A methodology for robust multi-train trajectory planning under dwell-time and control-point uncertainty

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Abstract

Methods to optimise train movements to maintain time and avoid excessive energy consumption are becoming widely applied, but outcomes remain sensitive to uncertainties within the system. In this paper variability in station dwell-times and the points of application of planned train control (traction, coasting or braking) are taken as examples of typical rail system uncertainties, and are used to demonstrate an approach to multi-train trajectory optimisation that is resilient to them.

Trade-offs are explored between highly optimised train trajectories that are vulnerable to perturbation, and less optimal trajectories that are robust to typical disturbances. Beyond dwell and control variations the method has application for the many other common rail network uncertainties, e.g. differences in the traction characteristics of nominally identical trains or variable train loading. Relative to optimisation without consideration of uncertainty,

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the approach is shown to find control strategies of substantially increased robustness for the test cases examined, and offers a more principled way to plan a network than the ad-hoc use of recovery time to mitigate everyday operational disturbances.
Keywords: multi-train trajectory optimisation, robust control, energy efficiency, trajectory planning, railway network optimisation, genetic algorithm

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1. Introduction

1.1. Background

Owing to increased traffic demands, railway systems in the UK and elsewhere have become increasingly busy and in some areas their capabilities restrict the volume of people and goods that can be moved[1]. Trains are operating at increased frequency but there is often also a desire to minimise travel time and, as a consequence, the system has little slack with an increased likelihood that small disruptions can propagate across large parts of the network with disproportionately severe effects.

Many uncertainties exist in rail networks, for example, small differences in the traction characteristics of nominally identical trains, varying dwell-times (DT) at stations, varying train loads, and variations in the locations at which train control switches between traction, coasting or braking. We refer to these locations as control-points (CP) for brevity. This is the case even where the planned trajectory is advised to the driver through a driver advisory system (DAS). For a busy network with little scope for recovery these uncertainties mean that a planned schedule that appears to be optimal may not be achievable in practice, i.e. it is not a robust. This has to be mitigated with recovery time built into the timetable, without which operation can become severely degraded as, for example, delays accumulate and cannot be recovered. An example taken from the UK shows a train making 14 station calls on a journey from London to Leeds having a scheduled extended stop of 480s (8 minutes) at its eighth station call. Analysis of 21 days of data[2] in the period 11$^{\text{th}}$ February to 12$^{\text{th}}$ March 2019 shows that actual dwell time here had a mean of 338s, maximum of 645s and minimum of 225s. The extended dwell was in some cases sufficient to enable recovery of delay that had accumulated by this mid stage of the journey but at other times was under or over generous. Beyond experience of the operator there is no certainty this stage of the journey or scheduled duration of extended stop is the optimum to maintain good service over the whole trip or for other trains routed in the vicinity.

In this paper a method is introduced that attempts directly to impose robustness in planned trajectories through probabilistic variation of CPs and DTs within a genetic algorithm (GA) based, multi-train optimisation process. This is in contrast to simple application of a GA to optimise train trajectory, which has been well covered by previous publications, for example by Chang and Sim, Yang et al. and Goodwin et al.[3,4,5]. The aim is to prove the concept of including uncertainties in the rail network optimisation process to support future application to an operational network. Trade-offs are explored between highly optimised trajectories that are vulnerable to disruption, and what may at first appear to be less optimal trajectories but ones that are robust to uncertainty. The newly developed procedure is shown to find network-wide driving strategies of increased robustness, and offers a more systematic way to plan a network than ad-hoc use of recovery time to mitigate everyday disturbances to operation.

1.2. Optimisation in uncertain systems

Most trajectory planning work (for both single and multiple trains) is focused on finding the optimal solution for a predefined timetable. Usually train control seeks to maintain the timetabled traverse time for a length of track, whilst minimising the energy consumption[6]. The problem is constrained by the dynamic performance limits of trains and restrictions on speed and headways. The traditional formulation of the multi-train trajectory planning problem can be formalised as:

$$f(\underline{\boldsymbol{X}}) \rightarrow min \tag{1}$$

where $\underline{\boldsymbol{X}}$ is a control strategy for all trains on the network, and $f(\underline{\boldsymbol{X}})$ is a cost function typically based on the traverse times and energy consumption of all train journeys.

In optimising a rail network it is often assumed that if the optimal control strategy is identified, then its implementation will result in optimal performance of the system. However, the optimum is likely to lie on the limit of the feasibility boundary and so will be very sensitive to parameter uncertainty or noise[7]. In this context noise refers to the many small uncertainties that most current models do not consider but which exist in reality, for example, variations in DT at stations dependent on passenger boarding rate[8,9]. These mean that if optimised control strategies (for a noiseless system) are applied to real operation it is unlikely they will perform as well as expected, and may in-fact result in severely sub-optimal outcomes. Most of these uncertainties fall into two different groups classified by Chen et al.[10] as:

Type 1 - variation (α) in uncontrolled parameters, e.g. variations between nominally identical vehicles, variations in station dwell times.

Type 2 - variation (δ) in control, caused by imperfect application, e.g. variation in locations at which a driver switches between traction, coasting and braking.

Including both Type 1 and 2 variations the problem becomes:

$$f(\underline{\boldsymbol{X}} + \delta, \alpha) \rightarrow min \tag{2}$$

By optimising this system, with noise included, the best result that could be achieved with Equation (1) is expected to be approached but not matched, this deviation being the cost of achieving a robust solution.

To explore the trade-off between cost and increased robustness, an existing multi-train trajectory optimisation study[5] was utilised with the control algorithm modified to include variations in DT, a Type 1 uncertainty, and CP, a Type 2 uncertainty. Other uncertainties could also be considered but DT and CP are taken here as examples with which to explore the method. The model developed by Goodwin et al.[5] uses a genetic algorithm (GA) based optimiser, although the same consideration of noise could be applied with alternative optimisation approaches – it is not specific to the GA implementation.

The uncertainties in DT and CP were quantified as statistical distributions from which values are sampled at each cost function evaluation (full details below). This implies that the value of the cost function will vary stochastically, and therefore a single evaluation could be misleading since it would only examine one combination of uncertainties. A more realistic picture is given by taking the mean of N evaluations for a particular network control strategy, i.e. considering the driving strategies for all trains movements that take place. This may be undertaken directly (Equation (3)), or in a GA optimiser may be achieved by increasing the population size. The larger the N used, the better the representation of how the network will really perform for that specific control strategy.

$$\hat{F}(\underline{\boldsymbol{X}}) = \frac{1}{N}\sum_{i=1}^{N} f(\underline{\boldsymbol{X}} + \delta, \alpha) \tag{3}$$

2. Methods

The existing multi-train trajectory planning optimisation method reported previously[5,11] was used as a starting point, focusing on movement of multiple trains on a simple railway network (Figure 1). The aim was to demonstrate the concept for robust multi-train trajectory planning by inclusion of uncertainties in the optimisation process, and this network is suited to that low technology readiness level (TRL) 2-3 objective, in preparation for later application at TRL 7-9 (operational demonstration and application to geographical networks).

2.1. Network and train schedule

The rail network studied (Figure 1) consists of three bi-directional single track lines, joining four stations. Three trains are considered with the journey schedule given below. Times for traverse and dwell are targets, i.e. maximum traverse times, and minimum DTs, but may be varied by the optimisation if (for example) an early arrival or extended dwell is beneficial to overall network behaviour. The network traverse times represent 10% slack relative to flat out running and aim to represent a mainline suburban railway operation with stops every 10-12 minutes. It should be noted that the simple network modelled here is being used to explore the optimisation approach, not to represent a specific network.

- Train 1: Mass 665 tonnes. Travels station 1 to 4 (traverse time 675s), calls at station 4 (planned dwell 60s, minimum 30s), terminates at station 2 (traverse time 747s)

- Train 2: Mass 600 tonnes. Travels station 2 to 4 (traverse time 701s), calls at station 4 (planned dwell 60s, minimum 30s), terminates at station 3 (traverse time 605s)

- Train 3: Mass 565 tonnes. Travels station 3 to 4 (traverse time 599s), calls at station 4 (planned dwell 60s, minimum 30s), terminates at station 1 (traverse time 637s)

At the beginning of the simulation all three trains depart their originating stations simultaneously. The motion was discretised in intervals of 1s, and over each individual time step it was assumed equations of linear acceleration apply. This allows non-uniform acceleration due to velocity dependence of forces controlling motion to be closely approximated[11] within a simple computational approach. Although an analytical calculation could be made for this simple network such an approach rapidly becomes unfeasible for a larger network, so it was decided to use the more scalable time discretisation approach here. The forces controlling motion are the maximum traction force available (360kN, irrespective of velocity), resistance to motion (g), and brake force (h). The velocity dependent quantities are summarised by Equations 4 and 5 in which v is the velocity[4].

$$g(v) = 11.4 + 0.101v + 0.001269v^2 \tag{4}$$

$$h(v) = \begin{cases} 300 - 0.2v & \text{if } 0 \leq v \leq 100\text{km/hr} \\ 280 - 1.2(v - 100) & \text{if } 100 < v \leq 200\text{km/hr} \\ 160 - 0.5(v - 200) & \text{if } 200 < v \leq 300\text{km/hr} \end{cases} \tag{5}$$

The lines of the network are each 30km long, with line-speed (V) of 300km/hr except for the restrictions given in Equation 6 in which subscripts indicate the stations connected, and s denotes the distance along the line measured from the originating station. For trains moving in the reverse direction the same line-speed restrictions were taken to exist at the same locations. Since they are single track bi-directional lines, a train must complete its traverse of a line before a train in the opposite direction can begin its motion. Stations are capable of accommodating multiple trains, and the network is taken to have no height variation (zero gradient) throughout although the method admits variations in gradient, curvature and any other deterministic features. In combination with the range of train weights these factors provide a variety of behaviour sufficient to test the optimisation of train trajectories to compare cases with and without noise.

$$\begin{aligned} V_{(1,4)}(s) &= 200\text{km/hr} \quad \text{if } 15\text{km} \leq s \leq 20\text{km} \\ V_{(2,4)}(s) &= 150\text{km/hr} \quad \text{if } 10\text{km} \leq s \leq 13\text{km} \\ V_{(3,4)}(s) &= 230\text{km/hr} \quad \text{if } 20\text{km} \leq s \leq 23\text{km} \end{aligned} \tag{6}$$

2.2. Optimisation approach

The optimisation process without consideration of noise is fully described by Goodwin et al.[5] so only a brief summary is provided here. Train movements are defined by a driving strategy consisting of traction and coasting pairs, so a journey is made up of a series of applications of power (traction) followed by coasting. At the end of the journey braking is triggered to bring the train to a stop at the required station. The locations of transitions between traction, coasting or braking are defined as the CPs. Terminology varies internationally and the same thing may also be referred to as a switching point, but is distinct from wayside timing points which may also be referred to as CPs in some rail operations. Optimisation is used to establish the best places to make these control changes, although as discussed later the planned positions may not be followed accurately during driving the train. The duration over which traction or coasting is applied determines the speed profile of the train, which must be kept within the line-speed-limit at any location.

With the driving strategy defined by CPs, their location can be varied to search for improvements in time keeping and reductions in network energy usage. During optimisation the performance of the driving strategies was tested by running a simulation of the network and scoring performance using the cost function Equation (7).

$$\begin{aligned}\text{cost function} =& \\ &\sum_{i=1}^{M} max(0, \text{arrival time - timetabled arrival}) \\ &\qquad + c\Big(\sum_{i=1}^{M} \text{energy consumed on journey}\Big) \quad (7)\end{aligned}$$

where M is the total number of journeys taken on the network across all trains. This trades off delays and energy consumption through the constant, c, by which the relative importance of these factors can be expressed. The value of c can be modified depending on the relative importance to the operator of timekeeping or energy use, but a fixed value was maintained during the current exploration focused on system variabilities. The cost function used here is an example through which the effects of uncertainties in the system can be explored; alternative cost functions have been developed that are able to account for wider issues than just time and energy, for example by Pavlides and Chow[12].

Genetic Algorithms are a popular heuristic search technique well suited to realistic (rather than mathematically well-behaved) problems expressed by non-convex, possibly non-smooth, cost functions and non-linear relationships. They model, in crude terms, the process of natural selection and empirically have been shown to be able to arrive at good solutions in reasonable times using reasonable computational resources. A population of individuals (phenotype) is initiated, e.g. representing a complete run of timetabled train movements on the network, each characterised by an arbitrary set of problem parameters (genotype), e.g. CP locations and accelerating/coasting/braking actions to achieve the train movements. Each evaluation of the phenotype using its current genotype is scored according to the cost function and the "fittest" (best scoring) individuals are selected for breeding, whereby the genotypes of pairs of fit individuals (parents) are recombined through the mechanisms of crossover (mimicking the combination of DNA strands in sexual reproduction) and mutation (mimicking the random variations that can take place in this process). The new generation of "children" then undergoes the same process of evaluation and breeding until a satisfactory solution is reached, determined for instance by a fall in cost below some pre-determined threshold or when local convergence is observed, e.g. via "small" changes in cost from generation to generation. There are many possible ways of implementing genetic operations and the specific choices for this work have been fully described by Goodwin et al.[5] and so are omitted here for brevity, other than to report that a particular feature is its very low risk of becoming "stuck" in cost function local minima rather than approaching the true minimum. It is worth noting that the means of embedding robustness/resilience in the GA optimisation process could also be applied to other optimisation techniques but here the focus is on GA as there is a large body of work indicating their suitability for addressing train control problems.

Without noise in the system, the best driving strategies across trains on the network would simply be those which when combined have the lowest cost function score at the defined end point of the optimisation. The "all-time best scoring” driving strategies for the network could be identified after a given number of potential strategies had been evaluated. However, in a noisy system a single evaluation of each driving strategy is very unlikely to identify the best performer under real-world conditions because it would only have considered one case of the uncertainties present in the system. To explore this issue it is useful to define *training noise* (the level of uncertainty considered during the optimisation process) and *utilisation noise* (the level of uncertainty in the system where the optimised driving strategy is applied).

To achieve a resilient control strategy the optimisation must be conducted over a range of possible variations. Algorithm 1 takes the example of dwell time to illustrate how training noise (i.e. training data-sets of dwell time around a network) is applied to give greatest likelihood of good performance for future utilisation noise which cannot be known in advance. For example, actual station DT on a specific future journey cannot be known when planning a timetable and the train control to achieve that timetable. The optimisation process therefore considers 50 different potential combinations of uncertainties, performing the optimisation process (step 2) for each case. The solutions generated (the best driving strategies for that particular realisation of uncertainty) are then re-evaluated against all 50 uncertainty cases to assess how they would respond to real-world variations in DT. A similar process is performed when considering uncertainty (noise) in the application of train control. The set of driving strategies that together offer the lowest average cost function score for the whole network (i.e. lowest delay and energy use for all trains combined) is then selected as the best solution. While it would be likely that this “best” solution could be improved upon for a

Algorithm 1 Robust genetic algorithm, illustrated for dwell time robustness

1. Select as training data 50 data-sets for dwell times across the network during running a timetable
2. For each training data-set n=1..50 simulate the timetable:
   (a) Run genetic algorithm (200 generations, population 100) to identify high performing train control strategies to achieve the timetable and energy objectives, defined by Equation (7)
   (b) Score each of the final population of train control strategies when applied to simulate the timetable for each of dwell data-sets 1..50, using Equation (3), N=50
   (c) Identify the best scoring train control strategy for training data-set n
3. Select the highest performing train control strategy across all 50 training data-sets

specific case of DT or CP uncertainty, on average it will be expected to offer the best network performance over a wide range of utilisation conditions without prior knowledge of exactly what these will be.

2.3. Representing DT uncertainty

Station DTs, measured as time spent in the station, excluding terminating trains, are inherently unpredictable largely owing to the boarding and alighting of passengers, the speed of which is affected by many uncontrollable variables such as the number and configuration of train doors, the number of passengers with large luggage, and variations in passenger personal mobility. Evidence demonstrating the distribution of DTs on the UK network is plotted in Figure 2, in which two distinct properties can be seen in data covering almost 400,000 station stops on a range of suburban, regional and inter-city routes. First, there is a distribution of DTs achieved around the nominal DT, so modelling just the nominal dwell would clearly be unrealistic. Second, shorter nominal dwells are associated with a more consistent (narrow) distribution of DTs achieved, whereas longer nominal dwells are associated with greater variability.

Variation in station DTs was introduced into the model so as to approximate real-world behaviour, with the stochastic component normally distributed, and DT truncated to avoid negative waiting times[a]. To enable the uncertainty in DT to be varied smoothly for investigation, the standard deviation in the stochastic component of DT was chosen to always be one third of the mean, resulting in the series of distributions shown in Figure 3, that closely resemble the real-world data in Figure 2. Since DT mean and standard deviation are related in the example cases explored in this paper, DT noise is quantified in the following sections by its mean value. The optimisation procedure is independent of this distributional choice, so, in real applications, the representation of DT could be tuned to the characteristics of a specific station, line, fleet or known passenger behaviour.

[a]A normal distribution was used for simplicity although the example data in Figure 2 have a skewed distribution. This has no impact on the consideration of robust trajectory planning, but developing an alternative distribution to represent DT stochastic component may be a useful area of future work.

2.4. Representing uncertain CP application

Adapting the model described by Goodwin et al.[5] to introduce uncertainty in CP application into the system was achieved by adding a zero-mean, normally distributed random distance to the position of each CP. Instead of the next control action being applied as soon as the train passed the distance specified in the control sequence the action was applied with the specified level of uncertainty (Figure 4). This distribution could be refined for a specific application, for example, through observation of drivers to understand the spread of early or late applications of control relative to DAS advice. However, as for the DT case, the optimisation procedure developed in this paper is independent of this distributional choice so a simple normal distribution is satisfactory. CP noise is quantified in the following sections by its standard deviation. Its inclusion in the representation of train control leads to two closely related situations becoming possible.

First, each time a driving strategy was simulated there was a chance that the trajectories taken do not maintain safe operation, for example by violating the line-speed-limits (Figure 4b). During the progress of the optimisation such driving strategies were discarded and replaced with a driving strategy probabilistically selected from the previous generation of the optimisation process, this being a safe but sub-optimal driving strategy.

Second, it is possible that the "best" control strategy found by the optimisation in fact relies on the specific deviations of CP application chosen to represent real-world variations during the optimisation. These deviations would be unlikely to reliably occur in reality, giving a high likelihood of poor network performance in a manner similar to optimising for a perfectly performing system with no uncertainties. To screen out these cases the final population of potential driving strategies produced by the GA was re-simulated without any noise and any candidates found to be invalid were discarded.

2.5. Optimisation parameters

The optimisation parameters used in this investigation are shown in Table 1, and are identical to those used by Goodwin et al.[5], except for the additions explained above to account for noise. Eiben[13] observes that parameters for a genetic process may have different optimum values throughout the optimisation process, and here it was found that a linear decrease in the size of mutation with each generation yielded improved performance.

Since GAs are not deterministic and the uncertainties being considered add further variability to the system, after completion of each optimisation a further analysis was conducted in which the performance of the chosen control strategy was explicitly estimated, using Equation (3) ($N = 500$).

3. Results and Discussion

3.1. Baseline: Optimising without noise

The first series of optimisations was carried out without any training noise during the optimisation process, the results from which are shown in Figure 5. This is equivalent to a conventional optimisation developed for application in ideal conditions (i.e. when there will be no uncertainties during utilisation). With that in mind, it would be expected that its performance when utilised with no CP or DT noise would be good, with a low score for delay and energy consumption. Performance could, however, deteriorate markedly under real conditions. For the system studied, the mean utilisation DT noise was found to have no effect on the validity of control strategies (no violations of the type shown in Figure 4 occured). This is expected since a DT variation should not adversely change driving behaviour after the train begins to move. Introduction of utilisation CP noise had negligible effect on the cost function scores, and this outcome held for all the different combinations of training noise.

From Figure 5(a) it can be seen that the probability of an optimised control strategy being valid, avoiding Figure 4 type violations, drops quickly as utilisation CP noise increased. The optimised solutions found are very close to the line-speed constraints, and when utilised in a system with no noise they always keep the safety constraints and are therefore considered valid solutions. However, as soon as a small amount of CP noise is introduced during the utilisation of the control strategies the probability of speed-limit violations becomes high, invalidating the control strategies. Far from being surprising, this lack of robustness is exactly the behaviour that would be expected from near optimal solutions to the noiseless problem[7]. By assuming certainty in CP application during the optimisation it has produced solutions that are sensitive to CP noise. In reality it would be expected that such speed-limit violations would be avoided by drivers, with the consequence that it would be very difficult or impossible to keep to the scheduled timetable.

Similarly, Figure 5(b) shows that, above a certain threshold, increased utilisation DT noise produced an almost linear increase in the averaged cost function score of a control strategy, i.e. increasingly poor performance. In seeking to minimise energy consumption the optimisation has selected driving strategies that make full use of the timetabled traverse time on each journey (since losses due to air resistance are reduced at lower speeds). If these strategies are followed recovery time is minimised so any late departure will cause a late arrival (with this effect amplified as delays propagate across the network). Below a DT noise with mean of 45s this lack of recovery time is not an issue because the utilisation DT has a very low probability of being greater than the planned DT of 60s. However, above a mean DT noise of 60s the majority of dwells are extended and, since there is minimal recovery time, any increase in DT causes delay and increases the Equation (3) cost function score. Again, by assuming certainty in DTs during the optimisation it has produced solutions that are sensitive to real-world conditions.

3.2. Optimising with CP variability

The second series of optimisations was carried out with different levels of training CP variability during the optimisation process, the results from which are illustrated in Figure 6.

Figure 6(a) shows that increasing the CP training noise leads to the optimisation finding control strategies that are substantially more robust to variation in CP application. However, from Figure 6(b) it can be seen that when utilised at a zero DT noise there is a small increase in cost function score associated with the rise in CP training noise. This is the cost of having an increase in robustness relative to that shown in Figure 5. In the case of a utilisation CP noise, raising its level from zero to a 100m standard deviation around nominal CP positions causes the probability of the control strategy being evaluated as valid to increase from 0.04 to 0.86 (>2000%), but the cost function score to increase by only 2.3%. This shows the benefit of retaining valid control under real conditions comes at a low cost.

3.3. Optimising with DT variability

The next series of optimisations was carried out with different levels of DT uncertainty during the optimisation process, but with no CP uncertainty. The results for training DT noise (mean) = 30, 45 and 135, 150s were found to be almost identical to training DT noise of 0 and 120s respectively so are omitted from Figure 7. For the first case, this is because the training noise level is too low to have a noticeable effect – the vast majority of DT instances are less than the nominal DT used in the system (60s) and therefore rarely affect the actual departure time. In the second case, this is because the training noise level is too high – the optimisation can no longer select genuine improvements in control above the noise. The overall behaviour is that a rise in DT training noise within the limits of what would constitute noise rather than a more major disruption in the real system increases the probability of developing driving strategies that remain valid in real-world conditions, i.e. avoiding Figure 4b type violations.

It can be seen from Figure 7(b) that increasing the DT training noise leads to solutions that have a lower cost function score when utilised at high DT noise (i.e. are more robust). The results in Figure 7(a) show a secondary benefit of slightly increased robustness to variation in CP application accuracy even though DT variability was the main consideration during optimisation. In this case, introducing one type of noise has led to the system becoming more robust to another type of noise. However, the increase in robustness is accompanied by an increase in average cost function score when utilised at low DT noise - seen in Figure 7(b).

The importance of the training noise matching the noise level at which the solution with be utilised is highlighted in Table 2. The modelling outcomes show that the cost function score when the utilisation DT noise matches the training noise is much lower than when the utilisation DT noise is fixed at zero but the training noise varied. In situations where the utilisation noise levels of a real system are not well known estimates will be needed in the choice of training noise. It follows that all non-robust optimisations make the (usually implicit) assumption that noise levels on all parameters are zero. For many situations, particularly metro applications, this may be an acceptable approximation but it is unlikely to hold in complex, interconnected, stochastic systems such as a busy mainline suburban, regional or intercity rail networks.

The trade-off between the cost function score and increased robustness could be summarised as a "cost" to be paid in terms of energy to build in recovery time to achieve increased robustness. The benefit of an optimisation process is in making this trade-off strategically, not through ad-hoc use of recovery time in the timetable with only limited understanding of its effect on the whole network. Referring to the London to Leeds train described in the Introduction, it would be possible to establish whether the current location and duration of recovery time included in its schedule could be improved upon, including considering the interaction of this train with others on the network. This can be explored by looking at the traverse times of the trains and the corresponding energy consumption. For convenience the convention CP_DT will be used to describe the training noise levels used during optimisation (e.g. 0_90 denotes a training CP noise standard deviation of 0 metres and a mean training DT noise of 90 seconds). The mean timetabled journey time in the system modelled is 660.7s, calculated across all legs of the timetable detailed in Section 2.1. From Table 3 it can be seen that the mean journey time with 0_0 is 658s, giving a mean recovery time per journey of just under 2.7s. Running with such low recovery time results in 0_0 having the lowest average speed and therefore the lowest energy consumption of the cases in Table 3. At first sight this may be considered a success, since the service is predicted to be is punctual and energy efficient, but when considering robustness this is actually a "brittle" solution, with utilisation noise rapidly leading to sub-optimal performance, as shown in Figure 5.

If the training DT noise is increased to 90s (training noise 0_90) then the mean speed of operation rises and journey traverse time falls to give a mean recovery time of just under 25s on each station to station traverse. This raises the proportion of journeys in which punctual operation can be maintained even where there is a significant probability that DT will be longer than timetabled, but at the cost of running faster and using slightly more energy. Mean recovery time is used here to summarise this behaviour, although the optimisation actually distributes it non-uniformly between journey legs to cope with interaction of services at station 4 (see Figure 1). Interestingly, fast running (in order to build up recovery time) is similar to typical driver behaviour[14] but in this case has been found by a direct optimisation, which has no prior knowledge of existing operational concepts.

3.4. Optimisation with both CP application and station DT variability

In real systems there may be uncertainty in many parameters simultaneously, so it is important to investigate the performance of the proposed method in this situation. The performance when high levels of CP and DT training noise are present simultaneously is shown in Figure 8. It can be seen that the robustness of control strategies to CP noise at utilisation, indicated by a high proportion of control strategies being valid during application, is predominantly influenced by the training noise level of CP used during optimisation. In terms of maximising robustness to CP variation the performance of the 100_90 optimisation is very close to the performance of the 100_0 optimisation, indicating that including DT noise at training makes only a marginal difference to performance under CP noise at utilisation. In contrast, Figure 8(b) shows the performance of the 100_90 optimisation is similar, but slightly more costly, than the 0_90 optimisation. Including both training noises simultaneously has led to higher cost function scores than applying DT training noise alone. However, the performance of the 100_90 optimisation is still an improvement over the 0_0 optimisation when utilised at a high level of DT noise. This indicates that the proposed approach for including uncertainty in the optimisation is capable of finding solutions that increase system robustness when two different types of uncertainty exist simultaneously.

4. Conclusion

When planning train trajectories it is important to consider the robustness of driving strategies if they are to be successful under real-world conditions. Uncertainty in location of control points (CPs) and length of dwell times (DTs) can otherwise nullify a planned control strategy, making it unlikely that the driving strategies found will be robust enough to perform as predicted in real operation. Non-systematic approaches may address this problem, such as ad-hoc addition of recovery time to timetables, or by driving trains aggressively in an effort to keep to the timetable, but at the cost of excess energy consumption and poor utilisation of trains, crew and network capacity.

A method to consider noise in a multi-train optimisation procedure has been described which seeks to find robust solutions to the multi-train trajectory planning problem. For a small demonstration network it is shown to be effective in finding robust control strategies in the presence of two different types of uncertainty: the accuracy of the CPs (i.e. differences in application point for traction, coasting or braking relative to a planned trajectory), and variation in station DTs. These uncertainties were first considered separately, before it was shown that they could be considered simultaneously in the optimisation, in which case it is predicted that the system will still achieve good levels of robustness. For both types of uncertainty a trade-off was observed between the robustness and the average cost function score at utilisation, which represents a combination of delay and energy costs for the train trajectory solution. The aim here was to explore the concept of including uncertainties in train trajectory planning using a simple network (technology readiness level, TRL, 2-3, proof of concept) in preparation for future application at TRL 7-9 (operational demonstration). The procedure for including training noise in the optimisation process is generalisable to include the many different uncertainties that can affect railway system operation.

It is expected that this approach to optimisation offers greatest benefit to mainline and suburban railways which are typically complex systems with high levels of uncertainties (large variety of vehicles, stations of different characteristics). These contrast with metro or underground operations in which there are lower levels of uncertainty (fewer vehicle types, often aiming for flat-out driving to maximise throughput of passengers). The predictions for the example network show that, for best network performance, the training noise used during the optimisation process should reflect the noise level that is expected when the optimised driving strategy is utilised. Planning for a level of uncertainty close to that experienced increases resilience and robustness of the driving strategy without excessive cost.

Directions for further development of the technique include considering different sources of noise (e.g. train resistance coefficients, traction efficiencies, rail-wheel adhesion levels), and different network topologies and timetables to understand how the method scales with increasing network size. For larger networks implementation using parallel computing on graphics processing units (GPUs) is expected to offer increased speed of computation.

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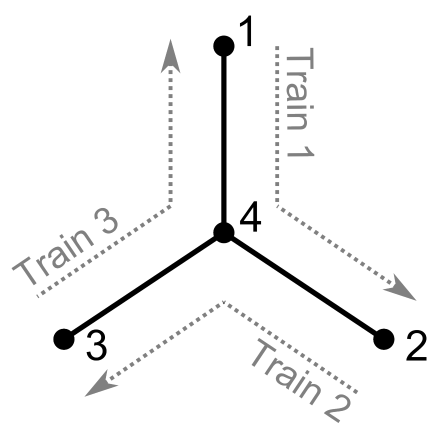


Figure 1: Network topology for concept exploration, previously investigated by Yang et al. and Goodwin et al.[4,5]. Edges represent single track line with bi-directional traffic, nodes represent stations.

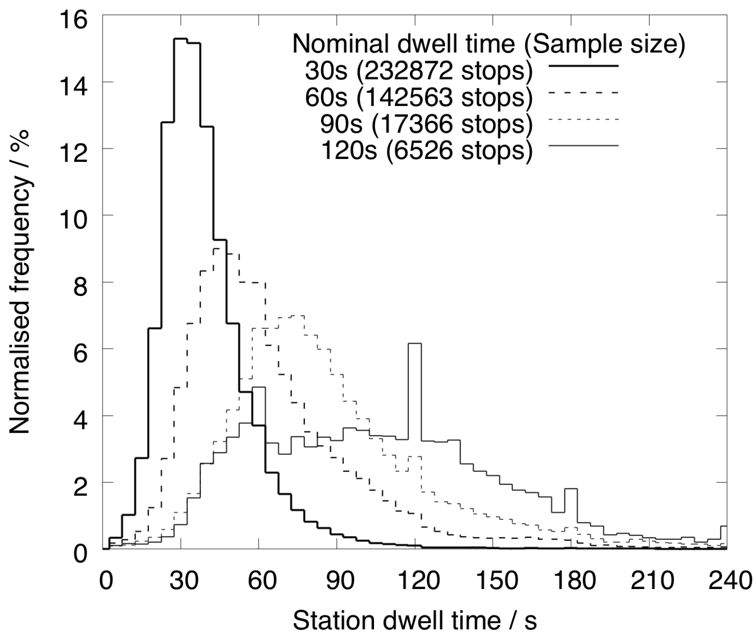


Figure 2: Distributions of DT achieved for planned nominal dwells of 30, 60, 90 and 120s, as percentages of total stops for each nominal time. Data were obtained from data feeds ([15],[16]) of arrival and departure times, pre-processed to remove erroneous zero and negative DTs. Data are for UK train operators East Midlands Trains (period 9th to 16th January 2016), Great Northern, and Thameslink (both for period 29th March to 3rd May 2016).

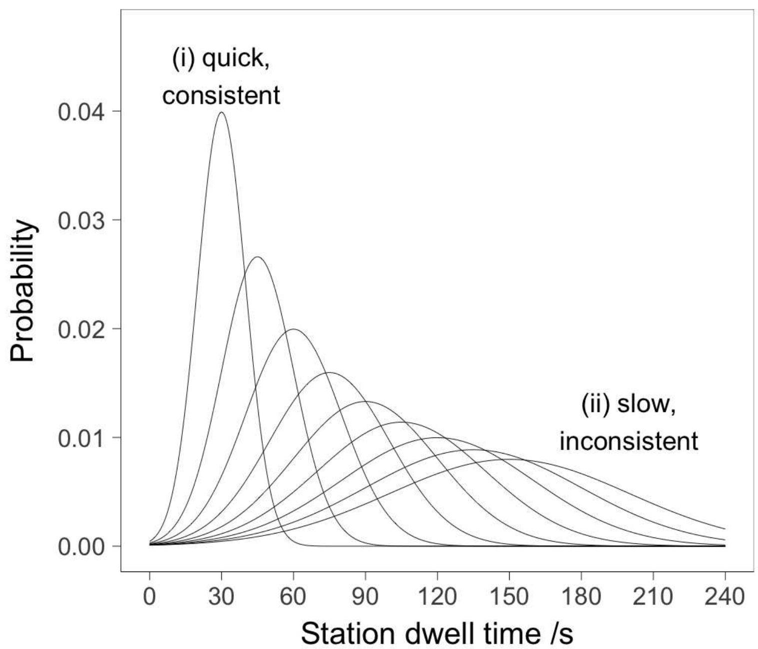


Figure 3: Distribution in stochastic DTs (DTs), used to represent DT variation in the optimisation process. (i) and (ii) illustrate the two extreme types of DT distribution, similar to those found for a large sample of real trains, Figure 2.

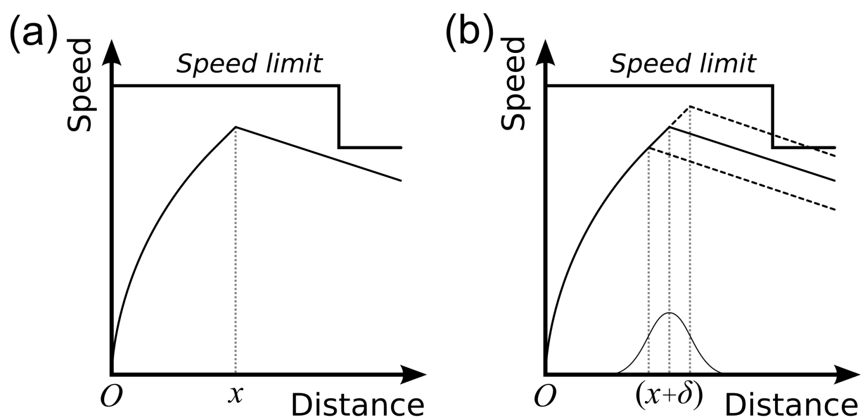
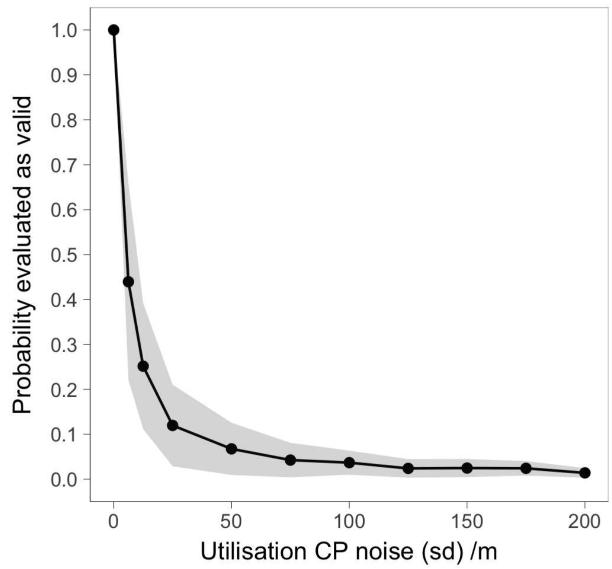


Figure 4: Effect of uncertainty in CP application on train velocity trajectories: (a) When applied without any noise, a control strategy may consistently pass close to a feasibility boundary without the possibility of violations arising. (b) When implemented with noise the same control strategy may break the safety constraints, which will result in it being evaluated as “invalid” for that simulation.



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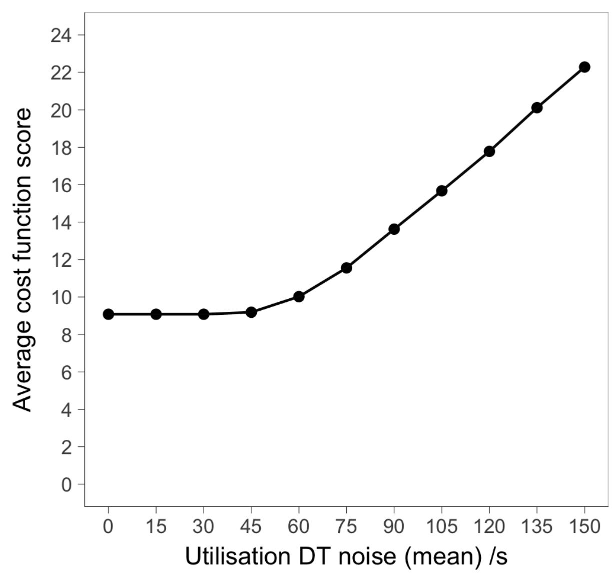
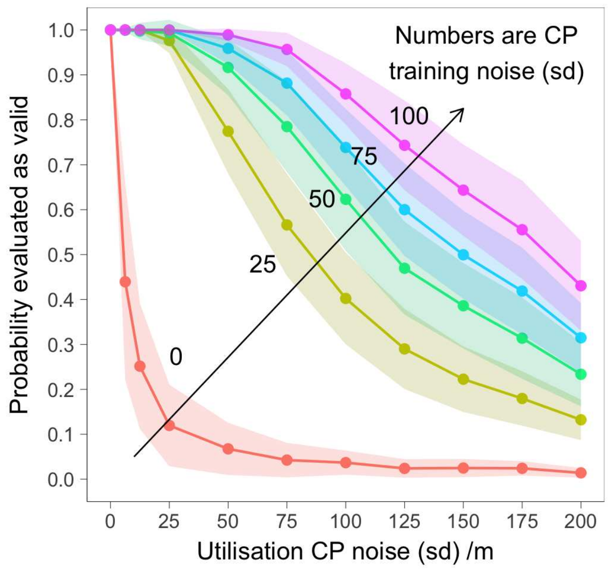


(b)

Figure 5: Predictions for utilisation of driving strategies optimised with no training noise. Shaded areas show one standard deviation. (a) Effect of utilisation CP noise on the probability the control strategy is valid (utilisation DT noise = 0s). (b) Effect of utilisation DT noise on the average cost function score (utilisation CP noise = 0m).



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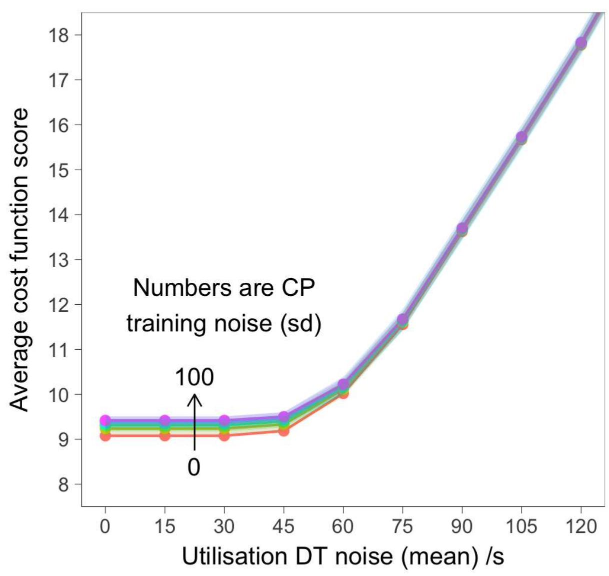
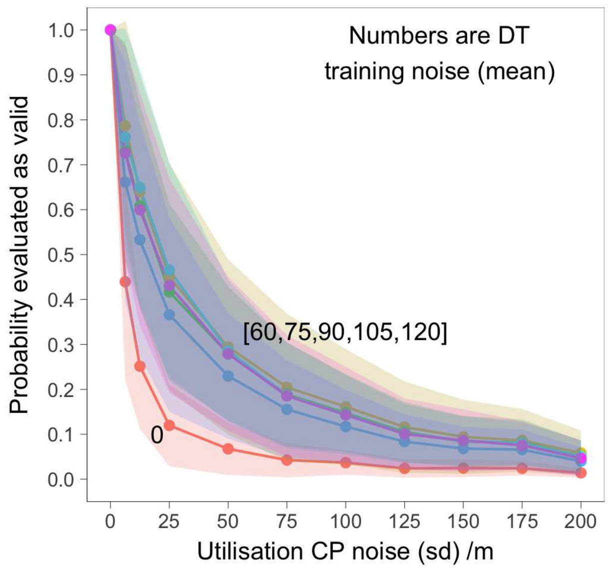


(b)

Figure 6: Predictions for utilisation of driving strategies optimised with zero DT training noise and a range of CP training noise. Shaded areas show one standard deviation. (a) Effect of variation in utilisation CP noise on the probability the control strategy is evaluated as valid (utilisation DT noise = 0s). (b) Effect of variation in utilisation DT noise on the average cost function score (utilisation CP noise = 0m).



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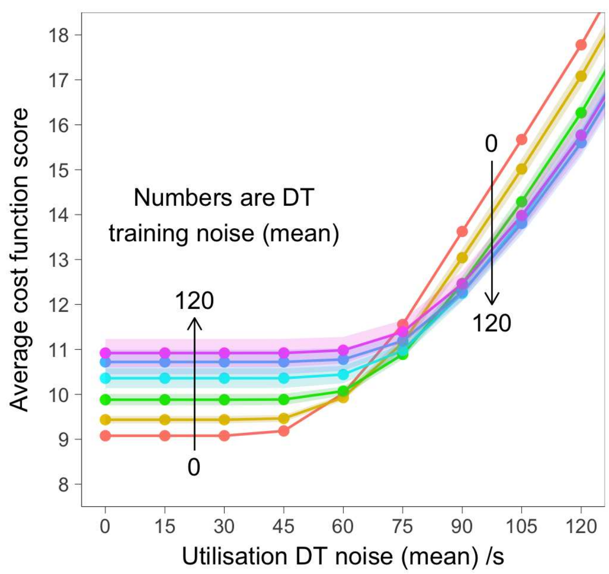
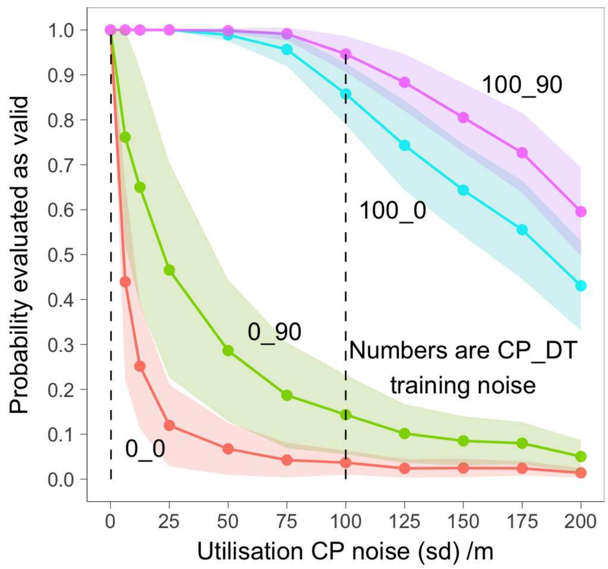
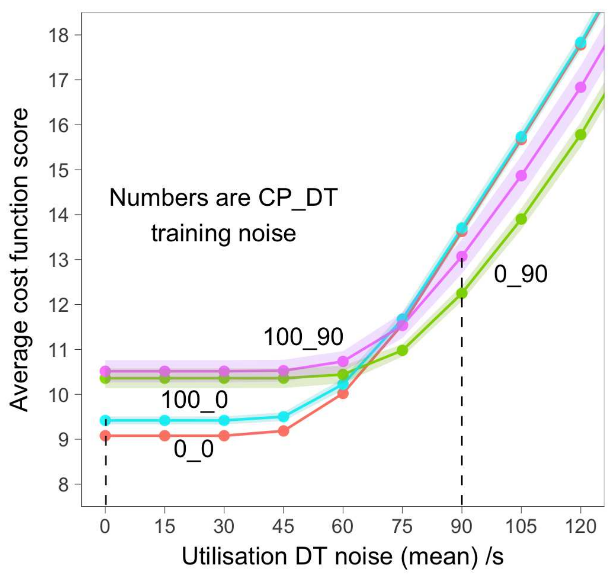


(b)

Figure 7: Predictions for utilisation of driving strategies optimised using different levels of training DT noise, with no CP (CP) training noise. Shaded areas show one standard deviation. (a) Effect of variation in utilisation CP noise on the probability the control strategy is evaluated as valid (utilisation DT noise = 0s). (b) Effect of variation in utilisation DT noise on the cost function score of operations (utilisation CP noise = 0m).



(a)



(b)

Figure 8: Predictions for utilisation of driving strategies optimised using combinations of DT (mean) and CP (sd) training noise. The dotted lines emphasise the level of the training noise used.(a) Effect of variation in utilisation CP noise on the probability the control strategy is evaluated as valid. (b) Effect of variation in utilisation DT noise on the cost function score of operations.

Table 1: Model and GA parameters used in this investigation.

Parameter	value
M (size of mutation, varied linearly during solution) (m)	start = 200 end = 0
Population size	100
Number of generations	200
c (value of energy relative to time) (min/kWh)	0.0015
N (for averaging final population)	50
Minimum dwell (s)	30

Table 2: Cost function score at utilisation relative to a base case for training without DT noise. CP noise zero in all cases. Higher cost function scores represent worse performance.

Training DT noise (mean) /s	Utilisation DT noise	Cost function % change
45	0	1
60	0	4
75	0	9
90	0	14
105	0	18
120	0	20
45	45	0
60	60	-1
75	75	-6
90	90	-10
105	105	-12
120	120	-11

Table 3: Average properties of the control strategies resulting from different combinations of training noise (all utilised at, CP noise = DT noise = 0).

	Training noise (CP_DT)			
	0_0	100_0	0_90	100_90
Mean speed /ms^{-1}	45.6	45.7	47.1	46.6
Mean journey time /s	658	656	636	644
Mean journey energy /kWh	1008	1038	1148	1152