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Cooperative Control of High-Speed Trains for Headway Regulation: A Self-Triggered Model Predictive Control Based Approach

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Abstract

The advanced train-to-train and train-to-ground communication technologies equipped in high-speed railways have the potential to allow trains to follow each with a steady headway and improve the safety and performance of the railway systems. A key enabler is a train control system that is able to respond to unforeseen disturbances in the system (e.g., incidents, train delays), and to adjust and coordinate the train headways and speeds. This paper proposes a multi-train cooperative control model based on the dynamic features during train longitude movement to adjust train following headway. In particular, our model simultaneously considers several practical constraints, e.g., train controller output constraints, safe train following distance, as well as communication delays and resources. Then, this control problem is solved through a rolling horizon approach by calculating the Riccati equation with Lagrangian multipliers. Due to the practical communication resource constraints and riding comfort requirement, we also improved the rolling horizon approach into a novel self-triggered model predictive control scheme to overcome these issues. Finally, two case studies are given through simulation experiments. The simulation results are analyzed which demonstrate the effectiveness of the proposed approach.

Keywords: Cooperative Train Control; Headway Adjustment; High-Speed Railways; Self-triggered Control.

1 Introduction

High-speed railway (HSR) is a type of rail transport that operates significantly faster than traditional rail traffic. Typically, the maximum speed of high-speed train is over 200 kilometers per hour(km/hr) for existing line and 250 km/hr for new lines. To operate such a HSR system safely and efficiently, several train control systems were developed around world, for instance the European Train Control System (ETCS) in Europe, Automatic Train Control (ATC) in Japan, Positive Train Control (PTC) in America and Chinese Train Control System (CTCS) in China. One of the most important features of these train control systems is the bi-directional train-to-ground communication technology that enables the frequent exchange of information between way-side control center and trains (Dong et al., 2016). Building upon this communication technology, the moving-block system can be implemented to enhance the system capacity, in which each train can follow the preceding train with a minimum head-to-head headway to the preceding train (see Fig. 1). With these new advanced communication and signalling technologies, the high-speed trains are operated evenly according to the planned timetable (Hansen and Pachl, 2014; Wang and Goverde, 2016).

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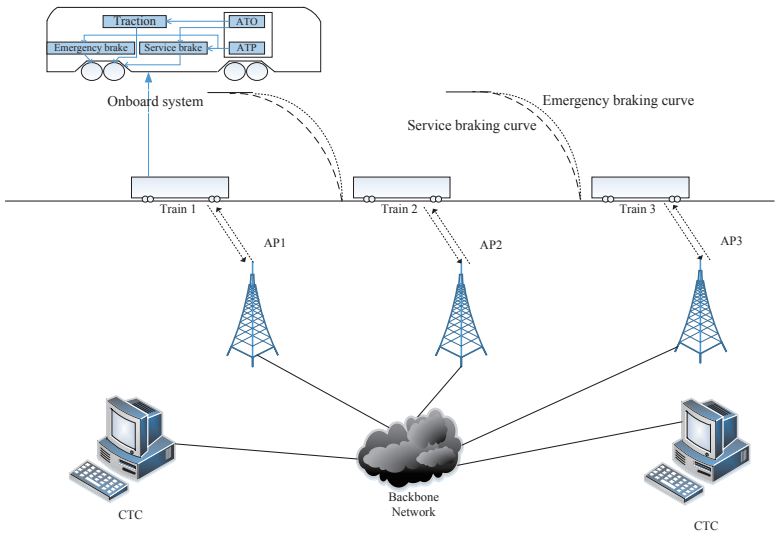


Figure 1: Illustration of train control systems with train-to-ground communication

In practice, train operations can be affected by unexpected disturbances, such as adverse to extreme weather, unsteady driving behaviors and infrastructure failures, etc. These lead to the irregularity in train following headway, deviation to schedule and reduced line capacity (Cacchiani et al., 2014). As an effective way to keep the line capacity under disturbances, headway adjustment is an effective train control to maintain a safe train following headway. For example, train following headway optimization can be utilized to enhance the system performance though:(i)improved the capacity at bottleneck area(i.e. junction area), and further on increased line capacity;(ii) absorbing the secondary delay and reduced delay propogation; and (iii) avoiding unnecessary train braking, decreased traction energy and reduced energy consumption (Xun et al., 2013).

Headway optimizing methods, as those commonly adopted in urban transit operations, typically involve holding control and station skipping (Osuna et al., 1972; Wegele et al., 2004; Sanchez-Martinez et al., 2016). Holding control was applied when a train is ahead of schedule, while station skipping strategy was applicable when a train fell behind schedule or was over crowded. Due to the short distance between stations in urban rail transit, these headway regulation methods essentially adjust the arrival and departure time of trains at stations (Yin et al., 2016, 2017). However, the distance between two stations in high-speed railway can be as long as 100 kilometers, much longer than that in urban rail transit. Therefore, there is scope in headway controls in HSR that capture not only the arrival & departure time at stations, but also the speed of trains between successive stations. As illustrated in Fig. 2, when a train runs from station i to station $i+1$, a buffer time is usually planed to allow small deviation from the schedule. Within the scope of buffer time, the train could run along one of several time-space (T-S) curves which correspond to different speed curves and running times. The decision on which T-S curve for train i to follow needs also to consider its headway to the train $j+1$ in front to ensure safety and passenger comfort. In this paper, we consider the train cooperative control problem for headway adjustment in HSR, i.e., the headway adjustment problem with train cooperative control is to coordinate train operations by proper train accelerating & braking to optimize train running headway with constrains of safety, punctuality and comfort.

Existing literature on real-time train headway control in HSR is limited. This may be due to the following two reasons. First, HSR operations are subjected to many complex constraints arising from the railway signalling systems, for example the moving-block and train speed limitation, minimum train following distance by train protection, etc. Due to these influencing factors, the traditional feedback controller is very difficult to guarantee

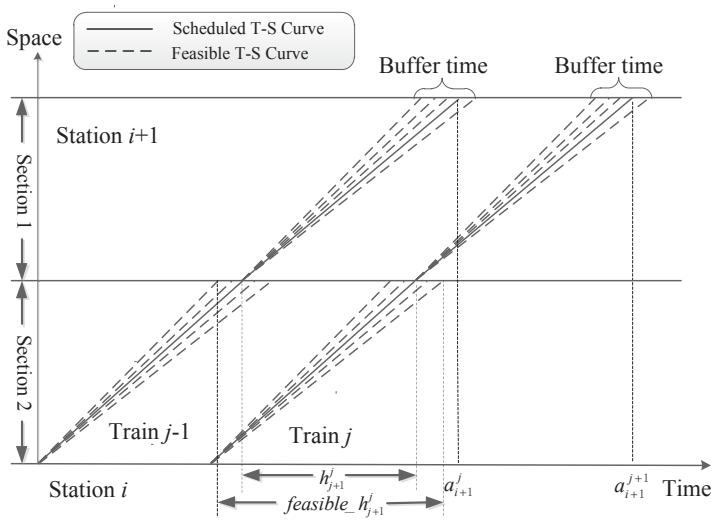


Figure 2: Train following scenario

the stability and performance of HSR train headway regulation problem. Second, the communication resources between control center and trains are usually very limited in HSR systems, making implementing most of the feedback controllers unpractical. The problem is more pronounced in CBTC systems, where the communication environment is complex between a fast moving HSR and ground. Therefore, it is a challenging task to realize efficient train control performance with complex signalling constraints and limited communication resources. To overcome these problems, this paper proposes a new cooperative train control, a self-triggered cooperative control. The method allows determining the optimal control output of trains (in our case the tractive and braking force of the trains), while taking account of the signalling constraints, as well as information update time. This is achieved by considering the next update time in advance based on the information at the current sampling instant. The method also allows smooth transition of control outputs, between sampling instants, which results in enhanced riding comfort for the passengers and improved the life time of the train's accelerating and braking systems.

The rest of this paper is organized as follows. In Section 2, we briefly review the literatures on the dynamic headway adjustment methods for rail traffic. In Section 3 and 4, we present the train cooperative control model with consideration of minimum headway which is a strict limitation for safety, and we design a self-triggered control scheme the considered problem. Then, the simulation and analyses are given in Section 5. This paper concludes in Section 6.

2 Literature review

Headway adjustment, also called automatic train regulation in metro, has attracted increased attention since 1990s (Van Breusegem et al., 1991; Lin et al., 2008; Li et al., 2016). Van Breusegem et al. (1991) proposed a state feedback control method for metro, which adopts a linear quadratic (LQ) regulator with Gaussian distribution. Ding et al. (2001) proposed a real-time headway control model which minimizes total headway variance at all stations. To describe the nonlinear characteristics of train control models, fuzzy expert system and genetic algorithm (GA) were applied to find the optimal regulation actions of the metro line in Wegele et al. (2004). Membership functions were developed to represent human expert experience for the regularity, headway, and congestion levels of a train platoon in metro system. The limitation of this method, however, is the computation inefficiency of getting the GA

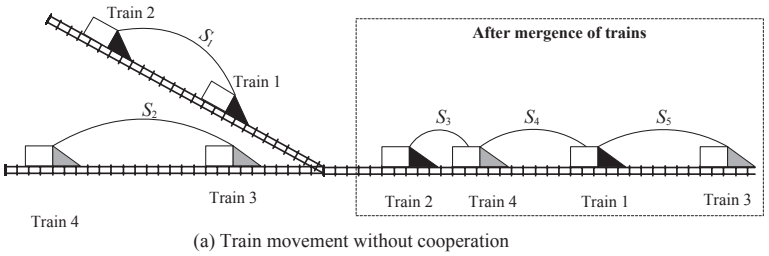
solutions for real-time operations. Lin et al. (2008) proposed dual heuristic dynamic programming (DHP) method to develop the adaptive critic automatic train regulation system. Moreover, to overcome the sensitivity of DHP with model errors, an adaptive optimal control (AOC) scheme was presented in Lin et al. (2009). Based on these previous studies, issues regarding energy saving were further considered and a DHP method for designing automatic train regulation with energy saving via coasting and station dwell time control was proposed in Lin et al. (2011). More recently, Sanchez-Martinez et al. (2016) considered the dynamic running time of trains and passenger demand in an urban transit line, and proposed a mathematical model for holding control optimization to regulate the train following headway. A set of simulation experiments verified the effectiveness of headway regulation for decreasing the passenger waiting time at transit stations.

For mainline railways, the dynamic headway adjustment is originally applied to change dispatching headway at railway terminals. In Zhao and Ioannou (2015), a dynamic headway regulation framework for positive train control system (PTC) was proposed. Based on its active communications between trains and ground equipment, this system could improve track capacity and safety in railway operation with the integration of a dynamic dispatching model. Recent studies have begun to address the headway adjustment on open tracks between two stations. For example, Emery et al. (2009) discussed the safe conditions for running with the shortest headway based on the emergency braking, which paves the way to reduce headways and increase capacity for high-speed railway lines. Wang and Goverde (2016, 2017) proposed an optimization approach that determines the trajectories of multiple trains under fixed-block signalling system for balancing tractive energy consumption and train delay time. Ye and Liu (2016, 2017) developed optimal train speed controls of multiple trains under both fixed-block and moving-block systems. In Takagi et al. (2012), a synchronization control of a train platoon was developed under moving block system. This synchronization control scheme could keep the successive trains following with a certain distance to ensure the minimum train following headway.

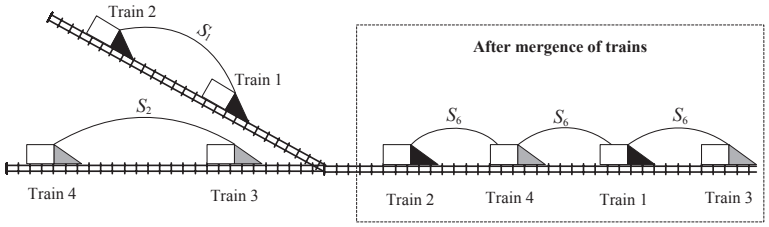
In practice, the train following headway varies due to some unexpected disturbances or the changing of line condition. An example of this is illustrated in Fig. 3(a) where irregular train following headway occurs when two rail lines merge into a single line, leading to the uneven distances S_3, S_4 and S_5 among these four trains. This kind of rail segment is called the critical block section (or bottleneck), which may significantly reduce line capacity (DB, 2008). Cooperative control approach has the potential to keep the consistency of trains in complex situations and improve the line capacity of bottlenecks(see Fig.3(b)), by keeping the trains operated in a regular distance S_6 with their former trains. Xun et al. (2015) proposed a cooperative control of multi-trains to solve headway adjustment problem. Dong et al. (2016) discussed the cooperative control synthesis and stability properties. However, such cooperative control systems are found to be difficult to implement in practice due to the constraints of train controller output limitation and train following safety headway. Furthermore, the practical train-to-ground communication resources are very limited that imposes more constraints on the control frequency of HSR trains. In this paper, we will formally address the above problems by firstly constructing a state-space model motivated by Cooperative Adaptive Cruise Control (CACC) form the field of automatous vehicles (Wang et al., 2014), and further developing a self-triggered control scheme to synchronize the running trains in a high-speed railway line. For balancing modelling complexity and practical usability, we first make the following assumptions throughout this paper:

Assumption 1. Our model simplifies the influencing factors of track gradient and track bends by using track speed limits to represent the affects of track gradients and track bends.

Assumption 2. The safe distance between each two adjacent trains is simplified as a set of linear functions associated with the positions and velocities of trains.



(a) Train movement without cooperation



(b) Train movement with cooperation

Figure 3: Headway adjustment for multiple train control problem. The boxed area after the merge shows the train sequence and headway.

3 State-Space Formulation for Train Cooperative Control

3.1 Symbols and notations

The relevant notations are listed below to describe and understand the problem more conveniently.

i	Train index, $i = 0, 1, 2, \cdots, I$
$s_i(t)$	Position of train i at time t;
$d_i(t)$	Distance headway between train i and train $i-1$;
$v_i(t)$	Running velocity of train i at time t;
$u_i(t)$	Controller output (i.e., acceleration and braking) of train i;
h^*	Desired train following headway time;
c_0, c_1, c_2	Resistance parameters;

Here, the first leading train is indexed by $i = 0$, and the following trains are respectively indexed by $i = 1, \cdots, I$. Essentially, the cooperative train control problem is to design a control scheme that treats these trains as a centralized platoon and determine the optimal controller output of each train $i \in \{1, \cdots, I\}$ in order to regulate the train following headway among the platoon and enhance the HSR line capacity.

Remark 3.1: In railway systems, the train control module and traction/braking motor are typically separate, where the train speed controller outputs a traction/braking acceleration u. Then, the traction/braking motor transfers the acceleration command to a traction/braking force F based the train mass. Since our study focuses the aspect of train control, our model implements u as the controller output in the following content, as shown in Figure 4.

Remark 3.2: In practice, the sequence of trains is determined by a high-level centralized train dispatching system. With the given sequence of trains, we can determine the leading train and the order of following trains for implementing the cooperative control approach. Especially, our modelling approach is highly suitable for train control systems with the emerging virtual coupling technology, where the vehicles are controlled by virtual couplers and cooperative control methods.

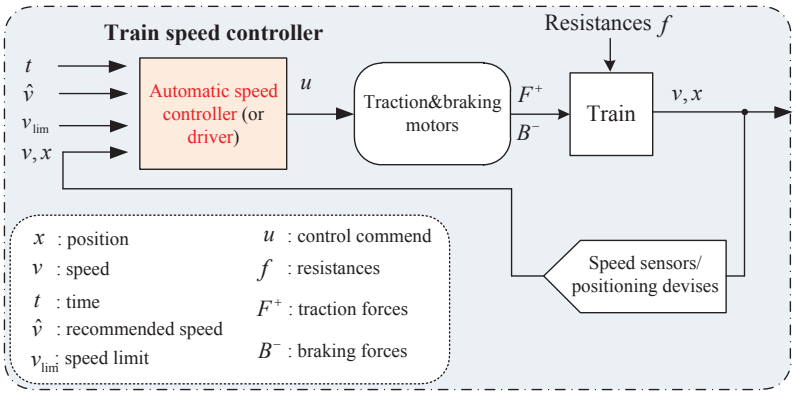


Figure 4: Illustration of railway train control model

3.2 State-space Formulation of Multiple trains

Following (Yin et al., 2017), the general dynamic state-space formulation of a single high-speed train can be represented by the following equations .

$$\begin{cases} \dot{s}_i(t) = v_i(t), \quad i = 0, 1, 2, \cdots, I \\ m_i \dot{v}_i(t) = m_i u_i(t) - (c_0 + c_1 v_i(t) + c_2 v_i^2(t)), \quad i = 1, 2, \cdots, I. \end{cases} \tag{1}$$

where m_i is the weight of train i , and $c_0 + c_1 v_i(t) + c_2 v_i^2(t)$ is the Davis equation that represents the resistances which typically involves track resistance, air resistance, etc. Here, we note that, for the cooperative train operations, the running speed profile of the first train (i.e., the first train $i = 0$) is pre-determined and is given as $v_0(t)$, and the desired train cruise speed is the equilibrium state when $v_0(t) = v_1(t) = \cdots = v_I(t)$. According to (1) and Taylor's expansion, the linearized dynamic equation around the equilibrium state is obtained by

$$\begin{cases} \dot{s}_i(t) = v_i(t), \quad i = 0, 1, 2, \cdots, I \\ \dot{v}_i(t) = u_i(t) - \frac{1}{m_i}(c_0 + c_1 v_i(t) + 2c_2(v_i(t) - v_0(t))v_0(t) + c_2 v_0^2(t)), \quad i = 1, 2, \cdots, I. \end{cases} \tag{2}$$

The state-space equation (2) can be rewritten as

$$\begin{cases} \dot{s}_i(t) = v_i(t), \quad i = 0, 1, 2, \cdots, I \\ \dot{v}_i(t) = b_i u_i(t) + a_i v_i(t) + p_i, \quad i = 1, 2, \cdots, I. \end{cases} \tag{3}$$

in which $a_i = -\frac{1}{m_i}(c_1 + 2c_2 v_0)$, $b_i = 1$ and $p_i = -\frac{1}{m_i}\left(c_0 - c_2 v_0^2\right)$ for each $i = 1, 2, \cdots, I$. For simplicity, we assume that, the trains are with identical mass m, and we use $a = -\frac{1}{m}(c_1 + 2c_2 v_0)$, $b = 1$ and $p = -\frac{1}{m}\left(c_0 - c_2 v_0^2\right)$ in terms of a_i, b_i and p_i here after.

In general, the class of longitudinal driving control problems can be described by the following general ordinary differential equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \quad \text{with} \quad \mathbf{x}(0) = \mathbf{x}_0, \tag{4}$$

in which $\mathbf{x}(t) \in \mathbb{R}^n$ and $\mathbf{u}(t) \in \mathbb{R}^m$ denote the vector of state and control input, respectively (Pontryagin et al., 1962; Wang et al., 2014). Since the aim of this study is to realize cooperative high-speed train operations in a train-to-train communication environment, we need to guarantee that each train i automatically keep constant headway with its former train $i - 1$. Similar the cooperative vehicle control problems in Wang et al. (2014) and Wang et al. (2014), we define $x_i(t) = (d_i(t) - v_i(t) * h^* - d_0, v_i(t) - v_{i-1}(t))$ for each $i = 1, 2, \cdots, I$, in which the

first term denotes the position deviation from the desired gap, and the second term represents the relative speed with the preceding train. h^* is the desired time headway between adjacent trains, $d_i(t)$ is the spacing of train i with its leading train $i-1$ at time t, and d_0 is the minimum safe distance. Hence, the system state for the cooperative train control problem is given as follows.

$$\begin{aligned} x(t) =&(x_1(t), x_2(t), \cdots, x_I(t)) \\ =&(d_1(t) - v_1(t) * h^* - d_0, v_1(t) - v_0(t), \\ &\cdots, d_I(t) - v_I(t) * h^* - d_0, v_I(t) - v_{I-1}(t))^T. \end{aligned} \tag{5}$$

For the operation HSR trains, the control input vector is consisting of the accelerating/braking of each train, i.e.,

$$u(t) = (u_1(t), u_2(t), \cdots, u_I(t)), \tag{6}$$

which is subject to constraints

$$u_i(t) \in [u_{min}, u_{max}], \quad \forall 1 \leq i \leq I. \tag{7}$$

Here, u_{min} and u_{max} represent the maximum train braking and maximum acceleration, respectively. From Eqs. (3)-(7), the deviation of system state $x(t)$ is replaced by

$$\begin{aligned} \dot{x}(t) =&(v_1(t) - v_0 - \dot{v}_1(t) \times h^*, \dot{v}_1(t) - \dot{v}_0, \cdots, v_I(t) - v_{I-1}(t) - \dot{v}_I(t) \times h^*, \dot{v}_I - \dot{v}_{I-1})^T \\ =&\big(v_1(t) - v_0 - (bu_1(t) + a_1 v_1(t) + p_1)h^*, bu_1(t), \cdots, \\ &v_I(t) - v_{I-1}(t) - (bu_I(t) + a_I v_I(t) + p_I)h^*, bu_I(t) - bu_{I-1}(t)\big)^T \\ =&(\hat{a}_1 v_1(t) - v_0 - u_1(t)bh^* - p_1 h^*, bu_1(t), \cdots, \\ &\hat{a}_I v_I(t) - v_{I-1}(t) - u_I(t)bh^* - p_I h^*, bu_I(t) - bu_{I-1}(t))^T \end{aligned} \tag{8}$$

where $\hat{a}_i = 1 - a \times h^*$ for each $i = 1, 2, \cdots, I$. Therefore, the state-space formulation for a cooperative train control system can be summarized as:

$$\dot{x}(t) = Ax(t) + Bu(t) + W(t), \tag{9}$$

where the coefficient matrices $A \in \mathbb{R}^{2I*2I}$, $B \in \mathbb{R}^{2I*I}$, and $W \in \mathbb{R}^{2I*1}$, which are respectively defined as:

$$A = \begin{bmatrix} 0 & \hat{a}_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \hat{a}_2 - 1 & 0 & \hat{a}_2 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots & & \\ 0 & \hat{a}_I - 1 & 0 & \hat{a}_I - 1 & \cdots & 0 & \hat{a}_I \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \tag{10}$$

$$B = \begin{bmatrix} -bh^* & 0 & \cdots & 0 & 0 \\ b & 0 & \cdots & 0 & 0 \\ 0 & -bh^* & \cdots & 0 & 0 \\ -b & b & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & -bh^* \\ 0 & 0 & \cdots & -b & b \end{bmatrix}, \tag{11}$$

$$W = \begin{bmatrix} (a_1 - 1)v_0 - p_1 h^* \\ 0 \\ (a_2 - 1)v_0 - p_2 h^* \\ \vdots \\ (a_{I-1} - 1)v_0 - p_{I-1} h^* \\ 0 \\ (a_I - 1)v_0 - p_I h^* \end{bmatrix}. \tag{12}$$

4 Self-Trigger Model Predictive Control Scheme

We can see that, the cooperative train control system in Eqs. (9)-(12) is essentially a multiple-input multiple-output (MIMO) linear time-invariant system. Note that, we also need to discretize to represent the continuous state-space system (9) in order to represent the control frequency of high-speed trains,. In this study, the minimum sampling time is set as $t_s = 1s$, and the matrices that describe the cooperative train control systems are discretized correspondingly, as:

$$A_s = e^{At_s} \tag{13}$$

$$B_s = \int_0^{t_s} e^{A\tau} d\tau \times B \tag{14}$$

$$W_s = t_s \times W. \tag{15}$$

For simplicity, we also use A, B and W to represent the discretized state-space matrices in the following description.

4.1 LQ optimal control problem

As we have shown that the cooperative train control problem can be formulated into a typical discrete-time state-space model with finite time-horizon, the standard optimal control problem is formulated as follows:

$$\min J_0(x, u) = \frac{1}{2} \sum_{k=k_0}^{k_N} \left[x_k^T Q_k x_k + u_k^T R_k u_k \right] \tag{16}$$

$$\text{s.t.,} \qquad x_{k+1} = Ax_k + Bu_k + W, \ k_0 \le k \le k_{N-1} \tag{17}$$

$$u_k \in [u_{min}, u_{max}], k_0 \le k \le k_N \tag{18}$$

$$q_k^i \times x_k \le d_{safe}^i, \forall 1 \le i \le I, k_0 \le k \le k_N \tag{19}$$

$$r_k^i \times x_k \le v_{safe}^i, \forall 1 \le i \le I, k_0 \le k \le k_N \tag{20}$$

in which $[k_0, k_N]$ is the control time horizon defined by set $\mathbb{N}$ (i.e., a set of discrete time units from k_0, $k_0 + t_s$, $\cdots$, k_N), constraint (18) is the output limitation constraint that determines the lower and upper bound of train controller, constraint (19) guarantees the train following distance, and constraint (20) ensures that the speed for each train is under the speed limit. For simplify, let $\mathbb{X}_f$ and $\mathbb{U}_f$ denote the feasible region of state x and control variable u, respectively. Here, we note that, both q_k^i and r_k^i for each $1 \le i \le I$, $k_0 \le k \le k_N$ are $2I \times 1$ vectors.

It can be seen that the above problem is a linearly constrained LQ control problem that can be solved by a Lagrangian-relaxation method combined with dynamic programming (see Lim et al. (1996)). Using this method, the linear constraints are first relaxed into the objective function by introducing a set of Lagrangian multipliers $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_{m_c})$, in which $m_c = 4 \times I \times N$ represents the number of linear constraints in this optimal control problem. The constrained LQ control problem is transformed into an unconstrained LQ problem

$$\min J(\lambda, x, u) = J_0(x, u) + \sum_{1}^{m_c} \lambda_i l_i(x, u), \tag{21}$$

in which

$$l_i(x, u) = \sum_{k=k_0}^{k_N} ((a_k^i)^T x_k + (b_k^i)^T u_{k-1}) \tag{22}$$

for each $1 \leq i \leq m_c$. It is clear that the reformulated (21) aims to find a control process that minimizes the cost function and satisfies $l_i(x, u) \leq q_i$ for each $1 \leq i \leq m_c$.

After introducing the Lagrangian multipliers λ, the original problem (9)-(12) is transformed into an unconstrained LQ problem. Given a set of Lagrangian multipliers $\lambda \geq 0$, the optimal controller output can be calculated by

$$u(\lambda) = \underset{u}{\operatorname{argmin}} J(\lambda, x, u).$$

As is indicated in Lim et al. (1996), this unconstrained LQ optimal control problem can be solved as follows.

$$u_k = -(R_{k+1} + B^T S_{k+1} B)^{-1} \times [B^T S_{k+1} A x_k + B^T h_{k+1}(\lambda^*) + b_{k+1}(\lambda^*)] \tag{23}$$

where matrix S is the solution of the Riccati equation that is calculated backwardly by

$$\begin{aligned} S_k =& Q_k + A^T S_{k+1} A - (B^T S_{k+1} A)^T [R_k + B^T S(k+1) B]^{-1} B^T S_{k+1} A, \\ S_I =& Q_I. \end{aligned} \tag{24}$$

In addition, h_k is calculated by

$$\begin{aligned} h_k(\lambda) =& a_k(\lambda) + A^T h_{k+1}(\lambda) - A^T S_{k+1} B (R_{k+1} + B^T S_{k+1} B)^{-1} \times (B^T h_{k+1}(\lambda) + b_k(\lambda)) \\ h_I(\lambda) =& a_I(\lambda), \end{aligned} \tag{25}$$

in which the constraint matrix are calculated from Eq. (22) as follows

$$a_k(\lambda) = \sum_{i=1}^{m_c} \lambda_i a_k^i, \quad \forall k \in [k_0, k_N]$$

$$b_k(\lambda) = \sum_{i=1}^{m_c} \lambda_i b_k^i, \quad \forall k \in [k_0, k_N].$$

Since the LQ control problem with Lagrangian multipliers guarantees the Kuhn-Tuck (KKT) condition (i.e., $u^* = u(\lambda^*)$, see Lim et al. (1996)), the optimal Lagrangian multipliers λ^* is given by

$$\lambda^* = \operatorname{argmax}(x_k^T S_{k_0} x_k + 2 h_{k_0}^T(\lambda) x_k + p_k(\lambda) - \lambda^T q). \tag{26}$$

Here, we note that, the optimal λ^* yields the optimal control law u^* for the cooperative train control problem. Since the probelm (26) is essentially a quadratic programming problem that can be solved efficiently by many optimization methods (Lim et al., 1996).

Remark 4.1 Here we note that several improvements are implemented in the developed LQ optimal control problem

for cooperative train control compared with Dong et al. (2016). First, the speed limits from signalling systems are embedded into the constraints of mathematical model, which are then transformed into the objective function with Lagrangian multipliers. In particular, the setting of speed limits can embed the information of track gradients and track bends, for example restrict to lower speed limitation on up-gradient segments and also at bends. Second, the moving-blocking system that separates the moving trains is handled by the constant time headway h^* and the minimum train following distance d^i_{safe} in the mathematical model. Specifically, by setting vectors $q^1_k = \{-1, 0, \cdots\}$, $q^2_k = \{0, 0, -1, 0 \cdots\}$, etc, the safe following distance under a relative-speed moving block system can be properly handled. In addition, our model can also be extended into relative-distance moving block systems by setting another set of q^i_k, and fixed (or quasi-moving) block systems by introducing a new sets of quadratic constraints.

4.2 Self-triggered MPC for efficient use of communication resources

With the development of advanced communication technologies in railway systems, many newly constructed HSR systems have been equipped with communication-based train control (CBTC) systems. Typically, a CBTC system involves Global System for Mobile communications - Railway (GSM-R) and Wireless Local Area Networks (WLANs) that provide the bidirectional train-ground communication for the safe and efficient operations of railway trains (Zhu et al., 2012). A practical issue of CBTC system is the limited wireless communication resources, especially for a complex environment like railway transportation networks.In addition, the end-to-end communication delay for transmitting an information is about 1.2 s with 99% confidence and at most 2.4 s with 99.9% confidence. Due to the limited bandwidth in wireless communications, it is actually impractical to receive the feedback information of trains and change the controller output within each second. To compensate on the above practical communication delay problem, we present an improved control method: the transformation of the LQ controller into a self-triggered model predictive control (SMPC) scheme to handle the communication delay issue (Dai et al., 2018). More specifically, SMPC simultaneously determines the actuator value as well as the next update time in advance based on the information at the current sampling instance (Mazo et al., 2009). For cooperative train control problem, SMPC has the following two main advantages. First, this kind of control scheme does not require wireless communications at all times resulting in a lower required sampling frequency. Second, the control scheme has the potential to improve the riding comfort of passengers as it does not change the controller output as frequently, making it similar to the manual driving by experienced drivers.

In the self-triggered scheme, the states x_k are only measured and transmitted to the controller at sample instants $\{t_l | l \in \mathbb{N}\} \subseteq \mathbb{N}$, which satisfy $t_{l+1} > t_l$, and

$$u_t = \bar{u}_l, t \in \mathbb{N}_{[t_l, t_{l+1})} \tag{27}$$

for all $l \in \mathbb{N}$. In each update time t_l, the aim of self-triggered control is to choose both the next control value $\bar{u}_l$ and the next update time t_{l+1} such that t_{l+1} is as large as possible while the system can still achieve the sub-optimality. In particular, the same control value $\bar{u}_l$ is used at times t_l, $t_l + 1$, $\cdots$, $t_{l+1} - 1$, and no communication nor computation are required on times $t_l + 1$, $\cdots$, $t_{l+1} - 1$. In order to guarantee the sub-optimality of self-triggered control, the following inequalities are appended

$$\sum_{t=t_l}^{t_{l+1}-1} (x_t^T Q_t x_t + \bar{u}_l^T R_t \bar{u}_l) \leq \beta \left[J(x_{t_l}) - J(x_{t_{l+1}}) \right] \tag{28}$$

where $\beta \geq 1$ is denoted to balance sub-optimality and the obtained saving in communication resources. In particular, when $\beta = 1$, the self-triggered control is degraded into a standard LQ optimal control problem defined in (16). Let N denote the prediction horizon in the MPC scheme, the SMPC for our problem is defined as follows.

Problem 4.1: At sample time t_l and state $x_{t_l} = x$, find the maximum M^*, such that

$$\min_{u \in \bar{\mathcal{U}}_M} \left(\sum_{k=0}^{M-1} (x_k^T(x,\mathbf{u}) Q x_k(x,\mathbf{u}) + u_k^T R u_k) + \sum_{k=M}^{M+N} \beta(x_k^T(x,\mathbf{u}) Q x_k(x,\mathbf{u}) + u_k^T R u_k) \right) \leq \beta J(x) \tag{29}$$

where

$$\bar{\mathcal{U}}_M(x) \triangleq \left\{ \mathbf{u} \in \mathbb{U}^M | \exists \bar{u} \in \mathbb{U}_f, u_i = \bar{u}, \forall i = 0, \cdots, M, \text{and } (x_i(x,\mathbf{u}) \in \mathbb{X}_f) \right\}. \tag{30}$$

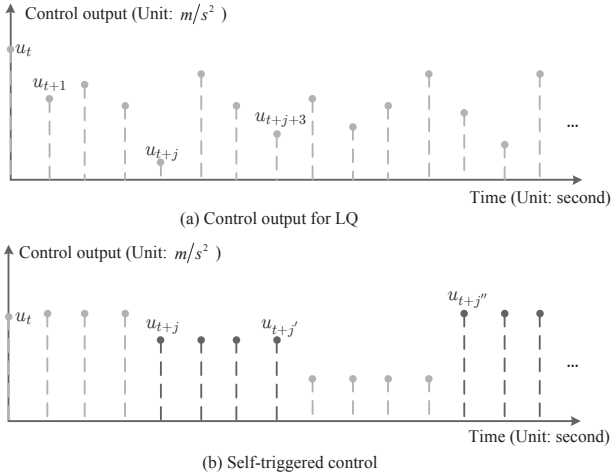


Figure 5: Control output illustration by SMPC

In the self-triggered setup, the goal at each sampling instant t_l is to decide not only $\bar{u}$ but also the next sampling instant t_{l+1}. To efficiently reduce the computation and communication, we need to find the largest M^* such that the sub-optimality is still satisfied. Define the SMPC problem on state x_{t_l} as

$$M_l^* = \max\{M \in \mathbb{N}_{[1,M_{max}]} | \bar{\mathcal{U}}_M \neq \varnothing\}. \tag{31}$$

Example 4.1 We illustrate the working of this in Fig.5. Fig. 4 shows example controller outputs under LQ control and using self-triggered control, where the x-axis refers to the time units in the considered time horizon and y-axis refers to the controller outputs on each time unit. With LQ control (Fig.4a), the controller outputs (in this study, the train' acceleration) is updated at every time interval (e.g. $t_s = 1$ sec), which could result in large variations in train accelerations and discomfort of passengers. With self-triggered control (Fig. 4b), however, the update of controller outputs is only made at required time units, and the controller outputs keep constant at other times. Moreover, SMPC still guarantees a good control performance.

Remark 4.2 For the standard model predict control of linear system with constraints, if the predict horizon is assumed to be infinity enough, i.e., $N = \infty$, the asymptotical stability can be guaranteed. Otherwise, the terminal state at $k+N$ should be selected to satisfy the terminal state constraint keep the stability of the closed-loop control (see proofs in Mayne et al. (2001, 2000); Cortes et al. (2012)). In that case, it guarantees that $J(x) \leq \alpha \|x\|$.

Theorem 4.1 *Recursive Feasibility and Constraint Satisfaction.* If state x_{t_0} is feasible at time t_0, then the feasibility of x_{t_l} can be ensured at every sampling instant t_l, $l \in \mathbb{N}$ by SMPC.

Proof. Consider two successive sampling instant t_l and t_{l+1}. Assume that x_{t_l} is a state with feasible solution M_l

and $\bar{u}_l$ at time t_l. It is obvious that $M_{l+1} = 1$ is a feasible solution of $x_{t_{l+1}}$ at time t_{l+1} since $M_{l+1} = 1$ actually transforms the self-trigger MPC into a standard LQ optimal control problem and the constraints (17) to (20) are trivially satisfied. Further by induction, x_{t_l} are feasible at all sampling instants t_l for $l \in \mathbb{N}$.

Theorem 4.2 *Quadratic Stability.* The SMPC scheme for the consider closed-loop linear system is asymptotically stable in $\mathbb{X}_f$.

Proof. As our considered problem is a discrete-time linear system, we can proof the theorem similar to Berglind et al. (2012). According to (28) for $l \in \mathbb{N}$, we have the following inequalities

$$\sum_{t=t_0}^{t_1-1} \left(\|x_t\|_Q^2 + \|u_t\|_R^2\right) \leq \beta[J(x_{t_0}) - J(x_{t_1})] \tag{32}$$

$$\sum_{t=t_1}^{t_2-1} \left(\|x_t\|_Q^2 + \|u_t\|_R^2\right) \leq \beta[J(x_{t_1}) - J(x_{t_2})] \tag{33}$$

$$\cdots$$

By summing above these above inequalities for $l = 0, 1, \cdots, +\infty$, we obtain the following inequality

$$\sum_{t=t_0}^{\infty} \left(\|x_t\|_Q^2 + \|u_t\|_R^2\right) \leq \beta[J(x_{t_0})]. \tag{34}$$

In addition, there exists some $\gamma_1 > 0$ that satisfies

$$\sum_{t=t_0}^{\infty} \left(\|x_t\|_Q^2 + \|u_t\|_R^2\right) >= \gamma_1 \sum_{t=0}^{\infty} \|x_t\|$$

and thus, we have $\lim_{t\to\infty} x_t = 0$. As our problem has limited the lower and upper bound of feasible region of the state and control variables $\mathbb{X}_f$ and $\mathbb{U}_f$ (e.g. as given in inequalities (18) - (20)), we can easily derive the upper bound of $J(x)$. Let $J(x)$ be the Lyapunov function and the asymptotical stability can be proved.

Remark 4.3 By employing the self-trigger control method in Eqs. (29) and (30), parameter β can be used to realise trade-off between the control performance and the frequency of control updates. In particular, if we choose $\beta = 1$, the optimal value function of SMPC can be no more than the value by MPC without self-trigger scheme. In such cases, the time sampling variable M^* would become very small. In contrast, M^* can be a larger one while the control performance might be decreased.

Based on the above descriptions, we summary the implementation procedure of SMPC for cooperative train control problem. In the self-triggered algorithm, the goal at each sampling instance t_l is to determine not only vector $\bar{u}_l$ but also the next sampling instance t_{l+1}. Moreover, the time sampling interval $t_{l+1} - t_l$ is the maximum value that guarantees the system performance by Eq. (28). Nevertheless, the minimization problem in (30) is not expressed in a convenient form that is typically hard to be solved directly. Therefore, in the algorithm implementing procedure, we initially set $M = 2$ at each sampling instance t_l and solve the resulted quadratic programming (QP) in (30). Then, we incrementally increases M until feasibility of (28) ceases to hold, and output $M-1$ as the optimal sampling interval that guarantees the system performance. The implementing procedure of self-triggered MPC is summarized in Algorithm 1.

5 Numerical Experiments

In order to verify the effectiveness of the proposed approach, we present two sets of numerical experiments in this section based on the operation data from Bejing-Shanghai high-speed railway. We consider the frequently

Algorithm 1 SMPC for the cooperative train operation

Step 1. Input the model parameters and coefficient matrices A, B and W. Initialize $k = k_0$ and $t_0 = k$.

Step 2. Measure the real-time train state x_k, including the positions and velocities of considered trains.

Step 3. Do for $M = 2, \cdots, M_{max}$;

Step 3.1 Solve the QP problem in (29);

Step 3.2 If an optimal solution is obtained, let $\bar{u}$ and $\bar{M}$ be the optimal solution;

Step 3.2 Otherwise, if no feasible solution is returned, set

$$M^* \leftarrow M - 1$$

$$k \leftarrow k + M^*$$

$$u^* \leftarrow \bar{u}$$

and break;

Step 4. Output u^* and M^* as the controller output and decision holding time interval.

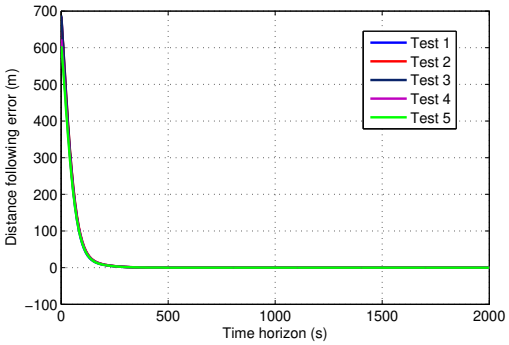
Step 5. If $k < k_N$, go to Step 3; otherwise terminate the algorithm.

used EMU along the route, the CRH3 whose Davis parameters are listed as follows: $c_0 = 0.7550$, $c_1 = 0.00636$, $c_1 = 0.000115$ (unit: N/kN) with respect to the train velocity unit Km/h. Each CRH3 consists of 16 vehicles, and the total weight is set as 980t. The maximum accelerating and braking are set as $1.0m/s^2$ and $-1.0m/s^2$, respectively. In the following experiments, we first conduct a set of experiments for the stability analysis of headway adjustment for the LQ controller. Then, we conduct more experiments to verify the effectiveness of self-triggered control scheme for the cooperation train operations in HSRs.

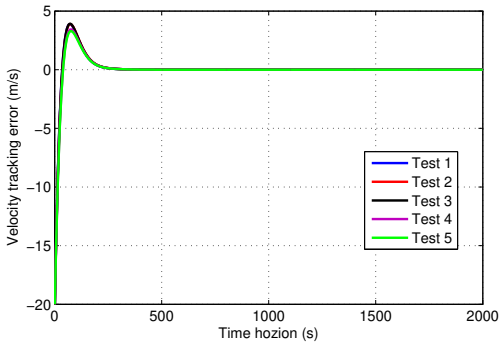
5.1 Experiment 1: Performance of CACC

In this experiment, we first consider a total number of 3 trains, where the first and leading train (i.e., train 1) is cruising at a speed $100m/s$. The cooperative control methods are applied the following two trains. The initial positions of trains 1,2 and 3 are randomly generated from a range $[3950, 4050]$, $[6950, 7150]$, $[9950, 10000]$ (unit: meter) respectively using a Monte Carlo simulation method. Since the length of a high-speed train is about 200m, parameter d_0 is set as 500m in our experiments, and the minimum train following distance d^i_{safe} is set as 1000m. The desired train following time headway h^* is defined as 30 (unit: *second*). The initial speeds of train 2 and train 3 are set as $80m/s$ and $120m/s$, respectively. It is clear that, these three trains are not operated in a stable state with this intial condition since the second train (i.e., train 2) is much slower and the third train (i.e., train 3) is faster with respect to the leading train (i.e., train 1).

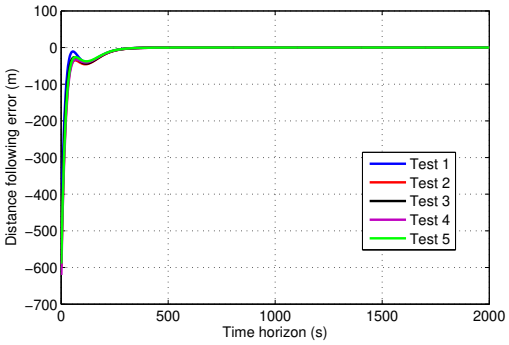
The LQ train control based on the state-space formulation of Section 3 is applied to the two following trains here. Fig. 6 shows the resulted relative distance following errors and velocity following errors between train 1, 2 and train 2, 3 for five test instances. First, we can see from Fig. 6a and Fig. 6b that, for all these five tests, the distance following error and velocity following error between train 1 and train 2 gradually converge to nearly zero. This indicates that, by using the headway adjustment, train 2 increases its speed, and these two trains are in stable state after about 400 seconds, which is very efficient and effective. Meanwhile. for train 2 and train 3, we can see from Fig. 6c and Fig. 6d that, the distance following error and velocity following error converge to the values that are nearly to be zeros. The velocity tracking error between train 2 and train 3 in Fig. 6d is a little larger than other performance indicators in Fig. 6a to Fig. 6c, which is possibly due to the limited predictive time horizon as well as the settings of matrices Q and R, although this does not affect the overall control performance. The results



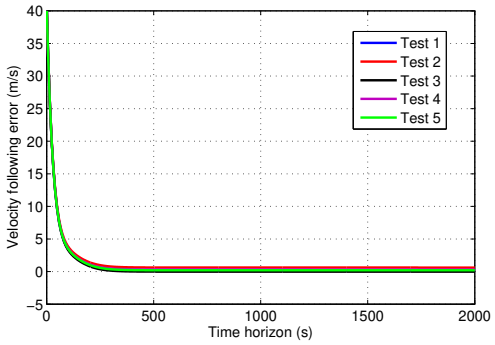
(a) Distance tracking error between train 1 and train 2



(b) Velocity tracking error between train 1 and train 2



(c) Distance tracking error between train 2 and train 3



(d) Velocity tracking error between train 2 and train 3

Figure 6: Distance and velocity tracking errors in the time-horizon

by this group of experiments suggest that, the headway adjustment method by train cooperative control can keep the high-speed trains into a stable state with fixed train following distance and coordinated velocities.

Then, we particularly consider a practical scenario to test the effectiveness of headway adjustment to improve the line and rail traffic capacity. In this case study, there are totally 14 trains running in a bottleneck segment with the speed of $100m/s$ in Beijing-Shanghai HSR. The headway time between adjacent trains is $120s$ and the train following distance is $1.2 \times 10^4 m$. Consider that the railway dispatcher wants to increase the traffic capacity at this bottleneck such that more trains can pass the section in a given time period due to some temporal requirements. Using the proposed headway adjustment method, we shall decrease the speed of the first train, and then compress the headway time among the trains in order to increase the line capacity.

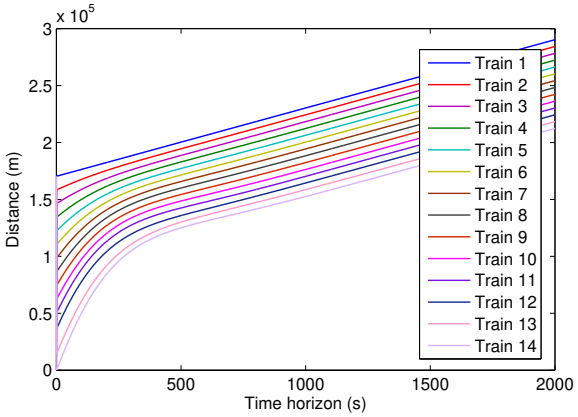


Figure 7: Illustration of time-space diagram of multiple trains

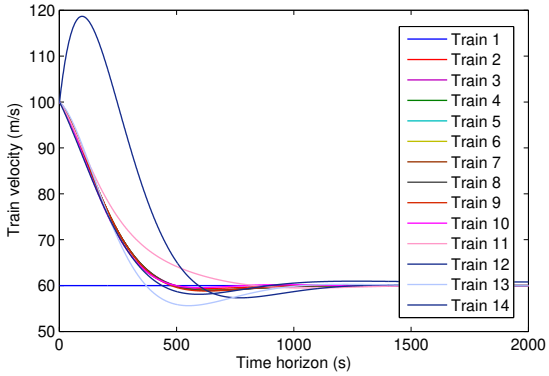


Figure 8: Time-velocity curves with train cooperative control

Fig. 7 illustrates the space-time diagram of these 14 trains after using the headway adjustment method. We can see that, with proper headway adjustment, all the trains follow the former train much closer with the identical speed, which illustrates the effectiveness of our proposed approach. In addition, Fig. 8 and Fig. 9 demonstrates the running speeds and accelerating & braking rates of these trains. We can see that, the speeds of these trains are decreased gradually to the convergence value, i.e., the speed of the first leading train. Moreover, the maximum accelerating rate is about $0.5m/s^2$ and the minimum braking rate is about $-0.3m/s^2$, which are proper values that guarantee the riding comfort and satisfy the output constraints.

In Table 1, the train following distance, train velocity, headway time, traffic flow volume and line capacity are recorded through the simulation experiments. Here, the traffic flow volume is essentially the number of trains that

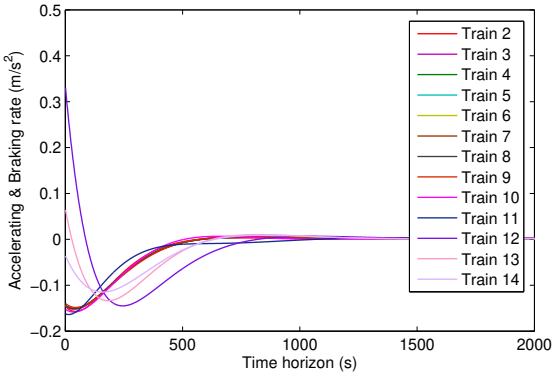


Figure 9: Curves of accelerating and braking rates

go through this segment in each 10 min. The line capacity is indicated by the number of trains in each 20 km segment. We can see that the traffic volume and line capacity are improved by 20% and 100% through headway time adjustment.

Table 1: Performance comparison with headway adjustment

Performance comparison	Initial status	Stable status
Train following distance (m)	12000	6000.1
Train velocity (m/s)	100.0	60.0
Headway time (s)	120	100
Traffic flow volume (tph)	5	6
Line capacity	1.67	3.33

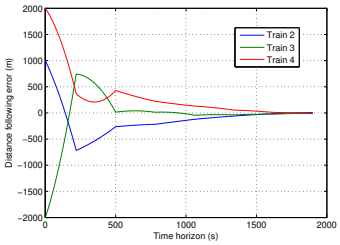
5.2 Experiment 2: Performance of LQ and SMPC controllers

In Experiment 1, we use LQ controller to verify the effectiveness of cooperative train operation to regulate the train following headway and increase the line capacity. We note that the LQ controller requires that the trains are entirely connected where the communication data among these trains are transformed during each time interval (i.e., 1 second). As can be seen from Fig. 9, the accelerating and braking rates of trains are continuous curves in the considered time horizon, which is not good for passengers' riding comfort. Thus, in this set of experiments, we conduct numerical experiments to test the performance of SMPC compared to the standard LQ controller. The parameter settings of trains, including the resistance coefficients, speed limit, controller output bounds, are the same as those in Experiment 1. Two cases with different train numbers are considered in this experiments. And the initial states (i.e., number of considered trains in the platoon, the position and velocity of each train) for the train platoon in these experiments are presented in Table 2.

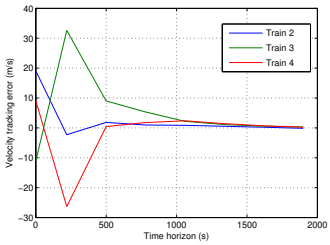
Figure 10 and 11 presents the computational results for Instance 1 and Instance 2, respectively. Specifically, the results for SMPC with four trains are shown in Figure 10a, Figure 10b and Figure 10c, which present the distance following error and velocity tracking with respect to the leading Train 1, and the controller output of each following train. For comparison, the results for LQ controller are presented in Figure 10d to Figure 10f. From these experiments, we can see that the train following states are gradually converged to the equilibrium state by both LQ and SMPC, as the distance tracking error and velocity tracking error are finally reduced to nearly 0 at the end of

Table 2: Initial states of each experiment

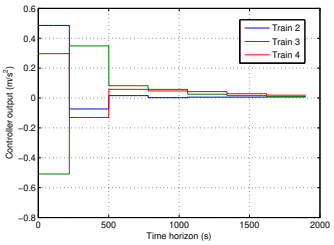
Instance	Number of trains	Initial positions states (unit: km)	Initial speed states (unit: m/s)
1	4	4, 13, 22, 36	100, 120, 110, 120
2	6	6, 13, 22, 36, 51, 67	100, 120, 110, 120, 130, 140



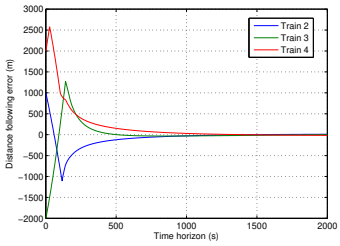
(a) Distance tracking error



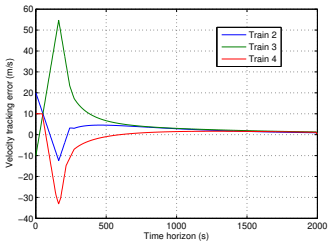
(b) Velocity tracking error



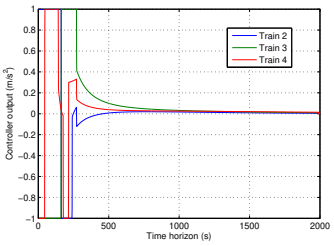
(c) Controller output



(d) Distance tracking error

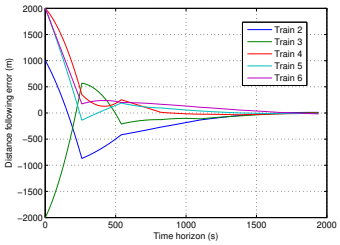


(e) Velocity tracking error

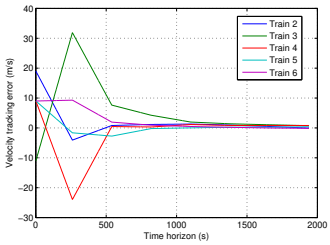


(f) Controller output

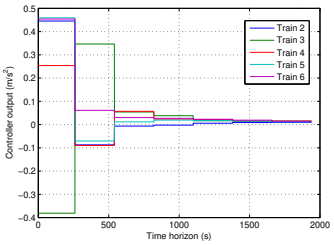
Figure 10: Comparison between the performance of SMPC (a-c) and LQ (d-f) for Instance 1.



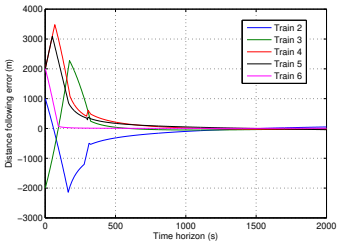
(a) Distance tracking error



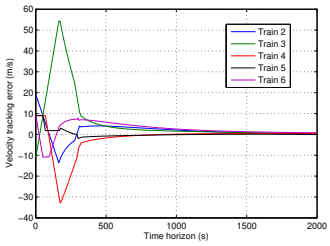
(b) Velocity tracking error



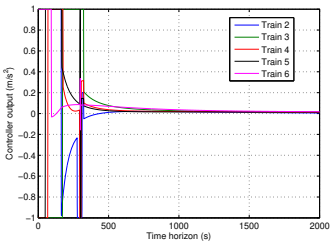
(c) Controller output



(d) Distance tracking error



(e) Velocity tracking error



(f) Controller output

Figure 11: Comparison between the performance of SMPC (a-c) and LQ (d-f) for Instance 2.

time horizon. The maximum accelerating and braking rates of trains by LQ and SMPC are strictly under (or equal to) $1.0m/s^2$. These observations indicate that these two control methods can realize the cooperative operation of high-speed trains for headway regulation. Meanwhile, the distance and velocity tracking errors by LQ controller are nearly equal to zero after about 1000 seconds, while it takes longer time but acceptable by SMPC. This indicates that the convergence rate of SMPC is not as good as LQ controller. Even though, we can see from Figure 10c and Figure 10f that, the controller output is very steady by SMPC, which only changes the controller output a few times, and the maximum accelerating rate is less than $0.6m/s^2$. This indicates that, the cooperative train operation with SMPC can be implemented in practice by using very few communication resources. For LQ controller, however, the accelerating and braking rates vary very frequently, and the maximum accelerating and braking rates reach $1.0m/s^2$ before time 400. These observations are consistent with our expectations by employing self-trigger control for the cooperative train operation problem to realize a trade-off between control performance and communication resources.

We also test these two control schemes by considering more trains in the platoon. The obtained results are presented in Figure 11a to Figure 11f. We can see from these figures that similar results are returned that verify the effectiveness of the proposed approaches.

6 Conclusion

In this paper, we developed a multi-train cooperative control method for dynamic headway adjustment in high-speed railways. We formulated the multi-train cooperative control problem as a time-invariant MIMO state-space system with a set of practical constraints, e.g., train controller output constraints, safe train following distance constraints. To solve the problem efficiently, a rolling horizon approach by calculating the Riccati equation with Lagrangian multipliers was developed. Due to the practical communication resource constraints and riding comfort requirement, we also improved the rolling horizon approach into a novel self-triggered model predictive control scheme to overcome these issues. Finally, two case studies based on the real-world data in Beijing-Shanghai HSR are given through simulation experiments that verify the effectiveness of self-triggered control scheme.

As more advanced communication and computer devices being deployed in railway systems, how to effectively use these new technologies to improve the performance of train control systems is an emerging issue from both theoretical and engineering aspects. Along this line, our further studies will address the following aspects. First although the practical influencing factors, e.g., track resistance and speed limits are considered in this paper, the real-world train dynamics in high-speed railway systems can be even more complex, for example the varying track gradients, track bends, and uncertainty of model parameters due to extreme weather and communication losses. Thus, further analysis of these nonlinear and random features of train control models, and consequently extending our work to a general framework with more accurate safe distance calculations can be one of future research directions. Second as the computational results have shown that, an appropriate scale of self-triggered time-interval can achieve better trade-off between train platoon control quality and communication resources, more analyzes on the performance indexes can be taken into consideration in the future researches. Third in this study, we have only considered the cooperative train control problem around a single bottleneck under the assumption that the train sequence is pre-determined. Our further research will also focus on the coordinated train control problem for a high-speed railway network, in which some other important indicators, such as the network capacity and resilience, can be also considered to realize a network-optimization-based operation strategy.

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