Proceedings Paper:

Stockdale, G, Milani, G and Sarhosis, V orcid.org/0000-0002-8604-8659 (2019) Irrecoverable collapse time for a fixed-hinge dry-stack arch under constant horizontal acceleration. In: AIP Conference Proceedings. 15th International Conference on of Computational Methods in Sciences and Engineering (ICCMSE 2019), 01-05 May 2019, Rhodes, Greece. AIP Publishing .

https://doi.org/10.1063/1.5138009

Irrecoverable Collapse Time for a Fixed-Hinge Dry-Stack Arch Under Constant Horizontal Acceleration

Gabriel Stockdale[1, a)] Gabriele Milani[1, b)] and Vasilis Sarhosis[2, c)]

[1] Department of Architecture, Built Environment and Construction Engineering (ABC),
Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milan, Italy
[2] School of Civil Engineering, University of Leeds, LS2 9JT, UK

[a)]Corresponding author: gabriellee.stockdale@polimi.it
[b)]gabriele.milani@polimi.it
[c)] vesilis.sarhosis@newcastle.ac.uk

Abstract. The collapse of dry-stack masonry arches results from the transformation of a static system to a mechanical state through the development of mechanical joints. The traditional failure condition is this mechanization through the formation of four-hinges in a kinematically admissible configuration. The first-order analysis of an arche's seismic capacity is obtained through limit analysis (LA) approaches. One approach is the equilibrium assessment of the kinematic theorem through the use of a kinematic collapse load calculator (KCLC). Utilizing a custom KCLC developed and validated from an experimental arch, with the added control of the single degree-of-freedom rotations, an analytic solution is developed between the applied acceleration and the minimum time duration required for collapse. The collapse multiplier and arch centroid data is recorded for all the admissible conditions that exist in the spatial deformation propagation. From this information, the work required to collapse the arch under kinematic equilibrium is established and utilized to decompose the static and kinematic energy contributions. The time-displacement domain is then defined from the resulting kinematic energy of the overloaded arch and used to evaluate the time where the kinematic energy exceeds the remaining work required for the loss of the kinematically admissible condition. This results in a simple analytical function linking excess static acceleration with a time limit of recovery.

INTRODUCTION

The seismic assessment and understanding of masonry arches is critical for the preservation of existing systems, but it is also vitally important for the reintroduction of dry-stack masonry arches as a modern structural alternative. In the modernization approach, the evaluation and understanding is isolated into two key components. The first component is the ability to design and define the failure, and the second is to quantify that designed failure. For dry stack masonry arches the ability to design and define has been proposed and statically tested [1,2]. In this process of modernization, the development of the Kinematic Collapse Load Calculator has begun, and its ability to efficiently analyze and adapt has been established [3,4]. The KCLC is a calculator that allows the user to define the boundary and load conditions. It then performs kinematic equilibrium-admissibility evaluations and displays the various results such as reactions, collapse value, and coordinates.

The KCLC provides static equilibrium analyses on a geometric configuration of the mechanical arch. Through the incorporation of the failure motion of the arch, the equilibrium analysis is applied to the specially deformed condition of motion. From this condition, the displacement domain of energy and work can be established.

The objective of this research is to establish the first time-domain analysis condition for the failure of a hinge-defined dry-stack masonry arch through the evaluation of excessive horizontal acceleration, kinematic equilibrium and standard arch-failure deformations.

KCLC AND DEFORMATION CAPACITY

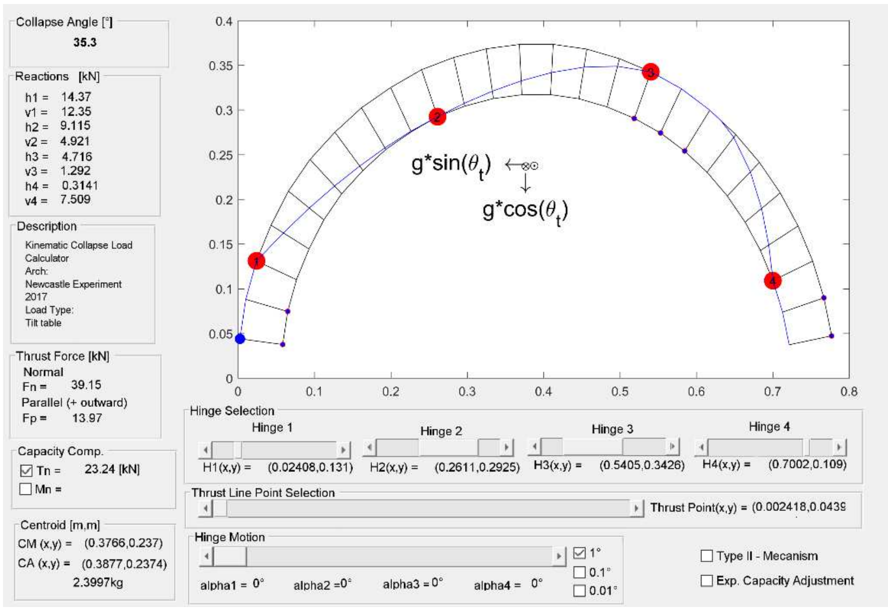

FIGURE 1. The custom KCLC, arch and hinge configuration used in this work.

Figure 1 shows the custom and validated KCLC for an experimental arch with the added hinge motion panel and centroid position display [3,4]. Considering the kinematic motion of rigid elements, the arch-hinge configuration can be represented by three fixed lengths connected by four pins as seen in Fig. 2. The deformation of the system can then be represented by the rotation of the pins and is bound by a SDOF of motion.

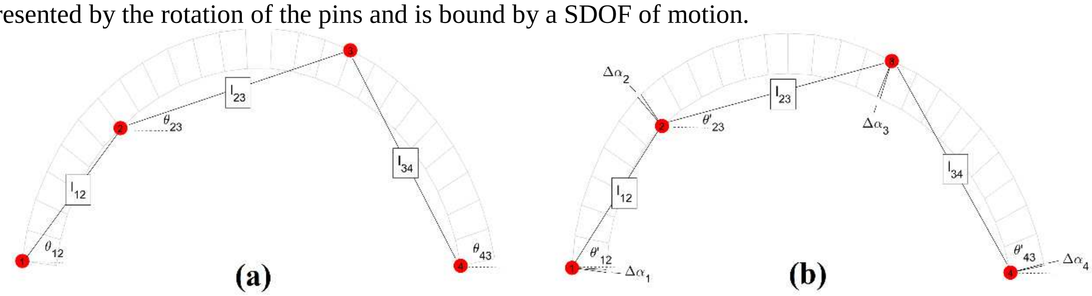

FIGURE 2. Four-pin arch mechanism (a) before and (b) after a SDOF deformation.

For a given rotation, $\Delta\alpha_1$, at H_1 (see Fig. 2) the remaining rotations can be determined by

$$\Delta\alpha_4 = \cos^{-1}\left(\frac{l_{12}}{l_{34}}[\cos(\theta_{12}+\Delta\alpha_1) - \cos(\theta_{12})] + \cos(\theta_{43})\right) - \theta_{43}$$
$$\Delta\alpha_2 = \Delta\alpha_1 + \theta'_{23} - \theta_{23} \tag{1}$$
$$\Delta\alpha_3 = \Delta\alpha_4 + \theta'_{23} - \theta_{23}$$

For each deformation applied to the arch-hinge configuration, the kinematic equilibrium and admissibility is evaluated, and the results displayed (see Fig. 3)

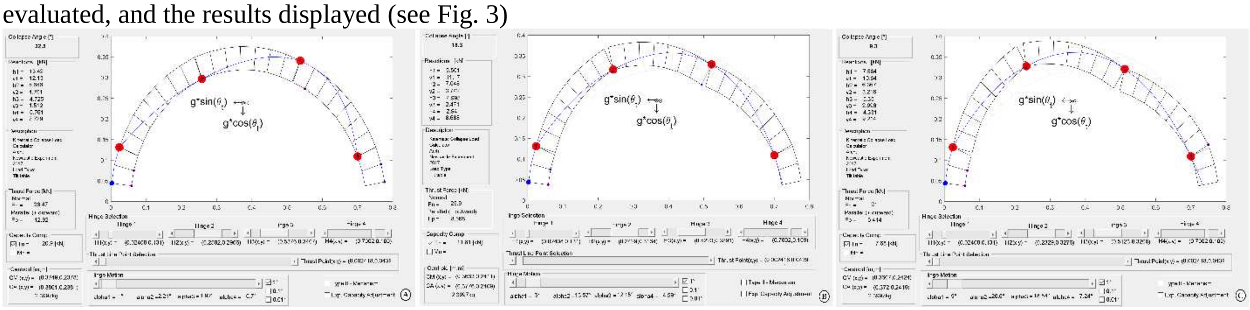

FIGURE 3. A (a) 1°, (b) 6°, and a (c) 9° α_1 rotation applied to the arch-hinge system shown in Fig. 1.

Using the alpha rotation parameter and the added CM coordinate display, the collapse force and center of mass (CM) position was recorded for the range of admissible deformations. The resulting force versus horizontal CM displacement and CM displacement slope verses horizontal displacements are shown in Figure 4.

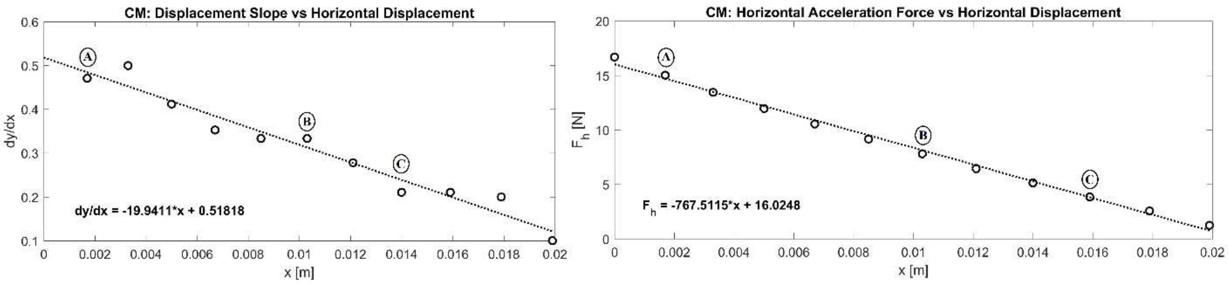

FIGURE 4. CM displacement slope and acceleration force versus horizontal displacement.

WORK AND ENERGY CONSERVATION

From the deformation capacities the work required to deform the arch to collapse can be determined as shown in Fig. 5. From Fig. 5 the required work to maintain equilibrium along the deformed path, W_h, is greater than the change in gravitational potential, U_g. It is postulated that the difference is the result of stored energy required to create and maintain the kinematic state itself. Nonetheless, the required work to maintain kinetic equilibrium establishes a cutoff condition where the conservation of energy dictates that any additional energy added to the system as work is in the form of kinetic energy. Starting from rest the change in kinetic energy becomes

$$\Delta K_E = W_{app} - W_h = \int \left(F_{app} - F_h\right) dx \tag{2}$$

for any applied work, W_{app}, from a horizontal acceleration force, F_{app} that is greater than F_h. Additionally, if at a displacement x' the applied force drops below F_h, with a resulting kinetic energy K_E' then the condition

$$K_E' < \int_{x'}^{x_c} \left(F_{app} - F_h\right) dx \tag{3}$$

will result in a recoverable system.

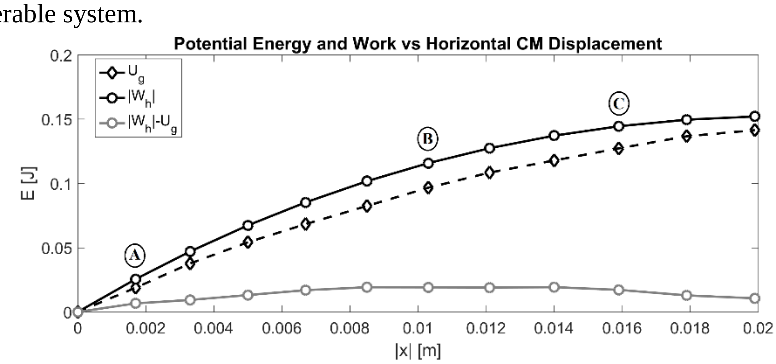

FIGURE 5. Potential Energy and Work required to deform the arch to a non-admissible configuration.

TIME DOMAIN

Starting from rest at time t_0 and a given load/displacement condition that results in a kinetic energy,

$$K_E(x) = \frac{1}{2} m\boldsymbol{v}^2 = \int \left(F_{app} - F_h\right) dx \tag{4}$$

the time domain can be introduced,

$$\left(\frac{dr}{dt}\right)^2 = \frac{1}{m} K_E(x) \tag{5}$$

From the linear slope-displacement representation if Fig. 4, and radial-Cartesian vector relations, the relationship between displacement and time becomes

$$\int_0^x \sqrt{\frac{m\left[A^2x^2 + 2A(1+B)x + (1+B^2)\right]}{K_E(x)}}\, dx = (t - t_0) \tag{6}$$

where A and B are the slope and constant fits to the linear slope-displacement equation in Fig. 4.

Constant Horizontal Acceleration

Consider a constant horizontal force for a duration of time t_d

$$F_{app} = \begin{cases} C' = C - F_h(0) \geq 0, & 0 \leq t \leq t_d \\ 0, & t > t_d \end{cases} \tag{7}$$

then the root function inside the integral of Eqn. 6 becomes

$$f_{KE}(x) = \sqrt{\frac{m[A^2x^2+2A(1+B)x+(1+B^2)]}{(C'-A'x)x}} \tag{8}$$

with A' as one-half the slop constant from the force-displacement linear fit equation. Figure 6 shows the plot of Eqn. 8 for the recorded displacements with the areas identified for the numerical evaluation of the integral in Eqn. 6. The results of the integration are also shown in Fig. 6 and the fitted result establishes a fifth-degree polynomial displacement-time relationship for the condition C' = 0.

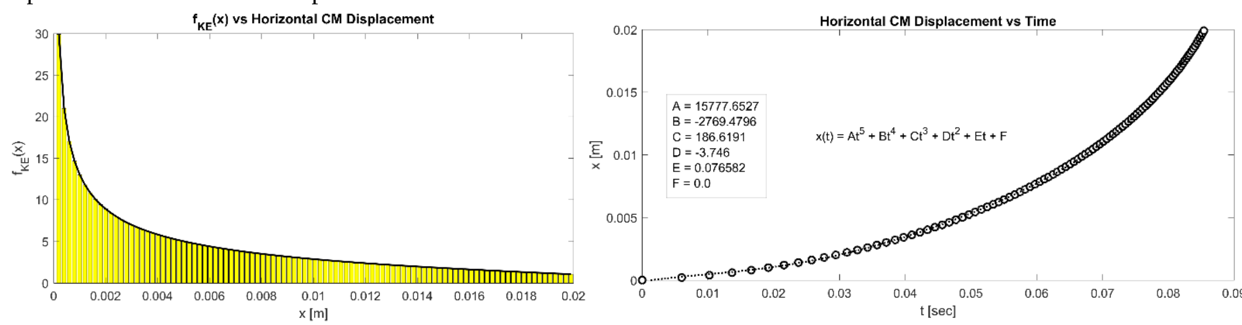

FIGURE 6. Plots of $f_{KE}(x)$ versus horizontal CM displacement and the resulting displacement-time plot for C' = 0.

Figure 7 shows the intersecting plots of kinetic energy and required work, W_{req}, for collapse verses time for variations of C'. This intersection of the energies is identified and establishes the time when the kinetic energy will exceed remaining work. Figure 7 also shows the establish irrevocable collapse time equation for the given arch-hinge configuration in Fig. 1 subjected to a constant acceleration that initiates mechanical deformation. That function is

$$t = 4.651 a_c^{0.7519} \tag{9}$$

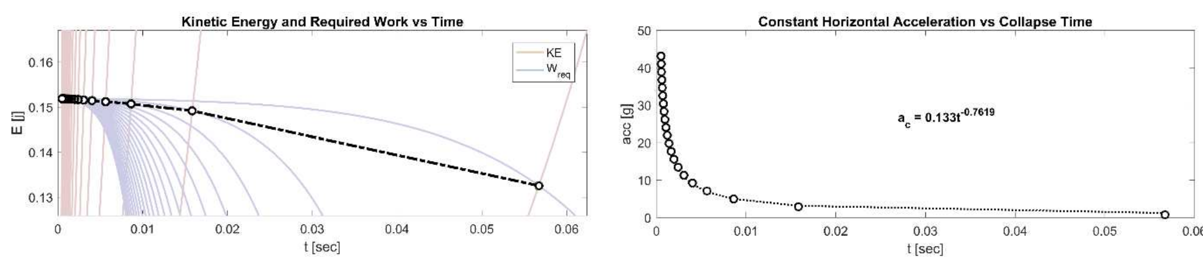

FIGURE 7. Kinematic-required work intersections and the resulting acceleration versus collapse time plot and fit.

REFERENCES

1. G. Stockdale, Reinforced stability-based design: a theoretical introduction through a mechanically reinforced masonry arch. International Journal of Masonry Research and Innovation, **1**(2), 101-142 (2016).
2. G. Stockdale, V. Sarhosis, and G. Milani, "Increase in seismic resistance for a dry joint masonry arch subjected to hinge control" in 10th IMC Conference Proceedings, International Masonry Society, (2018) pp. 968-981
3. G. Stockdale, S. Tiberti, D. Camilletti, G. Papa, A. Habieb, E. Bertolesi, G. Milani, S. Casolo, Kinematic collapse load calculator: Circular arches, SoftwareX, **7,** 174-179 (2018)
4. G. Stockdale, V. Sarhosis, G. Milani, Seismic Capacity and Multi-Mechanism Analysis for Dry-Stack Masonry Arches Subjected to Hinge Control, Bull Earthq Eng, DOI: 10.1007/s10518-019-00583-7 (2019) pp. 1-52