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The effect of soil non-linearity on mixed traffic railway lines: passenger vs freight loads

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Abstract. To add additional capacity to railway networks, freight services might be added to lines that have previously only be used for passenger services. Existing ballasted lines may have mixed subgrade conditions and thus the effect of increased axle loads on track behavior is unclear. Typically, such cases will result in elevated track deflections in comparison to passenger vehicles. As a result, the supporting subgrade experiences higher strain levels, which can fall into the large strain range. The related non-linear subgrade behavior plays an important role in track response but is challenging to model. As a solution, this paper presents a new semi-analytical numerical model, where the track is simulated analytically and allows for 1D wave propagation. The ground is modelled using a non-linear equivalent thin-layer finite element formulation. This allows for the subgrade stiffness to be updated in an iterative manner with minimal computational effort. A case study is presented to show that modest increases in axle load can have a marked effect on track deflections.

Keywords: Railway freight, railroad, non-linear soil

1 Literature Review

With the aim of adding additional capacity to existing railway lines, it may be desirable to add freight services to tracks that have previously only be used for passenger services. Some of these lines may have relatively low subgrade stiffness's and thus the effect of increased axle loads on track behavior is unclear.

To investigate and predict the track performance and ground response under various train loads and speeds, a number of modelling techniques have been proposed. The approaches include analytical models (Krylov, 1995) (Degrande & Lombaert, 2001)(Takemiya & Bian, 2005), semi-analytical models (Sheng, Jones, & Petyt, 1999)(Madshus & Kaynia, 2000)(Sheng, Jones, & Thompson, 2003)(Kaynia, Madshus, & Zackrisson, 2000)(Thompson, 2008)(Triepaischajonsak & Thompson, 2015). There are also numerical models: 2.5D models (Yang, Hung, & Chang, 2003)(P. Alves Costa, Calçada, & Silva Cardoso, 2012)(Pedro Alves Costa, Calçada, Silva Cardoso, & Bodare, 2010) and fully 3D models using finite element (FE) and possibly boundary element (BE) theories (Hall, 2003)(Kouroussis, Gazetas, Anastasopoulos, Conti, & Verlinden, 2011)(Arlaud, Costa D'Aguiar, & Balmes, 2015)(El Kacimi, Woodward, Laghrouche, & Medero, 2013).

For freight trains, the dominant frequency components of the vibration are within 4-30 Hz (Jones & Block, 1996). In order to study the vibrations induced by the freight trains, both dynamic and quasi-static generation mechanism, a track response model combined with transfer functions from sleeper to ground was utilized by (Jones & Block, 1996). Another numerical model was proposed for the studies of longitudinal dynamics of the trainset (Belforte, Cheli, Diana, & Melzi, 2008). On-site tests can be costly (Jones, 1994), meaning theoretical models are often used to examine the track performance and ground response from freight trains.

In modelling the ground vibrations from railways, linear elastic models of the soil are commonly used, because strains are small. Nonetheless, when axle loads increase and/or the train speed gets close to the critical velocity, the track deflections increase and non-linear soil response occurs (Madshus & Kaynia, 2000)(Pedro Alves Costa et al., 2010). To simulate this non-linear behavior, soil stiffness' can be artificially reduced (Madshus & Kaynia, 2000)(Kaynia et al., 2000). Alternatively, using an automated, equivalent non-linear approach, the shear modulus can be adjusted based on the maximum effective octahedral shear strain in each soil element. Then it can be updated element by element until a tolerance requirement is met (Pedro Alves Costa et al., 2010).

Since the supporting non-linear ground behavior plays a key role when modelling the vibrations generated by the freight trains, this paper provides a robust and efficient semi-analytical to model non-linear soil effects. The track is modelled analytically and allows for 1D wave propagation. The soil is modelled using a non-linear equivalent thin-layer method (TLM). The soil stiffness is updated in an iterative manner to simulate the non-linear behavior of the soil with the minimum computational effort.

2 Numerical Model Development

Freight trains subject railway tracks to heavy axle loads which result in elevated strains within the supporting subgrade. Large strains cause non-linear soil behavior, resulting in reduced support stiffness. Modelling non-linear soil behaviour is computationally intensive and thus difficult to include in a sensitivity analysis. Therefore, to reduce computational requirements, a thin-layer finite element model was developed, and then combined with an equivalent non-linear procedure. This soil model was then efficiently coupled with a track model that permitted 1D wave propagation.

2.1 Track Model

Ballasted track was modelled considering, rail, railpad, sleeper and ballast components, as shown in Fig. 1. One dimensional wave propagation was considered in the ballast and the track was coupled to the soil using an equivalent spring, using the approach outlined in (Dieterman & Metrikine, 1996).

$$\begin{bmatrix} EI_r k_1^4 + k_p^* - \omega^2 m_r & -k_p^* & 0 \\ -k_p^* & k_p^* + \frac{2\omega E_b^* b\alpha}{tan\left(\frac{\omega h}{C_p}\right)C_p} - \omega^2 m_s & \frac{-2\omega E_b^* b\alpha}{sin\left(\frac{\omega h}{C_p}\right)C_p} \\ 0 & \frac{-2\omega E_b^* b\alpha}{sin\left(\frac{\omega h}{C_p}\right)C_p} & \frac{2\omega E_b^* b\alpha}{tan\left(\frac{\omega h}{C_p}\right)C_p} + k_{eq} \end{bmatrix} \begin{Bmatrix} \tilde{u}_r(k_1,\omega) \\ \tilde{u}_s(k_1,\omega) \\ \tilde{u}_{bb}(k_1,\omega) \end{Bmatrix} = \begin{Bmatrix} \tilde{P}(k_1,\omega) \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

Where EI_r is the bending stiffness of the rail; m_r is the mass of rails per meter; m_s is the equivalent distributed mass of sleepers; k_p^* is the complex stiffness of the railpad; k_{eq} is the equivalent stiffness of the ground; E_b^* is the Young's modulus of the ballast; C_p is the compression wave speed in the ballast; h is the ballast layer height; α is the adimensional parameter, taken as 0.5; b is the half-width of the track.

The ballasted track model includes the coupling between the track and the soil, i.e., the complex equivalent stiffness of the ground k_{eq}. It was suggested by (Steenbergen & Metrikine, 2007) that the equivalent stiffness can be calculated using the ratio between the load and average displacement along the track-soil interface. Therefore, the equivalent stiffness can be mathematically represented in the wavenumber-frequency domain by the formula:

$$\tilde{k}_{eq}(k_1,\omega) = \frac{2\pi}{\int_{-\infty}^{+\infty} \tilde{u}_{zz}^G(k_1,k_2,0,\omega)\frac{\sin(k_2 b)^2}{(k_2 b)^2}dk_2} \quad (2)$$

Where u_{zz} is the Green's function of vertical displacement of the ground in the wavenumber-frequency domain, and k_1 and k_2 are the Fourier images of coordinate x and y, respectively. The Green function is computed by the Haskell-Thompson approach (Sheng et al., 1999).

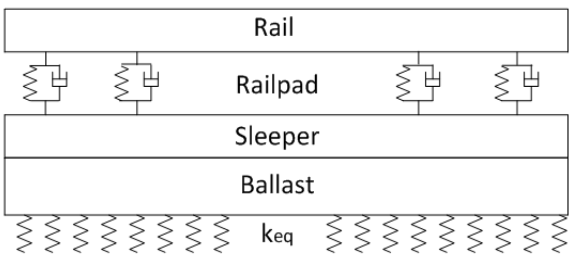

Fig. 1. Analytical ballasted track model layout

2.2 Soil Model

The soil is modeled using the Thin-Layer Method (TLM). The TLM is a semi-discrete numerical technique used for the analysis of wave motion in layered media. It is illustrated in the Fig. 2.

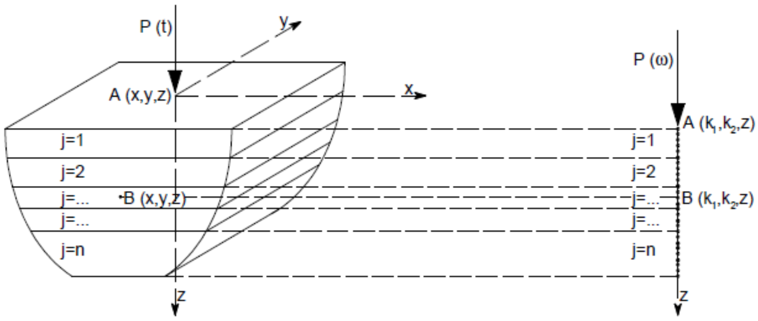

Fig. 2. Schematic diagram of Thin-Layer Method modelling process (Pedro Alves Costa, 2011)

It is worth noting that:

- The thickness of the thin layers was computed as $h = \frac{wavelength}{8} = \frac{2\pi}{8k_{max}}$, where k_{max} is the maximum wavenumber defined
- Quadratic elements were used for the soil model, as demonstrated in the Fig. 4
- After obtaining the displacement of each node, the strain/stress field inside the layer was then calculated using equations (3) and (4)

$$\{\boldsymbol{\varepsilon}\} = [\mathbf{B}]\{\mathbf{u}\} \qquad (3)$$

$$\{\boldsymbol{\sigma}\} = [\mathbf{D}]\{\boldsymbol{\varepsilon}\} = [\mathbf{D}][\mathbf{B}]\{\mathbf{u}\} \qquad (4)$$

Where $[\mathbf{B}] = [\mathbf{B_1} \quad \mathbf{B_2} \quad \mathbf{B_3}]$ and

$$[\mathrm{B_i}] = \begin{bmatrix} ik_1N_i & 0 & 0 \\ 0 & ik_2N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ ik_2N_i & ik_1N_i & 0 \\ 0 & \frac{\partial N_i}{\partial z} & ik_2N_i \\ \frac{\partial N_i}{\partial z} & 0 & ik_1N_i \end{bmatrix} \quad (5) \qquad \begin{aligned} N_1(\xi) &= \frac{1}{2}\xi^2 - \frac{1}{2}\xi \\ N_2(\xi) &= 1 - \xi^2 \\ N_3(\xi) &= \frac{1}{2}\xi^2 + \frac{1}{2}\xi \end{aligned} \quad (6)$$

- The inverse Fourier Transform was used to convert the results from the wavenumber-frequency domain back to the time-space domain

2.3 Equivalent Non-linear Model

For freight trains lines, the supporting subgrade is likely to experience high levels of strain. This can result in soil stiffness degradation, thus increasing the track displacements and causing track deterioration. To simulate this, a non-linear equivalent model, based on an iterative stiffness updating procedure, was used. This model was well-suited to the discretized nature of the TLM method and summarized using the following steps:

1) Assume low strain properties for all elements
2) Compute strain time histories and determine the maximum effective octahedral shear strain values for all elements
3) Use stiffness degradation curves, as shown in the Fig. 3, and calculated maximum effective octahedral strains to obtain the new stiffness for all elements
4) Use the same procedure to compute the new damping values for all elements
5) Repeat steps 2 – 4 until the established tolerance is met for all elements

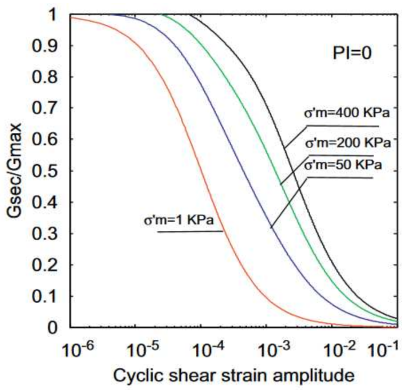

Fig. 3. Modulus reduction curves for non-plastic soil (Pedro Alves Costa et al., 2010)

3 Model Validation

(Chen et al., 2005) proposed an analytical approach that calculates the stresses in the ground using the equivalent stiffness on the basis of the model of an Euler beam resting on the half space subjected to a moving load. In order to validate the TLM model for the ground response, same case was studied and the stresses in the soil compared against Chen et al., 2005's simulation result.

3.1 Model Description

As depicted in the Fig. 4, the train-embankment-ground model contains an Euler beam resting on top of the half-space with a concentrated moving force acting on the beam. The half-space was modelled as thin layers and the coupling between the embankment and ground was represented by the equivalent stiffness. Assuming the load is at the centre of the embankment at the beginning, then it will move along the central line with a certain speed. The stresses generated by the contact force between the embankment and ground were calculated at 2m depth below the loading point.

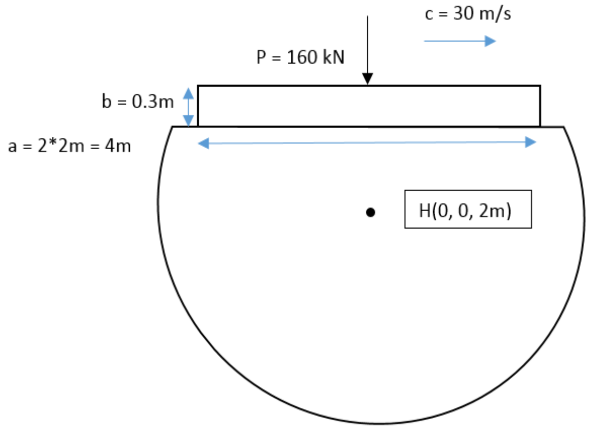

Fig. 4. Schematic diagram of Chen et al., 2005 validation model

Key embankment and ground properties related to the validation are listed in the Table 1 and Table 2 respectively. Aside from the listed parameters, the load speed was 30 m/s and the amplitude of the point load used in the simulation was 160 kN, without consideration of the irregularity of the contact surface.

Table 1. Properties of the embankment

Density (kg/m^3)	Young's modulus (MPa)	Width (m)	Height (m)	Mass (kg)	Second moment of area (m^4)
1900	30000	4	0.3	2280	0.009

Table 2. Properties of the ground

Shear modulus (MPa)	Poisson ratio	Density (kg/m^3)	Secondary wave speed (m/s)
10	0.45	1800	74.54

3.2 Simulation Result

Since the direction of the point load is vertical and forces on other directions are not considered, shear stress τ_{xy} and τ_{yz}, for soil elements beneath the moving point load, were zero. Therefore, apart from normal stresses σ_{xx}, σ_{yy} and σ_{zz}, only $\tau_{xz}(=\tau_{zx})$ was analysed. In this validation case, the dynamic stresses generated by the moving point load were computed using the TLM model and the response points for comparison were chosen at 2m depth directly underneath the motion line of the moving load.

Fig. 5 reveals good agreement for all dynamic stresses between Chen et al., 2005 and TLM's simulation result is found. Moreover, for a given soil, the strain can be calculated as $\varepsilon = \sigma/E$, where E is the Young's modulus of the soil layer. Therefore, the TLM is also applicable for the calculation of strains. This validation manifests that the TLM model is able to accurately describe the ground response induced by the moving load and also enables to predict the response in the soil at any given point.

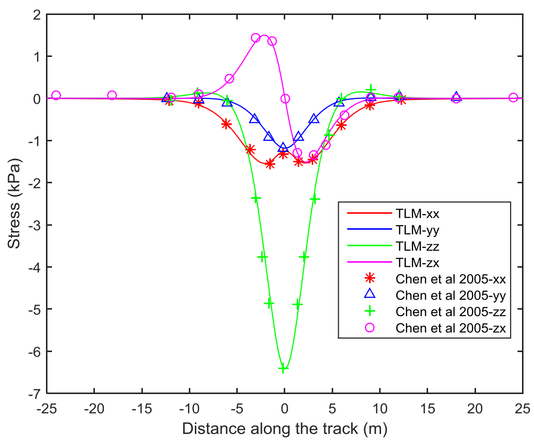

Fig. 5. Comparisons of the dynamic stresses of an element with 2m depth underneath the moving load

4 Analysis and Results

Simulations were run to determine the effect of adding 25 tonne fright axle loads to a previous passenger-only (17 tonne) ballasted line, with the aim of determining increases in track displacement and soil strain. To do so, the following track properties were assumed: $m_r = 120\ kg/m$, $\mathrm{m_s} = 490\ kg/m$, $\mathrm{k_p^*} = 5 \times 10^8 N/m^2$, $E_b^* = 125\ MPa$, $h = 0.35m$, $\mathrm{b} = 2.5\mathrm{m}$. The soil was modelled as a homogenous half-space using the following properties: $density = 2000\ kg/m^3$, $Young's\ modulus = 25\ MPa$, $Poissons\ ratio = 0.35$, $damping = 0.03$. The stiffness degradation profile was the same as that shown previously. Train speed for both the passenger and freight axle loads was 26 m/s.

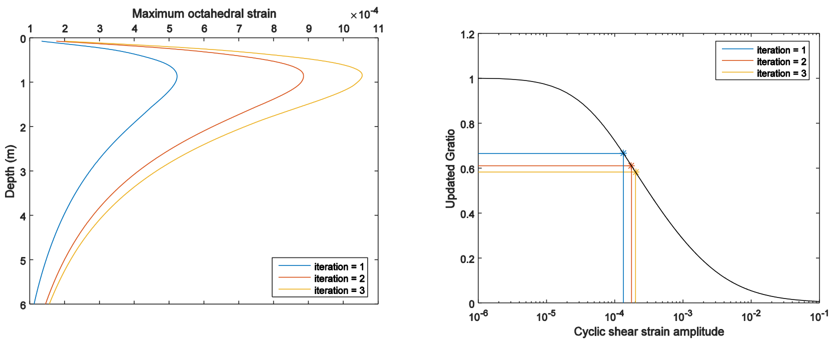

Fig. 6. Left: Octahedral strain vs soil depth, Right: Soil stiffness degradation during freight train passage

Fig. 6 (left) shows the variation of strain versus depth within the soil. It is observed that the maximum strain level is found approximately 1 m below the ground surface and decays rapidly with depth. Correspondingly, Fig. 6 (right) shows maximum strain levels and their corresponding effect on soil stiffness. After the first iteration, the soil drops to 67% of its original stiffness and by the third (and final) iteration, it has reached a value of 59%.

The resulting reduction in stiffness (Young's modulus) with depth is shown in Fig. 7 (left). For iteration 1, stiffness is constant with depth, however after strain updating, the subsequent iterations show large variations with depth, and are all lower than the starting value, particularly near the soil surface. For the passenger train, track displacements are 3.7mm, however for the freight train, the linear value is 5.5 mm displacement, and the non-linear (iteration 3) is 8.4 mm. Therefore, it can be seen that the soil behavior is significantly non-linear, and that traditional linear analysis would greatly underestimate track deflections. This would result in much faster loss of track geometry and require frequent tamping. In addition, it is interesting to note that as the soil stiffness decreases, dynamic effects become more prevalent, with iteration 3 displacements appearing less symmetric than iteration 1.

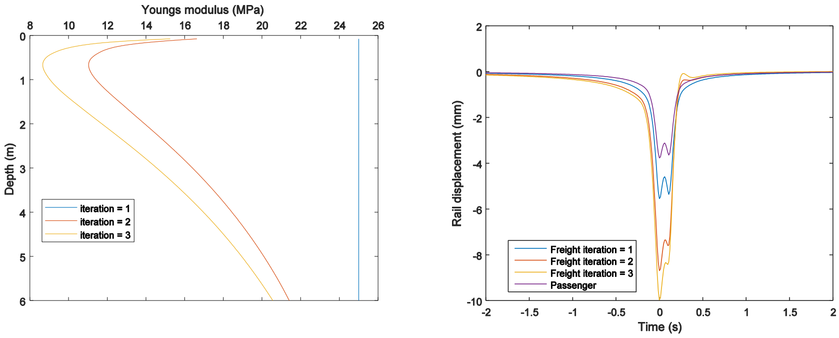

Fig. 7. Left: Young's modulus reduction with depth, Right: Track displacements

5 Conclusions

Under certain circumstances, it may be desirable to run freight trains on ballasted track originally designed for passenger services. In such cases, the track may have a relatively low subgrade stiffness, meaning the effect of freight axles loads can lead to non-linear behavior. To determine the effect of increased axle loads in such cases, an equivalent non-linear numerical model was developed, capable of quickly assessing soil stresses and strains, and resulting track displacements. The model was validated and then used to assess the behavior of freight axle loads on a low stiffness ballasted line. It was shown that the track displacements have the potential to become high, due to non-linear stiffness reduction and the resulting dynamic amplification.

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