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Detecting overreaction in the Bitcoin market: A quantile autoregression approach

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October 2018

Abstract

We examine the persistence of returns on Bitcoin at different parts on the return distributions through the use of the quantile autoregressive (QAR) models. We find lower quantiles of the daily return distribution and upper quantiles of the weekly return distribution to exhibit positive dependence with past returns. The evidence points to overreaction in the Bitcoin market: investors overreact during days of sharp declines in the Bitcoin price and during weeks of market rallies.

JEL classification: C21; C51; C53; G00

Keywords: Bitcoin; cryptocurrencies; quantile regression; overreaction

Acknowledgement: We would like to thank Narjess Boubakri (Editor) and two anonymous reviewers for their valuable comments.

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1 Introduction

Despite the massive growth of the cryptocurrency markets with more than 1,600 cryptocurrencies currently available, Bitcoin remains the largest virtual currency in circulation.¹ Our estimates based on data provided by <https://coinmarketcap.com> highlight that Bitcoin covers more than 40% of the capitalisation of the entire cryptocurrency markets. Such a supremacy became common knowledge particularly at the end of 2017 when media reported, almost daily, the news of Bitcoin hitting a new all-time high (Demir et al., 2018). The remarkable price swings observed – apparently not justified by any new pivotal information available to the market – motivate our interest to research on the existence of potential overreaction behaviour from the investors’ side.² Investors’ overreaction, if present, could lead to the formation of dependence patterns in Bitcoin returns, thus implying inefficiency.

The literature on Bitcoin – which has grown exponentially (Giudici and Abu-Hashish, 2018) – has already dealt with efficiency issues. Indeed, after the pioneering work by Urquhart (2016), a few other papers have highlighted how Bitcoin has improved its efficiency over time (Sensoy, 2018; Vidal-Tomás and Ibañez, 2018). Other authors, instead, have claimed the opposite, namely that Bitcoin is inefficient (Al-Yahyaee et al., 2018; Charfeddine and Maouchi, 2018; Jiang et al., 2018). Apart from efficiency, it is worth mentioning that scholars have researched around a wider range of issues. For instance, some authors have examined the existence of speculative bubbles in the Bitcoin market (Corbet et al., 2017); others have found that Bitcoin behaves more like a speculative investment rather than an alternative currency (Baur et al., 2017); some others have explored the hedging capabilities of Bitcoin (Dyhrberg, 2016; Bouri et al., 2017); and quite a few have speculated on its volatility (Dwyer, 2015; Aalborg et al., 2018; Ardia et al., 2018). More recent studies have also explored the predicting behaviour of the economic policy uncertainty index on Bitcoin returns (Demir et al., 2018), as well as the causal relationship between the attention on Bitcoin and Bitcoin returns (Dastgir et al., 2018).

Notwithstanding the growing attention around Bitcoin among academicians, to the best of our knowledge no one has investigated the issue of investors’ reaction to Bitcoin’s price movement.³ Our paper hence aims at enriching this emerging literature by examining the presence of investors’ overreaction to price movement at different points along the return distribution using the quantile autoregressive (QAR) model, first introduced by Koenker and Xiao (2006) and subsequently employed by Baur et al. (2012) to investigate stock market return autocorrelation.

The QAR technique is particularly attractive to our research because it allows for a more thorough investigation of market behaviour under different market conditions as proxied by the location of the return on its distribution. In other words, while more traditional methodologies would allow for the analysis of the impact of lagged returns on current returns at their conditional

¹<https://coinmarketcap.com> accessed on 5th July 2018.

²For instance, on 18th December 2017, Bitcoin hit the all-time high of USD19,783.06, having risen by more than 5% within 24 hours.

³So far, indeed, scholars have researched on the market reaction to the regime change in volatility – by finding signs of inverted leverage effect (Ardia et al., 2018) – but have not focused yet on the reaction from the returns perspective.

mean, the framework employed in this paper enables us to explore the influence of a lagged return on the various quantiles of the current return. Specifically, our hypothesis is that when returns are observed to be either very low or very high, investors in Bitcoin may not act in an entirely rational way, thereby introducing varying degrees of return persistence brought about by inefficiency. Moreover, in order to examine whether reaction is affected by investment horizons, the analyses in this paper are performed on Bitcoin data of different frequencies: daily, weekly, and monthly.

As a preview of our main findings, results from the QAR model show that investors indeed overreact to movement in the price of Bitcoin as returns are highly persistent at both the daily and the weekly frequencies when returns are located at the tails of the distribution. More specifically, investors appear to overreact when returns at the daily frequency are located at the lower quantiles of the distribution and when returns at the weekly frequency are located at the upper quantile of the distribution. Our interpretation of the former is that market participants rush to exit the market during days of negative sentiments when prices fall. As for the latter, the evidence points to investors' overreaction to favourable news during weeks of positive sentiment when prices are rising. At the monthly frequency, we find no evidence of overreaction.

The remainder of this letter is organised as follows. Section 2 explains the methodological approach while Section 3 describes the Bitcoin data used in the analysis. Section 4 discusses the main findings. Finally, Section 5 concludes.

2 Quantile Autoregressive Model (QAR)

We estimate the first-order conditional quantile autoregressive – QAR(1) – model:

$$q_\tau(R_t|\Omega_{t-1}) = \alpha_\tau + \beta_\tau R_{t-1} \quad (1)$$

where $q_\tau(\bullet)$ denotes the conditional quantile function at the τ th quantile with $\tau \in (0, 1)$, $R_t = \ln(P_t/P_{t-1}) \times 100$ is the Bitcoin return at the end of period t , calculated from the closing prices at t and $t - 1$, and Ω_{t-1} is the information set publicly available to the market participants at the end of period $t - 1$. Estimates of both α_τ and β_τ in Eq. (1) can be obtained by solving the following minimisation problem:

$$\min_{\alpha_\tau, \beta_\tau} \sum_{t=1}^T \rho_\tau(R_t - \alpha_\tau - \beta_\tau R_{t-1}) \quad (2)$$

where T is the total number of observations, $\rho_\tau(z) = z(\tau - \mathbf{1}_{[z \leq 0]})$ and $\mathbf{1}_{[z \leq 0]} = 1$ if $z \leq 0$ and 0 otherwise.

The QAR(1) model in Eq. (1) has several advantages over the counterpart conditional mean model, AR(1). The QAR(1) model can be used to investigate patterns of return dependence across the entire return distribution, thus allowing for insights into the persistence of returns during different market sentiments: when returns are either very low (negative sentiments) or

very high (positive sentiments). The estimated quantile regression parameters are also robust to the presence of outliers in the data – a prominent feature of our datasets which we will discuss below.

3 Data

We download the Bitcoin price data from <https://coinmarketcap.com>. The website hosts historical daily price data since 28th April 2013.⁴ Hence, our sample period starts from 28th April 2013 until 3rd July 2018. By computing the logarithmic price ratios, we obtain a total of 1,892 daily, 270 weekly, and 62 monthly return observations. We present the summary statistics of the Bitcoin returns for various frequencies in Table 1. The time series plots and histograms are shown in Figure 1.

We observe that the distributions of Bitcoin returns are leptokurtic and appear to have very large standard deviations at all the frequencies. Extreme price movement is evident in the data. For example, a daily loss of around -26% was observed between 17th and 18th December 2013 and a monthly gain of 171% was realised between October and November 2013. The return distribution is slightly negatively skewed at the daily frequency while it is highly positively skewed at the monthly frequency. The p -values for the Jarque-Bera test statistics for the daily, weekly, and monthly Bitcoin returns are all zero, indicating rejection of the null hypotheses of normality.

As a preliminary check for the presence of return autocorrelation, we perform a data-driven Portmanteau test, introduced by Escanciano and Lobato (2009), on the time series of Bitcoin returns.⁵ The Escanciano-Lobato statistics for the daily, weekly, and monthly returns are 0.001, 0.989, and 0.018 with the p -values of 0.922, 0.320, and 0.893, respectively. The evidence points to the absence of autocorrelation in all the return series, suggesting efficiency at all the frequencies under investigation.

4 Analysis & Discussions

We present results for the QAR(1) models in Table 2. For comparisons, we also present results for the AR(1) models along the first two rows of the table. Firstly, in line with the results of the Portmanteau tests reported in the previous section, results for the AR(1) models suggest that the behaviour of returns on Bitcoin at the means at all the frequencies under examination is compatible with a white noise process. These findings are consistent with that of Urquhart (2016) who, among others, finds the Bitcoin market to be efficient after 2013.

⁴<https://coinmarketcap.com/currencies/bitcoin/historical-data/>. This data source has the merit of reporting the Bitcoin price as the volume weighted average of approximately 400 currency cross pairs converted to USD (Wei, 2018), thus saving us from having to retrieve data individually from hundreds of exchanges globally.

⁵Unlike the conventional Portmanteau test originally proposed by Ljung and Box (1978), the lag length for the Escanciano-Lobato is automatically selected by the data. In addition, the test is robust to conditional heteroskedasticity, making it especially suitable for financial time series.

We now turn to the results for the QAR(1) models. According to the estimates of β_τ , presented in the third column of Table 2, Bitcoin returns appear to be predictable at the 10th quantile and around the median at the daily frequency – a reflection of the overreaction of prices in an inefficient market (Lehmann, 1990). More specifically, at the 10th quantile, where the magnitude of the daily return is approximately -4.00% , the positive return dependence indicates that, during periods of negative market sentiments, tumbling Bitcoin prices cause investors to overreact, rushing for exit, and thereby causing prices to fall further. The estimates of β_τ around the median indicate statistically significant negative return autocorrelation although the size of the estimated parameter is small and therefore not economically relevant.

With regard to the Bitcoin return autocorrelation at the weekly frequency, the estimates of β_τ and their standard errors shown in the fifth column of Table 2 indicate the absence of persistence when returns are located below the median. The values of $\hat{\beta}_\tau$ when $\tau \geq 0.50$ point to the presence of positive return dependence whereby the relationship becomes stronger as we move from the median towards the higher quantiles. The pattern of monotonically increasing degree of return autocorrelation at the weekly frequency suggests that, as positive market conditions become discernible to market participants during periods of optimism, investors overreact to price rallies, causing the Bitcoin price to rise further. As far as the results for the monthly frequency are concerned, returns on Bitcoins are found to follow a white noise process. This is not surprising as any mispricing during the month would have been arbitrated away. Finally, it is worth noting however that results for monthly Bitcoin returns should be interpreted with caution due to the small sample size.

To illustrate the pattern of return dependence graphically, the quantile processes for $\hat{\beta}_\tau$ are presented in Figures 2b, 2d, and 2f. In each figure, the dotted black line shows the estimate of β_τ at $\tau = 0.05, 0.08, \dots, 0.95$ while the grey shade portrays the 90% confidence interval calculated by solving a parametric linear programming problem by inverting a rank test as described in Koenker (1994). The red solid line along with the red dotted lines, drawn across the figure, are the estimate of β_{Mean} and its corresponding 90% confidence interval, respectively. We can see from the estimated quantile processes in both Figures 2b and 2d that the statistically significant positive return autocorrelation is visible at the left tail of the daily return distribution and at the right tail of the weekly return distribution. At the monthly frequency, however, we find no evidence of return dependence as can be seen from the quantile process in Figure 2f. The red solid lines and their corresponding confidence intervals indicate the absence of return persistence at all the frequencies when the analysis is performed at the means.

Finally, we employ a variant of the Wald test described in Koenker and Bassett (1982) to investigate the hypotheses that the slope coefficients are identical at the different quantiles. The null hypotheses for the tests are as follows: $\beta_{0.10} = \beta_{0.90}$, $\beta_{0.10} = \beta_{0.50}$, $\beta_{0.50} = \beta_{0.90}$, $\beta_{0.25} = \beta_{0.75}$, and $\beta_{0.10} = \beta_{0.25} = \beta_{0.50} = \beta_{0.75} = \beta_{0.90}$. We report the F -statistics along with their corresponding p -values in Table 3. The results suggest that differences between β_τ estimated at the 10th against the 90th quantiles and the 10th against the median are statistically

significant at both the daily and the weekly frequencies. Furthermore, at the weekly frequency, the difference at the 2nd and the 3rd quantiles is also statistically significant. Taken together, the Wald test results point to the varying degrees of return persistence in the Bitcoin market across the different parts of the return distributions at the daily and the weekly frequencies. None of the differences are statistically significant at the monthly frequency, however.

5 Conclusions

This paper contributes to the expanding literature on cryptocurrencies. We are the first to offer evidence of investors' overreaction to Bitcoin price movements at different points along the return distribution. More specifically, using the quantile autoregressive (QAR) model, we show that, at the daily frequency, investors overreact to sharp declines in the Bitcoin price: days of extreme negative returns are likely to be followed by periods of negative returns. We interpret this finding as market participants, being alarmed by negative sentiments, rushing to exit the market, causing the Bitcoin market to fall further. On the contrary, investors appear to overreact under optimism when returns are positive at the weekly frequency: weekly positive returns appear to lead to even more bullish sentiments, causing the Bitcoin price to continue to rise. Economically, our work supports the view that the Bitcoin market is inefficient (Al-Yahyaee et al., 2018; Charfeddine and Maouchi, 2018; Jiang et al., 2018), while contradicting other studies that point towards improved efficiency over time (Sensoy, 2018; Vidal-Tomás and Ibañez, 2018).

Our results are also relevant from a policy perspective. Indeed, as highlighted by Al-Yahyaee et al. (2018) and Jiang et al. (2018), policymakers should work to strengthen the supervision of Bitcoin trading. Even though Bitcoin occupies only a small corner of the financial markets compared with more traditional asset classes, light touch regulatory oversight coupled with little understanding of this highly complex digital currency have a potential to destabilise the financial markets. Future research should therefore focus on the design of an effective regulatory framework which better protects investors from very aggressive price movement and contains potential spillovers of risk spreading from the cryptocurrency markets.

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Table 1
Descriptive Statistics

This table reports the means, standard deviations, minima, maxima, skewness, kurtosis and quantiles for daily, weekly, and monthly returns on Bitcoin. The sample period is between 29/4/2013 and 3/7/2018.

	Daily	Weekly	Monthly
Number of observations	1,892	270	62
Mean	0.21	1.43	6.18
Standard deviation	4.47	12.04	30.97
Skewness	-0.19	0.20	2.37
Kurtosis	10.79	4.43	14.17
Minimum	-26.62	-35.35	-41.28
25th quantile	-1.27	-4.85	-9.40
Median	0.20	1.42	4.18
75th quantile	1.95	7.26	18.02
Maximum	35.75	45.32	171.14

Table 2
Quantile Regression Results

This table reports the quantile regression results for the model shown in Eq (1). We set $\tau = 0.1, 0.2 \dots, 0.9$. Numbers in parentheses are standard errors calculate by inverting a rank test as described in [Koenker \(1994\)](#). Statistical significance at the 10%, 5% and 1% levels are indicated by *, **, *** respectively.

τ	Daily		Weekly		Monthly	
	$\hat{\alpha}_\tau$	$\hat{\beta}_\tau$	$\hat{\alpha}_\tau$	$\hat{\beta}_\tau$	$\hat{\alpha}_\tau$	$\hat{\beta}_\tau$
Mean	0.21** (0.10)	0.01 (0.02)	1.43* (0.80)	0.09 (0.06)	6.16 (4.00)	0.02 (0.13)
0.1	-4.07*** (0.29)	0.10*** (0.04)	-12.83*** (1.56)	-0.14 (0.12)	-23.15*** (5.72)	-0.10 (0.18)
0.2	-1.94*** (0.14)	0.03 (0.03)	-6.91*** (1.19)	-0.15 (0.10)	-16.39*** (4.73)	-0.14 (0.16)
0.3	-0.80*** (0.08)	-0.02 (0.02)	-2.67*** (0.97)	-0.03 (0.07)	-5.64 (4.17)	-0.20 (0.18)
0.4	-0.26*** (0.06)	-0.03*** (0.01)	-0.27 (0.74)	0.02 (0.03)	-2.18 (3.91)	-0.12 (0.17)
0.5	0.21*** (0.06)	-0.03*** (0.01)	1.58*** (0.65)	0.08*** (0.02)	4.81 (4.06)	-0.12 (0.18)
0.6	0.74*** (0.07)	-0.04*** (0.01)	3.54*** (0.70)	0.14*** (0.03)	8.97** (3.72)	-0.02 (0.20)
0.7	1.47*** (0.10)	-0.02 (0.02)	5.94*** (0.76)	0.18*** (0.06)	16.78*** (3.79)	0.03 (0.22)
0.8	2.57*** (0.14)	-0.04 (0.03)	9.07*** (1.13)	0.23*** (0.09)	23.33*** (4.75)	0.20 (0.25)
0.9	4.62*** (0.17)	-0.02 (0.04)	14.60*** (1.82)	0.21* (0.12)	37.01*** (9.29)	0.22 (0.40)

Table 3
Slope Equality Test Results

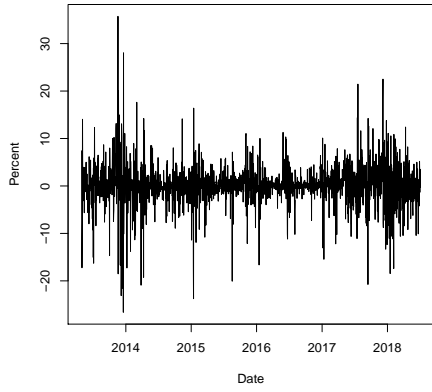
This table reports the results from the slope equality test of [Koenker and Bassett \(1982\)](#). Statistical significance at the 10%, 5% and 1% levels are indicated by *, **, and *** respectively.

H_0	Daily			Weekly			Monthly		
	F	Df	p -value	F	Df	p -value	F	Df	p -value
$\beta_{0.10} = \beta_{0.90}$	5.20	1	0.02	4.84	1	0.03	0.25	1	0.62
$\beta_{0.10} = \beta_{0.50}$	11.43	1	0.00	3.58	1	0.06	0.01	1	0.93
$\beta_{0.50} = \beta_{0.90}$	0.04	1	0.85	1.24	1	0.27	0.31	1	0.58
$\beta_{0.25} = \beta_{0.75}$	1.36	1	0.24	10.86	1	0.00	1.83	1	0.18
$\beta_{0.10} = \beta_{0.25} = \beta_{0.50} = \beta_{0.75} = \beta_{0.90}$	3.10	4	0.02	2.97	4	0.02	0.59	4	0.67

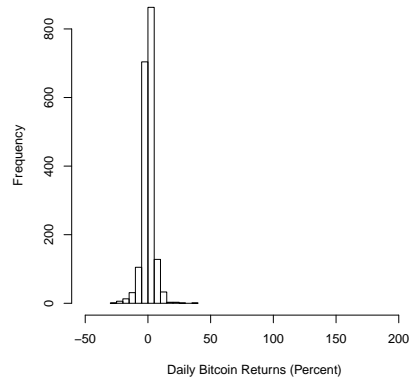
Figure 1

Time Series Plots and Histograms of Returns on Bitcoin

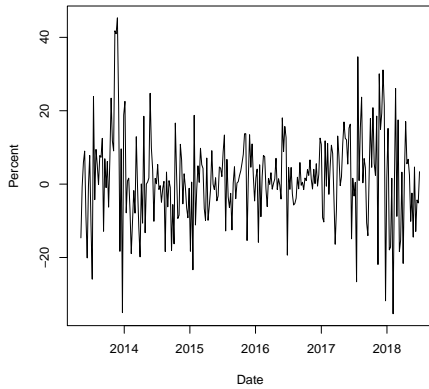
Figures 1a-1f below show time series plots and histograms of daily, weekly, and monthly returns on Bitcoin during a period of 29/4/2013 and 3/7/2018.



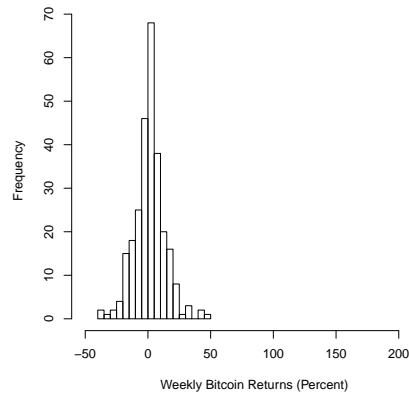
(a) Daily returns on Bitcoin



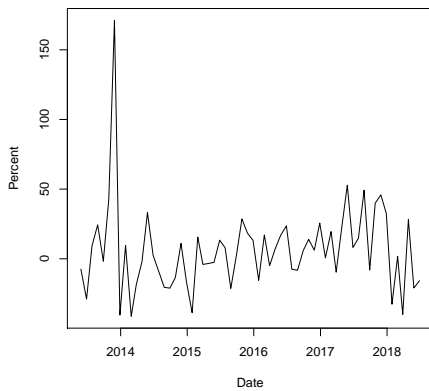
(b) Histogram of daily returns on Bitcoin



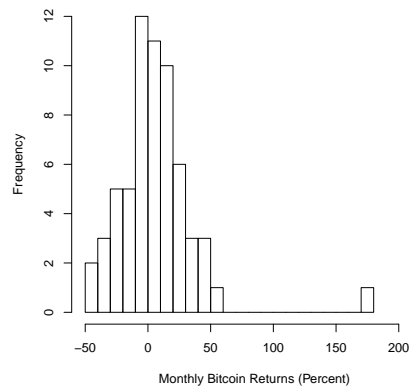
(c) Weekly returns on Bitcoin



(d) Histogram of weekly returns on Bitcoin



(e) Monthly returns on Bitcoin

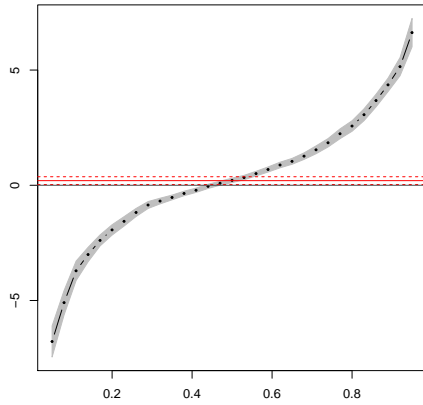


(f) Histogram of monthly returns on Bitcoin

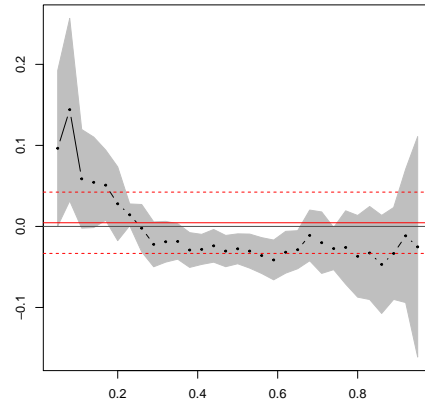
Figure 2

Quantile Process Estimates

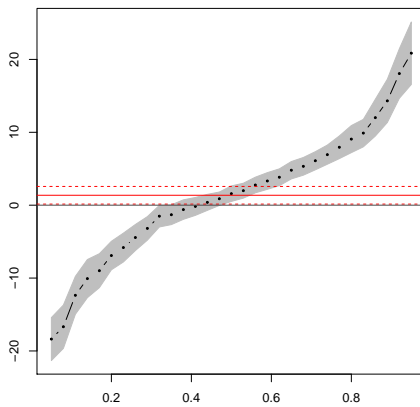
Figures 2a-2f show plots of quantile process estimates of α_τ and β_τ for Eq. (1) where $\tau = 0.05, 0.08, 0.11, \dots, 0.95$. The 90% confidence intervals of the estimated quantile regression parameters are depicted by the shade areas in the plots. In each subfigure, the red solid line represents the estimated parameter for the AR(1) model along with the corresponding 90% confidence interval shown by the dotted lines.



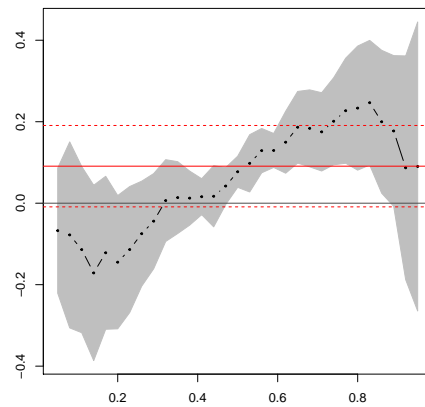
(a) Estimate of α_τ (daily)



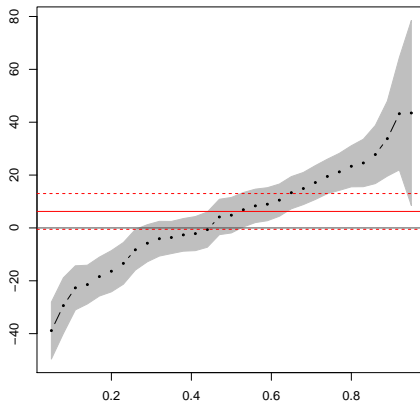
(b) Estimate of β_τ (daily)



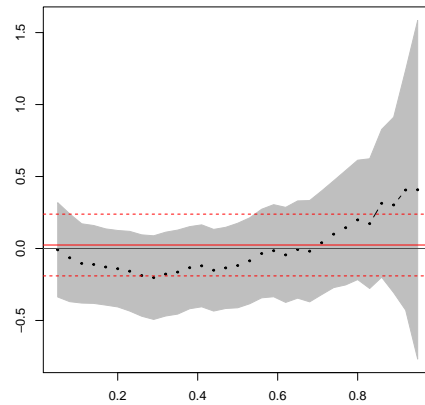
(c) Estimate of α_τ (weekly)



(d) Estimate of β_τ (weekly)



(e) Estimate of α_τ (monthly)



(f) Estimate of β_τ (monthly)