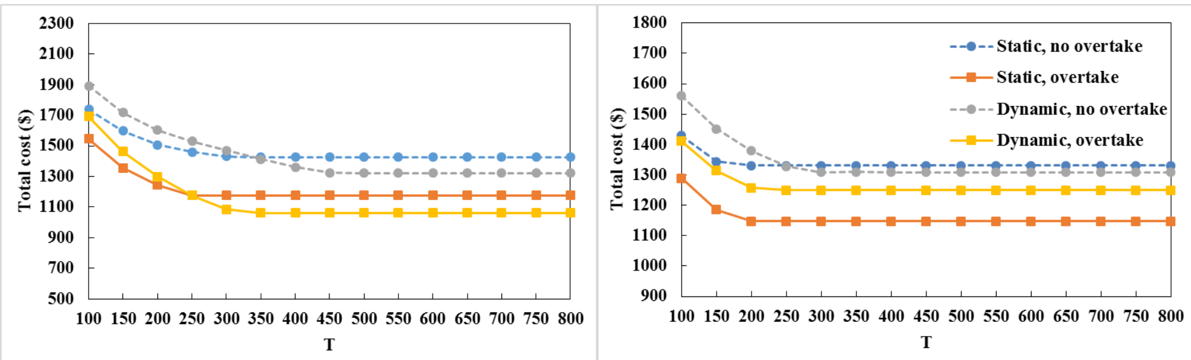
Fig. 10 presents the number of skipped stops under different risk threshold values T. The threshold corresponds to the maximal value of standard deviation. We observe that the number of skipped stops and stop patterns of four cases (i.e., static demand with overtaking; dynamic demand with overtaking; static demand without overtaking; dynamic demand without overtaking) are quite distinctive. This suggests that neglecting overtaking and demand dynamics may lead to potential planning errors. In particular, the number of skipped stops with overtaking is generally smaller than that without overtaking. This results from cost trade-offs between passengers and operators. Evidently, serving more stops could reduce the average passenger waiting time cost at the expense of increasing operation cost. Because overtaking reduces the moving bottleneck effect along the route and allows for running considerably more frequent service, the in-vehicle travel time will be less affected by serving more stops in the case with overtaking. Consequently, fewer skipped stops can be expected to reduce the passengers' waiting time and total cost.

In addition, the number of skipped stops with dynamic demand is generally smaller than that of static demand, particularly under high risk threshold. This suggests that, when dynamic demand is incorporated, fewer skipped stops are required to reduce the passengers' waiting time. This also indicates that the assumption of static demand is likely to undermine the "real" demand volume. Table 3 shows the stop patterns for four schemes corresponding to the threshold $T = 100$, where "1" and "0" denote the stops being served or skipped by each vehicle, respectively. This reinforces the message that the stop patterns of four cases are quite distinct.

Table 3 Stop patterns of four schemes ($T = 100$)

Trip	Static, no overtake	Static, overtake	Dynamic, no overtake	Dynamic, overtake
	8:00-9:00 am			
1	1110011010111000	1111000110110110	1111001011011111	111100000 1001101

2	1100001101001100	0001111011101000	1100001000111011	1110010001110100
3	1100111110010110	1110111001101101	1100010111010001	1101011000011111
4	1011001011010000	0110010011110100	1011000111110110	1100010100011110
5	1010101001000100	1101100100101111	1011100011101001	1110000001011001
6	1001001001000110	0011101111110100	1100110010011100	0010110100101111
		9:00-10:00 am		
1	1111111110010010	1111111111111001	1111010100001100	1111101000011011
2	1101100101111110	0010010001001100	1100010001110000	1110111001001001
3	1101001101100001	1111111111111110	1011101000110111	1101100110110101
4	1011110000010011	0111010010110101	1011001010011001	1001001110010110
5	1011010000111000	1101110110100101	1011000111000010	1111001110000111
6	0111001011101110	0100011101011111	1101000011101000	0000100000011001



(a) 8:00-9:00 am (b) 9:00-10:00 am

Fig. 11. Pareto-optimal solutions of total costs for different scenarios

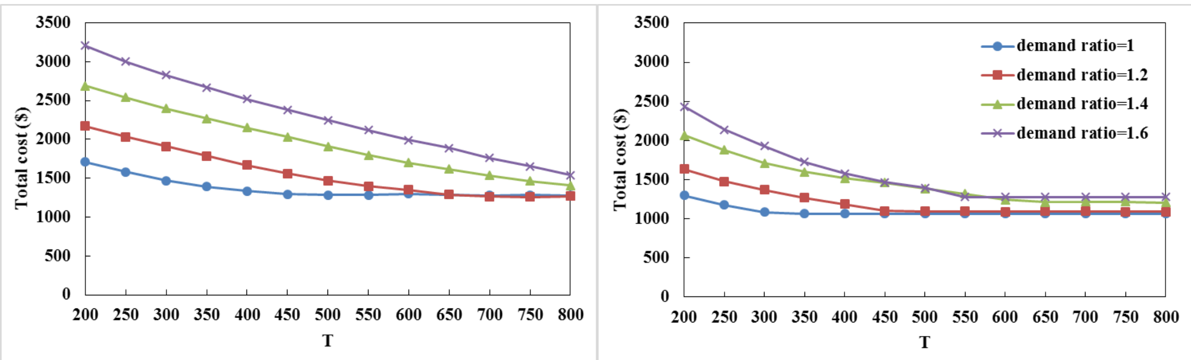
Fig. 11 presents the total costs for four cases under various threshold T. It can be seen that the total cost decreases with the increase of T but at a decreasing rate, and they do not change after reaching a critical value. The reason is that the variance could be analogous as the risk of robust schedule, and higher risk indicates smaller cost, which validates the model properties as discussed in Section 3.5. This indicates that there exists a trade-off between the smallest cost and the lowest risk in the design of stop-skipping scheme, which provides a practical insight that, to achieve a small expected cost, we should endure a relatively large variance. More importantly, the system costs with overtaking are smaller than those of without overtaking.

In addition, the costs with dynamic demand are generally higher than those with static demand, particularly under low risk threshold. This indicates that ignoring the demand dynamics may overestimate the benefits of limited-stop service. The reason is that, in a static model, the excess demand and the resulting overcrowding and failed boarding may be overlooked, which leads to an underestimation of average waiting time and dwell time. Since the in-vehicle travel time and operation cost are closely related to the dwell times, higher costs could be expected in a dynamic demand framework. Therefore, to be more realistic, we next analyse the performance of the limited-stop service under dynamic demand. The period of interest is 8:00-9:00 am.

5.2.2 Sensitivity to demand

Demand is an important factor influencing the vehicle loads and system performance. Fig. 12 presents the total costs under different demand levels, distinguished by overtaking and no overtaking. As expected, the total costs with overtaking are consistently lower than those without overtaking under the same demand level. The total costs decrease as the increase of risk threshold T before reaching a critical value. However, marginal increase of demand contributes less to the cost growth. The gaps between different demand levels tend to decrease as T increases.

In comparison, the total cost appears to be less sensitive to demand variations in the case of overtaking, particularly when the risk threshold is sufficiently high. The reason is that when the passenger demand grows, the vehicles tend to be overcrowded at some stops. Such side effect would be amplified when the vehicle is blocked by its preceding vehicle, which leads to uneven passenger distribution along the route and under-utilization of vehicles. However, when overtaking is allowed, the buses will be able to serve more waiting passengers and smooth the load along the route, such that the available in-vehicle space could be utilized more efficiently by offering the same bus supply. This also explains why the savings from overtaking generally get larger with higher demand (Fig. 13).



(a) No overtake (b) Overtake

Fig. 12. Pareto-optimal solutions of total costs under various demand levels

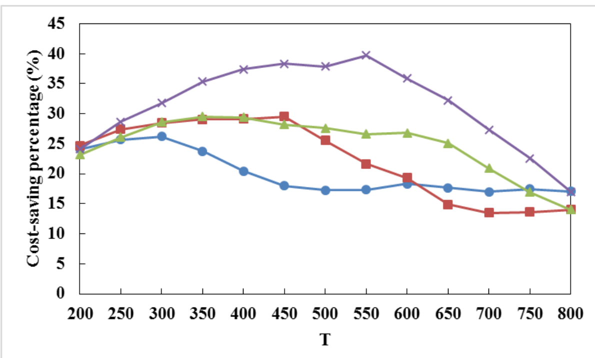


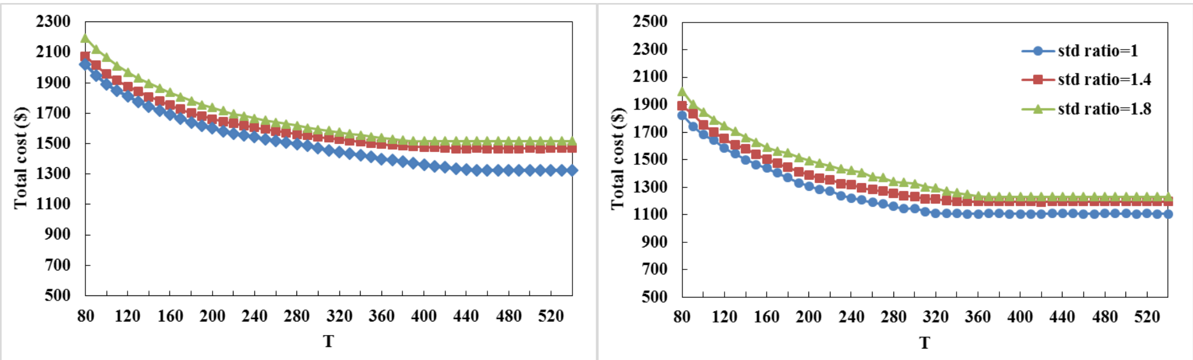
Fig. 13. Pareto-optimal solutions of cost-saving percentages from overtaking operation under various

demand levels

Alternately, a higher risk threshold indicates a larger solution space and enhanced schedule flexibility. Typically, the positive infinity threshold corresponds to the pure cost-efficient solution. As a result, when the risk threshold is larger, less influence of demand variation can be expected (Fig. 12).

5.2.3 Sensitivity to travel time variability

As discussed, travel time uncertainty is prevalent during transit operation. The purpose of this section is to investigate the performance improvement resulting from overtaking operation under various levels of travel time variability. The travel time variability can be represented by the standard deviation of link travel time. To this end, the base standard deviation is amplified by the ratios 1.4 and 1.8. The performance improvement is defined as the relative errors between the results with and without overtaking. As shown in Fig. 14, when bus overtaking is allowed, the resulting performance is always better than that without overtaking. The cost savings generally increase as the threshold T increases; when the threshold T is large enough, the cost savings become trivial. The total cost is less sensitive to travel time variability when overtaking is allowed, which suggests that overtaking can mitigate the adverse effects of travel time variations.



(a) No overtake (b) Overtake

Fig.14. Pareto-optimal solutions of total costs under various travel time variability

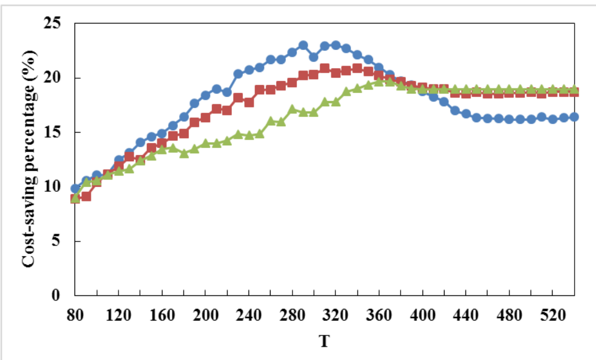
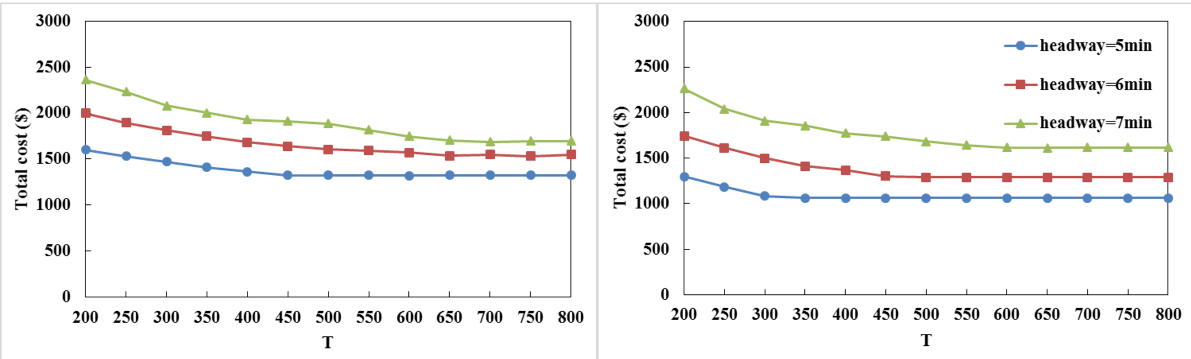


Fig.15. Pareto-optimal solutions of cost savings from overtaking operation under various travel time variability

As shown in Fig. 15, when the risk threshold is larger than a critical value (380 in this example), more cost savings can be observed as the travel time variability increases. There are two possible reasons for this: First, bus bunching is more likely to occur with higher travel time variability, under which circumstance overtaking becomes more effective. Because vehicle overtaking is a contributor to reducing the passengers' waiting time and in-vehicle travel time (Wu et al., 2017), more cost savings could be expected with higher travel time variability. Second, as discussed in Section 5.2.2, a higher risk threshold indicates a larger solution space and enhanced schedule flexibility. Thus, when the risk threshold is larger, the improvement from overtaking and travel time variability becomes more significant.

5.2.4 Sensitivity to headways



(a) No overtake (b) Overtake

Fig.16. Pareto-optimal solutions of total costs under different headways

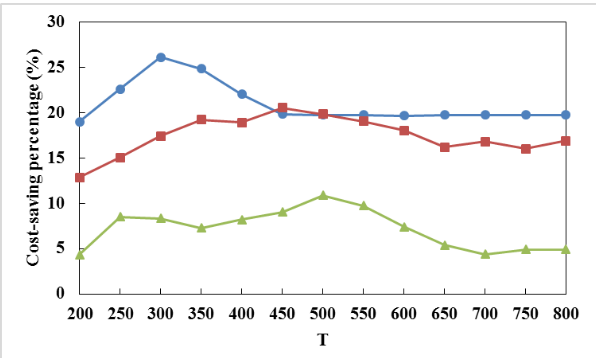


Fig. 17. Pareto-optimal solutions of total cost savings from overtaking operation under different headways

Departure headway or frequency setting is a main activity in the tactical planning of bus operation. In this section, we investigate the effect of departure headway size on system performance. To isolate from the effect of demand congestion, the base headway is amplified by 1 and 2 min. The results for total costs are shown in Fig. 16. Similar to the effect of travel time variability, the total cost decreases with the increment of the threshold T before reaching a critical value. As shown in Fig. 17, the cost savings generally decrease with longer headway. This is because, with longer headway, it is more difficult for the bus trips with different stop patterns to catch up with each other, which reduces the likelihood of bunching and overtaking. This implies that, to achieve the benefit of overtaking in the limited-stop bus service, the departure headways should be carefully determined and set as short as possible. These results also suggest that the limited-stop service is preferably applied in high-frequency transit service.

6. Concluding remarks

Limited-stop bus service can improve the operational efficiency while also aggravating headway irregularity and bus overtaking effect. However, the effects of vehicle overtaking and dynamics on limited-stop bus service, and the issue of schedule robustness, have not previously been fully addressed. This paper presents a stop-skipping optimization model considering both vehicle overtaking and dynamics in the context of both robust and cost-efficient fleet allocation. To cope with the expensive-to-evaluate objective function, we propose an efficient solving method to this stop-skipping design problem: simulation-based optimization framework integrating the response surface methodology (RSM), which only needs to fit an initial input-output dataset before solving to optimality. In particular, according to the characteristics of this problem, we propose an effective coding method to handle the binary decision variables for adapting to the RSM.

The model is implemented in a real-world bus line in Guangzhou, China. With different uncertainty levels, we investigate the Pareto frontier for robust solutions. It shows that the stop patterns are quite distinctive when overtaking and dynamics are considered, particularly under low uncertainty levels. In particular, the number of skipped stops with overtaking is generally lower than that without overtaking as a result of cost trade-offs between passengers and operators. We also found that the system cost can be reduced as a result of overtaking operation, under which circumstance higher travel time variability and shorter departure headway are more beneficial. Therefore, these findings indicate that ignoring vehicle overtaking and dynamics for the stop-skipping problem would induce idle capacity in the transit system, including higher operation costs and additional fleet size, thus underestimating the efficacy of the stop-skipping scheme and leading to potential planning errors. The results revealed in this study could help transit operators determine the best trade-off between fleet allocation and the level of robustness in the design of limited-stop bus service.

In subsequent research, an extended list of operational strategies may be developed considering more practical constraints. For example, while this paper focuses on the planning of stop patterns for bus trips, future research could be extended to joint optimization of departure headways and stop patterns. In addition, this study focuses on the planning for a single bus route, and thus it would be interesting to extend the model under common-line conditions considering passengers' route choice behaviour.

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Appendix A. Validation and test of the surrogate models

To test the accuracy of the surrogate models, the leave-one-out cross-validation method is used for the initial sample data (Chen et al., 2014). Specifically, the i-th input variable point is deleted one by one, and then other quadratic polynomial models are fitted, leaving out the i-th input variable point. Based on this, the i-th output variables is estimated using the fitted surrogate models. Then, the primary simulation results are compared to the estimated results. The overall performance of the surrogate models is evaluated using three accuracy measures:

1. Mean absolute percent error (MAPE)

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{f(x^{(i)})-\hat{f}(x^{(i)})}{f(x^{(i)})}\right| \times 100 \quad (44)$$

2. Mean absolute error (MAE)

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|f\left(x^{(i)}\right) - \hat{f}\left(x^{(i)}\right)\right| \quad (45)$$

3. Root mean square error (RMSE)

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[f(x^{(i)}) - \hat{f}(x^{(i)})\right]^2} \quad (46)$$

where $x^{(i)}$ denotes the i-th input variable point; $f\left(x^{(i)}\right)$ and $\hat{f}\left(x^{(i)}\right)$ denote the simulation output and predicted result of the i-th experiment point, respectively. N denotes the number of initial sample data points.

Table 4 Prediction performance of the models using the leave-one-out cross-validation method

Scenarios	MAPE (%)	MAE	RMSE
No overtake, static	2.57	58.7	76.1
Overtake, static	2.21	47.2	60.3
No overtake, dynamic	2.75	65.7	83.5
Overtake, dynamic	2.76	62.3	78.6

Table 4 presents the prediction performance of expected total costs of the training samples under four scenarios. MAPEs are within the range of 2.2% - 2.8%. MAEs are within 58-66$, and RMSEs are within 60-84$. The results under static demand are slightly better than those under dynamic demand. The possible reason is that the incorporation of demand dynamics complicates the objective function and produces more simulation noise.

To further verify the generalization ability of the surrogate models, 20 samples of stopping patterns (instances) are randomly generated for testing. Then, the corresponding predicted total costs are compared to the costs measured using simulation experiments, and the results are shown in Table 5. As we can see, MAPEs are within the range of 3.1% - 4.9%. The prediction performance under the dynamic scenarios, which is comparable to that under the static scenarios, is acceptable from a practical viewpoint. The results on the test set verify the model's generalization ability. Therefore, we can conclude that the developed surrogate models are quite adequate for use as a decision support tool.

Table 5 Prediction performance for the test set

Scenarios	MAPE	MAE	RMSE
No overtake, static	3.72	82.30	92.24
Overtake, static	4.20	92.53	104.90
No overtake, dynamic	3.14	75.03	84.71
Overtake, dynamic	4.85	112.30	131.93

Appendix B. Fitting results of the quadratic polynomial models

Because the stopping patterns of 6 bus trips are considered in our example, there are in total 28 coefficients to be calibrated in the quadratic polynomial model for each scenario. Table 6 presents the coefficients of determination R^2 for linear and quadratic polynomial models. As we can see, the coefficients of determination R^2 of quadratic models are significantly higher than those of the linear models, with the values of higher than 0.85. This suggests that the quadratic relationship is significant. Table 7 shows the coefficients of the fitted linear and quadratic polynomial models for all scenarios.

Table 6 Coefficients of determination R^2 of linear and quadratic models

Models	Scenarios			
	No overtake, static	Overtake, static	No overtake, dynamic	Overtake, dynamic
Linear	0.2116	0.5041	0.2505	0.4561
Quadratic	0.8515	0.8979	0.8490	0.8666

Table 7 Coefficients of the fitted quadratic polynomial models

Coefficients	Scenarios			
	No overtake, static	Overtake, static	No overtake, dynamic	Overtake, dynamic
β_0	3619.03	3573.19	4010.77	4017.50
β_1	0.021095	0.024584	0.018606	0.025136
β_2	-0.019337	-0.021738	-0.018059	-0.026205
β_3	-6.64227$\times 10^{-3}$	-6.68111$\times 10^{-3}$	7.98180$\times 10^{-4}$	-7.53763$\times 10^{-3}$
β_4	-0.019877	-0.018270	-0.027800	-0.024313
β_5	-0.023874	-0.021377	-0.024725	-0.019333
β_6	-0.025222	-0.025993	-0.034742	-0.032364
β_{12}	-7.60959$\times 10^{-10}$	4.58322$\times 10^{-8}$	2.26425$\times 10^{-10}$	8.00105$\times 10^{-8}$

β_{13}	-1.06670$\times 10^{-8}$	-8.32594$\times 10^{-9}$	1.35173$\times 10^{-8}$	-8.33402$\times 10^{-9}$
β_{14}	-2.53754$\times 10^{-8}$	-2.37064$\times 10^{-8}$	-3.81597$\times 10^{-9}$	-3.74093$\times 10^{-8}$
β_{15}	-7.43366$\times 10^{-9}$	3.17937$\times 10^{-10}$	-1.48660$\times 10^{-8}$	6.27096$\times 10^{-9}$
β_{16}	8.54853$\times 10^{-9}$	9.47916$\times 10^{-9}$	5.13306$\times 10^{-9}$	1.91554$\times 10^{-8}$
β_{23}	1.38905$\times 10^{-7}$	1.39855$\times 10^{-7}$	1.14996$\times 10^{-7}$	1.60785$\times 10^{-7}$
β_{24}	-7.16856$\times 10^{-8}$	-9.78192$\times 10^{-8}$	-5.44519$\times 10^{-8}$	-1.19619$\times 10^{-7}$
β_{25}	-1.46139$\times 10^{-8}$	-3.27625$\times 10^{-9}$	1.62544$\times 10^{-8}$	-1.50083$\times 10^{-8}$
β_{26}	3.02200$\times 10^{-8}$	3.85789$\times 10^{-8}$	5.06386$\times 10^{-8}$	4.95962$\times 10^{-8}$
β_{34}	-1.05533$\times 10^{-7}$	-5.02707$\times 10^{-8}$	-1.43158$\times 10^{-7}$	-5.98435$\times 10^{-8}$
β_{35}	-2.77933$\times 10^{-8}$	-3.20454$\times 10^{-8}$	-2.34374$\times 10^{-8}$	-4.91650$\times 10^{-8}$
β_{36}	-1.32936$\times 10^{-8}$	2.65501$\times 10^{-8}$	1.25465$\times 10^{-8}$	-9.59836$\times 10^{-9}$
β_{45}	1.08226$\times 10^{-7}$	6.95829$\times 10^{-8}$	1.36119$\times 10^{-7}$	9.69563$\times 10^{-8}$
β_{46}	1.12103$\times 10^{-8}$	4.51008$\times 10^{-9}$	2.74252$\times 10^{-8}$	-3.05906$\times 10^{-9}$
β_{56}	1.32352$\times 10^{-7}$	1.87377$\times 10^{-7}$	1.42443$\times 10^{-7}$	2.29639$\times 10^{-7}$
β_{11}	-3.27803$\times 10^{-7}$	-4.28119$\times 10^{-7}$	-3.24379$\times 10^{-7}$	-4.70645$\times 10^{-7}$
β_{22}	1.87314$\times 10^{-7}$	2.04357$\times 10^{-7}$	1.46632$\times 10^{-7}$	2.40262$\times 10^{-7}$
β_{33}	7.97491$\times 10^{-8}$	4.71825$\times 10^{-8}$	-4.50024$\times 10^{-9}$	6.96232$\times 10^{-8}$
β_{44}	3.49590$\times 10^{-7}$	3.54365$\times 10^{-7}$	4.38521$\times 10^{-7}$	4.60948$\times 10^{-7}$
β_{55}	1.21350$\times 10^{-7}$	5.32389$\times 10^{-8}$	6.58310$\times 10^{-8}$	-3.20653$\times 10^{-8}$
β_{66}	2.46440$\times 10^{-7}$	2.17745$\times 10^{-7}$	3.39132$\times 10^{-7}$	2.75295$\times 10^{-7}$

REFERENCE

Ceder, A., Stern, H.I., 1981. Deficit function bus scheduling with deadheading trip insertion for fleet size reduction. Transportation Science, 15(4), 338-363.

Chen, J., Liu, Z., Zhu, S., Wang, W., 2015. Design of limited-stop bus service with capacity constraint and stochastic travel time. Transportation Research Part E, 83, 1-15.

Chen, J., Liu, Z., Wang, S., Chen, X., 2018. Continuum approximation modeling of transit network design considering local route service and short-turn strategy. Transportation Research Part E, 119, 165-188.

Chen, X., Hellinga, B., Chang, C., Fu, L., 2015. Optimization of headways with stop-skipping control: a case study of bus rapid transit system. Journal of Advanced Transportation. 49, 385-401.

Chen, X., Zhang, L., He, X., Xiong C., 2014. Surrogate-based optimization of expensive-to-evaluate objective for optimal highway toll charges in transportation network. Computer-aided Civil and Infrastructure Engineering, 29, 359-381.

Chen, X., Xiong, C., He, X., Zhu, Z., Zhang L., 2016. Time-of-day vehicle mileage fees for congestion mitigation and revenue generation: A simulation-based optimization method and its real-world application. Transportation Research Part C, 63, 71-95.

Eberlein, X.J., Wilson, N.H.M., Barnhart, C., Bernstein, D., 1998. The real-time deadheading problem in transit operations control. Transportation Research Part B, 32(2), 77-100.

Freyss, M., Giesen, R., Munoz, J.C., 2013. Continuous approximation for skip-stop operation in rail transit. Transportation Research Part C, 36, 419-433.

Fonzone, A., Schmöcker, J-D., Liu, R., 2015. A model of bus bunching under reliability based passenger arrival patterns. Transportation Research Part C, 59, 164-182.

Furth, P.F., 1986. Zonal route design for transit corridors. Transportation Science, 20(1), 1-12.

Furth, P.G., 1987. Short turning on transit routes. Transportation Research Record, 1108, 42-52.

Furth, P.G., Day, F.B., 1985. Transit routing and scheduling strategies for heavy demand corridors. Transportation Research Record, 1o11, 23-26.

Furth, P.G., 1985. Alternating deadheading in bus route operations. Transportation Science, 19(1), 13-28.

Gordon, J., Koutsopoulos, H., Wilson, N., Attanucci, J., 2013. Automated inference of linked transit journeys in London using fare-transaction and vehicle location data. Transportation Research Record: Journal of the Transportation Research Board, 17-24.

Hadas, Y., Ceder, A., 2010. Optimal coordination of public-transit vehicles using operational tactics examined by simulation. Transportation Research Part C, 18, 879-895.

Nesheli M.M, Ceder A. 2014. Optimal combinations of selected tactics for public-transport transfer synchronization. Transportation Research Part C, 48, 491-502.

Jung, J., Sohn, K., 2017. Deep-learning architecture to forecast destinations of bus passengers from entry only smart-card data, 11(6), 334-339.

Kleijnen, J.P.C., 2008. Tilburg University, In: 2008-70, N. (Ed.), Working Paper Serieses. Tilburg University, Thlburg, The Netherlands.

Larrain, H., Muñoz, J.C., Giesen, R, 2015. Generation and design heuristics for zonal express services. Transportation Research Part E, 79, 201-212.

Larrain, H., Muñoz. J.C., 2016. When and where are limited-stop bus services justified? Transportmetrica A, 12(9), 811-831.

Leiva, C., Muñoz, J.C., Giesen, R., Larrain, H., 2010. Design of limited-stop services for an urban corridor with capacity constraints. Transportation Research Part B, 44, 1186-1201.

Li, S., Liu, R., Yang, L., Gao, Z., 2019. Robust bus controls considering delay disturbances and passenger demand uncertainty. Transportation Research Part B, 123, 88-109.

Liu, R., Sinha, S., 2007. Modelling urban bus service and passenger reliability. Paper presented at the International Symposium on Transportation Network Reliability, The Hague, July 2007.

Liu, Z., Yan, Y., Qu, X., Zhang, Y., 2013. Bus stop-skipping scheme with random travel time. Transportation Research Part C, 35, 46-56.

Milla, F., Cortés, C.E., Cipriano, A., 2012. Bus-stop control strategies based on fuzzy rules for the operation of a public transport system. IEEE Transactions on Intelligent Transportation Systems, 13(3), 1394-1403.

Moreira-Matias, L., Cats, O., Gama, J., Mendes-Moreira, J., Sousa, J., 2016. An online learning approach to eliminate bus bunching in real-time. Applied Soft Computing, 47, 460-482.

Sun, A., Hickman, M., 2005. The real-time stop-skipping problem. Journal of Intelligent Transportation System, 9(2), 91-109.

Sun, L., Tirachini, A., Axhausen, K. W., Erath, A., and Lee, D. H., 2014. Models of bus boarding and alighting dynamics. Transportation Research Part A, 69, 447-460.

Sorratini, J., Liu, R. and Sinha, S., 2008. Assessing bus transport reliability using micro-simulation. Transport Planning & Technology, 31(3), 303-324.

Soto, G., Larrain, H., Muñoz. J.C., 2017. A new solution framework for the limited-stop bus service design problem. Transportation Research Part B, 105, 67-85.

Wang, D.Z.W., Nayan, A., Szeto, W.Y., 2018. Optimal bus service design with limited stop services in a travel corridor. Transportation Research Part E, 111, 70-86.

Wu, W., Liu, R., Jin, W., 2016. Designing robust schedule coordination scheme for transit networks with safety control margins. Transportation Research Part B, 93, 495-519.

Wu, W., Liu, R., Jin, W., 2017. Modelling bus bunching and holding control with vehicle overtaking and distributed passenger boarding behaviour. Transportation Research Part B, 104, 175-197.

Wu, W., Liu, R., Jin, W., 2018. Integrating bus holding control strategies and schedule recovery: Simulation-based compassion and recommendation. Journal of Advanced Transportation, Volume 2018, Article ID 907801, 13 pages.

Wu, W., Liu, R., Jin, W., Ma, C., 2019. Stochastic bus schedule coordination considering demand assignment and rerouting of passengers. Transportation Research Part B, 121, 275-303.

Xiong, C., Zhu, Z., Chen, X., Zhang, L., 2018. Optimal travel information provision strategies: an agent-based approach under uncertainty. Transportmetrica B, 6(2), 129-150.

Yalçınkaya, Ö., Bayhan, G.M., 2009. Modelling and optimization of average travel time for a metro line by simulation and response surface methodology. European Journal of Operational Research, 196(1), 225-233.

Yao, B., Hu, P., Lu, X., Gao, J., Zhang, M., 2014. Transit network design based on travel time reliability. Transportation Research Part C, 43, 233-248.

Yu, B., Lam, W.H.K., Tam, M., 2011. Bus arrival time prediction at bus stop with multiple routes. Transportation Research Part C, 19, 1157-1170.

Yu, B., Yang, Z., Jin, P., Wu, S., Yao, B., 2012a. Transit route network design-maximizing direct and transfer demand density. Transportation Research Part C, 22, 58-75.

Yu, B., Yang, Z., Li, S., 2012b. Real-time partway deadheading strategy based on transit service reliability assessment. Transportation Research Part A, 46(8), 1265-1279.

Zhang, W. Xu, W., 2017. Simulation-based robust optimization for the schedule of single-direction bus transit route: The design of experiment. Transportation Research Part E, 106, 203-230.

Zheng, L., Xue, X., Xu, C., Ran, B., 2019. A stochastic simulation-based optimization method for equitable and efficient network-wide signal timing under uncertainties. Transportation Research Part B, 122, 287-308.