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Influence of Gear Ratio on the Performance of Fractional Slot Concentrated Winding Permanent Magnet Machines

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***Abstract*—Fractional slot concentrated winding (FSCW) permanent magnet synchronous machines (PMSMs) have been a research hotspot over the past few decades. Recently, the magnetic gearing effect in FSCW PMSMs is revealed along with its gear ratio which is a function of slot/pole number combination. At the design stage of FSCW PMSMs, one of the key issues is the selection of the slot/pole number combination. This paper shows that the gear ratio can contribute to a proper slot/pole number selection in any multi-phase FSCW PMSMs by acting as a unified index for a quick overall performance comparison. In the paper, firstly, the magnetic gearing effect in FSCW PMSMs is explained and the gear ratio is further discussed. The advantages of adopting gear ratio as the overall performance index over other indices are revealed. The influence of the gear ratio on the winding factor, torque output, cogging torque, inductance and rotor losses of 3- and 6-phase FSCW PMSMs are analyzed and validated by experiments, thus proving the analyses.**

***Index Terms*— concentrated winding, fractional slot, gear ratio, inductance, losses, magnetic gearing effect, permanent magnet machines, winding factor.**

I. Introduction

Permanent magnet synchronous machines (PMSMs) with fractional slot concentrated windings (FSCW) have been gaining popularity with academics and industries in the past few years due to their compact size, high torque and power density[1]-[3].

At the initial design stage of FSCW PMSMs, one of the key issues is to make a rapid performance comparison to select the most promising slot/pole combinations since it significantly affects the machine's performance [4]. In [5]-[8], the winding factors for FSCW PMSMs with several slot/pole combinations, as well as the influence of slot/pole combination on cogging torque, are clearly given. In [9] and [10], it is found that the slot/pole combination has an impact on the mutual and air-gap leakage inductances, which are related to the fault tolerant [11] and flux weakening performance [12], respectively. In [13] and [14], the influence of the number of slot per pole per phase (q) and number of phases on the rotor losses are investigated. The unbalanced magnetic force in FSCW PMSMs is investigated and some suggestions are made on the choice of the slot/pole number [8] [15]. The influence of the slot/pole combination on the radial force and vibration mode was shown in [16]. The influence of q on the irreversible demagnetization risk was shown in [17].

Although the performance indicators for different slot/pole number combinations have been provided in some of the previous papers, they focus more on how precisely the indicators can predict the relevant performance metrics. It is inconvenient to make a rapid overall performance comparison among FSCW PMSMs because different performance metrics use different indicators. Therefore, when slot/pole number changes, a unified performance index for all the performance metrics is desirable for sake of rapid comparison. To achieve this, slot per pole per phase (q) is adopted. The adoption of q makes the discontinuous slot/pole number be a continuous function. If the relationship between q and various machine performance metrics was established, people would be able to make a rapid comparison for slot/pole number selection by just comparing q. However, only [13] shows the relationship between q and the iron loss index even though it contributes to a rapid comparison. Moreover, one problem for adopting q is that it is also a function of the number of phases: q changes when the number of phase changes. Since multi-phase FSCW PMSMs have also been a hotspot, using q as the index for comparison could bring about inconvenience when different phase numbers are considered. Up to now, there is no paper providing an index which does not change with phase number and can be used for a quick overall performance comparison.

Recently, the magnetic gearing effect [18]/modulation effect [19] in FSCW PMSMs was revealed [20]. The parameter used to represent the gearing effect in FSCW PMSMs is defined as the gear ratio, which is a function of the slot and pole numbers. [19] and [20] focus on revealing the magnetic gearing/flux modulation effect as the torque production mechanism in different machine types. In this paper, the influence of gear ratio on different machine performance metrics will be investigated for machines with different slot/pole number combinations and particularly different phases. It shows that gear ratio can contribute to a proper slot/pole number selection in any multi-phase FSCW PMSMs by acting as a unified index for a quick overall performance comparison.

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This paper is organized as follows: The magnetic gearing effect in FSCW PMSMs is explained and gear ratio is further discussed in section II. It is found that the gear ratio is not only a representation of the gearing effect but also an inherent parameter of the FSCW PMSMs regardless of the phase number. Hence, it is more convenient than other performance indicators in that it can be a unified index for all the performance metrics and it does not change with the phase number. Then, the influence of the gear ratio on the winding factor, torque output, cogging torque, inductances and rotor losses of 3- and 6-phase FSCW PMSMs are demonstrated. In section IV, prototypes are built and tested, thus proving the claims.

II. Magnetic Gearing Effect and Gear Ratio in FSCW PMSMs

In this section, the magnetic gearing effect in FSCW PMSMs is explained and the gear ratio is further discussed.

A. Magnetic Gearing Effect in FSCW PMSMs

For a N_s-slot/2p_r-pole FSCW PMSM, where N_s is the slot number and p_r is the pole pair number, due to the winding distribution, the p_rth and (N_s-p_r)th armature reaction harmonics always exist in pairs, their amplitudes are comparable and larger than any other armature reaction harmonics. Their speeds of rotation are shown in Fig. 1, where Ω_r it the mechanical speed of the rotor. From [20], useful torque can be generated under the following circumstances.

(a) the p_rth PM fundamental and original p_rth armature reaction harmonic share the same order and speed of rotation, they can interact with each other and produce useful torque. Here "original" means that the harmonic is not modulated by the stator slots. In this case, the torque is produced by the principle of the conventional PMSM as shown in Fig 1 (a).

(b) the (N_s-p_r)th armature reaction harmonic has different order and speed of rotation with the p_rth PM fundamental harmonic. However, the (N_s-p_r)th harmonic can be modulated by the N_s-pole stator teeth, which results in the space harmonic with the order of p_r and the speed of Ω_r. The modulated harmonic can interact with the p_rth PM harmonic to produce a steady torque as shown in Fig 1 (b). This phenomenon is called magnetic gearing effect and it has been proven in [20] that it can contribute up to 30% to the total torque in FSCW PMSMs.

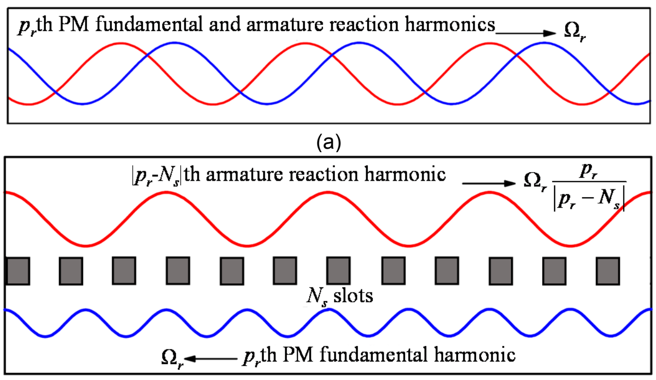


Fig. 1. Schematic of torque production mechanism. (a) Principle of conventional PMSMs. (b) Principle of magnetic gearing effect.

B. Gear Ratio in FSCW PMSMs

In a magnetic gear, the change of speed between high and low speed rotors is called the gear ratio [18]. For FSCW PMSMs, the magnetic gearing effect exists between the p_rth and (N_s-p_r)th air gap field harmonics. From Fig. 1 (b), the gear ratio in FSCW PMSMs can be defined as the ratio of their speeds [20]:

$$G_r = \frac{p_r - N_s}{p_r} \quad (1)$$

The denominator of G_r is the order of the fundamental harmonic and the nominator is the order of the harmonic which makes the main contribution to the torque via magnetic gearing effect. As mentioned above, these two harmonics are critical harmonics in FSCW PMSMs since they always exist in pairs, have larger amplitude than any other harmonics and are highly involved in the torque production, flux weakening operation and losses generation. Their behavior has decisive influence on the related machine performance, and therefore, G_r, as a function of their orders can not only reflect the magnetic gearing effect in FSCW PMSM as shown in [20] but also be used as an index for the performance comparison between FSCW PMSMs with different slot/pole number combinations. The variation of the performance can be shown directly with respect to G_r. It is noteworthy that G_rs in FSCW PMSMs always have negative values because the p_rth and (N_s-p_r)th air gap field harmonics always rotate in opposite direction.

III. Influence of Gear Ratio on Machine Performance

In this section, the gear ratio is compared with other machine performance indicators. The relationship between the gear ratio and winding factor, output torque, cogging torque, inductance and rotor losses will be analyzed in 3- and 6-phase FSCW PMSMs with double-layer windings. After the clarification of the relationships, the gear ratio can be used for performance comparison at the machine design stage.

A. Comparison of G_r and q

There are various aspects of machine performance needed to be considered at the design stage. Regarding the relationship between the slot/pole combination and different motor performance metrics, a number of researches have been reviewed in the introduction. However, different performance metrics use different indicators which increases the calculation and makes a rapid overall performance comparison and proper slot/pole number selection impossible. To have a more generalized and direct insight into the relationship between the slot/pole combination and the machine performance, slot per pole per phase (q) is usually used. Compared with slot/pole combination which is discontinuous, q has a continuous value. By using q, a direct and continuous relationship between machine performance and slot/pole combination can be established. Comparison can be made by just comparing q.

In this paper, the gear ratio, which is also a function of slot/pole combination is used instead of q to carry out the machine performance comparison. Compared with q, G_r can also establish a direct and continuous relationship between various machine performance metrics and slot/pole combination, apart from this, the major advantage of using G_r is that it shows directly the harmonic behavior in the FSCW PMSMs regardless of the phase number. It can eliminate the influence of the phase number and provide a unified reference value for performance comparison for machines with any phase number. For example, for winding factor comparison which is shown in Figs. 2 (a) and (b), if G_r is used as the index, for both 3- and 6- phase machines, the reference value is -1: the closer gear ratio to -1, the higher winding factor it will be. However, if q is used, the reference value varies with the phase number, which is 1/6 for the 6-phase and 1/3 for the 3-phase machine. Table I provides a comparison of reference values when q and G_r are used as the performance index respectively, where m is the phase number. It shows that if q is used as the index, different reference values should be calculated and used for performance comparison for different phase numbers. In comparison, G_r is more convenient for performance comparisons not only because it reduces redundant performance indicator calculations but also provides fixed reference of -1 when the phase number changes. Moreover, a reference value should be easy to compare with. It is easier to compare a fraction with -1 than with $1/m$. More importantly, G_r is not only the index for winding factor comparison, it also applies to other performance metrics such as torque, cogging torque and inductance as will be seen in the sections below. Its advantage over q also applies to the aforementioned performance metrics: for all the performance comparisons and all phase numbers, the reference value is unified as -1 instead of a varying $1/m$.

TABLE I
COMPARISON OF REFERENCE VALUE BETWEEN Q AND GR

Index	q	G_r
Reference value	$1/m$	-1

B. Selection of Slot/pole Combination and Phase Numbers

Several FSCW PMSM models with different slot/pole combinations were built to investigate the influence of gear ratio on the machine performance. For sake of simplicity, of all the feasible slot/pole combinations and phase numbers, only 12- and 24-slot machines with 3- and 6- phase designs are considered. The conclusion drawn from 3- and 6-phase machines can also be applied to other multi-phase machines, which is proven in the appendix. For the 12- and 24-slot machine candidates, the feasible pole numbers and the corresponding gear ratios for both 3- and 6-phase machines are given in Table II. Although there are many feasible slot/pole combinations, the present investigation is limited to those have G_r ranging from -0.5 to -2. Designs with G_r falling out of this range are proven to have inferior performance such as low torque and power density, and therefore were eliminated from consideration.

To make a fair comparison, all the machine candidates are globally optimized. The optimization is carried out based on Maxwell software using genetic algorithm to achieve the maximum torque under the following constrains: (1) Fixed stator outer radius r_o=50mm; (2) Fixed air-gap length l_a=1mm; (3) Fixed stack length l_{ef}=50mm; (4) Fixed shaft radius=10mm; (5) Fixed copper loss at 75W considering the end winding. It should be noted that copper loss is the main loss and source of heating for the machine of this size at low speed. 75W is selected within a reasonable copper loss range for the machine of this size. Therefore, all the machine candidates are optimized under the same thermal loading; (6) Fixed slot packing factor; (7) Fixed overall PM volume, the PM used in the simulation has remanence of 1.2T with the relative permeability of 1.05. The related dimensional parameters are illustrated in Fig. 9 in the appendix. During the optimization, the stator yoke radius r_y, stator tooth width w_t, stator inner radius r_i, slot opening angle s_o and the PM height l_{pm} are variables. The optimized machine parameters are provided in Table V in the appendix.

C. Influence of Gear Ratio on Winding Factor

In FSCW PMSMs, the fundamental winding factor is one of the important metrics that directly influences the machine's torque density. It is therefore important to check the fundamental winding factor k_w at the machine design stage. The fundamental winding factors of FSCW PMSMs can be calculated using the back EMF phasors [7].

The winding factors for the 3 and 6-phase machines with different G_rs are provided in Fig. 2. Since the machines with 3-slot/2-pole(G_r=-2) and 4-pole(G_r=-0.5) sub-motors have the lowest fundamental winding factor of 0.866 among double-layer FSCW PM machines [6], the fundamental winding factor is above 0.866 when G_r is between -0.5 and -2 for a double-layer FSCW PM machine. For both 3- and 6-phase machines, the winding factor approaches to 1 when G_r approaches -1, and the winding factor decreases monotonically when G_r is either less than or greater than -1. This can be explained from (1), since G_r approaches to -1 means that the pole number is closer to the slot number, which then leads to a higher winding factor in FSCW PMSMs. The machines with the same G_r, same phase number but different slot number, have the same winding factor. For the machines with the same G_r but different phase number, the ones with higher number of phases have higher winding factors.

D. Influence of Gear Ratio on Torque Output

The average electromagnetic torque of a FSCW surface mounted PM machine fed with sinusoidal currents under I_d=0 control is given by

$$T = \frac{1}{4\pi} k_w n_l n_t N_s B_\delta A_\delta I \quad (2)$$

where n_l is the number of layers in each slot. n_t is the number of turns around each tooth, B_δ is the peak value of the fundamental no-load air gap flux density, A_δ is the air gap area, I is the peak value of the phase current.

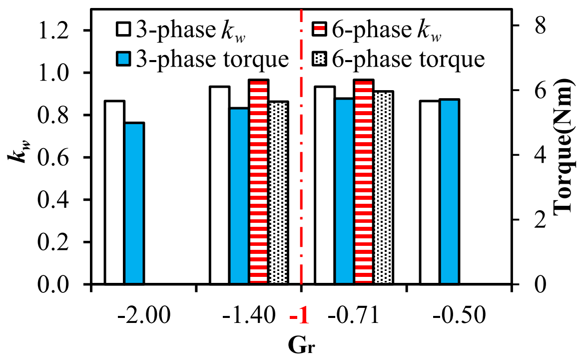
The output torque of the machine candidates under 75W copper loss are calculated by FEA and shown in Fig. 2. It can be seen that the torque trend with respect to the gear ratio is almost the same with the winding factor, which is consistent with (2). Hence, within a certain phase number, a larger torque output can be obtained when the gear ratio approaches -1. For the machines with the same G_r but different phase number, the ones with higher number of phases will have higher torque.

TABLE II
GEAR RATIOS IN FSCW PM ELECTRICAL MACHINES
(A) 12-SLOT

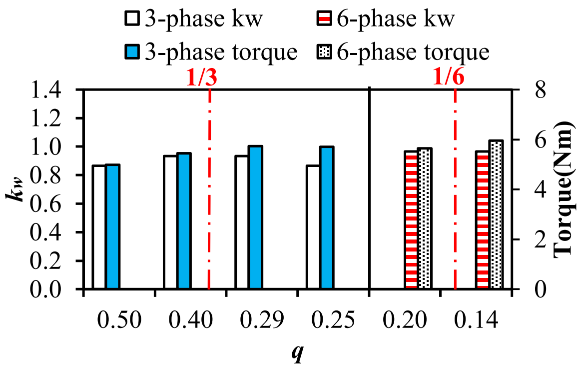
$2p_r$	8	10	14	16
3-phase	-2.00	-1.40	-0.71	-0.50
6-phase	-	-1.40	-0.71	-

(B) 24-SLOT

$2p_r$	16	20	22	26	28	32
3-phase	-2.00	-1.40	-1.18	-0.85	-0.71	-0.50
6-phase	-	-1.40	-1.18	-0.85	-0.71	-



(a)



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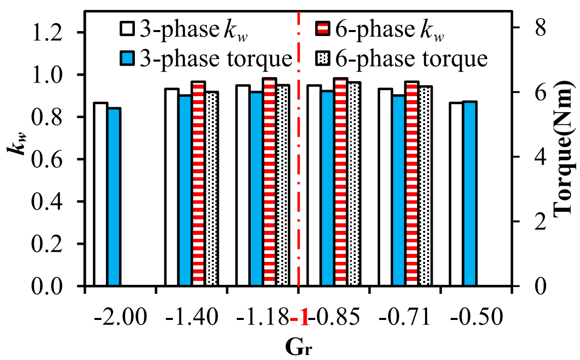


(c)

Fig. 2. Winding factors and torques for different machines (75W copper loss). (a) 12-slot. (b) Comparison between G_r and q as reference. (c) 24-slot.

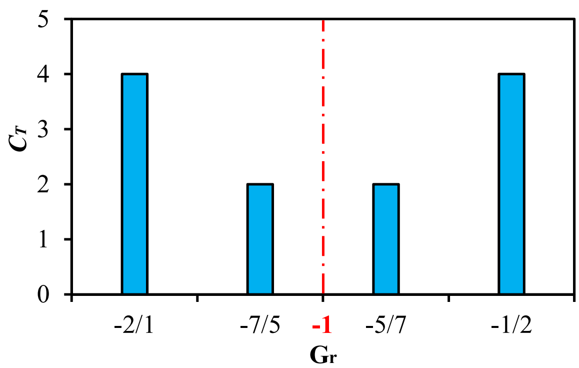
E. Influence of Gear Ratio on Cogging Torque

In the slotted FSCW PMSMs, cogging torque is produced by the interaction between the PM MMF and the air-gap permeance. Cogging torque can cause speed ripples and induces vibration at low load and speed [21]. There has been intensive research on the reduction of cogging torque. Although there are many methods to reduce the cogging torque, such as rotor skewing, employing auxiliary slots, magnet shaping, etc., they require a more complicated structure, thus making them more difficult to manufacture and assemble. Therefore, it is important to select a proper slot/pole combination with inherent low cogging torque at the design stage. In [21], "goodness" of slot/pole combination, which is defined as

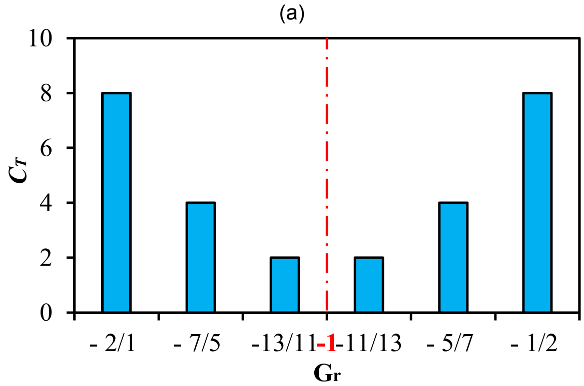
$$C_T = \frac{2p_r N_s}{N_c} \tag{3}$$

is used as an index in evaluating the cogging torque amplitude, where N_c is the least common multiple of the $2p_r$ and N_s. It has been analytically proven in [22] that for a given slot number, the smaller the "goodness" is, the smaller the cogging torque will be. It should also be noted that C_T can also be used to show the vibration mode of the machine.

The relationship between C_T and G_r for different machines are provided in Fig. 3. It can be seen that for the FSCW PMSMs in this paper, C_T has a decreasing trend when Gr approaches -1. It should be noted that this trend is only applicable to the FSCW PMSMs with sub-motors whose G_rs are within -2~-0.5 and slot/pole number combinations satisfy $N_s=2p_r\pm1$(or 2). For other slot/pole number combinations, G_r can still be used for C_T comparison among machines with the same slot but different pole numbers if it is represented as a fraction. As shown in Fig. 3, for gear ratios less than -1, C_T increases as the denominator decreases; while, for gear ratios greater than -1, C_T increases as the numerator decreases. For any given slot number, the machines with G_r equal to -2 or -0.5 have greater cogging torque than the others. For the machines with the same G_r but different slot number, a higher number of slot means a higher C_T, lower cogging torque and larger vibration mode.



(a)



(b)

Fig. 3. "Goodness" for different machines. (a) 12-slot. (b) 24-slot.

F. Influence of Gear Ratio on Inductance

It has been proven that FSCW machines with surface mounted PMs can achieve good flux weakening performance. The reason for this can be found by considering the characteristic current [12], defined as

$$I_{ch} = \frac{\psi_m}{L_d} \tag{4}$$

where ψ_m is the rms magnet flux linkage and L_d is the d-axis inductance, which is almost equal to the q-axis inductance in electrical machines with surface-mounted PMs.

Since the phase inductance of surface-mounted electrical machines increases when concentrated fractional-slot stator windings are introduced, thus reducing the machine's characteristic current sufficiently to match its rated current. The FSCW machines with surface-mounted PMs have a better flux-weakening performance than the integral slot distributed winding surface-mounted PMSMs.

The d-axis inductances of the different machines are calculated by FEA. The relationships between G_r and L_d for the 12- and 24-slot machines of different phases are provided in Fig. 4. It shows that for both 3- and 6-phase machines, L_d becomes larger when G_r approaches -1, and L_d decreases monotonically when G_r is either less than or greater than -1. The reason is that there are two types of armature reaction flux in FSCW PMSMs [23]. Type 1 flux links $2N$ turns of windings, whereas Type 2 flux links N turns of windings in a phase. For FSCW PMSMs, when the PM pole number approaches the slot number, the proportion of Type 1 flux increases while Type 2 flux decreases, which indicates that L_d will increase. For the machines with the same G_r but different phase number, the ones with higher number of phases have smaller L_d. This is because with the increase of the number, the number of turns per phase decreases.

Another important application for FSCW PMSMs is as fault tolerant machines. The magnetic coupling between the phases should be as weak as possible so that the healthy phases can work properly when the other phases are under faulty condition. This addresses the importance of the mutual inductance between phases, which acts as an index for evaluating the magnetic coupling between phases. A small mutual inductance can even provide an opportunity to build modular segmented stators with physically separate phase coil sections [7].

The absolute values of mutual inductance $|L_m|$ of the different machines are calculated by FEA. The relationships between G_r and $|L_m|$ for the 12-slot and 24-slot machines are given in Fig. 4. It can be seen that $|L_m|$ decreases when G_r approaches to -1, and $|L_m|$ increases monotonically when G_r is either less than or greater than -1. The reason for this is that for FSCW PMSMs, when the PM pole number approaches the slot number, the armature reaction flux becomes more concentrated in one phase whereas the flux linking two phases reduces, thus indicating a decrease in $|L_m|$.

G. Influence of Gear Ratio on PM Losses

For FSCW PMSMs, there are a large amount of harmonics in the air gap due to the winding distribution and slotting effect. In surface mounted PMSMs, the PMs are exposed to the air-gap harmonics directly. Hence, the PM eddy current losses cannot be neglected because the asynchronous harmonics can induce a large amount of eddy current in the PMs. This results in joule loss in the magnets, particularly in NdFeB magnets due to their relatively high electric conductivity. The PM losses can heat up the magnets and cause irreversible demagnetization of the PMs.

Eddy-current losses in the rotor surface PMs are caused by two kinds of harmonics: slotting permeance harmonics and armature reaction harmonics. The losses caused by slotting harmonics depend mainly on the structure of the teeth, although this is generally less important when the slot opening is not large. Thus, in this paper, only the relationship between G_r and PM losses caused by armature reaction harmonics are considered.

As stated in Section II, for a N_s-slot/2p_r-pole FSCW PMSM, apart from the fundamental p_rth harmonic, the major armature reaction harmonic which contributes to the torque production via magnetic gearing effect is the (N_s-p_r)th harmonic. For a FSCW PMSM with a certain number of slots, electrical loading and phases, with the increase of G_r, the order of the (N_s-p_r)th harmonic decreases whereas its amplitude increases, as shown in Fig. 5 (a). When the machine is rotating at the speed of $n$rpm, the frequency of the current induced by the (N_s-p_r)th harmonic at the rotor side will be

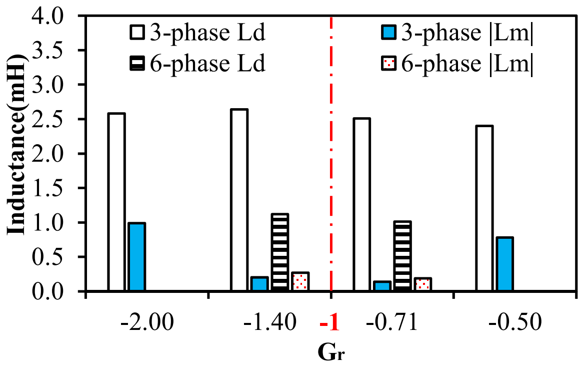
$$f = nN_s \tag{5}$$

Generally, an index of the rotor losses can be used to evaluate the PM losses in FSCW PMSMs of different slot/pole combinations at the design stage [13]:

$$I_{rl} = \sum_{v} \frac{\xi^4}{\sqrt[4]{(\xi^4+\pi^4)^3}} \left(\frac{k_{wv}}{k_w}\right)^2 \frac{v}{p_r} k_{gap} \tag{6}$$

where ξ is the specific wave length, k_{wv} is the winding factor of the harmonic of order v and k_{gap} is the air-gap coefficient.

The indices of PM losses in the 3- and 6 phase, 12- and 24-slot machines rotating at 100rpm under 75W copper loss are calculated and given in Fig. 5. FEM is also used to calculate PM losses in the different machines for validation. The result shows that for both phase numbers, the PM eddy current loss caused by armature reaction harmonics increases with the increase of G_r for a given slot number. This is because under the same slot number, the frequency of the induced current for all feasible pole numbers is the same when the speed of rotation is fixed as shown in (5). Thus, the eddy current only depends on the amplitude of the (N_s-p_r)th harmonic which increases with the increase of G_r as shown in Fig. 5 (a).



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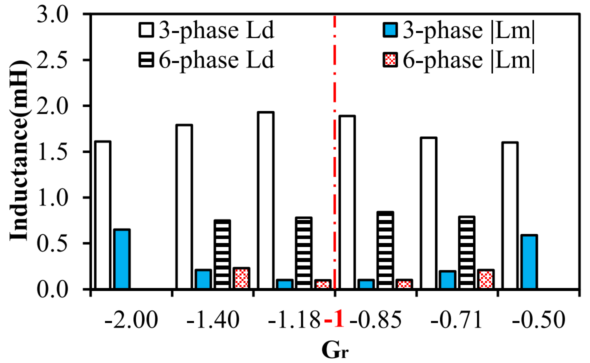
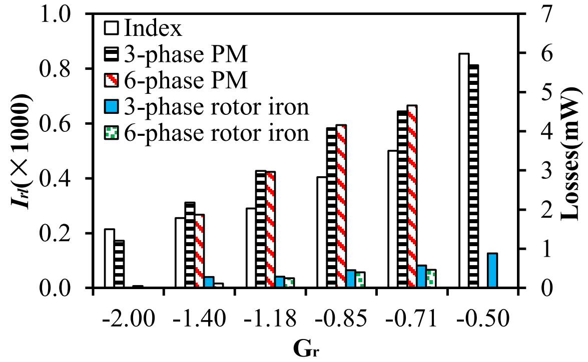


(b)

Fig. 4. D-axis and mutual inductances for different machines. (a) 12-slot. (b) 24-slot.



(c)

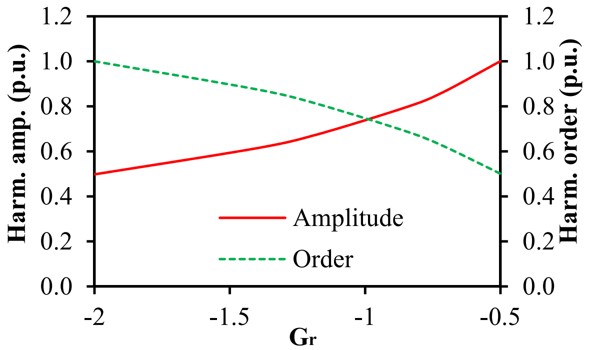
Fig. 5. PM losses index, PM and rotor iron losses for different machines (75W copper loss, 100rpm). (a) Variation of the major harmonic. (b) 12-slot. (c) 24-slot.

H. Influence of Gear Ratio on Rotor Core Losses

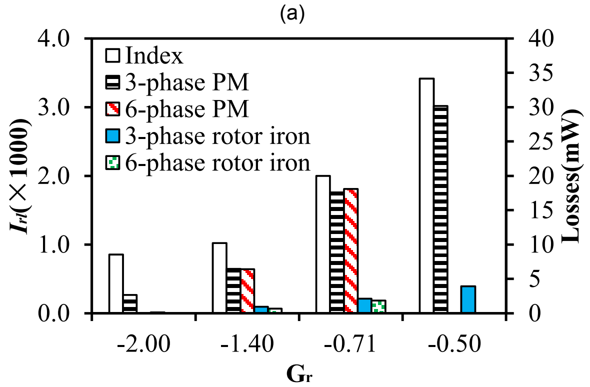
In FSCW PMSMs, the rotor iron losses may also make a significant contribution to the total losses. It is particularly important when the rotor thermal condition is considered. The instantaneous rotor iron losses can be calculated by the Bertotti losses equation, which is given by (7).

$$\mathrm{d}P(t)=k_f\left\{k_h B_m^2 f+\sigma\frac{\mathrm{d}^2}{12}\left(\frac{\mathrm{d}B}{\mathrm{d}t}(t)\right)^2+k_e\left(\frac{\mathrm{d}B}{\mathrm{d}t}(t)\right)^{1.5}\right\} \quad (7)$$

where d is the differential operator, k_f is the lamination factor, B is the value of flux density on the rotor core, B_m is its maximum value, σ is the conductivity of the rotor core, k_h and k_e are the hysteresis and excess loss coefficients respectively. The three parts at the right side of the equation represent hysteresis losses, classical eddy current losses and excess losses respectively.



(a)



(b)

The rotor iron losses in different machines of different phases were calculated by FEM under the same copper loss of 75W, as shown in Fig. 5. It can be seen that the rotor iron loss increases with the increase of G_r for both 3- and 6-phase machines under fixed slot numbers. This can be explained by the same reason with the PM loss. Since the (N_s-p_r)th armature reaction harmonic still plays an important role in the rotor iron loss. Under the same slot number, the frequency of flux density variation on the rotor core for all feasible pole numbers is the same when the speed of rotation is fixed as shown in (5). Thus, the rotor iron loss will highly depend on the amplitude of the (N_s-p_r)th harmonic which increases with the increase of G_r.

IV. Experimental Validation

In this section, 8-pole-12-slot and 10-pole-12-slot FSCW PMSMs are built and their performances are tested to validate the analysis in the above sections, Fig. 6. The main dimensional parameters of the prototypes are given in Table III, except that the pole arc of the 8-pole PM is optimized as 155 electrical degrees. For the machine performance analyzed above, the correctness of the winding factor and “goodness” can be validated by [7] and [21]. The correctness of the influence of G_r on the rotor losses has also been validated by the index I_{rl} proposed in [13] and the FEA validation. Hence, only the torque output and inductance, which were only calculated by FEA, need to be validated by experiments.

First, the static torques of the prototypes are measured. The test rig is shown in Fig. 7. For the 3-phase 8-pole-12-slot and 10-pole-12-slot machines, the experiments are done by applying DC current to phase A in series connection to the parallel phase B and phase C, i.e. $-2I_b=-2I_c=I_a=5$A. For the 6-phase 10-pole-12-slot machine, the experiment is done by applying one DC current to phase A in series connection to the parallel phase B and phase C, i.e. $-2I_b=-2I_c=I_a=5$A and another DC current to phase D in series connection to phase F, i.e. $-I_f=I_d=5$A, $I_e=0$A. The static torques within 0-180 electric degrees of the prototype machines can subsequently be measured by rotating the stator housings in 180 electric degrees with fixed steps. The measured results are given and compared with the 2-D FEA results in Fig. 8. Although the FEA predicted torques are slightly higher than the measured results, good agreements are obtained. The experimental results show that the output torque of the 8-pole-12-slot machine is smaller than that of the 10-pole-12-slot machine. The 6-phase 10-pole-12-slot machine has larger output torque than the 3-phase one. Hence, the analyses in the above section have been proven by the experiment.

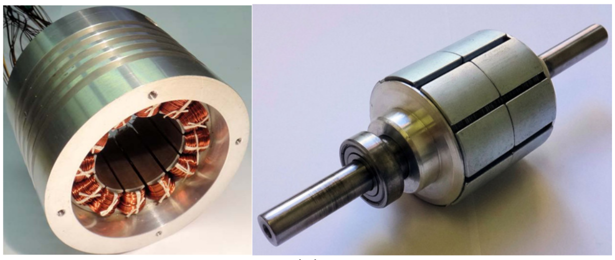
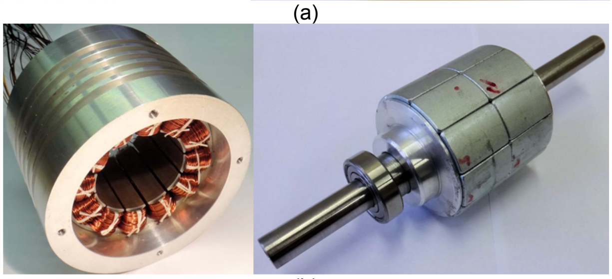

(a)

(b)

Fig. 6. Photos of prototypes. (a) 8-pole-12-slot. (b) 10-pole-12slot.

TABLE III
DIMENSIONAL PARAMETERS

Parameters	Values
Stator outer diameter	100mm
Stator inner diameter	57mm
Rotor outer diameter	55mm
Turns per phase	184
PM thickness	3mm
PM pole arc	180°
Axial length	50mm

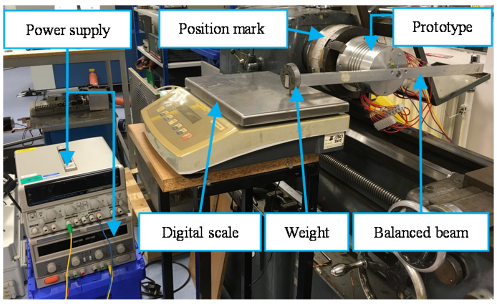


Fig. 7. Photo of the test rig.

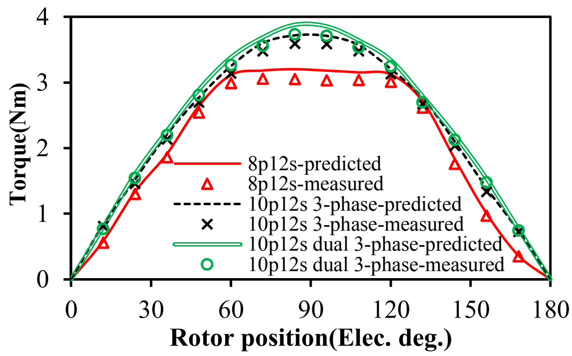


Fig. 8. Static torque waveforms (I=5A).

The d-axis and mutual inductances of the two 3-phase prototypes are tested by a LCR meter in the following way [23]:

Step 1: During the experiment, the winding inductance of A-phase is tested by the LCR meter, when the inductance of A-phase achieves its minimum value L_{Ad}, the d-axis coincides with the axis of A-phase.

Step 2: Lock the rotor position, test the B-phase inductance L'_{Bd} and the line inductance between A-phase and B-phase L_{ABd}.

Step 3: The d-axis and mutual inductance L_d and L_m can be calculated by (8) and (9), respectively.

$$L_{d}=\frac{L_{ABd}+L_{Ad}-L'_{Bd}}{2} \tag{8}$$

$$L_{m}=\frac{L_{ABd}-L_{Ad}-L'_{Bd}}{2} \tag{9}$$

The d-axis and mutual inductances of the 6-phase prototype are tested by a LCR meter in the following way:

Step 1: During the experiment, the winding inductance of A-phase is tested by the LCR meter, when the inductance of A-phase achieves its minimum value L_{Ad}, the d-axis coincides with the axis of A-phase.

Step 2: Lock the rotor position, test the D-phase inductance L'_{Dd}

Step 3: Connect phase A and D in series, test the line inductance between A-phase and D-phase L_{ADd}.

Step 4: The mutual and d-axis inductance L_d and L_m can be calculated by (10) and (11), respectively.

$$L_{m}=\frac{\left|L_{ADd}-L_{Ad}-L'_{Dd}\right|}{2} \tag{10}$$

$$L_{d}=L_{Ad}+\sqrt{3}L_{m} \tag{11}$$

The experimental measured and FEA results are given in Table IV. It can be seen that the FEA results match well with the experimental results. The measured results also show that L_d of the 8-pole-12-slot machine is smaller than that of the 10-pole-12-slot machine, whereas $|L_m|$ of the 8-pole-12-slot machine is larger than that of the 10-pole-12-slot machine, the 6-phase 10-pole-12-slot machine has smaller L_d and $|L_m|$ than the 3-phase one, which validates the analyses in the above section.

TABLE IV
INDUCTANCE OF THE PROTOTYPES

Slot/pole combination		8p12s	10p12s	
			3-phase	6-phase
L_d(mH)	FEA	3.61	4.36	2.24
	Measured	3.81	4.33	2.33
$\|L_m\|$(mH)	FEA	1.17	0.41	0.38
	Measured	1.16	0.40	0.35

V. CONCLUSIONS

In this paper, the magnetic gearing effect and the gear ratio in FSCW PMSMs are introduced and employed to investigate its relationship with the various machine performance of FSCW machines with different phases and slot/pole number combinations. It is found that the gear ratio is not only a representation of the gearing effect, but, once the slot and pole numbers are fixed, also an inherent parameter of the FSCW PMSMs regardless of the phase number. Hence, it can provide a unified reference for FSCW PMSMs of different phase numbers to evaluate the performance and select the proper correct slot/pole combination at the machine design stage. For a given slot number, the following conclusions can be drawn for FSCW PMSMs of all phases: under the same slot number, (a) when the gear ratio approaches -1, the winding factor and d-axis inductance increase whereas the mutual inductance and vibration mode decrease monotonically; (b) the FSCW PM machine will have larger torque output and smaller cogging torque with the gear ratio close to -1; and (c) the rotor losses, including the PM eddy current losses and the rotor iron losses, increase with the increase of the gear ratio. In this way, gear ratio can contribute to a proper slot/pole number selection in any multi-phase FSCW PMSMs by acting as a unified index for a quick overall performance comparison. This paper can also provide a good in-depth understanding of FSCW machines.

APPENDIX

Without loss of generality, 24-slot FSCW PM machines with 4-phase designs are also considered in this paper. The feasible pole numbers and the corresponding gear ratios of the 24-slot 4-phase machines are provided in Table VI. All the 4-phase machine candidates are globally optimized under the same conditions with the 3- and 6-phase machines in Section III. The optimized machine parameters are provided in Table V.

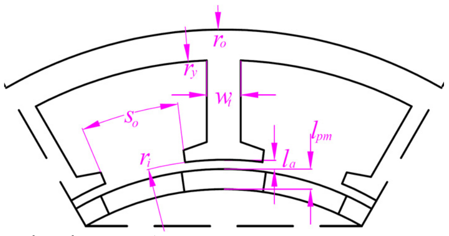


Fig. 9. Dimensional parameters.

TABLE V
OPTIMIZED DIMENSIONAL PARAMETERS.

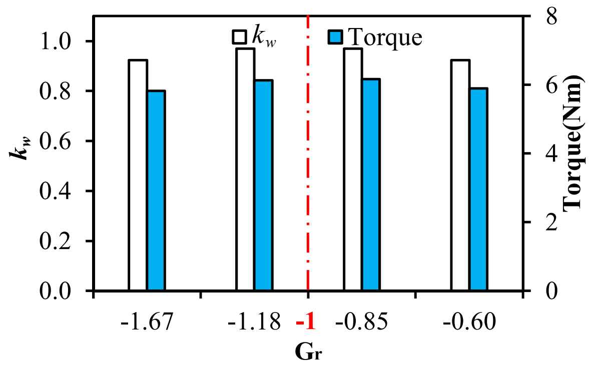
Phase no.	N_s	$2p_r$	r_y (mm)	w_t (mm)	s_o (Mech. deg.)	r_i (mm)	l_{pm} (mm)	TPC
3	12	8	46.3	7.2	4.8	28.6	3	46
		10	46.6	6.6	3.7	29.5	2.9	
		14	47.3	5.4	10.4	31.0	2.7	
		16	47.7	4.6	12.7	33.1	2.5	
6	12	10	46.7	6.6	3.8	29.6	2.9	
		14	47.5	5.4	10.5	31.1	2.7	
3	24	16	47.8	4.0	3.1	32.3	2.6	23
		20	48.0	3.8	3.4	33.4	2.5	
		22	48.1	3.5	3.8	33.8	2.5	
		26	48.4	3.1	3.6	34.3	2.4	
		28	48.5	2.9	3.3	34.7	2.4	
		32	48.8	2.4	3.4	35.4	2.4	
6	24	20	48.1	3.8	3.3	33.4	2.5	
		22	48.2	3.5	3.7	33.8	2.5	
		26	48.5	3.1	3.5	34.3	2.4	
		28	48.6	2.9	3.2	34.7	2.4	
4	24	18	48.0	3.8	3.3	32.5	2.6	
		22	48.2	3.5	3.6	33.8	2.5	
		26	48.4	3.1	3.6	34.3	2.4	
		30	48.6	2.6	3.8	35.0	2.4	

*TPC-number of turns per coil

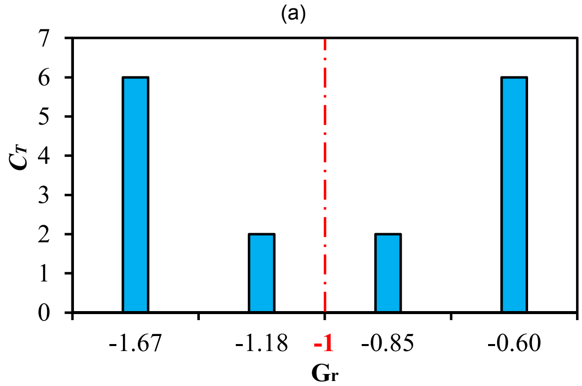
TABLE VI
GEAR RATIOS IN 24-SLOT 4-PHASE FSCW PM MACHINES

$2p_r$	18	22	26	30
4-phase	-1.67	-1.18	-0.85	-0.6

The winding factor, torque, C_T, d-axis inductance, mutual inductance, rotor PM and iron losses for the 4-phase FSCW PM machines are calculated and the results are shown in Fig.10. They show that G_r can still be used for performance comparison and the reference is always -1 no matter how the phase number changes: when the gear ratio approaches -1, the winding factor, torque and d-axis inductance of the 4-phase machines increase whereas the mutual inductance and C_T decrease monotonically as shown in Figs. 10 (a)-(c); the rotor losses, including the PM eddy current losses and the rotor iron losses, increase with the increase of the gear ratio as shown in Fig. 10 (d).



(a)



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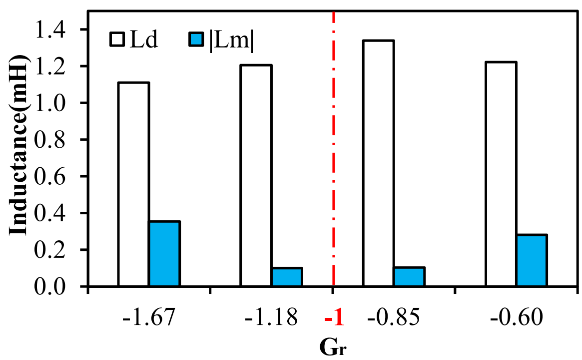
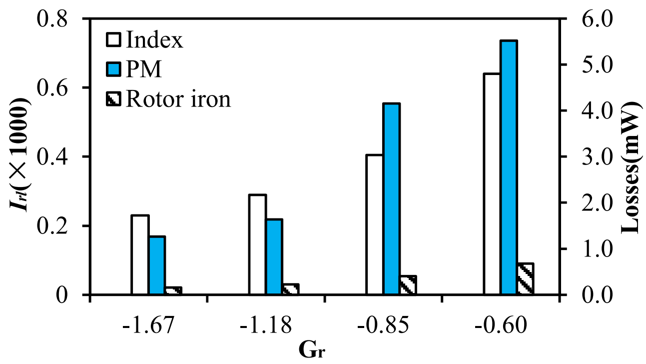


(c)



(d)

Fig. 10. Influence of Gr on 4-phase FSCW PMSM performance. (a) Winding factor and torque (75W copper loss). (b) C_T. (c) Inductance. (d) Rotor losses (75W copper loss, 100rpm).

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