Modeling strategies for the computational analysis of unreinforced masonry structures: Review and classification

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Abstract

Masonry structures, although classically suitable to withstand gravitational loads, are sensibly vulnerable if subjected to extraordinary actions such as earthquakes, exhibiting cracks even for events of moderate intensity compared to other structural typologies like as reinforced concrete or steel buildings. In the last half-century, the scientific community devoted a consistent effort to the computational analysis of masonry structures in order to develop tools for the prediction (and the assessment) of their structural behavior. Given the complexity of the material, different approaches and scales of representation of the mechanical behavior of masonry, as well as different strategies of analysis, have been proposed. In this paper, a comprehensive review of the existing modeling strategies for masonry structures, as well as a novel classification of these strategies are presented. Although a fully coherent collocation of all the modeling approaches is substantially impossible due to the peculiar features of each solution proposed, this classification attempts to make some order on the wide scientific production on this field. The modeling strategies are herein classified into four main categories: block-based models, continuum models, geometry-based models, and macroelement models. Each category is comprehensively reviewed. The future challenges of computational analysis of masonry structures are also discussed.

Keywords: Masonry mechanics, Numerical modeling, Computational analysis, Seismic response

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1. Introduction

Masonry structures represent a large part of the existing constructions in the world. A great part of the historic architectural heritage consists of monumental masonry structures (buildings, towers, castles, churches, mosques, temples, etc.). Furthermore, ordinary residential buildings are typically made of masonry in several countries. As it can be noted in Figure 1, considerable differences appear between monumental and ordinary buildings, in terms of material, geometry and structural details.

(a) (b)

Figure 1: Examples of (a) monumental and (b) ordinary masonry structures.

It is well known that unreinforced masonry (URM) structures, although classically suitable to withstand gravitational loads, are sensibly vulnerable if subjected to extraordinary actions such as earthquakes. Indeed, the structural response to this kind of actions is often characterized by the arising of cracks in the masonry and/or partial (or even full) collapses even for seismic events of moderate intensity if compared to other structural typologies like as reinforced concrete or steel buildings. Given the heterogeneity of masonry, made of blocks usually bonded with mortar, cracks usually run along the mortar joints, even if the case of cracked blocks is possible as well depending on the relative strength properties of the two basic components (i.e. mortar and blocks). Indeed, alternative solutions to the unreinforced one have been developed over the centuries, aimed at improving the properties of ductility and dissipation as well as the strength, as the confined or reinforced masonry. Despite that, the paper focuses only to the unreinforced masonry solution.

In the last half-century, the scientific community devoted a consistent effort to the computational analysis of masonry structures. The main objective at the basis of this topic is that, if a mechanical model is found to be able to simulate the structural response of masonry structures, it can be used to predict the structural response to extraordinary loads and, therefore, to evaluate the main weaknesses and safety of a masonry building. This approach, although new masonry buildings can be designed and computationally analyzed, has been mainly oriented to the assessment of the near-collapse behavior of existing masonry buildings, given their widespread dissemination and their weak structural response.

However, given the deep complexities and uncertainties which characterize the geometry of buildings (especially for the historic ones) and the mechanical response of masonry, the computational analysis of masonry structures is still a challenging task.

In this paper, a comprehensive review of the existing modeling strategies for masonry structures is presented and a classification of these strategies is proposed. This classification of modeling strategies for masonry structures consists of the four following categories (Figure 2): block-based models (BBM), continuum models (CM), geometry-based models (GBM), and macroelement models (MM). This classification, although a fully coherent collocation of all the modeling approaches is substantially impossible due to the peculiar features of each solution proposed, attempts to make some order on the wide scientific production on this field.

Firstly, the main mechanical and geometrical challenges of masonry structures are briefly discussed in Section 2. Then, the limitations and possibilities of analysis approaches (i.e. evolutionary analysis and limit analysis) for masonry structures are pointed out in Section 3. The proposed classification of modeling strategies for masonry structures is presented in Section 4. Each category is then comprehensively reviewed (BBM in Section 5, CM in Section 6, GBM in Section 7, and MM in Section 8) and the limitations and possibilities of each strategy are deeply discussed. In the conclusions (Section 9), a summary of the pros and cons and of the fields of application of each category is given and a discussion on future challenges of computational analysis of masonry structures is held.

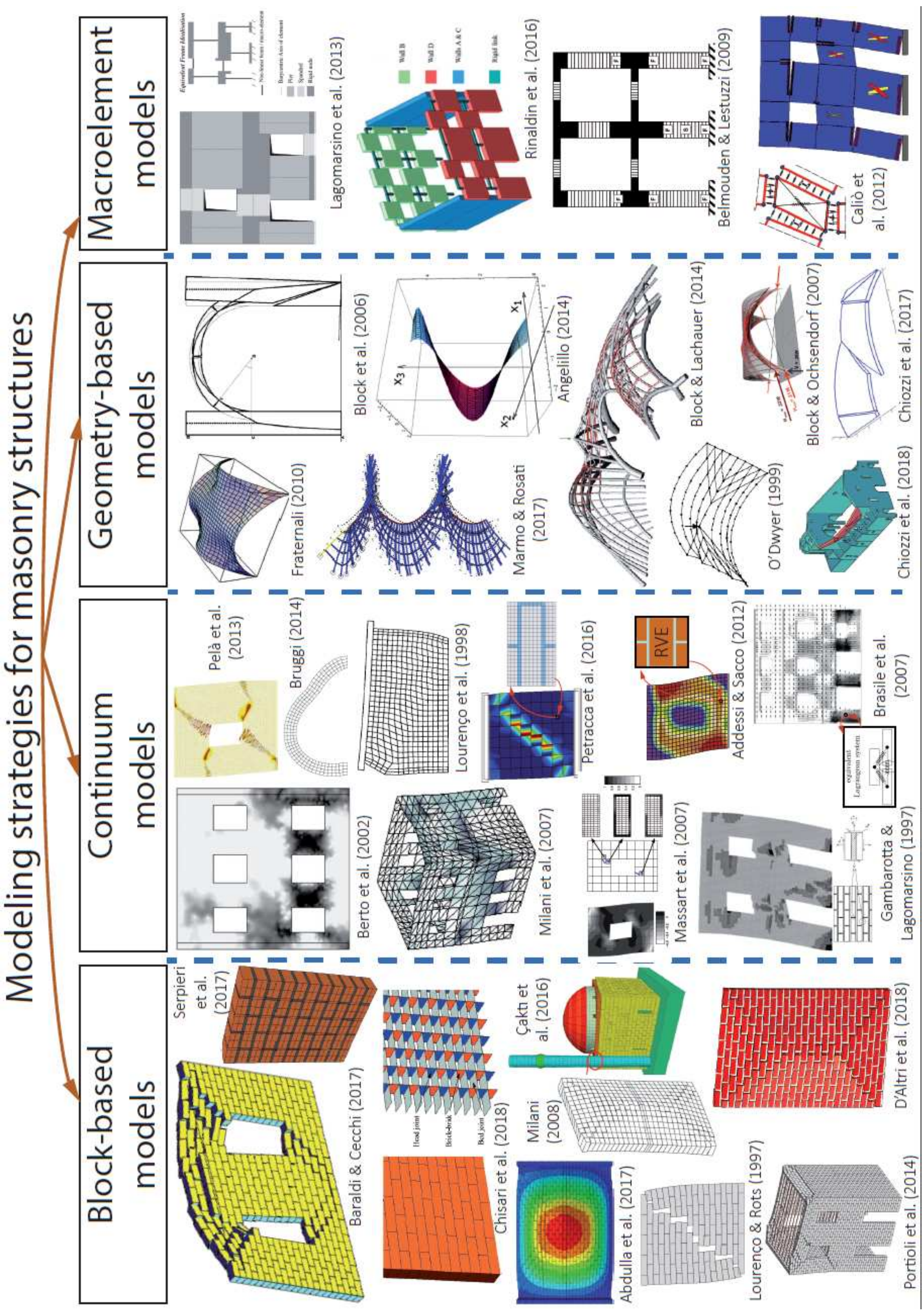


Figure 2: Modeling strategies for masonry structures.

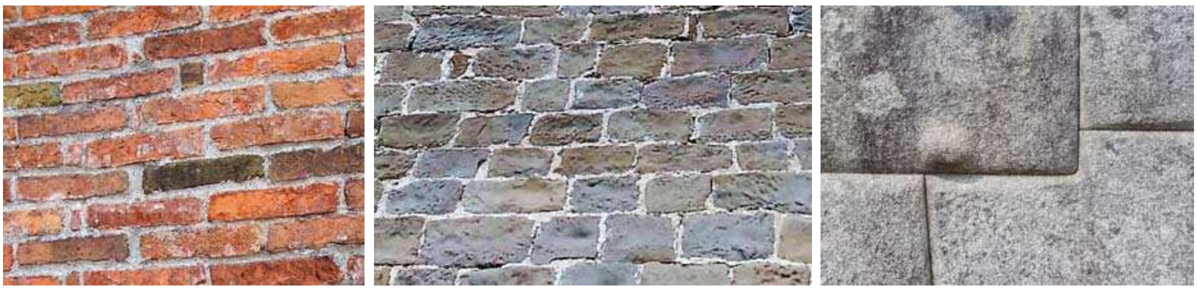
2. Mechanical and geometrical issues

A reliable simulation of the mechanical response of an existing masonry structure should be based on reliable mechanical properties characterized through experimental tests and on detailed geometrical and structural surveys.

This section aims to briefly highlight the main mechanical and geometrical challenges which arise when dealing with masonry structures. Further aspects on this topic can be found in [1, 2].

2.1. Masonry mechanical behavior

Masonry is a very complex material from a mechanical point of view. It is composed of blocks usually bonded with mortar. Blocks are typically made of quasi-brittle materials such as building stones, fired and non-fired bricks. Blocks are assembled with a certain pattern, which is called "bond". This makes masonry an heterogeneous material. As highlighted in [1], the term "masonry" actually refers to a very wide category of building materials (Figure 3), with different mechanical features and peculiarities.



(a) (b) (c)

Figure 3: Examples of masonry: (a) brick masonry, (b) stone masonry and (c) Inca's masonry (dry stone masonry).

The overall masonry response is governed by the mechanical properties of its components (block and mortar) and the bond between them. Masonry components are generally characterized by a quasi-brittle response in tension and compression. In particular, the compressive behavior is characterized by higher values of strength and fracture energy with respect to the tensile behavior. Beyond the nonlinearity showed by the masonry components, the bond between blocks and mortar is usually very weak, characterized by a cohesive-frictional normal stress-dependent behavior in shear and a cohesive behavior in tension (with essentially irrelevant cohesion in case of dry stone masonry) [2]. Therefore, the overall response of masonry is highly nonlinear.

Masonry is an anisotropic material [3]. Anisotropy can be observed in the elastic behavior (elastic anisotropy), in the strength properties (beyond the difference between compressive and tensile strengths, distinctive of quasi-brittle materials, it shows also different strengths along with different directions, i.e. strength anisotropy), and in the post-peak response (brittleness anisotropy). In particular, regular brick masonry usually shows significant anisotropic properties. Conversely, anisotropy in random stone masonry, although a significant difference in compressive and tensile strengths is always observed, could be less significant (e.g. in terms of elasticity, strengths, and brittleness) than in regular brick masonry, given the lack of periodicity in the material.

The interpretation of the mechanical behavior of masonry could be based on different scales, typically the scale of the material [3, 4, 5] and the scale of the structural element [6, 7, 8, 9, 10]. For both cases, the description of the mechanical behavior has to be generally defined in terms of stiffness, strength and ductility. Figure 4 shows the limit strength domains of masonry at the scale of the material (Figure 4(a)) and at the scale of the pier (Figure 4(b)) for plane stress states.

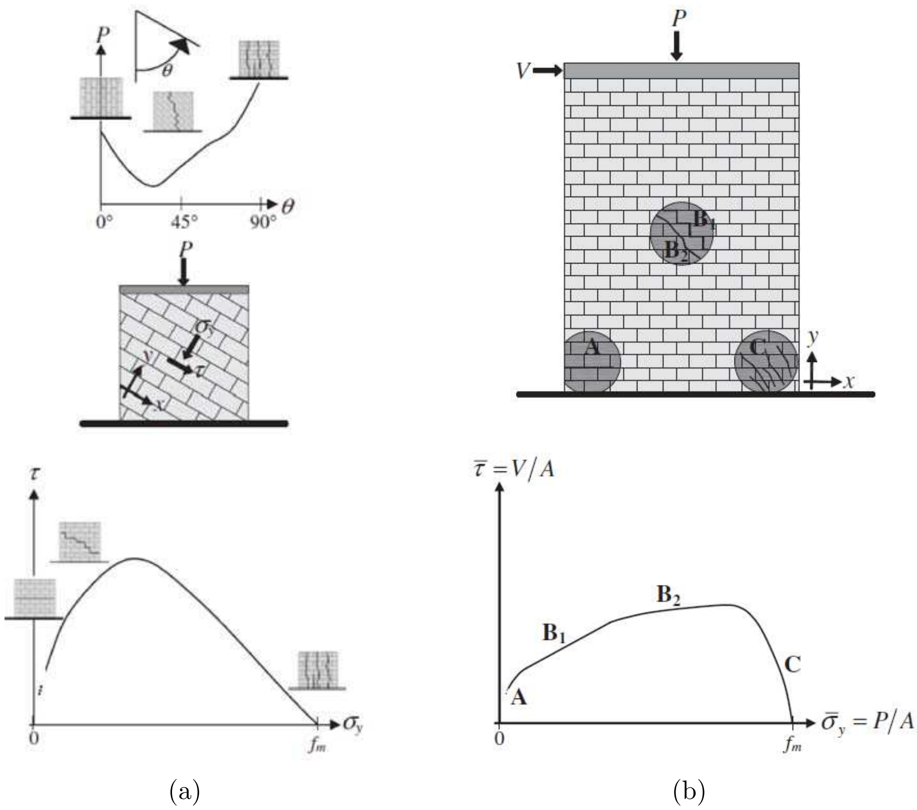
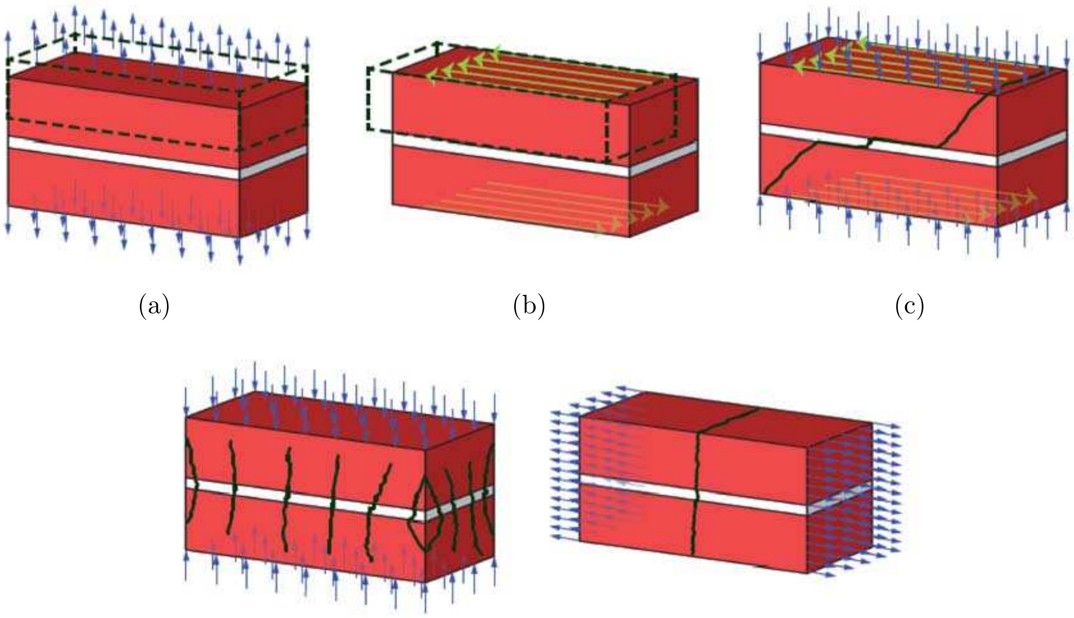


Figure 4: Failure modes and limit domains of masonry: (a) scale of the material and (b) scale of the pier, from [7].

Failure mechanisms in masonry are usually complex and articulated. Typical failure of masonry at a two-block masonry assemblage scale are sketched in Figure 5. At a structural scale, some examples of masonry failure are depicted in Figure 6.



(a) (b) (c)

(d) (e)

Figure 5: Masonry failure mechanisms (at a two-block masonry assemblage scale, from [11]): (a) block-mortar bond tensile failure, (b) block-mortar bond shear sliding, (c) diagonal masonry cracking, (d) masonry crushing, and (e) block and mortar tensile cracking.

2.2. Experimental characterization of masonry

The experimental characterization of masonry mechanical properties is still a challenging task. Indeed, although several experimental tests and set-ups have been proposed in the last decades, their reliability and reproducibility are still object of debate [13, 14].

Basically, the experimental characterization of masonry could be done at different scales, as shown in Figure 7: masonry components (block, mortar and block-mortar bond), wallets (small masonry assemblages), panels (real-scale masonry walls), and buildings (full-scale masonry structures).

When dealing with existing masonry buildings, in-situ tests should be used to mechanically characterize the structure [16, 17]. However, in-situ testing is usually characterized by larger difficulties and limitations than laboratory testing. This leads, in general, to greater uncertainties on the characterized mechanical properties. Even, merely non-destructive tests

(a) (b) (c) (d)

Figure 6: Masonry failure mechanisms (at a structural scale): (a) diagonal cracking, (b) sliding, (c) crumbling, and (d) crushing (from [12]).

(a) (b) (c) (d)

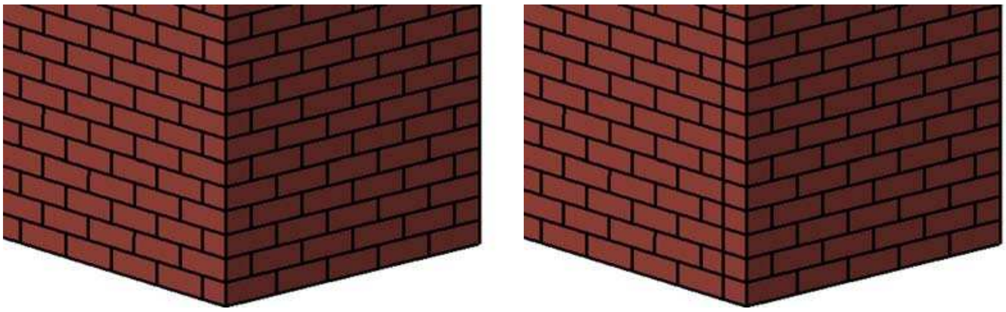
Figure 7: Experimental characterization of masonry at different scales: (a) masonry components testing (from [13]), (b) wallets testing (from [15]), (c) panels testing (from [15]), and (d) building testing (from [15]).

could be used in historic monumental buildings to guarantee their conservation and authenticity [18, 19]. To limit the invasiveness, together with experimental tests, also indirect methods have been proposed in the literature [20] to assign mechanical properties to masonry which are based on a qualitative interpretation of its main features (such as quality of mortar joints, effectiveness of in-plane and transversal interlocking, bond). Anyway, a very limited mechanical information can be generally obtained on this kind of masonry structures.

2.3. Structural details

In masonry structures, structural details play a fundamental role in the mechanical response. Indeed, the toothing between orthogonal walls (Figure 8), the quality of connection with horizontal diaphragms, the flexibility of horizontal diaphragms, the interaction with adjacent buildings, etc., could considerably affect the structural behavior of masonry buildings

[21].



(a) (b)

Figure 8: Example of corner between two orthogonal masonry walls (one leaf running bond walls): (a) toothing texture, and (b) without-toothing texture of the corner.

In general, the structural details also depend on the historical evolution of the building, in terms of restorations, additions of parts, destination changes, damages and repairs, etc. The knowledge of these aspects could be challenging for historic structures, as they are the result of a subsequent superimposition of modifications. Indeed, the setting up of an effective knowledge procedure when dealing with masonry cultural heritage assets is related not only to the cost-benefit optimization (with respect to the reliability of the final outcome), but also to the minimization of invasiveness on the construction, with the aim of its conservation [22]. Beyond the traditional approaches proposed in standards or guidelines for the seismic assessment of existing buildings (e.g. at international levels, Eurocode 8 - Part 3 [23] and ASCE/SEI 41/06) or, more specifically, of heritage structures [24, 25], literature proposals to improve the knowledge phase have been recently developed [26, 27].

2.4. Geometrical challenges

In some cases, the definition of the geometry of the structure could be challenging as well, especially for historic monumental buildings characterized by complex and irregular geometries. In these cases, an accurate geometrical and structural survey is required.

One first issue concerns the identification of the structure (i.e. the load-bearing system) within the building geometry. This non-trivial operation has to be carried out by the analyst basing on the knowledge of the building.

Another issue regards the employability of the geometry in structural analysis purposes. The geometry of these structures can be manually drawn on a computer-aided design (CAD) environment basing on the geometric survey. The CAD-based geometry can be directly used within simplified structural analysis frameworks, such as the one proposed in [28]. However, the employability of this CAD-based geometry in mesh-based structural analysis could be problematic. Indeed, the discretization process of these geometries is usually accompanied by mesh errors, compatibility problems, excessively refined meshes, etc. Several approaches which use as input 3D point clouds for the automatic mesh generation of historic building have been recently proposed in [29, 30, 31, 32] to deal with the aforementioned issues. The development and the optimization of these methods is still an on-going process.

3. Analysis approaches

The collapse or near-collapse response of masonry structures can be investigated following two main ways: (i) evolutionary analyses and (ii) limit analysis-based solutions. In this section, the main features of these two analysis approaches are briefly recalled.

3.1. Evolutionary analyses

In evolutionary analysis procedures, the evolution of the equilibrium conditions of a structure subjected to certain actions is investigated step-by-step. The loading and the structural response are divided into a sequence of intervals or "steps". These procedures allows to account for mechanical nonlinearity, which is fundamental and mandatory to be considered for a reliable assessment of the collapse and near-collapse behavior of masonry structures. Although few examples of linear elastic models have been developed for the preliminary assessment of historic masonry structures [33, 34], their effectiveness in investigating the failure mode and the safety of these structure is substantially limited.

As the aim of these analyses consists in studying the collapse behavior of masonry structures, large displacements could occur and, therefore, geometrical nonlinearity could play a non-marginal role and should be included in the computations.

Evolutionary analyses could be classified in nonlinear static and nonlinear dynamic (time history) analyses:

(i) *Nonlinear static analysis.* In nonlinear static analyses, the structure is step-by-step subjected to certain actions until its critical and post-critical conditions. The pseudo-time in which the structural response evolves does not represent any physical characteristics.

Given the mechanical nonlinearity assumed for the material, nonlinear differential equations have to be solved. These equations can be transformed in nonlinear algebraic equations and solved within a numerical framework. Typically, the nonlinear equations are step-wise linearized and resolved following an iterative procedure. Among the most famous iterative procedures are: the Picard iteration (or direct iteration) method, the Newton-Raphson iteration methods, and the Riks methods (the interested reader is referred to [35] for more information about iterative procedures).

These kind of analyses are typically used to simulate quasi-static experimental tests on masonry structures and to perform the so-called pushover analysis. Pushover analysis is a very common and standardized procedure to assess the seismic behavior of a masonry structure, which is subjected to a monotonically increasing displacement of a control node given a load pattern of horizontal forces kept constant in shape during the analysis.

(ii) *Nonlinear dynamic (time history) analysis.* In nonlinear time history analysis (also called transient nonlinear analysis), the structure is step-by-step subjected to time-dependent actions and the structural response evolves in the actual time, accounting for inertial and damping effects as well.

Time integration methods are employed to approximately satisfy the equations of motion during each time step of the analysis. These methods may be classified as either explicit or implicit [36]. An explicit method is labeled as one in which the new response values calculated at each step depend only on quantities obtained in the previous step. Conversely, in an implicit method the expressions giving the new values for a given step include values which pertain to that same step. Therefore, trial values of the unknowns must be assumed and refined by successive iterations. Among the most famous time integration methods are the following: Euler-Gauss procedure, Newmark Beta methods, second central difference formulation, linear acceleration procedures [36]. In any case, a large body of literature has been written on this topic and the interested reader is referred to [36] for more details.

Nonlinear time history analyses can simulate the effects of dynamic actions (e.g. earthquakes, impacts, explosions, etc.) on masonry structures. Indeed, the possibility to account for time-dependent loads allows to simulate the response of the structure against, for instance, a real accelerogram. Shaking table experimental tests on masonry structures can be analyzed as well. Occasionally, dynamic analysis can be also used for simulating quasi-static tests and processes, by applying, for example, loads in a very slow way.

3.2. Limit analysis-based solutions

Heyman [37] firstly applied limit theorems of plasticity to masonry structures, adopting the following three hypotheses:

(i) masonry has no tensile strength,

(ii) the compressive strength of masonry is infinite,

(iii) sliding of one masonry block upon another cannot occur.

These hypotheses, together with the negligibility of elastic strains, allowed the formulation of the static theorem (lower-bound limit analysis) and the kinematic theorem (upper-bound limit analysis) for masonry structures.

The smart Heyman's rigid no-tension model has been widely used and fruitfully applied in analyzing the stability of masonry systems [38]. Firstly, these assumptions allowed simple graphic statics solutions for the stability analysis of masonry vaults [39], and kinematic analysis of common seismic failure modes of masonry buildings [40]. Secondly, the Heyman's hypotheses established a solid base for the formulation of modern computational limit analysis-based methods. These numerous methods (that will be discussed in the following) are based on either the static theorem [41] or the kinematic theorem [42], and the problem can be formulated as solution of an optimization problem (using or not genetic algorithms), of nonlinear differential equations, of linear or sequential linear programming, etc.

One of the main disadvantages of limit analysis-based solutions consists in the fact that their output is limited to the collapse multiplier and the collapse mechanism, and no information is available on the ultimate displacement and/or post-peak response, which appear fundamental in widely adopted displacement-based seismic assessment procedures for masonry structures.

4. Modeling strategies

In this section, a classification of the modeling strategies for masonry structures is proposed. This classification is focused on the ways masonry and/or masonry structures are modeled. Therefore, the analysis approaches discussed in Section 3 can be, in principle, applied to each modeling strategy category.

Each modeling strategy has some peculiar appealing features, which, in general, could have a specific area of application. Furthermore, depending on the scale of representation conceived in the numerical strategy, different scales of material testing (Figure 7) could be used to calibrate the mechanical parameters of the model, see Section 2.2.

Although each modeling solution which can be found in the scientific literature presents original and peculiar features, making the classification non-trivial and non-fully coherent, our aim consists in trying to make some order on the wide scientific production on this field [43, 44, 45].

The present classification proposes four main categories of modeling strategies for masonry structures (Figure 2):

(i) *Block-based models.* Masonry is block-by-block modeled and, therefore, the actual masonry bond can be accounted for. The block behavior can be considered rigid or deformable, whereas their interaction can be mechanically represented by means of several suitable formulations, that are reviewed in Section 5.

(ii) *Continuum models.* The masonry material is modeled as a continuum deformable body. The nonlinear constitutive law adopted for the material can be defined either through (i) *direct approaches*, i.e. by means of constitutive laws calibrated, for example, on experimental tests, or through (ii) *homogenization procedures and multi-scale approaches*, where the constitutive law of the material (considered as homogeneous in the structural-scale model) is deduced from an homogenization process which relates the structural-scale model to a material-scale model (representing the main masonry heterogeneities) of a representative volume element (RVE) of the structure. In this cases, the solution of structural-scale problems could be formulated in a multi-scale framework. These continuum models are reviewed in Section 6.

(iii) *Geometry-based models.* The structure is modeled as a rigid body. The geometry of the structure represents the main (or even the only) input of these modeling strategies. The structural equilibrium and/or collapse are investigated through different procedures. Typically, these methods implement limit analysis-based solutions (see Section 3.2), which can be based on either static or kinematic theorems. Although these models could, in some respects, be considered as continuum models (see category (ii)), it should be remarked that the present category is based on the assumption of rigid body. The geometry-based models are reviewed in Section 7.

(iv) *Macroelement models.* The structure is idealized into panel-scale structural components (macroelements) with a phenomenological or mechanical-based nonlinear response. Typically, two main structural components may be identified: piers and spandrels. The subdivision of the structure into panel-scale portions is an *a priori* operation made by the analyst who interprets the structural conception of the building. Although these models could, in some respects, be considered continuum approaches, the main difference with the models in (ii) is that the constitutive law of macroelements attempts to reproduce the mechanical response of panel-scale structural components, while the constitutive law of the models in (ii) tries to reproduce the mechanical behavior of the masonry material. Macroelement models are reviewed in Section 8.

In the following, each category is comprehensively reviewed, showing the limitations and possibilities of each approach, accounting for new and recently proposed solutions. In this spirit, the following sections could be seen as an updating of well-known review papers [43, 44] on this field.

5. Block-based models

Block-based models represent the masonry behavior at the scale of the main heterogeneity of the material, characterized by blocks assembled by mortar (or dry) joints, which governs the main aspects of its mechanical and failure response. Indeed, these models can account for the actual masonry bond, which substantially controls the anisotropicity and the failure pattern of the material.

The first example of nonlinear block-based model dates probably back to 1978, thanks to the pioneering work by Page [46], where masonry is considered as an assemblage (that will be called "textured continuum" in the following) of elastic brick elements acting in conjunction with linkage elements simulating the mortar joints which have limited shear strength depending upon the bond strength and the level of compression. From that work, several block-based models have been developed and proposed.

The main positive features of the block-based modeling strategy category can be summarized as:

- Representation of the actual masonry bond and many structural details (e.g. toothing of corners between orthogonal walls, see Figure 8);
- Easy mechanical characterization from small-scale experimental tests;
- Clear representation of the failure modes, which do not need demanding interpretation. Indeed, detailed insights on the weakest parts of the structure can be found, helping the designing of strengthening devices;

- Anisotropy intrinsically accounted for in the definition of the actual masonry bond;
- 3D models can account for, at the same time, the in-plane and out-of-plane responses of masonry walls (and their interactions [47]);
- The interaction between orthogonal walls if subjected to horizontal loads (in terms, for example, of vertical reaction transfer) is intrinsically accounted for in 3D models.

Conversely, the main negative features of the block-based models can be summarized as:

- The main issue of these models resides in their huge computational demand. This well-known problem [43, 44], typically limits the applicability of these modeling strategies to panel-scale structures. Indeed, few examples of applications on full-scale masonry structures can be found in the literature [48, 49]. However, given the continuous power increment of the computational facilities, this problem could be less significant in the near future;
- 2D models unlikely show a reliable out-of-plane response;
- The actual bond of existing masonry structures is often non-completely know. Therefore, the block-by-block discretization could be approximated in those cases;
- The assembly of the model is usually a time-consuming and complex operation, which limits the use of these modeling strategies to academic studies and very few high-level consultancy groups.

In this section, block-based models are classified into different subcategories depending on the way the interaction between blocks is formulated (Figure 9):

1. Interface element-based approaches;
2. Contact-based approaches;
3. Textured continuum-based approaches;
4. Block-based limit analysis approaches;
5. Extended finite element approaches.

Each subcategory is then exhaustively reviewed in the following.

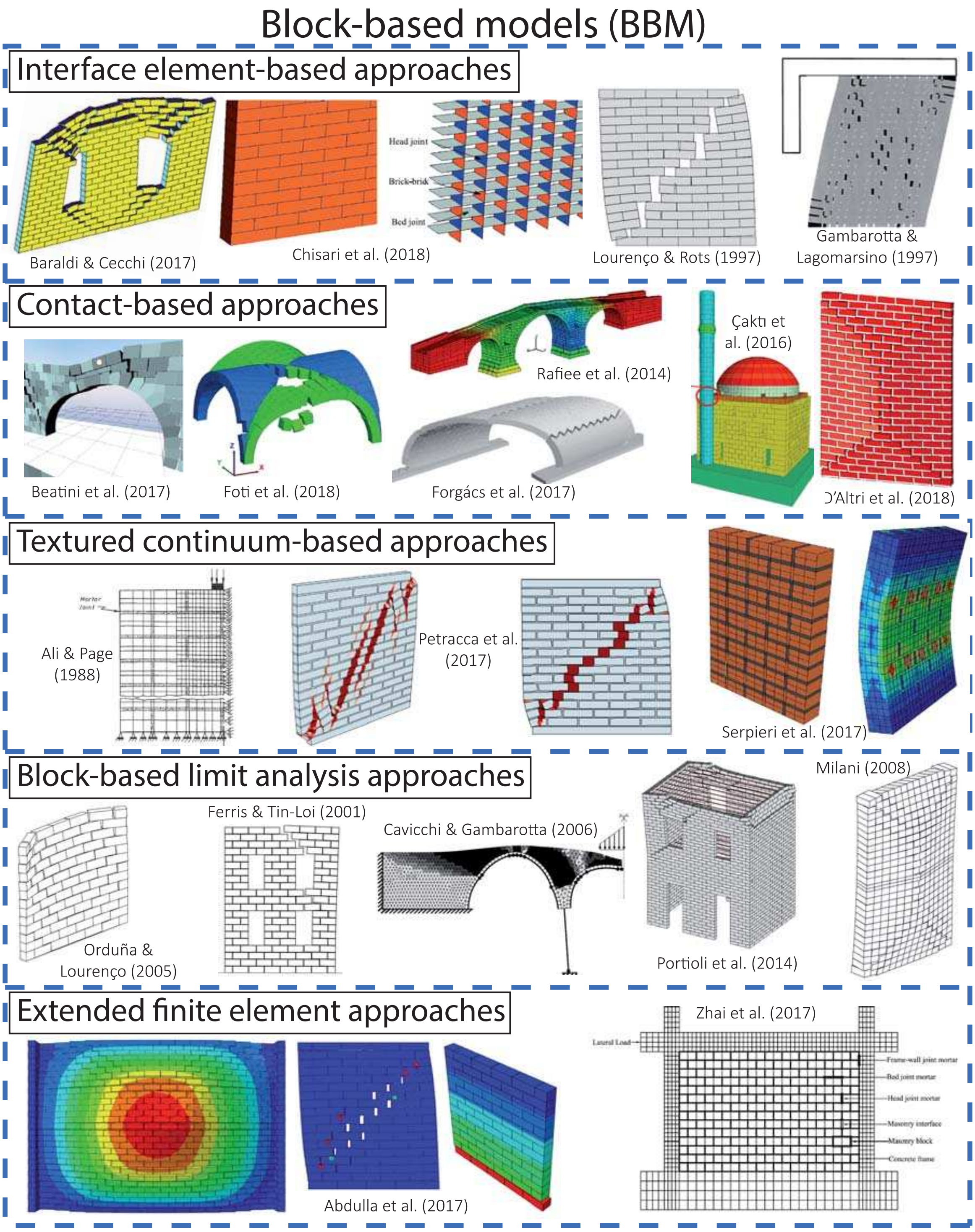


Figure 9: Examples of block-based models.

5.1. Interface element-based approaches

A first nonlinear interface-based model to simulate the collapse behavior of masonry structures appeared in [50], where the mortar joints were modeled with zero-thickness interface elements and the masonry units (which were considered as expanded to account for the geometry of the mortar joints) were modeled with smeared crack elements, within a FE approach (Figure 10). In particular, a dilatant interface plasticity-based constitutive model capable of simulating the initiation and propagation of interface fracture under combined normal and shear stresses was developed.

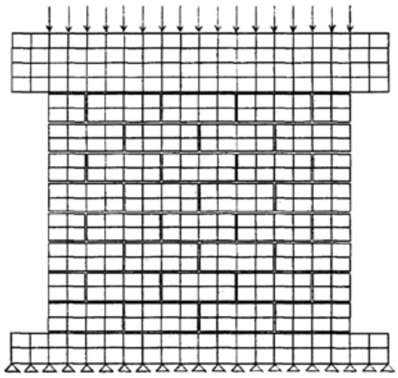


Figure 10: Example of a pioneering interface-based model [50].

An important improvement of this approach has been proposed by Lourenço & Rots [51]. In particular, they developed a multi-surface interface-based model in which all the nonlinearities (including also crushing) were concentrated in the interfaces. This permitted to increase the efficiency of the model, in the framework of softening plasticity. Such a model [51] has been diffusely used in the years that followed, and is still today utilized for many applications on masonry structures [52, 53]. For example, an interesting application of this interface model has been conducted in [54] for historic non-regular stone masonry shear walls. Furthermore, an extension of the interface model developed in [51] to the cyclic behavior of masonry shear walls has been presented and validated in [55], fully-based on the plasticity theory.

A cyclic mortar joint interface model based on damage mechanics has been developed by Gambarotta & Lagomarsino [56]. In particular, the constitutive equation of the interface is postulated in terms of two internal variables representing the frictional sliding and the mortar joint damage. The interface model exhibits a brittle response under tensile stresses and is characterized by frictional dissipation together with stiffness degrading under compressive stresses (Figure 9).

Other approaches, based on cohesive interfaces with damage and friction have been presented in [57, 58, 59], which were suitable for the simulation of masonry shear walls.

Additionally, several strategies have been based on the assumption of rigid blocks which interact through nonlinear springs simulating the response of masonry joints as well as crushing. This is the case, for example, of the model developed by Malomo et al. [60] within the framework of the so-called applied element method. Although similar, in principle, to the rigid body spring model (RBSM) developed by Casolo [61] (which is, however, used without accounting for the actual masonry bond and, so, the spring linear and nonlinear properties have to be homogenized), in [60] the block-by-block modeling is pursued for the analysis of the in-plane cyclic behavior of masonry walls.

All references described up unto this point are conceived for the analysis of 2D problems, typically in-plane problems. This aspect, as discussed above, considerably limits the applicability of the modeling strategies to real problems. To overcome this issue, several 3D models have been developed [62, 63, 64] to deal with real case studies as well. Primarily, two different interface elements have been developed specifically for 3D analysis of masonry structures.

Firstly, an extension of the Lourenço & Rots [51] multi-surface interface model to the 3D case, accounting also for geometrical nonlinearity, has been developed by Macorini and Izzudin [65]. In particular, a co-rotational approach has been employed in [65] for the interface element, which shifts the treatment of geometric nonlinearity to the level of discrete entities, and enables the consideration of material nonlinearity within a simplified local framework employing first-order kinematics (Figure 9). This approach has been extensively used for real applications [66, 67] by using partitioning routines [68, 69]. Moreover, the interface model presented in [65] has been further developed for simulating the cyclic response of masonry structures [48] by using a damage-plasticity approach.

Secondly, another interface constitutive model has been developed in [70] and coupled with elasto-plastic block elements for the explicit cyclic analysis of 3D masonry walls. This interface model has been broadly used for studying several aspects of the mechanics of masonry walls [71, 72, 47, 73].

5.2. Contact-based approaches

Block-based modeling strategies based on contact mechanics are widely used for the accurate modeling of masonry structures. Basically, rigid or deformable (linear or nonlinear) blocks interact following a frictional or cohesive-frictional contact definition. Although several in-house formulations have been developed and validated (see for instance [74, 75]), three main families of contact-based approaches can be found.

Firstly, a wide family of modeling approaches has been based on the distinct element method (DEM), also called discrete element method in the literature [76], originally proposed by Cundall & Stack [77] for the analysis of granular assemblies and implemented in the UDEC code [78]. DEM approaches are based on contact penalty formulations and explicit integration schemes. In this context, several applications have been conducted on real masonry structures [79, 80, 81, 82, 83, 84, 85, 86, 87, 88] using rigid or linear elastic blocks (Figure 9).

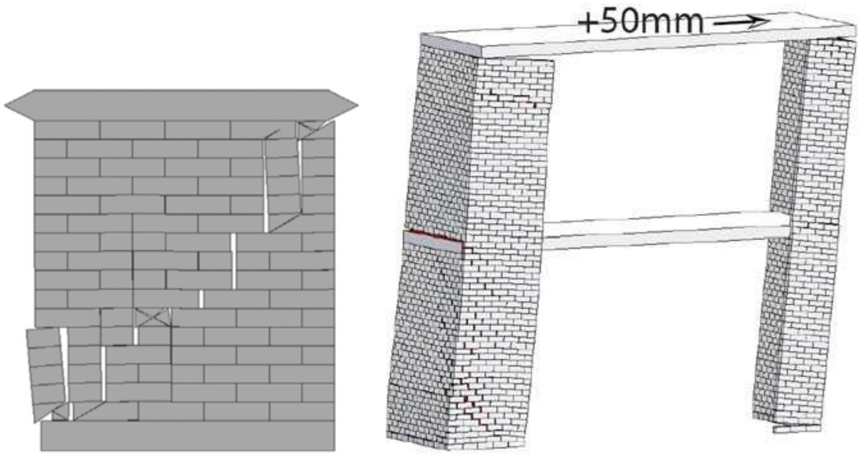
Secondly, an implicit approach which considers the deformability of blocks is the so-called discontinuous deformation analysis (DDA) [89]. DDA fulfills constraints of no tension between blocks and no penetration of one block into another. Also, Coloumb's law is fulfilled at all contact positions for both static and dynamic computations [90].

Thirdly, another family is based on the non-smooth contact dynamics (NSCD) method, developed by Jean [91] and Moreau [92] and characterized by a direct contact formulation, in its non-smooth form, implicit integrations schemes, and energy dissipation due to blocks' impacts. This approach, although successfully applied to several real case studies [93, 94, 95, 96], appears limited to dry stone masonry structures, as it seems still not capable in representing cohesive responses of the mortar joints.

Although the approaches belonging to the aforementioned three families are generally rather fast and permit full-scale applications as well, they cannot properly account for masonry crushing, which can be, in some cases, crucial in the mechanical response of masonry structures. To this aim, other approaches have been developed to account for block nonlinearity in tension and compression (Figure 11).

In the framework of the so-called finite-discrete element method (FDEM) [97], Smoljanović et al. [98] developed a code for the computational analysis of dry stone masonry structures [98] and extended it to 3D structures in [99]. Additionally, they implemented the nonlinear response of blocks in [100] to account for masonry crushing and block fragmentation (Figure 11(a)).

Finally, a very recent 3D block-based model with contacting damaging blocks has been developed and validated in [11], where the mortar layers are explicitly modeled in the block mesh. This model, based on implicit integration schemes, contact penalty method, compressive and tensile damage for the blocks, and rigid-cohesive-frictional contact behavior, provided very accurate results for the in-plane and out-of-plane response of masonry panels. Moreover, the model presented in [11] has been extended to the cyclic behavior of full-scale masonry structures (Figure 11(b)) in [49].



(a) (b)

Figure 11: Examples of contact-based approaches which include masonry crushing [100, 49].

5.3. Textured continuum-based approaches

The main idea of block-based textured continuum models [46] is to have, in a FEM framework with nonlinear elements, blocks and joints modeled separately without any interface between them, allowing for nonlinear deformation characteristics of the two materials as well as failure of the blocks, the mortar, or the mortar joints by bond.

An example of a pioneering mesh discretization of this kind of approaches is shown in Figure 9 (see Ali & Page [101]), in which the FEs with block properties are distinguished from the ones with mortar (or more correctly mortar joint) properties. In particular, the model used in [101] uses a strength criterion for crack initiation and propagation, and the smeared crack modeling technique for reproducing the effects of the crack.

More recently, a block-based textured continuum model which discretizes both units and mortar-joints with continuum elements, making use of a tension/compression damage model, has been developed in [102]. Particularly, in [102] the damage model has been refined to properly reproduce the nonlinear response under shear and to control the dilatancy. Another solution, based on a enriched kinematic damage model, has been proposed in [103].

A very innovative approach to mechanically model the nonlinear response of mortar joints has been lately presented in [104], where a microstructured 3D composite interphase formulation based on a multiplane cohesive-zone model has been proposed. Basically, a multiscale modeling strategy for the constitutive law of mortar joints has been adopted, allowing to conduct a consistent and reproducible calibration procedure of the mortar joint parameters.

5.4. Block-based limit analysis approaches

Block-based limit analysis represents an accurate and robust approach for the prediction of collapse load and failure mechanism of masonry structures. Several 2D and 3D approaches have been developed along the last two decades (Figure 9), generally based on either static or kinematic theorems of limit analysis, even if the implementation of friction in the computations is usually non-conservative with respect to the limit analysis theorems.

The first block-based limit analysis approach applied to masonry assemblages is probably the one developed by Baggio & Trovalusci [105], where the solution of the limit analysis problem in the presence of friction at interfaces between rigid blocks, i.e. a nonlinear programming problem, is obtained by solving a preliminary problem of linear programming, corresponding to a linearized limit analysis in the presence of dilatancy at the interfaces [106].

Another approach has been developed by Ferris & Tin-Loi [107], where the computation of the collapse loads of discrete rigid block systems, characterized by nonassociative friction and tensionless contact interfaces, is formulated and solved as a special constrained optimization problem, i.e. the so-called mathematical program with equilibrium constraints.

Furthermore, Sutcliffe et al. [108] developed a technique for computing the lower bound limit loads in unreinforced masonry shear walls under conditions of plane strain. By using a Mohr–Coulomb approximation of the yield surfaces, the numerical procedure proposed in [108] computes a statically admissible stress field via linear programming and finite elements. By imposing equilibrium, an expression of the collapse load is formed by imposing equilibrium, and the solution obtained is a rigorous lower bound on the actual collapse load.

Later, Orduña & Lourenço [109, 110] proposed a solution procedure for the non-associated limit analysis of rigid block masonry assemblages, incorporating non-associated flow rules and a coupled yield surface.

Moreover, a formulation for limit analysis of masonry block structures with non-associative frictional joints, using linear programming, has been proposed in [111], extended to 3D structures accounting for torsional effects in [112], and optimized using cone programming in [113]. In these approaches, rigid blocks interact via no-tension contact surfaces with Coulomb friction.

Conversely, the approach proposed and developed by Milani [114], based on 3D FE upper bound limit analyses of in- and out-of-plane loaded masonry walls, implements interfaces with a Mohr–Coulomb failure criterion with tension cut-off and cap in compression for mortar joints, whereas a Mohr–Coulomb failure criterion is adopted for bricks. Therefore, mortar joint cohesion and masonry crushing are accounted for in this approach. Other direct applications of this model can be found in [115, 116], whereas applications within homogenization procedures are going to be discussed in the following section.

Although block-based limit analysis approaches have been also applied to real structures, e.g. masonry bridges in [117], their computational demand appears particularly high, preventing their use for large-scale masonry structures.

5.5. Extended finite element approaches

Very recently, few block-based models formulated in the framework of the extended finite element method (XFEM) have been proposed [118, 119] (Figure 9).

Particularly, Abdulla et al. [118] proposed a 3D model which includes surface-based cohesive behavior to capture the elastic and plastic behavior of masonry joints and a Drucker-Prager plasticity model to simulate crushing of masonry under compression (Figure 9).

Furthermore, XFEM is adopted in [119] to model the cracking behavior and the compressive failure of masonry in infill panels, and the discrete interface element is employed to simulate the behavior of the masonry mortar joints and the joints at the frame-to-infill interface (Figure 9).

Although only two models have been proposed so far in this subcategory, these approaches can represent a powerful alternative for block-based analysis of masonry structures.

6. Continuum models

In continuum approaches, masonry is modeled as a continuum deformable body (Figure 12). This category of modeling strategies has the advantage that the mesh discretization does not have to describe the main heterogeneities of masonry, and, hence, can have dimensions which can be significantly greater than the block size. So, the computational effort of these approaches is, in general, lower than block-based approaches. However, given the complexities of masonry from a mechanical point of view (Section 2), the definition of suitable homogeneous constitutive laws for masonry is a challenging task, and can be pursued either through (i) *direct approaches*, i.e. by means of constitutive laws calibrated, for example, on experimental tests, or through (ii) *homogenization procedures and multi-scale approaches*, where the constitutive law of the material (considered as homogeneous in the structural-scale model) is derived from an homogenization process which relates the structural-scale model to a material-scale model (representing the main masonry heterogeneities). The homogenization process is typically based on refined modeling strategies (e.g. block-based models) of a representative volume element (RVE) of the structure.

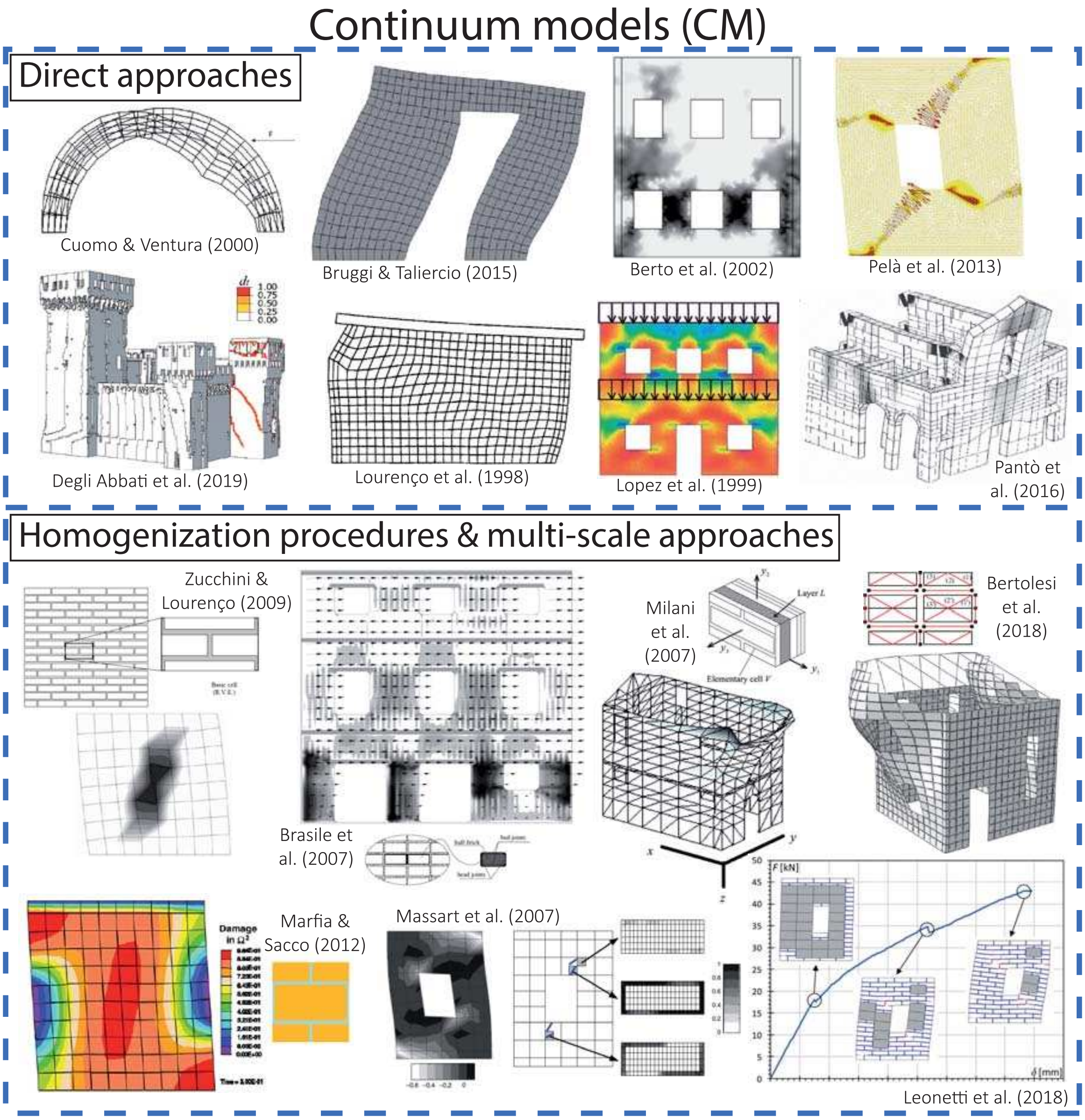


Figure 12: Examples of continuum models.

6.1. Direct approaches

Direct continuum models rely on continuum constitutive laws which can, somehow, approximate the overall mechanical response of masonry. In these approaches, the mechanical properties (elastic parameters, strength domain, etc.) could be calibrated through experimental tests or other data (e.g. experimentally-derived analytical strength domains), without resorting to RVE-based homogenization procedures.

Several formulations, with different levels of approximation, have been developed and tested on real applications. Indeed, although the mechanical properties of the homogeneous model should be, in theory, rigorously deduced from homogenization theories, many simplified approaches have been successfully applied on interesting case studies.

One first family of direct approaches consists in a drastic idealization of the masonry mechanical behavior, i.e. masonry is conceived as a perfectly *no-tension* material. Generally, perfectly no-tension material means an isotropic medium incapable of sustaining tensile stresses but, otherwise, linear-elastic [120]. This radical hypothesis, although sustained by the fact that the mechanical characterization of masonry is very challenging especially in the tensile regime, can be a valuable basis for preliminary structural analyses [121]. Nevertheless, the hypothesis of no-tension material has been widely used in the analysis of the stability of masonry vaults and domes [37, 38], in the framework of geometry-based models (Section 7).

In [121], an approximate, piecewise-linear description of perfectly no-tension material behavior has been developed, leading to a very simple formulation of the discretized boundary value problem in finite terms. Later, Angelillo [122] proposed a FE solution based on a complementary energy theorem for elastic no-tension bodies. The solution relies on a problem of minimization of a quadratic function with equality and inequality constraints. Starting from an elementary stress field, an optimal approximate solution (safe in the spirit of limit analysis) is reached. Other solutions of the FE analysis of no-tension structures can be found in [123, 124, 125]. More recently, Bruggi [126] proposed a FE analysis of no-tension structures as a topology optimization problem. Then, Bruggi & Taliercio [127] proposed a non-incremental energy-based algorithm to define the distribution and the orientation of an equivalent orthotropic material, minimizing the potential energy so that to achieve a compression-only state of stress.

Although the cited no-tension approaches represent elegant solutions for such a complex problem, their applicability to real case studies is still limited. Indeed, all the aforementioned approaches are limited to 2D problems and only very recently 3D no-tension structures have been investigated [128]. However, these approaches cannot simulate the post-peak behavior of masonry structures, which is a strong limitation in the field of seismic assessment of structures. Moreover, although the assumption of null tensile strength can be considered, in general, conservative, this could lead to failure mechanisms which are not coherent with the ones experimentally observed, given that in reality the tensile strengths of masonry are non-zero.

Other direct continuum models for masonry structures rely on continuum nonlinear constitutive laws based either on fracture mechanics (smeared crack models), on damage mechanics, or on plasticity theory. Several smeared crack [129, 130], plastic [131], damage [132], and plastic-damage [133, 134] models have been primarily developed for the FE analysis of concrete structures. However, their usability for the simulation of the collapse or near collapse behavior of masonry structures presents some limitations, mainly due to the multi-level anisotropy (elastic, strength and brittleness anisotropies, see Section 2) of masonry and its heterogeneity introduced by mortar joints. A pioneering test of the accuracy of smeared crack models for masonry structures is reported in [135]. While the model adopted in [135] showed good performance with respect to flexure-dominated behavior, it showed problems in capturing the brittle shear behavior of masonry panels.

Although non-fully coherent with masonry mechanics, smeared crack and isotropic damage and plastic-damage models have been extensively used for analyzing masonry structures [136], mainly due to their efficiency, their diffusion in commercial FE codes, and the relatively few mechanical parameters to characterize.

Particularly, the utilization of these nonlinear models has been found especially indicated for the analysis of historic monumental structures, given their limited computational effort and their capability to represent complex and large-scale 3D geometries. In addition, historic buildings are usually characterized by multi-leaf irregular randomly-assembled masonries, which are often impossible to represent block-by-block and to mechanically characterize, given also the strict limitations for destructive in situ tests on historic buildings [137]. Indeed, poor information is usually available on the mechanical properties of historic masonries, favoring the use of isotropic nonlinear models. Many applications of isotropic smeared crack, damage and plastic-damage models have been successfully conducted on historic towers [138, 139, 140, 140], churches and temples [141, 142, 143, 144], palaces [145, 146, 30, 147], and masonry bridges [148, 149]. In particular, most of the applications on historic monumental structures rely on 3D models (Figure 13), as the structural behavior is rarely representable by 2D models, given the complex and irregular geometries of these buildings (Section 2).

Although each reliable damage model has to conceive a regularization of the fracture en-

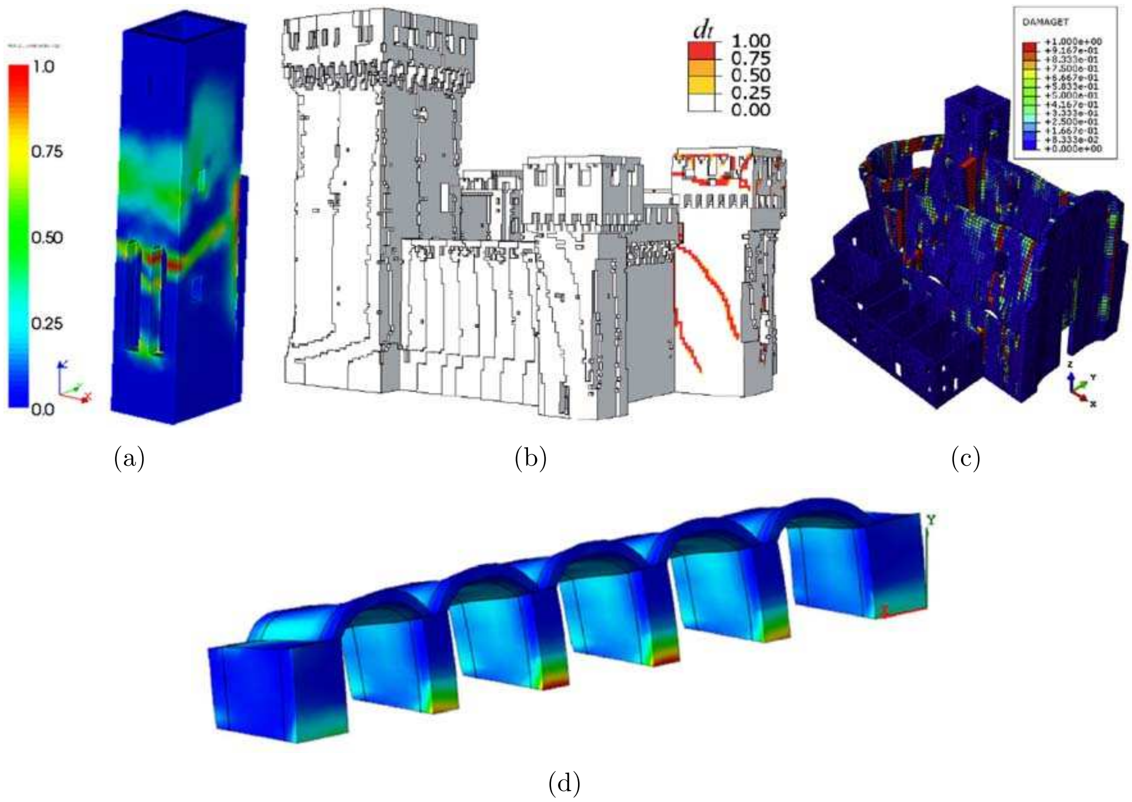


Figure 13: Examples of direct continuum isotropic approaches applied on historic monumental structures [138, 147, 142, 149].

ergy, which is usually normalized on a characteristic dimension of the element, very coarse meshes could lead to unsafe results as they are not able to essentially represent the damage pattern and the stress redistribution. An enhancement of the aforementioned constitutive models could be represented by the use of crack-tracking algorithms, originating from the analysis of localized cracking in quasi-brittle materials, which ensure mesh-bias independency of the numerical results and the realistic representation of propagating cracks in the numerical simulation of fracture in quasi-brittle materials [150, 151].

However, when dealing with periodic well-organized masonry, the assumption of only one tensile strength value (that governs the tensile response in each direction) risks to be too simplistic. To this aim, some orthotropic nonlinear constitutive laws have been developed and applied on masonry structures.

A first example of orthotropic plasticity model with softening has been proposed in [152], while in [153] the ability of that continuum model to represent the inelastic behavior of orthotropic materials is shown, and a set of experimental tests to characterize the constitutive behavior of masonry is proposed, demonstrating the capability of the model to reproduce the strength behavior of different masonry types.

Successively, the effect of anisotropy has been introduced in [154] by means of fictitious isotropic stress and strain spaces. The material properties in the fictitious isotropic spaces are mapped into the actual anisotropic space by means of a consistent fourth-order tensor. The advantage of the model is that the classical theory of plasticity can be used to model the non-linear behavior in the isotropic spaces.

Later, an orthotropic damage model specifically developed for the analysis of brittle masonry subjected to cyclic in-plane loading has been described in [155]. Different elastic and inelastic properties have been assumed along the two natural axes of the masonry (i.e. the bed joints and the head joints directions) also as principal axes of damage.

More recently, Pelà et al. [156, 157] proposed an orthotropic damage model for the analysis of masonry structures, in which the orthotropic behavior is simulated through the concept of mapped tensors from the anisotropic field to an auxiliary workspace. The model affords the simulation of orthotropic induced damage, while also accounting for unilateral effects, thanks to a stress tensor split into tensile and compressive contributions. The damage model has also been combined with a crack-tracking technique [158] to reproduce the propagation of localized cracks in the FE problem.

Although the described direct continuum anisotropic approaches (Figure 12) represent scientifically sound solutions, their application on real case study has been limited by the fact that their computational effort and the number of material properties to be mechanically characterized is substantially higher than isotropic approaches.

Additionally, other solutions adopt an homogeneous FE model of the structure, but, instead of a proper continuum, they use alternative solutions to describe the nonlinear behavior of masonry. For example, Reyes et al. [159] proposed a numerical procedure for fracture of brickwork masonry based on the strong discontinuity approach, accounting for the anisotropy of the material.

Other approaches, based on FE limit analysis, conceive the homogeneous structural-scale model made of rigid or deformable elements, placing nonlinear interfaces in between the elements, where plastic dissipation can occur. Dealing with historic full-scale buildings, FE limit analysis approaches have been successfully applied [160, 32] by using averaged mechanical properties, without using a rigorous homogenization procedure.

Finally, other approaches based on systems of springs [161, 162] can be fully characterized through a suitable calibration of linear and nonlinear spring properties.

These latter approaches (FE limit analysis and spring-based approaches) can be considered borderline in the context of continuum models (as they have interfaces between elements or spring systems instead of a proper continuum). However, given that they eventually behave as a continuum (where all the deformabilities and nonlinearities are lumped in the interfaces/springs) and the structure is effectively discretized by means of a continuum mesh, their classification in this category could be considered legitimate.

6.2. Homogenization procedures & multi-scale approaches

The constitutive law of the homogeneous structural-scale model which tries to represent masonry can be deduced from homogenization processes, typically based on RVEs. The definition of a proper RVE is essential, as it should be statistically representative of the material-scale heterogeneity under study, embodying the characteristic material heterogeneities. To this aim, several RVEs geometries have been proposed, to account for different periodic and non-periodic patterns of masonry (Figure 14).

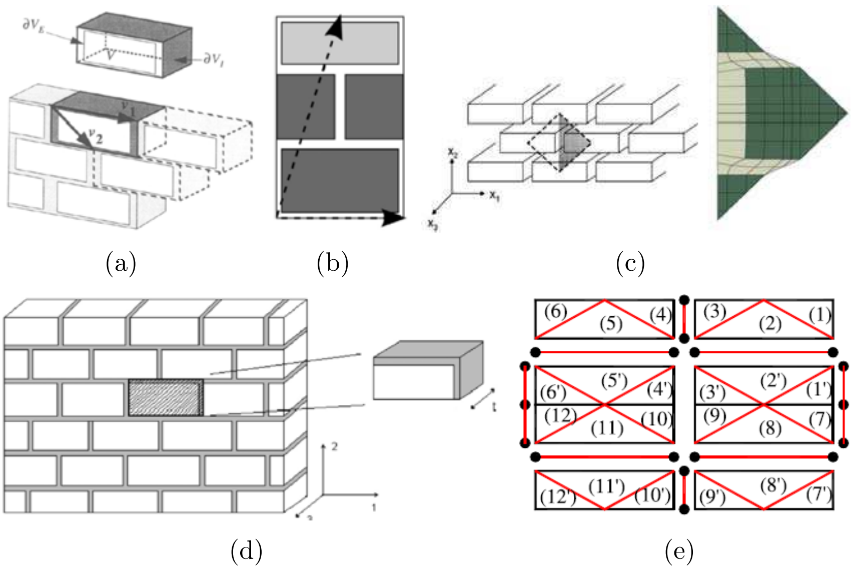


Figure 14: Examples of RVEs adopted for the derivation of homogenized masonry mechanical properties [163, 164, 165, 166, 167].

Given the mechanical complexity of masonry, in terms, for example, of anisotropy, a very wide family of continuum approaches rely on homogenization procedures and multi-scale approaches [168]. Basically, three main families of approaches could be distinguished (Figure 15):

(i) *A priori homogenization approaches* (Figure 15(a)), which typically rely into two steps: in the first step, (*a priori*) RVE-based homogenization is performed to deduce the mechanical properties of the structural-scale material; the second step relies into the introduction in the structural-scale model of the homogenized mechanical properties.

(ii) *Step-by-step multi-scale approaches* (Figure 15(b)), in which the overall behavior at the structural scale is step-by-step determined by solving a boundary value problem (BVP) on the RVE for each integration point of the structural-scale model. In this way, an estimation of the expected average response to be used as constitutive relations in the structural-scale model is step-by-step obtained. In these approaches, the heterogeneity of masonry is not directly accounted for in the structural-scale model, being explicitly accounted for into the material-scale RVE.

(iii) *Adaptive multi-scale approaches* (Figure 15(c)), in which the material-scale model is adaptively inserted and resolved on the structural-scale model, thus establishing a strong coupling between the two scales.

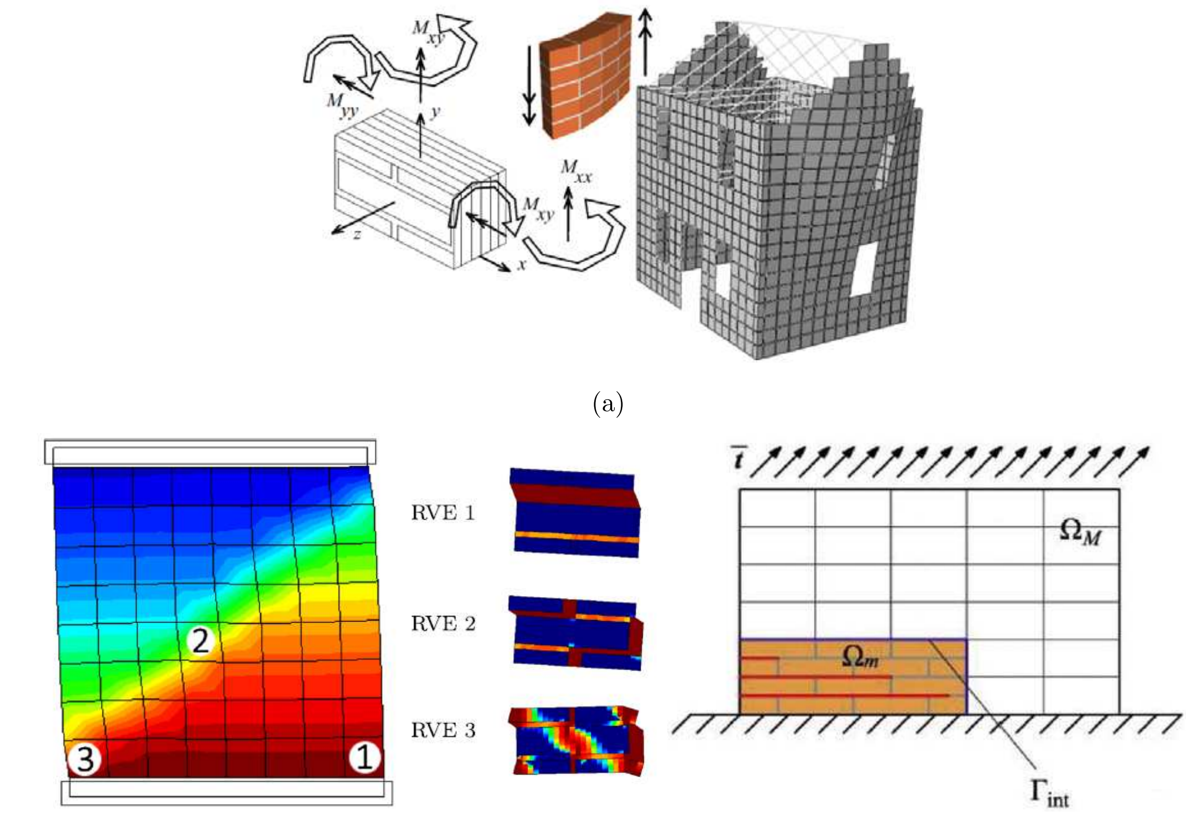


Figure 15: Homogenization procedures and multi-scale approaches: (a) *a priori* homogenization [169], (b) step-by-step multi-scale [170], and (c) adaptive multi-scale [171] approaches.

6.2.1. A priori homogenization approaches

Although *a priori* homogenization approaches typically consists of two steps (i.e. in the first step the mechanical properties are deduced through an homogenization process, and in the second step homogenized properties are introduced in the structural scale model), most of the solutions provided in the literature focused on the first step, while only few approaches dealt with both steps.

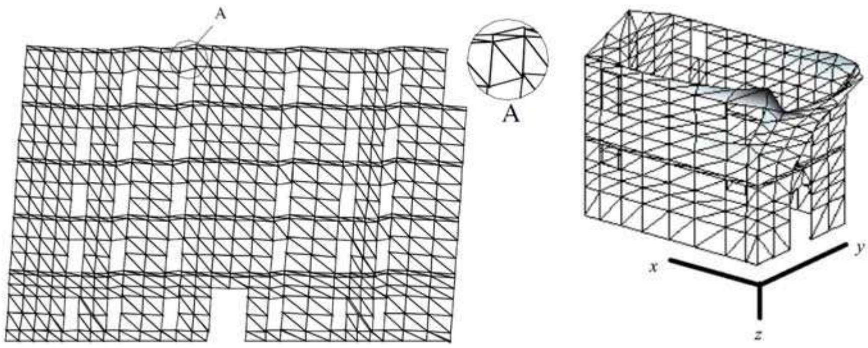
The deduction of homogenized constitutive laws for the analysis of heterogeneous quasi-brittle materials, such as masonry, can be based on closed-form (analytical), quasi-analytical, and numerical methods.

A pioneering contribution on the mathematical description of the macroscopic behavior of brick masonry has been given in [172]. Successively, Anthoine [163] rigorously derived the in-plane elastic characteristics of masonry through homogenization theory. Briccoli Bati et al. [173] applied a material-scale model for the determination of the overall linear elastic mechanical properties of a simple texture of brick masonry. In the framework of the Cosserat continuum models, Masiani & Trovalusci [174] studied the case of 2D periodic rigid block assemblies joined by linear elastic mortar joints, deducing the structural-scale model characterization of the equivalent medium by equating the virtual stress power of the coarse model with the virtual power of the internal actions of the discrete fine model. An extension to the 3D case has been analyzed in [175]. Further approaches for the derivation of homogenized elastic properties of masonry can be found in [176, 177, 178, 179, 180, 165].

Other approaches, beyond the definition of elastic properties, attempted to derive masonry strength domains (both in-plane and out-of-plane) [181]. For example, in [182], a structural-scale strength criterion for in-plane masonry response is derived through a continuum model. Zucchini & Lourenço [183, 184] derived both elastic moduli and failure surfaces through a linear and nonlinear homogenization procedures. Wei & Hao [185] develop a continuum damage model for masonry accounting for the strain rate effect, using a homogenization theory implemented in a numerical algorithm. Stefanou et al. [166] provided a straightforward methodology for the estimation in closed-form of the overall strength domain of an in-plane loaded masonry wall by accounting for the failure of its bricks.

Most of the existing models for masonry concerned periodic material-scale textures. Cecchi & Sab [186] analyzed non-periodic masonries, typical of historic buildings, by means of a perturbation approach, while Cavalagli et al. [164, 187] used a random media material-scale approach.

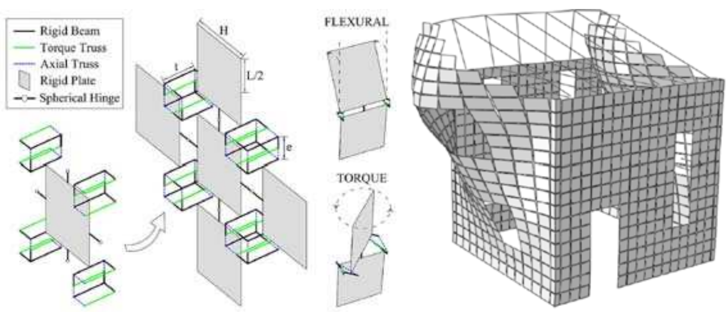
Moreover, several approaches for the derivation of the homogenized failure surfaces for masonry have been based on FE limit analysis [188, 189, 190, 167, 191, 192]. For example, in [188] a simple material scale model for the homogenized limit analysis of in-plane loaded masonry has been proposed. In particular, a linear optimization problem is derived on the RVE in order to recover the homogenized failure surface of the brickwork, under plane stress conditions. One of the main benefits of these approaches relies on the fact that, once homogenized the masonry properties in terms of elastic moduli and strength domain (so, they are *a priori* defined), they can be directly implemented in structural-scale models (Figure 16), to solve real case studies [193, 194].



(a) (b)

Figure 16: Examples of homogenized FE limit analysis approaches [193, 194].

The same benefit can be observed in RBSM approaches [195, 196, 197, 169], where the linear and nonlinear properties of the springs between rigid elements, which do not represent the actual masonry bond, can be *a priori* homogenized (Figure 17). Once determined the homogenized properties, they can be directly used for structural applications [169].



(a) (b)

Figure 17: Examples of homogenized RBSM approaches [197, 169].

6.2.2. Step-by-step multi-scale approaches

Plenty of step-by-step multi-scale approaches can be found in the scientific literature, which may differ in terms of:

- Continuum type adopted in the structural-scale model (Cauchy continuum, Cosserat continuum, etc);
- Type of homogenization procedure (first or second order computational homogenizations, transformation field analysis (TFA), etc);
- Type of modeling of the RVE (i.e. modeling strategy adopted for the material-scale model, e.g. block-based models).

These approaches typically relies on step-by-step and point-by-point transitions between the structural-scale model and the material-scale model, and vice-versa. Multi-scale computational homogenization methods are traditionally implemented within the FEM framework and, so, also called FE^2 approaches. Most of these approaches are based on FE first-order homogenization schemes.

In this context, Cauchy continuum models have been classically adopted in structural-scale models, which are recovered applying periodic homogenization techniques for the simulation of in-plane behavior of masonry structures (Figure 15(b)).

A pioneering computational homogenization method has been proposed by Papa [198], where a unilateral damage model for masonry based on a homogenization procedure has been developed, and by Luciano & Sacco [199, 200], where a damage model for periodic masonry has been developed from a material-scale heterogeneity analysis. Around that time, Gambarotta & Lagomarsino [201] considered an equivalent stratified medium made up of mortar joints and brick units layers, adopting the damage constitutive laws both for the bricks and the mortar joints developed in [56]. Successively, a continuum framework has been developed for modeling of inelastic behavior of structural masonry in [202]. This formulation incorporated the anisotropic material characteristics and addressed both stages of the deformation process, i.e. those associated with homogeneous as well as localized deformation mode. Calderini & Lagormarsino [203] obtained homogenized in-plane constitutive equations, in terms of mean-stress and mean-strain. Different in-plane damage mechanisms have been considered, being the damage process governed by evolution laws based on an energetic approach and on a non-associated Coulomb friction law. Later, Zucchini & Lourenço [204] proposed an improved material-scale model for masonry homogenization in the nonlinear domain, incorporating suitably chosen deformation mechanisms coupled with damage and plasticity models.

Sacco [205] proposed a multi-scale procedure based on a micromechanical analysis of the damaging process of the mortar material, assuming linear elastic blocks. In this case, a nonlinear homogenization procedure based on TFA has been proposed, making use of the superposition of the effects and the FE method. An improvement of this approach has been developed by Marfia & Sacco [206], where an extension of the TFA-based homogenization procedure to the case of nonuniform eigenstrain, as well as the use of nonlinear behavior of blocks in the material-scale model has been implemented.

In first-order computational homogenization schemes, where the formulation relies on the first gradient of the kinematics field, two main limitations could arise.

The first limitation is linked to the principle of separation of scales, which enforces the assumption of uniformity upon the structural-scale fields attributed to each RVE. Indeed, this assumption is not totally effective in structural-scale parts where high deformation gradients are present in the relative RVE.

The second limitation derives from the cohesive (quasi-brittle) response of masonry, i.e. due to the fact that softening effects arise in the stress–strain relationships. Being the characteristic lengths of the structural- and material-scales non-intrinsically accounted for in classical Cauchy continuum models, mesh-sensitivity issues tend to arise when material softening behavior appears. In order to overcome such a drawback, nonlocal approaches, higher-order continuum models, as well as regularization processes can be adopted to guarantee problem objectivity.

A simple way to overcome localization problems consists in following a regularization process, for example, in terms of fracture energy. A classical first order computational homogenization together with a regularization procedure based on the fracture energy of the material-scale model has been proposed in [170]. In this approach, a generalized geometrical characteristic length takes into account the size of the structural-scale element as well as the size of the RVE, ensuring objectivity of the dissipated energy at the structural-scale.

Massart et al. [207] proposed an enhanced multi-scale model using nonlocal implicit gradient isotropic damage models for both the constituents, describing the damage preferential orientations and employing at the macroscopic scale an embedded band model.

A second-order computational homogenization of periodic masonry has been proposed by Bacigalupo & Gambarotta [208, 209]. This computational procedure has been derived assuming an appropriate representation of the material-scale displacement field as the superposition of a local structural-scale displacement field and an unknown material-scale fluctuation field accounting for the effects of the heterogeneities.

Other approaches have been based on the adoption of Cosserat continuum models at the structural-scale. Generally, this allowed to account for a internal length of the material and to overcome localization problems [210]. Salerno & de Felice [211] investigated on the accuracy of various identification schemes for Cauchy and Cosserat continua, showing that micro-polar continuum better reproduces the discrete solutions, in the case of non-periodic deformation states, due to its capability to take scale effects into account. Alternatively, Casolo [212] considered isotropic linear elastic models both for the brick and the mortar and used a computational approach to identify the homogenized elastic tensor of the equivalent Cosserat medium. In addition, Addessi et al. [213] developed a structural-scale Cosserat continuum, which automatically accounts for the absolute size of the masonry components, derived by a rational homogenization procedure based on TFA. Another homogenization method for the Cosserat continuum has been presented by De Bellis & Addessi [214]. Finally, Addessi & Sacco [215] developed a nonlinear constitutive law for the material-scale model, which includes damage, friction, crushing and unilateral contact effects for the mortar joints. The nonlinear homogenization has been performed employing the TFA technique, properly extended to the structural-scale Cosserat continuum.

Although the multi-scale approaches mentioned earlier where focused on the in-plane response of masonry walls, also the out-of-plane analysis of masonry structures is an interesting issue, especially from a earthquake engineering point of view. To this aim, Mercatoris & Massart [216] presented a multi-scale framework for the failure of periodic quasi-brittle thin planar shells, using a shear-enhanced element with the Reissner- Mindlin description and employing it for the failure of out-of-plane loaded masonry walls. Furthermore, a computational homogenization approach for the analysis of general heterogeneous thick shell structures, with special focus on periodic brick-masonry walls has been proposed in [217].

A very efficient multilevel approach has been developed by Brasile et al. [218, 219]. Although this approach could be considered borderline in a multi-scale framework (being rather a multilevel approach), the strategy proposed in [218, 219] is based on an iterative scheme which uses two different (local and global) masonry models simultaneously. The former is a fine block-based model and describes the nonlinear mechanical response including damage evolution and friction toughness phenomena. The latter is a linearized FE approximation of the previous model, defined at the rough scale of the wall and used to accelerate the iteration. The proposed iterative scheme proved to be efficient and robust for in-plane nonlinear analysis of masonry façades.

6.2.3. Adaptive multi-scale approaches

A second multi-scale strategy (CMM) consists in the use of the so-called adaptive multi-scale methods [220, 221, 171, 222] (Figure 15(c)). In these approaches, a first-order homogenized model initially represents the masonry response until a threshold criterion is reached. For instance, such a criterion could be able to account for the onset of damage propagation. After reaching the threshold, the area of interest is replaced by an heterogeneous material-scale description able to represent the high localized deformation without the mesh-dependency of the first-order theory.

7. Geometry-based models

In geometry-based models, the structure is modeled as a rigid body. The geometry of the structure represents the main (or even the only) input of these modeling strategies. These approaches typically investigate the structural equilibrium and/or collapse through limit analysis-based solutions (Figure 18), which can be based on either static or kinematic theorems. Although typically based on limit analysis and on the Heyman's rigid no-tension model [37], these approaches have been formulated following several innovative solutions.

7.1. Static theorem-based approaches

As shown by Heyman in [37], applications of the static theorem of limit analysis on real masonry structures were possible by simple graphic statics [37, 39]. Particularly, static theorem-based approaches (Figure 18) appear specially suitable for the investigation of the equilibrium states in masonry arches, vaults and domes (i.e. masonry vaulted structures). In general, these approaches can provide the range of possible equilibrium states of the vaulted structure, bounded between two extreme equilibrium conditions.

A first computational development for the equilibrium analysis of masonry vaults has been proposed by O'Dwyer [223], where, after the decomposition of the vault into an optimized system of arches in equilibrium, a procedure for the application of the static theorem to vaults and domes has been presented. Another computational approach, called funicular model, for the assessment of masonry structures based on the well-known analogy between the equilibrium of arches and that of hanging strings has been presented in [224]. Further, a computational tool for the real-time limit analysis of 2D vaulted masonry structures has been presented by Block et al. [225].

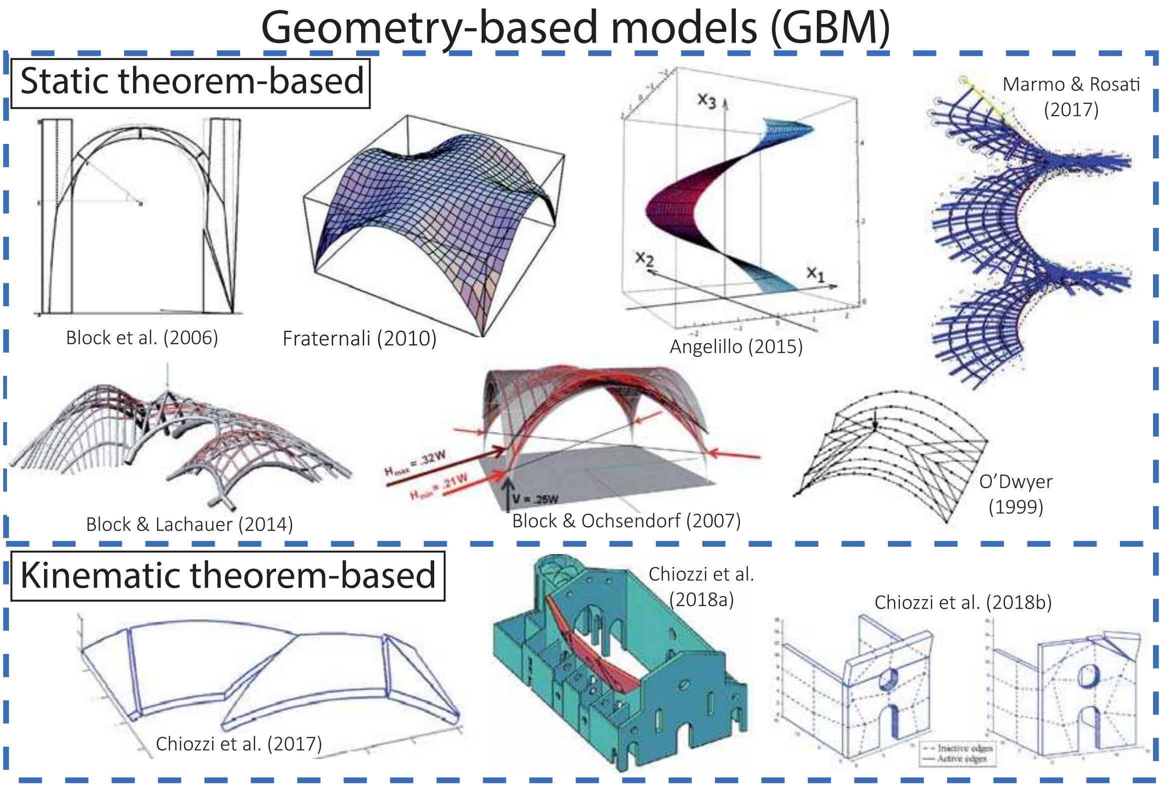


Figure 18: Examples of geometry-based models.

An innovative approach for the equilibrium analysis of vaulted masonry structures, called thrust network analysis (TNA), has been proposed by Block & Ochsendorf [226]. The TNA method, based on a duality between geometry and in-plane forces in networks, finds possible funicular solutions under gravitational loading within a defined envelope, generating compression-only vaulted surfaces and networks. In this way, the range of possible equilibrium states of the vault, bounded by a minimum and maximum thrust, can be obtained. A nonlinear extension of TNA has been presented in [227] for the application on Gothic masonry vaults, while in [228] TNA is extended with the use of structural matrix analysis and efficient optimization strategies. Finally, an extension of TNA with joints consideration has been provided in [229].

Another interesting thrust network approach has been developed by Fraternali [230], where the equilibrium problem of unreinforced masonry vaults is investigated through polyhedral stress functions. The masonry vault is conceived as a no-tension membrane carrying a discrete network of compressive singular stresses, through a non-conforming variational approximation of the continuous problem. The geometry of the thrust surface and the associated stress field are determined by means of a predictor–corrector procedure based on polyhedral approximations of the thrust surface and membrane stress potential. Another approach which considers masonry vaulted structures as unilateral membrane has been proposed by Angelillo et al. [231] and by Angelillo [232], where the discrete network of singular stresses has been defined basing on the Airy's stress formulation [233].

Finally, a reformulation of the original version of the TNA [226] by discarding the dual grid and focusing only on the primal grid, thus significantly enhancing the computational performances, has been proposed by Marmo & Rosati [234]. In [234], TNA is also extended by including horizontal forces in the analysis as well as holes or free edges in the vault. A further application on masonry helical staircases has been presented in [235].

In summary, static theorem-based approaches appear particularly attractive for the assessment of the statical safety of masonry vaulted structures. Indeed, if compression-only networks can be found within the boundaries of a vault, then the vault will stand in compression. Moreover, if the solution lie within the middle third of the section, any tension (and, therefore, any hinges) will be present in the section. This easy and powerful concept for understanding the stability and proximity to collapse of such structures has been formerly expressed by Heyman [37]. However, only few of the above-mentioned approaches can account for horizontal actions (such as seismic actions [234]), and no one could account for the interaction with the bearing structures (e.g. bearing walls), whose deformations could induce damage and equilibrium changes in the vaulted structure, as evidenced in [236] for earthquake actions.

7.2. Kinematic theorem-based approaches

Kinematic theorem-based limit analysis approaches have been widely used in the last decades for the fast and effective assessment of existing masonry buildings. Giuffrè [237] proposed a kinematic limit analysis approach for studying the seismic vulnerability of masonry buildings based on their decomposition into rigid blocks, following failure mechanisms actually observed in existing masonry buildings in Italy. Given the simplicity and effectiveness of the approach proposed by Giuffrè, it has been adopted in the Italian code [238, 24, 239, 240]. Figure 19 shows few examples of collapse mechanisms to be accounted for in the seismic assessment of masonry churches through kinematic limit analysis, from [24]. Kinematic linear and nonlinear (in which the displacement capacity of the structure until collapse is also evaluated) are commonly used in the professional practice for the safety assessment of existing masonry buildings [239].

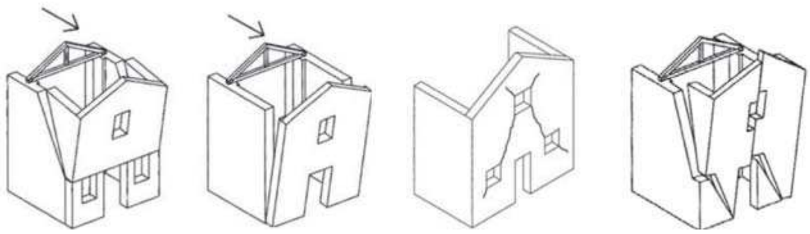


Figure 19: Examples of collapse mechanisms to be accounted for in the seismic assessment of masonry churches through kinematic limit analysis [24].

Basically, in all these cases, the collapse mechanisms to be analyzed are *a priori* determined, on the basis of recurring failure mechanisms actually observed. However, in the context of static theorem-based approaches, the collapse multiplier evaluated in this way is not necessarily the lower one, given, for instance, peculiar features of the geometry of the structure.

To this aim, more advanced computational static theorem-based approaches have been developed to precisely evaluate the collapse multiplier and the collapse mechanism of masonry structures (Figure 18). Milani [241] developed a simple discontinuous upper bound limit analysis approach with sequential linear programming mesh adaptation to analyze the actual failure mechanisms of masonry double curvature structures. Very recently, Chiozzi et al. [242] proposed a genetic algorithm for the limit analysis of masonry vaults based on an upper bound formulation. Given a masonry vault geometry, that can be represented by a non-uniform rational B-spline (NURBS) parametric surface, and a NURBS mesh of the given surface, each element of the mesh is a NURBS surface itself and can be idealized as a rigid body. The initial mesh is adjusted by means of a genetic algorithm in order to enforce that element edges accurately represent the actual failure mechanism. This approach has also been validated for the out-of-plane collapse behavior of masonry walls [243]. Finally, an automatic upper bound adaptive limit analysis program for masonry churches, called UB-ALMANAC, has been proposed in [28]. A NURBS mesh is directly prepared within a CAD environment based on the 3D geometrical model of the whole church. Limit analysis is then performed automatically under the desired horizontal loads distribution, using the kinematic theorem of limit analysis with dissipation allowed only along interfaces and progressive adaptation of the mesh through a genetic algorithm, leading to a quick estimation of the first activating failure mechanism and the most vulnerable part of the church.

Although these approaches cannot provide the displacement capacity of a masonry structures, they are very powerful for the fast and effective evaluation of the main vulnerabilities of a masonry building.

8. Macroelement models

In macroelement models (Figure 20), the structure is idealized into panel-scale structural components with a phenomenological or mechanical-based nonlinear response. Typically, two main structural components may be identified: piers and spandrels.

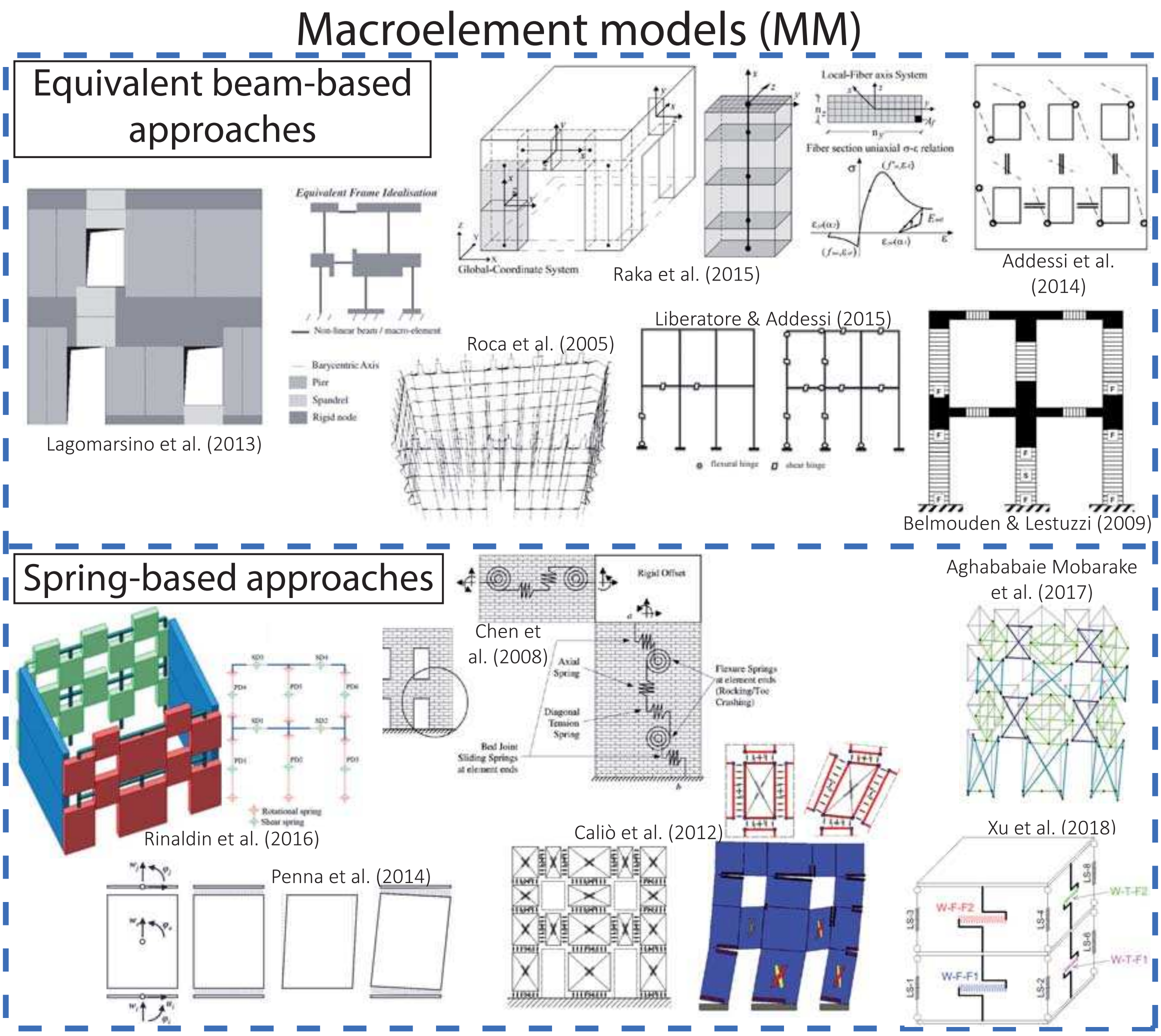


Figure 20: Examples of macroelement models.

These approaches are mainly focused on the analysis of the global seismic response of masonry buildings. Indeed, macroelement models are generally based on the assumption that any activation of local failure mode, mainly associated with the out-of-plane response of masonry walls, is prevented [244]. In this framework, the global seismic response is, therefore, strictly related either to the in-plane capacity of walls or to the load transfer due to the presence of diaphragms. In these approaches, global analyses (evolutionary static and/or dynamic) are typically conducted on 3D models, to account for load transfer between the bearing walls due to an horizontal action.

In these modeling approaches, the structural components (piers and spandrels) need to be *a priori* identified, on the basis of damage observations on real buildings. Indeed, earthquake-damage observations showed that cracks and damages are usually concentrated in piers and spandrels. Piers are the vertical resisting elements which carry either vertical or horizontal loads. Conversely, spandrels are the horizontal parts of the structure between two vertically aligned openings, which couple the response of contiguous piers when horizontally loaded. Although the identification of masonry piers and spandrels [245, 246, 247, 248, 249, 250, 251, 252, 253] may result easy and rather trivial in case of masonry façades with regularly distributed openings (e.g. for regular ordinary masonry structures, see Figure 1(b)), it becomes more complex in case of irregularly arranged openings, being substantially impossible for very complex geometries (e.g. for historic monumental masonry structures, see Figure 1(a)).

Macroelement models are the most widely diffused modeling strategies particularly for the seismic assessment of masonry structures, substantially the only one used by practitioners. Indeed, their very limited computational effort (also in case of 3D structures), coupled with the easy and quick definition of the model and mechanical properties, leaded their widespread dissemination.

However, being the macroelement models one of the most simplified approaches to analyze masonry structures (Figure 2), they present, together with their manageable computational effort, also some drawbacks. In particular, they usually assume that any activation of local (out-of-plane) failure mode is prevented. This decoupling assumption, although local failure modes can be separately assessed through kinematic limit analysis (see Section 7.2), could lead to conventional estimate of the seismic capacity, as in reality out-of-plane and in-plane damages can simultaneously arise [47]. Additionally, macroelement models cannot meticulously account for structural details, such as the toothing between orthogonal walls. Finally, the *a priori* idealization of the structure in piers and spandrels could lead to the definition of a mechanical system that could be far from the actual one, particularly for the case of very irregular opening layouts. Therefore, a certain level of expertise is anyway requested to the analyst.

Although most of macroelement models are equivalent beam-based [254], several spring-based approaches have also been recently developed. Either equivalent beam-based or spring-based approaches (Figure 20) are reviewed in the following.

8.1. Equivalent beam-based approaches

The idealization of masonry panels as nonlinear beams represent the most common assumption in the so-called "equivalent frame models". A pioneering equivalent beam-based model has been proposed by Tomaževič [255]. The so-called POR method [255] was based on crude mechanical assumptions, i.e. in-plane damage for horizontally loaded masonry façades was only due to shear forces in the piers, while both spandrels and nodal regions were considered rigid and fully resistant. This simple mechanical description, based on simplified elasto-plastic relationships to describe beam nonlinearity, provided sufficient reliability only in the case of buildings with weak piers and strong spandrels. Successively enhancements were presented in [248], implementing the flexibility and the limited strength of masonry spandrels.

Other more advanced equivalent beam-based models [256, 257, 258, 259, 260, 261, 262] proposed the idealization the masonry structure as an assemblage of pier and spandrel beam elements, linked by rigid links (Figure 20) which represent the nodes between piers and spandrels (i.e. zones in which seismic damage is rarely observable). These models rely on the phenomenological nonlinear elasto-plastic constitutive laws adopted for the beam elements.

Later, Grande et al. [263] proposed a simple beam FE for the nonlinear analysis of masonry structures, based on three parts: two rigid offsets, able to simulate the very stiff behavior of the masonry pier-lintel intersections, and a flexible central part. Furthermore, special shear interfaces were also introduced in the model to account for the shear failure. Another 2-node force-based beam FE has been formulated in [264], where the resultant stress components were exactly interpolated along the beam axis, performing analytical integration (without resorting to a fiber approach). The beam FE was composed of a central flexible element, characterized by a no-tension constitutive relationship, and a lumped nonlinear shear hinge. A further beam FE has been proposed in [265], where both flexural and shear plastic lumped hinges were inserted at the two nodes of the beam, following a classical elastic-plastic constitutive relationship. Finally, Liberatore & Addessi [266] developed a 2-node force-based beam FE consisting of a central linear elastic element, two flexural hinges and a shear link with elastic-perfectly plastic behavior, determined by a predictor–corrector method.

A 2D nonlinear beam with lumped plasticity that assumes a bi-linear relation with cut-off in strength (without hardening) and stiffness decay in the nonlinear phase has been proposed in [249], as implemented in the Tremuri software [267]. Being the latter particularly efficient for monotonic actions, more recently the formulation of this nonlinear beam has been refined by Cattari and Lagomarsino [268] through a piecewise-linear behavior. In particular, such refined constitutive law allows the description of the nonlinear response until very severe damage levels (from 1 to 5), through progressive strength degradation in correspondence of assigned values of drift.

The model includes also an accurate description of the hysteretic response formulated through a phenomenological approach, to capture the differences among the various possible failure modes (flexural type, shear type or even hybrid) and the different response of piers and spandrels, which revealed particularly efficient in performing nonlinear dynamic analyses [269].

Finally, a very advanced equivalent beam-based macroelement has been recently proposed by Raka et al. [270] for the nonlinear static and dynamic analysis of masonry buildings. The beam formulation considered axial, bending, and shear deformations within the framework of the Timoshenko beam theory. In particular, a phenomenological cyclic law for the beam section, accounting for the shear panel response, has been coupled with a fiber-based model that accounts for the axial and bending responses. Although the model accuracy is strongly dependent on the fiber and shear constitutive laws adopted, the formulation proposed in [270] is general and versatile.

8.2. Spring-based approaches

Alternatively to the use of equivalent beam elements, several macroelement models have been formulated by implementing nonlinear springs (Figure 20), within a fictitious frame, to approximate the in-plane nonlinear response of masonry walls and façades.

A pioneering application of a spring-based macroelement model has been presented in [271], adapting a model with nonlinear shear springs in series with rotational springs originally developed, in the 1980s, for the in-plane analysis of reinforced concrete walls. The proposed formulation for the analysis of masonry structures included an axial spring, three shear springs, and two rotational springs to simulate the axial, bed joint sliding, diagonal tension, and rocking/toe crushing failure modes experimentally observed on masonry pier tests.

In [272] and [273] a two-node element capable to represent the in-plane cyclic behavior of a whole masonry panels has been proposed aimed to describe both the shear behavior and the coupled axial-flexural one at the two nodes thanks to a bed of spring and two additional internal degree of freedom. In particular, the shear stress-strain cyclic relation has been derived by the macroscopic integration of the continuum model developed in [201]. Some aspects of this original formulation were further improved by Penna et al. [274] including a nonlinear degrading model for rocking damage, which permits to keep into account the effect of limited compressive strength. The latter model has also been implemented in the Tremuri software [249].

An interesting advance in the context of spring-based macroelement models has been developed by Caliò et al. [275], where piers and spandrels where idealized through equivalent discrete elements made of nonlinear springs to simulate the in-plane nonlinear response of masonry walls. The basic panel element is represented by an articulated quadrilateral constituted by four rigid edges connected by four hinges and two diagonal nonlinear springs. Each side of the panel can interact with other panels by means of a discrete distribution of nonlinear springs. The reliability of the proposed approach has been evaluated by means of nonlinear evolutionary static analyses performed on masonry structures. In [275] (and also in [276] for infilled frame structures), such a modeling approach has been used to directly represent piers and spandrels through basic panel elements. Nevertheless, given the versatility of the approach, such a modeling strategy has been used in [161, 162, 277] to simulate the masonry material response (and, so, not only the structural components response), see Section 6.1.

Another spring-based approach has been presented in [278], where each structural component has been described through multi-spring nonlinear elements connected by rigid links. In particular, nonlinear springs were placed at the two ends of the piers and spandrels for describing the flexural behavior and in the middle for representing the response in shear. The other parts were constituted of rigid links. Specific hysteretic rules for the degradation of stiffness and strength were also used for modeling the structural response under cyclic loading.

Aghababaie Mobarake et al. [279] proposed a basic panel element made-up of six sub-elements including upper and lower rigid beams and right, left (bilateral) and X-bracing nonlinear trusses, with four nonlinear zero-length sub-elements between the upper and lower beams and truss sub-elements. Each pier, spandrel and node between them is idealized by using a single proposed basic panel element. The approach proposed in [279] provided a rather simple and efficient platform for nonlinear static and dynamic analyses by considering the in-plane behavior of masonry panels.

Finally, a very recent and simplified solution has been presented by Xu et al. [280], where the masonry façade is considered as an integral unit, rather than composed of independent piers and spandrels. According to the strategy proposed in [280], the masonry façade is modeled by means of two vertical springs and a horizontal nonlinear spring that governs the wall shear response. The hysteretic behavior is governed by a group of control parameters, that depend on the distribution of openings and/or confining elements as well as on the dimensions, material properties and boundary conditions of the façade. The extremely simplified modeling strategy proposed in [280] could represent a complementary approach for the analysis of masonry structures subjected to horizontal cyclic loadings.

9. Conclusions

In this paper, a comprehensive review of the existing modeling strategies for masonry structures, as well as a classification of these strategies, has been presented. The classification of modeling strategies for masonry structures consisted of four categories (Figure 2): block-based models, continuum models, geometry-based models, and macroelement models. Although a fully coherent collocation of all the modeling approaches was substantially impossible due to the peculiar features of each solution proposed, this classification attempted to make some order on the wide scientific production on this field.

From the comprehensive review of modeling strategies for masonry structures carried out in this paper, the following conclusions can be drawn:

- Block-based models could represent the most accurate strategy to analyze the mechanical response of masonry structures. Several applications showed the potentialities of BBM to investigate the structural behavior of large-scale structures (specifically for contact-based approaches), with irregular and complex geometries as well. However, although the area of application of BBM appears theoretically large, their high computational demand strictly limits their employment to very important case studies and academic works. Anyway, they could be adopted to gain in-depth insights on specific features of the mechanics of masonry structures, and to provide reference solutions for more simplified approaches (e.g. MM).

- Continuum models represent widely used solutions for the structural analysis of masonry buildings. Concerning direct approaches, isotropic smeared crack and plastic-damage constitutive laws have been widely used for the structural assessment of historic monumental structures. Indeed, these approaches often represent the only suitable strategy to deal with such complex structures. However, the results obtained should be carefully interpreted, as they could sensibly overestimate, for example, the ultimate displacement capacity. Although no-tension continuum approaches seem to fail in a proper mechanical analysis of masonry structures, other simplified approaches, such as homogenized FE limit analysis and homogenized discrete approaches, appear particularly suitable for the structural assessment of full-scale masonry structures, even though the difficulties in the homogenization processes. Concerning multi-scale approaches, although very smart solutions have been proposed, they present some limitations. In particular, most of them have been tested only on 2D panel-scale masonry structures, with very few exceptions (see for example [218, 219]). Eventually, the so called FE^2 methods appears computational demanding. Indeed, although theoretically more efficient than BBM, the fact that their are usually implemented in homemade codes sensibly limits their efficiency and optimization. So far, no example of 3D computational homogenization method exists, being all the approaches developed in the last decades limited to 2D problems. Furthermore, being these approaches based on the mechanical response of the periodic RVE, the possibility of accurately represent specific structural details appears rather limited.

- Geometry-based models, although typically based on limit analysis solutions, can provide very useful outcomes. On the one hand, static theorem-based computational approaches represent effective solutions (substantially the only ones) for the investigation of the equilibrium states (and, therefore, the safety) in masonry vaulted structures. On the other hand, static theorem-based computational approaches appear especially suitable to predict the collapse mechanism (and the collapse multiplier) in complex masonry structures. These results, although non-comprehensive, represent a fundamental information in the mechanical analysis of masonry structures.

- Macroelement models mostly represent the only modeling strategy manageable by practitioners. Nevertheless, their reliability should be further improved by accounting for structural details (e.g. toothing between orthogonal walls) and the interaction between out-of-plane and in-plane damages. Anyway, MM are limited to the seismic assessment of ordinary masonry structures.

In summary, although significant advances have been made in the context of modeling strategies for masonry structures, each computational solution shows peculiar limitations and a specific area of application. Therefore, the choice of the most suitable modeling strategy should be formulated depending on the features and the complexity of the structure under investigation, the output required, the data available, and the expertise level.

Finally, the use of 3D models, which can represent the 3D features of a masonry structure, appears particularly indicated for the seismic assessment of masonry buildings to account for the geometric irregularities and the structural details which usually characterize ordinary and monumental buildings.

Acknowledgments

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