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Displaced Thinned Coprime Arrays with an Additional Sensor for DOA Estimation

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***Abstract*—A new sparse array structure based on the recently proposed thinned coprime arrays is proposed to maximize the number of unique lags. The design process involves two stages: the first stage displaces one subarray from its original position for an increase in the number of lags; as the displacement results in the minimum interelement spacing equal to integer multiples of half-wavelength, an additional sensor at a distance of half-wavelength is then added in the displaced subarray to avoid spatial aliasing. The strategic location of the additional sensor results in a significant increase in the overall unique lags which can be utilized for direction-of-arrival estimation (DOA) using compressive sensing based methods. Furthermore, the new structure has excellent performance in the presence of mutual coupling as shown by simulation results.**

***Index Terms*—Coprime array, direction-of-arrival estimation, maximum unique lags, sparse arrays.**

I. INTRODUCTION

Sparse arrays can resolve more sources than the number of sensors through exploitation of their difference co-array model. Some representative examples include minimum redundancy array (MHA), minimum hole array (MHA), nested arrays and super nested arrays [1–6].

Another example is the coprime arrays, which consists of two uniform linear subarrays. One subarray has M sensors with Nd inter-element spacing, while the other subarray has N sensors with Md inter-element spacing with M and N as coprime integers and d as the unit spacing set to $\frac{\lambda}{2}$, i.e. half wavelength of the signal. This structure is referred to as the prototype coprime array with $M+N-1$ sensors [7], and provides $2(M+N)-1$ consecutive lags. Conventional coprime arrays with $2M$ sensors in the second subarray provide significantly larger consecutive lags to the tune of $2MN+2M-1$ with $2M+N-1$ sensors [8], and can be exploited using subspace based DOA estimation methods such as MUSIC [8–11]. This structure also generates $3MN+M-N$ unique lags which can all be exploited using compressive sensing (CS) based DOA estimation methods [12]. Recently, thinned coprime arrays (TCA) have been proposed which retain all the properties of conventional coprime arrays with $\lceil\frac{M}{2}\rceil$ fewer sensors by removing a series of redundant sensors from one subarray [13], resulting in a structure with excellent sparsity and robustness to counter mutual coupling [14]. Generalized coprime arrays in the form of coprime arrays with displaced subarrays (CADiS) were recently proposed [15, 16] which increase unique lags through displacment of subarrays.

In this paper we propose a displaced thinned coprime array with an additional sensor (DiTCAAS) based on TCA with a two step design, where the first step involves a displacement of $(2M-2)N$ of the 2nd and 3rd subarrays $\mathbb{X}_2$ and $\mathbb{X}_3$. This displacement maximizes the number of unique lags. Due to the minimum inter-element spacing equal to integer multiples of half-wavelength, an additional sensor at a distance of half-wavelength from one of the sensors in displaced subarray $\mathbb{X}_3$ is added in the second stage. Two locations are found for the placement of the additional sensor, due to which significantly higher number of unique lags can be obtained. The resulting structure has more unique lags than other notable arrays structures for the same number of sensors, and due to its higher unique lags and sparsity has the best estimation performance in the presence of mutual coupling.

This paper is organized as follows. The coprime array model is reviewed in Sec. II and the proposed DiTCAAS is given in Sec. III. The degrees of freedom (DOFs) comparison is presented in Sec. IV. Simulations results are provided in Sec. V, followed by conclusions in Sec. VI.

II. CONVENTIONAL COPRIME ARRAY

For a conventional coprime array with $2M+N-1$ sensors, the array sensors are positioned at

$$\mathbb{P}=\{Mnd \mid 0\leq n\leq N-1\}\cup\{Nmd \mid 1\leq m\leq 2M-1\} \tag{1}$$

The positions of the sensors are given by the set $\mathbf{p}=[p_1,...,p_{2M+N-1}]^T$, where $p_i\in\mathbb{P}, i=1,...,2M+N-1$. The first sensor in both subarrays is co-located at the zeroth position with $p_1=0$.

With Q uncorrelated impinging signals from angles $\Theta=[\theta_1,...,\theta_Q]$ and their sampled baseband waveforms

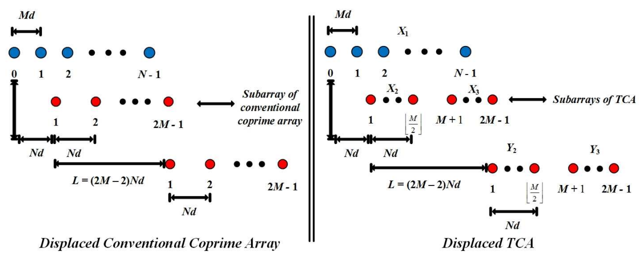

Fig. 1: Displaced conventional coprime array and TCA

$s_q(t), t = 1, ..., T$, for $q = 1, ..., Q$, the received data vector is given by

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \mathbf{a}(\theta_q) s_q(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (2)$$

where

$$\mathbf{a}(\theta_q) = [1, e^{-j\frac{2\pi p_2}{\lambda} sin(\theta_q)}, ..., e^{-j\frac{2\pi p_{2M+N-1}}{\lambda} sin(\theta_q)}]^T \quad (3)$$

is the steering vector, $\mathbf{A} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_Q)]$ and $\mathbf{s}(t) = [s_1(t), ...s_Q(t)]^T$. The entries of the noise vector $\mathbf{n}(t)$ are assumed to be spatially white Gaussian with a distribution $CN(0, \sigma_n^2 \mathbf{I}_{2M+N-1})$. The covariance matrix is given by

$$\mathbf{R}_{\mathbf{xx}} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R}_{\mathbf{ss}}\mathbf{A}^H + \sigma_n^2 \mathbf{I}_{2M+N-1} \quad (4)$$

$$\mathbf{R}_{\mathbf{xx}} = \sum_{q=1}^{Q} \sigma_q^2 \mathbf{a}(\theta_q)\mathbf{a}^H(\theta_q) + \sigma_n^2 \mathbf{I}_{2M+N-1} \quad (5)$$

where $\mathbf{R}_{\mathbf{ss}} = E[\mathbf{s}(t)\mathbf{s}^H(t)] = \text{diag}([\sigma_1^2, ..., \sigma_Q^2])$ is the source covariance matrix, with σ_q^2 denoting the signal power of the qth source.For the antennas located at the mth and nth positions in $\mathbf{p}$, the correlation $E[\mathbf{x}_m(t)\mathbf{x}_n^*(t)]$ results in the (m, n)th entry in $\mathbf{R}_{\mathbf{xx}}$ with lag $p_m - p_n$. All the values of m and n, where $0 \le m, n \le 2M + N - 1$, yield the lags or virtual sensors of the following difference co-array:

$$\mathbb{C}_{\mathbb{P}} = \{z \mid z = u - v, u \in \mathbb{P}, v \in \mathbb{P}\} \, . \quad (6)$$

III. Theoretical foundations for DiTCAAS

A. Stage 1 - Displaced thinned coprime array

Definition 1 (Displaced thinned coprime arrays)**.** *Assume M and N are coprime integers with $M \ge 4$ and $N \ge 3$, then the displaced thinned coprime arrays are specified by the integer set* $\mathbb{X}$, defined by

$$\mathbb{X} = \mathbb{X}_1 \cup \mathbb{Y}_2 \cup \mathbb{Y}_3,$$

where

$$\begin{cases} \mathbb{X}_1 = \{nMd \mid 0 \le n \le N-1\}, \\ \mathbb{Y}_2 = \{(2M-2+m)Nd \mid 1 \le m \le \lfloor \frac{M}{2} \rfloor\}, \\ \mathbb{Y}_3 = \{(3M-1+m)Nd \mid 0 \le m \le M-2\}. \end{cases} \quad (7)$$

where $\mathbb{Y}_2$ and $\mathbb{Y}_3$ represent the displaced versions of $\mathbb{X}_2$ and $\mathbb{X}_3$ in TCA respectively. Next we present some properties of displaced thinned coprime arrays.

Lemma 1. *For displaced TCA, no repitition in cross lags exist between the 1st subarray and the latter two subarrays at displacement $L = (2M-2)N$.*

Proof: We first consider the displaced coprime array as shown in the left half of Fig. 1. By displacing the $2M-1$ element subarray by $L = (2M-2)Nd$, the new sensor positions of displaced coprime array are given by

$$\mathbb{E} = \mathbb{C} \cup \mathbb{D} \quad (8)$$

$$\mathbb{C} = \{Mnd \mid 0 \le n \le N-1\} \quad (9)$$

$$\mathbb{D} = \{(2M-2+m)Nd \mid 1 \le m \le 2M-1\} \quad (10)$$

As shown in [13, 17], the repeated lags in the cross difference co-arrays Diff($\mathbb{D}$, $\mathbb{C}$) are additive inverses of each other, where Diff($\mathbb{D}$, $\mathbb{C}$) represents the differences in sensor positions of $\mathbb{C}$ from $\mathbb{D}$. These repeated lags exist due to colocation of the two subarrays. By displacing the 2nd subarray sufficiently, the conjugate pairs of cross lags cease to exist and the only repitition of lags occurs when some cross lags equal to self lags.

We only analyze the positive lags for convenience. The self lags of the two subarrays $\mathbb{C}$ and $\mathbb{D}$ themselves are of the form

$$\text{Diff}(\mathbb{C}, \mathbb{C}) = nM \quad (11)$$

$$\text{Diff}(\mathbb{D}, \mathbb{D}) = m'N \quad (12)$$

where $0 \le n \le N-1$ and $0 \le m' \le 2M-2$. Then we take the cross differences of the last two sensors of $\mathbb{C}$ from the first two sensors of $\mathbb{D}$, expressed as

$$\text{Diff}((2M-1)N, (N-1)M) = (M-1)N + M \quad (13)$$

$$\text{Diff}((2M-1)N, (N-2)M) = (M-1)N + 2M \quad (14)$$

$$\text{Diff}(2MN, (N-1)M) = M(N+1) \quad (15)$$

$$\text{Diff}(2MN, (N-2)M) = M(N+2) \quad (16)$$

With (13) and (14), cross differences of sensor at $(2M-1)N$ with sensors in $\mathbb{C}$ are of the form $(M-1)N + sM$, $1 \le s \le N$. As the two coprime numbers M and N cannot be a factor of $(M-1)N + sM$, self lags in (11) and (12) are not generated. Similarly, for lags in (15) and (16), cross differences related to the sensor at $2MN$ are of the form $M(N+s)$, which proves that all cross lags from sensors beyond $2MN$ in the 2nd subarray with sensors in $\mathbb{C}$ will be greater than the aperture of subarray $\mathbb{C}$ and therefore are unique compared with the self lags in (11) and (12), proving the unique nature of cross lags. As TCA is a redundant version of coprime array, Lemma 1 is equally applicable to displaced TCA, thus completing the proof. ■

Theorem 1. *The total number of unique lags for a displaced TCA with* $M \geq 4$ and $N \geq 3$ *is given by*

$$T_{umax} = \begin{cases} 3MN + 4M - 5, & \textit{for even M} \\ 3MN + 4M - N - 5, & \textit{for odd M} \end{cases} \tag{17}$$

Proof: Consider displaced TCA as shown in the right half of Fig. 1 where the first sensor of $\mathbb{Y}_2$ starts from $(2M-1)Nd$. First we start with the even M case where $\mathbb{X}_1$ has N sensors while $\mathbb{Y}_2$ and $\mathbb{Y}_3$ have a total of $\frac{M}{2} + M - 1 = \frac{3M-2}{2}$ sensors. A total of N sensors in $\mathbb{X}_1$ generate $N-1$ unique self positive lags for non-zero positions. As shown in [13], $\frac{3M-2}{2}$ sensors of $\mathbb{Y}_2$ and $\mathbb{Y}_3$ are able to generate all of the $2M-2$ unique lags like the $(2M-1)$-element subarray in conventional coprime array. As the cross lags between displaced subarrays $\mathbb{Y}_2$, $\mathbb{Y}_3$ and $\mathbb{X}_1$ are all unique as per Lemma 1, the total number of positive unique lags for displaced TCA with even M are given by

$$T_{ulep} = (N-1) + (2M-2) + \frac{3M-2}{2}N = \frac{3MN}{2} + 2M - 3 \tag{18}$$

Then the total number of unique lags (adding negative lags and zero lag) for a displaced TCA with even M is

$$T_{ule} = 3MN + 4M - 5 \tag{19}$$

which proves the first part of (17).

For odd M, we can prove it in a similar way. ∎

B. Stage 2 - Additional sensor at half-wavelength

Although displaced TCA results in increased unique lags, the minimum interelement spacing becomes an integer multiple of half-wavelength, leading to the well-known spatial aliasing problem. To mitigate this problem, we investigate the addition of another sensor at half-wavelength from a sensor in the displaced TCA to make sure that the minimum interelement spacing of displaced TCA remains $\frac{\lambda}{2}$. The additional sensor also needs to be placed so that the overall structure has significantly higher number of unique lags. The new structure will be termed as displaced thinned coprime array with an additional sensor (DiTCAAS).

We first analyze the conventional coprime array to find out the positions of sensors in one subarray which are separated from their nearest sensor in the other subarray by a given distance for an arbitrary M and N. A general result in this direction is presented in Lemma 2.

Lemma 2. *The sensor of* $(2M-1)$*-element subarray leading/lagging the nearest sensor of* $N-$*element subarray by distance* n *where* $1 \leq n \leq M-1$*, is located at index* i and k*, given by the relationships* (20) *and* (21) *respectively*

$$i \bmod (N, M) = n + jM \tag{20}$$

$$M - k \bmod (N, M) = n - jM \tag{21}$$

where $1 \leq i, k \leq 2M-1$, $j \geq 0$ *and* $\mathrm{mod}(N, M)$ *refers to the modulo operator and returns the remainder of* $\frac{N}{M}$.

Proof: The distance between a sensor of $(2M-1)$-element subarray located at iN and its nearest sensor of N-element subarray lesser in value than iN is given by

$$S_i = \mathrm{mod}(iN, M) = \mathrm{mod}(i \bmod (N, M), M) \tag{22}$$

As $\mathrm{mod}(n, M) = \mathrm{mod}(n + jM, M)$ where $1 \leq n \leq M-1$ and $j \geq 0$, index i for a particular n can be found by solving

$$i \bmod (N, M) = n + jM \tag{23}$$

Similarly, the distance of a sensor of $(2M-1)$-element subarray located at kN relative to the nearest sensor of N-element subarray greater in value than kN is given by

$$\hat{S_k} = M - \mathrm{mod}(kN, M) = M - \mathrm{mod}(k \bmod (N, M), M) \tag{24}$$

As $\mathrm{mod}(n, M) = \mathrm{mod}(n - jM, M)$, index i for a particular n can be found by solving

$$M - k \bmod (N, M) = n - jM \tag{25}$$

∎

Please note that for TCA, (20) and (21) represent index of physical sensors for index range $1 \leq i, k \leq \lfloor \frac{M}{2} \rfloor$. For $n = 1$ corresponding to half-wavelength distance, (20) and (21) change to

$$i \bmod (N, M) = 1 + jM \tag{26}$$

$$M - k \bmod (N, M) = 1 - jM \tag{27}$$

Now we show that index i and k are related to each other. Equating S_i with $\hat{S_k}$ and rearranging the terms, we have

$$\mathrm{mod}(iN, M) + \mathrm{mod}(kN, M) = M \tag{28}$$

Applying modulo on both sides yields

$$\mathrm{mod}(iN + kN, M) = \mathrm{mod}(M, M) = 0 \tag{29}$$

Since M and N are coprime, the solution is given by

$$i + k = pM, p \in \mathbb{Z} \tag{30}$$

where $p = 1$ since $1 \leq i, k \leq \lfloor \frac{M}{2} \rfloor$:

$$i + k = M \tag{31}$$

In the next step, we present two suitable locations for the additional sensor which can significantly increase unique lags.

Theorem 2. *The total number of unique lags for* DiTCAAS *with additional sensor located at* $3M - 2 + iN - 1$ *or* $3M - 2 + kN + 1$ *with* $M \geq 4$ and $N \geq 3$ *is given by*

$$T_{umax} = \begin{cases} 3MN + 7M + 2N - 9, & \textit{for even M} \\ 3MN + 7M + N - 10, & \textit{for odd M} \end{cases} \tag{32}$$

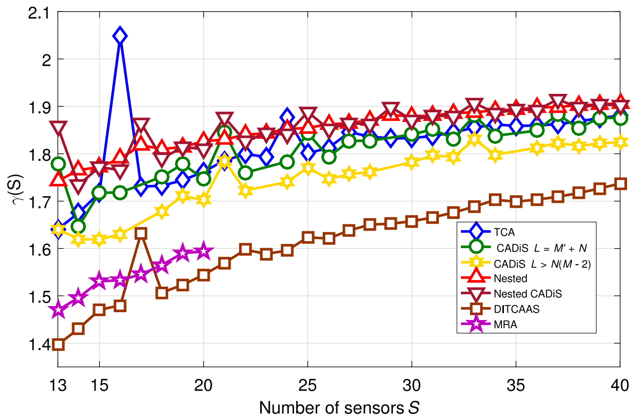

Fig. 2: Unique lags capacity comparison for sparse arrays

Proof: For the two proposed locations $(3M-2)N+iN-1$ and $(3M-2)N+kN+1$, $(3M-2)N$ represents the redundant sensor at MN in TCA after displacement of $(2M-2)N$. This reference position is chosen to maximize the number of unique lags for additional sensor as shown later.

The starting sensor of $\mathbb{Y}_2$ at $(2M-1)N$ is equidistant from the additional sensor and a respective sensor of $\mathbb{X}_1$ which will be shown as follows. The differences in position of the additional sensor placed at $(3M-2)N+iN-1$ or $(3M-2)N+kN+1$ relative to $(2M-1)N$ (the first sensor in $\mathbb{Y}_2$) denoted by S_1 and S_2 are given by

$$S_1 = (M+i-1)N-1 \tag{33}$$

$$S_2 = (M+k-1)N+1 \tag{34}$$

Then by taking the difference of S_1 and S_2 from $(2M-1)N$, denoted by S_3 and S_4 respectively and according to (31), we have

$$S_3 = (M-i)N+1 = kN+1 \tag{35}$$

$$S_4 = (M-k)N-1 = iN-1 \tag{36}$$

For index i and k corresponding to $n=1$, $iN-1$ and $kN+1$ represent the positions of the sensors of $\mathbb{X}_1$ in TCA which proves that the sensor at $(2M-1)N$ is equidistant from the additional sensor and sensor of $\mathbb{X}_1$. The additional sensor will contribute the same set of lags by interacting with $\mathbb{Y}_2$ and $\mathbb{Y}_3$ as the sensor in (35) or (36) of $\mathbb{X}_1$ will do with $\mathbb{X}_2$ and $\mathbb{X}_3$ in TCA. As TCA and displaced TCA differ from each other only by the displacement $(2M-2)N$ for the displaced subarrays, their cross difference coarrays also differ from each other by a factor of $(2M-2)N$. As a result, with the exception of one repitition of the equidistant lag, the interaction between the additional sensor and $\mathbb{Y}_2$ and $\mathbb{Y}_3$ will generate unique lags. Now we consider the interaction of additional sensor with $\mathbb{X}_1$. As the additional sensor is placed at $iN-1$ or $kN+1$ respectively from $(3M-2)N$, and represents displacement equal to multiples of M, it will generate part of the set of lags generated by the position $(3M-2)N$ relative to $\mathbb{X}_1$ given by

$$S_5 = (3M-2)N - lM, 0 \le l \le N-1 \tag{37}$$

in addition to i or k lags equal to $S_5 + qM$ where $1 \le q \le i$ or $1 \le q \le k$. Since $(3M-2)N$ in displaced TCA represents the displaced position of a redundant sensor in conventional coprime array at MN, missing in TCA, all the set of lags generated by the additional sensor through interaction with $\mathbb{X}_1$ will be unique. This proves that the additional sensor at these two locations through interaction with the displaced TCA generates only one repeated lag with all remaining lags as unique lags. As a result, this extra sensor brings $2(H-1)$ new unique lags for a displaced TCA with H sensors, for a total number of $H+1$ sensors for DiTCAAS. Now we calculate the total number of unique lags for DiTCAAS for cases of even and odd M. For even M, the total number of sensors for displaced TCA is given by $\frac{3M}{2}+N-1$. The contribution of unique lags for additional sensor is

$$S_{add_{even}} = 2 \times (\frac{3M}{2}+N-2) = 3M+2N-4$$

Then, the total number of unique lags for DiTCAAS with even M for $\frac{3M+2N}{2}$ sensors is given by

$$S_{DiTCAAS_{even}} = 3MN + 7M + 2N - 9 \tag{38}$$

Similarly for odd M, the total number of sensors for displaced TCA is given by $\frac{3M+2N-3}{2}$. The contribution of unique lags for additional sensor is given by

$$S_{add_{odd}} = 2 \times (\frac{3M+2N-3}{2} - 1) = 3M+2N-5$$

Then the total number of unique lags for DiTCAAS with odd M for $\frac{3M+2N-1}{2}$ sensors is

$$S_{DiTCAAS_{odd}} = 3MN + 7M + N - 10 \tag{39}$$

■

IV. DOF Comparison of Sparse Arrays

We consider the proposed DiTCAAS, TCA, nested array, nested CADiS, MRA and sparse CADiS. Among them, nested array, nested CADiS and MRA generate hole-free co-arrays while sparse CADiS, TCA and DiTCAAS, all generate co-arrays with holes. As far as the availability of sparse arrays for arbitrary number of sensors is concerned, MRA in literature is available for a maximum of 20 sensors [2], while sparse CADiS is not available for specific number of sensors. On the other hand, nested array, nested CADiS, TCA and DiTCAAS can all be generated for any number of sensors.

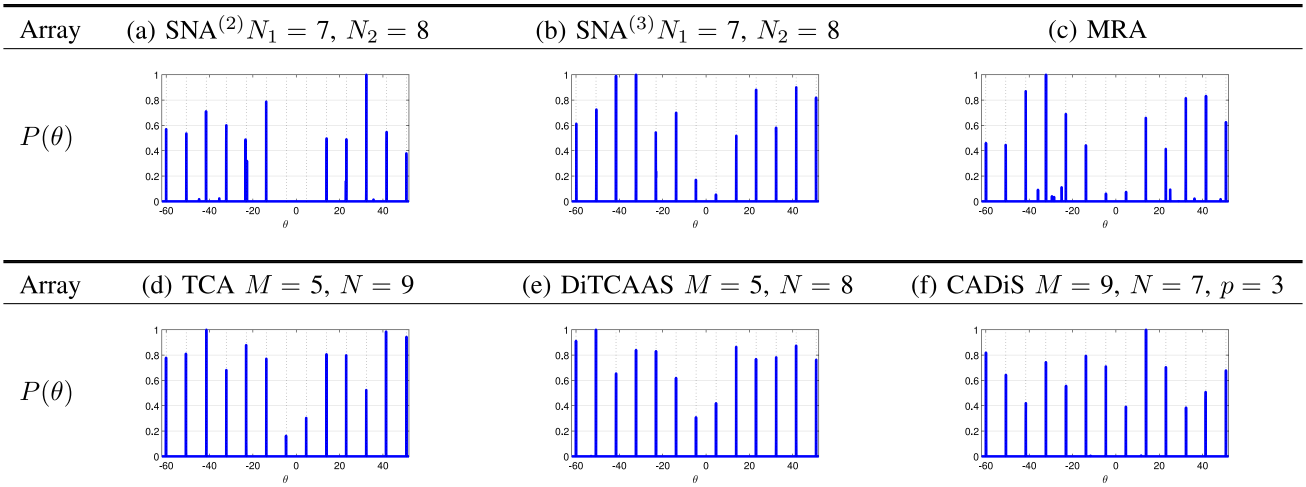

Fig. 3: DOA Spectrum comparison among 15 sensors SNA, MRA, TCA, DiTCAAS and CADiS with $|c_1| = 0.4$.

To compare the sparsity of these array structures, the DOF capacity beyond the redundancy is analyzed, defined as [1]

$$\gamma(S) = \frac{S^2}{DOFs} \tag{40}$$

where S represents the total number of sensors in an array and $DOFs$ represents the two-sided unique lags based on the difference co-array. The results are plotted in Fig. 2, where the smaller the value of $\gamma(S)$, the higher the DOF capacity. It is clear that DiTCAAS has the highest DOF capacity compared to other sparse arrays, thus generating the highest number of unique lags for a fixed number of sensors. The proposed DiTCAAS holds strong potential to achieve significantly lower DOA estimation error with CS based methods than other sparse arrays.

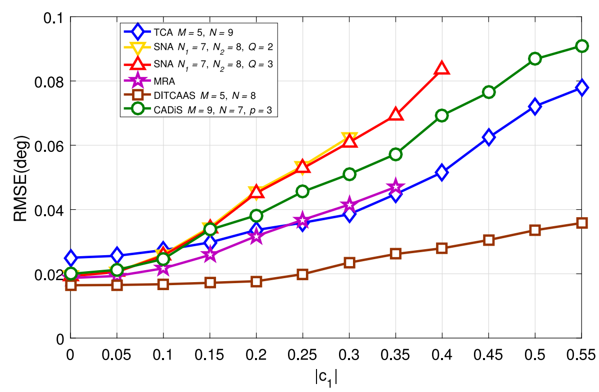

Fig. 4: RMSE versus mutual coupling coefficient

V. Simulation Results

In this section we investigate the performance of different sparse arrays in the presence of mutual coupling, where the CS-based method is employed for DOA estimation. 15-sensor sparse arrays are considered including the second and third order super nested arrays $N_1 = 7$, $N_2 = 8$, TCA $M = 5, N = 9$, sparse CADiS $M = 9, N = 7, p = 3$, MRA as $\{0, 1, 6, 14, 22, 30, 38, 46, 54, 62, 64, 66, 69, 71, 73\}d$ [2] and DiTCAAS $M = 5$, $N = 8$ with additional sensor at $(3M - 2)N + kN+$ 1 where $k = 3$ and represented as $\{0, 5, 10, 15, 20, 25, 30, 35, 72, 80, 112, 120, 128, 129, 136\}d$. The characteristics of these sparse arrays including aperture, consecutive lags, unique lags and weight functions $w(1)$, $w(2)$ and $w(3)$, defined in [5] are shown in Table I.

Although DiTCAAS generates the lowest number of consecutive lags at 20 compared to other sparse arrays, it generates the highest number of unique lags at 153, even more than the MRA and has excellent sparsity with only $w(1) = 1$. For the simulation, mutual coupling model is incorporated from the work in [5]. First we present the DOA spectrum for 13 sources with considered parameters as 1000 snapshots, 10 dB SNR and mutual coupling coefficient $|c_1| = 0.4$ in Fig. 3, where it can be clearly seen that the second order super nested array fails to resolve the sources and is severely affected by mutual coupling. Although MRA is significantly sparser than the second order super nested array, it still suffers from a degraded spectrum with lots of spurious peaks. The third order super nested array is able to resolve all the sources but with a degraded spectrum for two sources. On the other hand, TCA, sparse CADiS and DiTCAAS detect all the 13 peaks with a clean spectrum. In the next step, root mean square error (RMSE) curve for DOA estimation against varying $|c_1|$ is presented. The parameters chosen are 13 sources, 10 dB SNR, 1000 snapshots with $|c_1|$ varied from 0 to 0.55 and the results are presented in Fig. 4, where each point on the curve is an average of 200

Array	SNA (7, 8, 2)	SNA (7, 8, 3)	MRA	CADiS (9, 7, 3)	TCA (5, 9)	DiTCAAS (5, 8)
Aperture	63	63	73	77	81	136
Con. Lags	127	127	147	54	99	20
Uni. Lags	127	127	147	131	131	153
$w(1)$	1	1	1	0	1	1
$w(2)$	6	3	4	0	1	0
$w(3)$	1	2	1	6	1	0

TABLE I: Sparse array characteristics for 15 sensors.

independent simulation runs. It can be seen that DiTCAAS has the lowest RMSE compared to other sparse arrays due to its excellent sparsity and higher number of unique lags. Even at $|c_1| = 0.55$, DiTCAAS incurs half the error of TCA, which showcases the potential of DiTCAAS. Overall, DiTCAAS has proved itself to be a very robust array for CS-based DOA estimation with mutual coupling.

VI. Conclusion

In this paper, a new sparse array structure called DiTCAAS based on TCA is proposed which provides a significantly higher number of unique lags. Due to its excellent sparsity, availability for any number of sensors, systematic construction and very high number of unique lags, DiTCAAS achieves the lowest RMSE and robustness to heavy mutual coupling compared to super nested arrays, MRA, TCA and sparse CADiS wth CS-based DOA estimation.

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