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Propagation of leaky surface waves on contact magnetohydrodynamic discontinuities in incompressible plasmas

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We study the wave propagation on a magnetohydrodynamic contact discontinuity. Using the Laplace transform, we obtain the solution to the initial value problem describing the evolution of a perturbation of the discontinuity. We use this solution to study the leaky modes that determine the asymptotic behaviour of the solution for large time. We find the approximate expressions describing the leaky modes for a small inclination angle of the magnetic field. We also discuss the transition to the tangential discontinuity as the inclination angle tends to zero. We show that there is no continuous transition from the leaky modes on a contact discontinuity to the surface modes on a tangential discontinuity. However, such a transition exists if we take the average quantities describing the leaky modes. *Published by AIP Publishing.* https://doi.org/10.1063/1.5050591

I. INTRODUCTION

Observations on board space satellites as early as the Skylab mission clearly showed that the solar atmosphere is highly inhomogeneous and dynamic. The results of these observations were presented in numerous reviews.[1–9] In particular, the magnetic field in the photosphere and lower part of chromosphere is concentrated in magnetic flux tubes where it has the magnitude up to a few kilogauss. Typical examples of such magnetic flux concentrations are sun spots.

In the higher parts of the solar atmosphere (the upper part of the chromosphere and the corona), the magnetic pressure strongly dominates the plasma pressure. This prevents the strong concentration of the magnetic flux. However, the plasma density in the upper part of the solar atmosphere is highly inhomogeneous, which results in the existence of narrow layers with fast variation of the Alfvén speed. Plasmas with highly variable Alfvén speed are called magnetically structured.[10]

The discovery of magnetic structuring of the solar atmosphere boosted the interest of theorists in studying waves in magnetically structured plasmas. It was further enhanced by the discovery of the ubiquitous presence of waves and oscillations in the solar atmosphere made on board of space missions over the last two decades.[11–24] A new branch of solar physics called solar atmospheric seismology started to emerge fast.[25–35]

The simplest magnetic structure is a single magnetic interface, which is a particular case of tangential magnetohydrodynamic (MHD) discontinuity with the zero plasma velocity at both sides. Recall that in tangential MHD discontinuity, the magnetic field is parallel to the surface of the discontinuity at both sides. The wave propagation on a magnetic interface was extensively studied in both the linear[10,36–38] and nonlinear[39–44] regimes.

However, in the solar atmosphere, there are discontinuities with the magnetic field not parallel to their surfaces. These are contact MHD discontinuities with the properties very much different from those of tangential MHD discontinuities. The only two quantities that are allowed to have jumps at a contact MHD discontinuity are the density and temperature. Contact MHD discontinuities can be considered as simplified models of, for example, sunspot penumbra, solar prominences, and transition region.

The wave propagation at contact discontinuities was studied by Malara *et al.*[45] However, these authors considered the interaction of waves incoming to a contact discontinuity from infinity and calculated the coefficients of transmission and reflection. Recently, the MHD wave propagation along a contact discontinuity has been studied by Vickers *et al.*[46] using an eigenmode technique. It was found that because of the inclination of the magnetic field, only leaky waves are supported by the interface, so surface waves are attenuated without any damping mechanism present. In this paper, we study the temporal development of propagating surface waves on MHD contact discontinuities, after an initial perturbation of the interface.

This paper is organised as follows. In Sec. II, we formulate the problem and present the governing equations and boundary conditions. In Sec. III, we use the Laplace transform to obtain the solution to the initial value problem describing the evolution of the discontinuity perturbation. In Sec. IV, we calculate the leaky modes describing the asymptotic behaviour of the solution to the initial value problem at large time. In Sec. V, we consider the case of small inclination angle and study the transition to the tangential discontinuity as the inclination angle tends to zero. The time evolution of initial perturbations is investigated in Sec. VI. Section VII contains the summary of the obtained results and our conclusions.

II. PROBLEM FORMULATION AND GOVERNING EQUATIONS

We consider surface waves on a contact magnetohydrodynamic discontinuity in an incompressible ideal plasma. In

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the equilibrium state, the plasma is at rest and the magnetic field is unidirectional and has constant magnitude. In Cartesian coordinates x, y, z, the magnetic field is defined by $\mathbf{B} = B(\cos\theta, 0, \sin\theta)$. Without loss of generality, we can assume that $\theta > 0$. The density is piece-wise constant and equal to ρ_1 in $z < 0$ and ρ_2 in $z > 0$. The plasma motion is described by the system of linearised ideal MHD equations

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\mathbf{b}, \tag{2}$$

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{B}\cdot\nabla)\mathbf{v}, \quad \nabla\cdot\mathbf{b} = 0, \tag{3}$$

where $\mathbf{v}$ is the velocity, $\mathbf{b}$ is the magnetic field perturbation, μ_0 is the magnetic permeability of free space, $P = p + \mathbf{B}\cdot\mathbf{b}/\mu_0$ is the total pressure perturbation, and p is the perturbation of plasma pressure. Taking the divergence of Eq. (2), we obtain

$$\nabla^2 P = 0. \tag{4}$$

The variables $\mathbf{v}$, $\mathbf{b}$, and P must be continuous at $z = 0$. Below we consider a planar problem and assume that the perturbations are independent of y, and the y-components of the velocity and magnetic field perturbation are zero, so $\mathbf{v} = (u, 0, w)$ and $\mathbf{b} = (b_x, 0, b_z)$.

It is worth noting that the approximation of incompressible plasma is definitely not applicable to waves in the chromosphere and corona where the plasma-beta is either moderate or small. It only can be applied, with some reservations, to waves in the solar photosphere. However, the aim of this article is not to obtain results directly applicable to solar physics, but rather to study the main properties of waves propagating on a contact MHD discontinuity, and also to clarify similarities and differences in properties of waves propagating on contact and tangential MHD discontinuities.

III. SOLUTION TO THE INITIAL VALUE PROBLEM

Since the domain where we consider the wave propagation is unbounded in the x-direction, and the equilibrium quantities are independent of x, we can take the perturbations of all quantities proportional to $\exp(ikx)$, where k is real and positive. Then, the system of Eqs. (1)–(4) reduces to

$$\rho\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial z} + \frac{B}{\mu_0}\left(\frac{\partial b_z}{\partial z}\sin\theta + ikb_z\cos\theta\right), \tag{5}$$

$$\frac{\partial w}{\partial z} + iku = 0, \tag{6}$$

$$\frac{\partial b_z}{\partial t} = B\left(\frac{\partial w}{\partial z}\sin\theta + ikw\cos\theta\right), \tag{7}$$

$$\frac{\partial b_z}{\partial z} + ikb_x = 0, \tag{8}$$

$$\frac{\partial^2 P}{\partial z^2} - k^2 P = 0. \tag{9}$$

We introduce the Laplace transform with respect to time

$$\hat{f}(\omega) = \int_0^\infty f(t)e^{i\omega t}dt. \tag{10}$$

It is defined in the upper half of the complex ω-plane. Applying the Laplace transform to Eqs. (5)–(9) yields

$$i\omega\hat{w} = \frac{1}{\rho}\frac{\partial\hat{P}}{\partial z} - \frac{V_A^2}{B}\left(\frac{\partial\hat{b}_z}{\partial z}\sin\theta + ik\hat{b}_z\cos\theta\right) - w_0, \tag{11}$$

$$\frac{\partial\hat{w}}{\partial z} + ik\hat{u} = 0, \tag{12}$$

$$i\omega\hat{b}_z = -B\left(\frac{\partial\hat{w}}{\partial z}\sin\theta + ik\hat{w}\cos\theta\right) - b_{z0}, \tag{13}$$

$$\frac{\partial\hat{b}_z}{\partial z} + ik\hat{b}_x = 0, \tag{14}$$

$$\frac{\partial^2\hat{P}}{\partial z^2} - k^2\hat{P} = 0, \tag{15}$$

where $V_A = B/\sqrt{\mu_0\rho}$ is the Alfvén speed, and $w_0(z)$ and $b_{z0}(z)$ are the values of w and b_z at $t = 0$. Note that $\rho_1 V_{A1}^2 = \rho_2 V_{A2}^2 = \rho V_A^2$, where the subscripts 1 and 2 indicate that the quantity is calculated in $z < 0$ and $z > 0$, respectively. The solution to the system of Eqs. (11)–(15) must vanish as $|z| \to \infty$.

Eliminating b_z from Eqs. (11) and (13) yields

$$V_A^2\frac{\partial^2\hat{w}}{\partial z^2}\sin^2\theta + ikV_A^2\frac{\partial\hat{w}}{\partial z}\sin 2\theta + (\omega^2 - k^2V_A^2\cos^2\theta)\hat{w}$$
$$= i\omega w_0 - V_A^2 F(z) - \frac{i\omega}{\rho}\frac{d\hat{P}}{dz}, \tag{16}$$

$$F(z) = \frac{1}{B}\left(\frac{\partial b_{z0}}{\partial z}\sin\theta + ikb_{z0}\cos\theta\right). \tag{17}$$

To simplify the analysis, we take $b_{z0}(z) = 0$, so $F(z) = 0$.

The solution to Eq. (15) vanishing at infinity and continuous at $z = 0$ is

$$\hat{P} = A(\omega)\begin{cases} e^{kz}, & z < 0, \\ e^{-kz}, & z > 0, \end{cases} \tag{18}$$

where $A(\omega)$ is an arbitrary function. Now, we look for the solution to Eq. (16). This solution must be continuous at $z = 0$. Moreover, since u must be continuous at $z = 0$, it follows from Eq. (12) that $\partial\hat{w}/\partial z$ also must be continuous at $z = 0$. Using Eq. (18) and the method of variation of arbitrary constants, we obtain the general solution to Eq. (16)

$$\hat{w} = \frac{1}{2V_A\sin\theta}\int_0^z w_0(z')\left(e^{i\lambda_+(z-z')} - e^{i\lambda_-(z-z')}\right)dz'$$
$$+A_+e^{i\lambda_+z} + A_-e^{i\lambda_-z} \mp \frac{ik\omega Ae^{\pm kz}}{\rho(\omega^2 - k^2V_A^2e^{\mp 2i\theta})}, \tag{19}$$

where $A_+(\omega)$ and $A_-(\omega)$ are arbitrary functions, the upper and lower signs correspond to $z < 0$ and $z > 0$, respectively, and

$$\lambda_\pm = \frac{-kV_A\cos\theta \pm \omega}{V_A\sin\theta}. \tag{20}$$

Since we assume that ω is in the upper part of the ω-plane, it follows that $\Re(i\lambda_+) < 0$ and $\Re(i\lambda_-) > 0$, where $\Re$ indicates the real part of a quantity.

Now, we use the condition that $\hat{w} \to 0$ as $|z| \to \infty$. For simplicity, we assume that $w_0(z)$ has finite support meaning that there is such z_m that $w_0(z) = 0$ for $|z| \ge z_m$. This condition guarantees the convergence of the integral in Eq. (19). When $z<0$, the asymptotic behaviour of $\hat{w}$ for large $|z|$ is

$$\hat{w} \sim e^{i\lambda_{1+}z}\left(A_{1+} - \frac{1}{2V_{A1}\sin\theta}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}dz\right). \tag{21}$$

It follows from this result that, in order to have $\hat{w} \to 0$ as $z \to -\infty$, we must take

$$A_{1+} = \frac{1}{2V_{A1}\sin\theta}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}dz. \tag{22}$$

In a similar way, using the condition that $\hat{w} \to 0$ as $z \to \infty$, we obtain

$$A_{2-} = \frac{1}{2V_{A2}\sin\theta}\int_0^\infty w_0(z)e^{-i\lambda_{2-}z}dz. \tag{23}$$

It follows from Eqs. (19), (22), and (23) that

$$\begin{aligned}\hat{w} = e^{i\lambda_{1-}z}&\left(A_{1-} - \frac{1}{2V_{A1}\sin\theta}\int_0^z w_0(z')e^{-i\lambda_{1-}z'}dz'\right)\\ &- \frac{e^{i\lambda_{1+}z}}{2V_{A1}\sin\theta}\int_{-\infty}^z w_0(z')e^{-i\lambda_{1+}z'}dz'\\ &- \frac{ik\omega Ae^{kz}}{\rho_1(\omega^2 - k^2V_{A1}^2e^{-2i\theta})}\end{aligned} \tag{24}$$

for $z<0$, and

$$\begin{aligned}\hat{w} = e^{i\lambda_{2+}z}&\left(A_{2+} + \frac{1}{2V_{A2}\sin\theta}\int_0^z w_0(z')e^{-i\lambda_{2+}z'}dz'\right)\\ &+ \frac{e^{i\lambda_{2-}z}}{2V_{A2}\sin\theta}\int_z^\infty w_0(z')e^{-i\lambda_{2-}z'}dz'\\ &+ \frac{ik\omega Ae^{-kz}}{\rho_2(\omega^2 - k^2V_{A2}^2e^{2i\theta})}\end{aligned} \tag{25}$$

for $z>0$. Using the condition that w and $\partial w/\partial z$ must be continuous at $z=0$, and Eqs. (24) and (25) yield

$$\begin{aligned}A_{2+} &+ \frac{ik\omega A}{\rho_2(\omega^2 - k^2V_{A2}^2e^{2i\theta})} + \frac{1}{V_{A2}\sin\theta}\int_0^\infty w_0(z)e^{-i\lambda_{2-}z}\,dz\\ &= A_{1-} - \frac{ik\omega A}{\rho_1(\omega^2 - k^2V_{A1}^2e^{-2i\theta})}\\ &\quad + \frac{1}{2V_{A1}\sin\theta}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}\,dz,\end{aligned} \tag{26}$$

$$\begin{aligned}\lambda_{2+}A_{2+} &- \frac{k^2\omega A}{\rho_2(\omega^2 - k^2V_{A2}^2e^{2i\theta})}\\ &+ \frac{\lambda_{2-}}{2V_{A2}\sin\theta}\int_0^\infty w_0(z)e^{-i\lambda_{2-}z}\,dz\\ &= \lambda_{1-}A_{1-} - \frac{k^2\omega A}{\rho_1(\omega^2 - k^2V_{A1}^2e^{-2i\theta})}\\ &\quad + \frac{\lambda_{1+}}{2V_{A1}\sin\theta}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}\,dz.\end{aligned} \tag{27}$$

Equations (26) and (27) constitute a system of linear algebraic equations for A_- and A_+. Solving this system, we obtain

$$\begin{aligned}A_{1-} = \frac{kAV_{A1}V_{A2}\sin\theta}{V_{A1}+V_{A2}}&\left(\frac{i\lambda_{2+}+k}{\rho_2(\omega^2 - k^2V_{A2}^2e^{2i\theta})}\right.\\ &\left.+ \frac{i\lambda_{2+}-k}{\rho_1(\omega^2 - k^2V_{A1}^2e^{-2i\theta})}\right)\\ &+ \frac{1}{(V_{A1}+V_{A2})\sin\theta}\left(\frac{V_{A1}}{V_{A2}}\int_0^\infty w_0(z)e^{-i\lambda_{2-}z}\,dz\right.\\ &\left.+ \frac{V_{A2}-V_{A1}}{2V_{A1}}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}\,dz\right),\end{aligned} \tag{28}$$

$$\begin{aligned}A_{2+} = \frac{kAV_{A1}V_{A2}\sin\theta}{V_{A1}+V_{A2}}&\left(\frac{i\lambda_{1-}+k}{\rho_2(\omega^2 - k^2V_{A2}^2e^{2i\theta})}\right.\\ &\left.+ \frac{i\lambda_{1-}-k}{\rho_1(\omega^2 - k^2V_{A1}^2e^{-2i\theta})}\right)\\ &+ \frac{1}{(V_{A2}+V_{A1})\sin\theta}\left(\frac{V_{A2}}{V_{A1}}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}\,dz\right.\\ &\left.+ \frac{V_{A1}-V_{A2}}{2V_{A2}}\int_0^\infty w_0(z)e^{-i\lambda_{2-}z}\,dz\right).\end{aligned} \tag{29}$$

We still must satisfy the condition that the magnetic field perturbation is continuous at $z=0$. Since both w and $\partial w/\partial z$ are continuous at $z=0$, it follows from Eq. (7) that b_z is continuous at $z=0$. Then, since b_x is continuous at $z=0$, it follows from Eq. (8) that $\partial b_z/\partial z$ is continuous at $z=0$. Using Eqs. (24) and (25), we obtain from Eq. (13) the expressions for b_z

$$\begin{aligned}\hat{b}_z = \frac{ib_{z0}}{\omega} &+ \frac{ik^2BAe^{kz-i\theta}}{\rho_1(\omega^2 - k^2V_{A1}^2e^{-2i\theta})} + \frac{Be^{i\lambda_{1-}z}}{V_{A1}}\\ &\times\left(A_- - \frac{1}{2V_{A1}\sin\theta}\int_0^z w_0(z')e^{-i\lambda_{1-}z'}dz'\right)\\ &- \frac{Be^{i\lambda_{1+}z}}{2V_{A1}^2\sin\theta}\int_{-\infty}^z w_0(z')e^{-i\lambda_{1+}z'}dz',\end{aligned} \tag{30}$$

when $z<0$, and

$$\begin{aligned}\hat{b}_z = \frac{ib_{z0}}{\omega} &- \frac{ik^2BAe^{-kz+i\theta}}{\rho_2(\omega^2 - k^2V_{A2}^2e^{2i\theta})} - \frac{Be^{i\lambda_{2+}z}}{V_{A2}}\\ &\times\left(A_+ + \frac{1}{2V_{A2}\sin\theta}\int_0^z w_0(z')e^{-i\lambda_{2+}z'}dz'\right)\\ &+ \frac{Be^{i\lambda_{2-}z}}{2V_{A2}^2\sin\theta}\int_z^\infty w_0(z')e^{-i\lambda_{2-}z'}dz',\end{aligned} \tag{31}$$

when $z>0$. Then, the condition of continuity of $\partial b_z/\partial z$ at $z=0$ is written as

$$\frac{\lambda_{1-}A_{1-}}{V_{A1}} + \frac{\lambda_{2+}A_{2+}}{V_{A2}} + k^3 A\left(\frac{e^{-i\theta}}{\rho_1(\omega^2 - k^2V_{A1}^2 e^{-2i\theta})}\right.$$
$$\left. - \frac{e^{i\theta}}{\rho_2(\omega^2 - k^2V_{A2}^2 e^{2i\theta})}\right) + \frac{iw_0(0)}{\sin\theta}\left(\frac{1}{V_{A2}^2} - \frac{1}{V_{A1}^2}\right)$$
$$= \frac{1}{2\sin\theta}\left(\frac{\lambda_{1+}}{V_{A1}^2}\int_{-\infty}^{0} w_0(z)e^{-i\lambda_{1+}z}\,dz\right.$$
$$\left. + \frac{\lambda_{2-}}{V_{A2}^2}\int_0^{\infty} w_0(z)e^{-i\lambda_{2-}z}\,dz\right). \tag{32}$$

Using Eqs. (28) and (29), we obtain from this equation

$$A(\omega) = \frac{H(\omega)G(\omega)}{kD(\omega)}, \tag{33}$$

where

$$D(\omega) = (\rho_1+\rho_2)\omega^2 + 2ik\omega(\rho_1V_{A1}+\rho_2V_{A2})\sin\theta - 2\rho V_A^2 k^2, \tag{34}$$

$$H(\omega) = \rho_1\rho_2(V_{A2}-V_{A1}) \times (\omega - kV_{A1}e^{-i\theta})(\omega + kV_{A2}e^{i\theta}), \tag{35}$$

$$G(\omega) = \frac{i\omega}{\sin\theta}\left(\int_{-\infty}^{0}\frac{w_0(z)}{V_{A1}^2}e^{-i\lambda_{1+}z}\,dz + \int_0^{\infty}\frac{w_0(z)}{V_{A2}^2}e^{-i\lambda_{2-}z}\,dz\right)$$
$$+ w_0(0)\left(\frac{1}{V_{A1}} + \frac{1}{V_{A2}}\right). \tag{36}$$

We introduce the notation $Q(t) = P(t, z=0)$. It follows from Eq. (18) that $A(\omega)$ is the Laplace transform of $Q(t)$. Then

$$Q(t) = \frac{1}{2\pi}\int_{i\varsigma-\infty}^{i\varsigma+\infty}\frac{H(\omega)G(\omega)}{kD(\omega)}e^{-i\omega t}d\omega, \tag{37}$$

where ς is chosen in such a way that the integration line is above all singularities of the integrand. Using Eqs. (12)–(14) and (19), we can calculate the Laplace transforms of the velocity and magnetic field perturbations and determine their dependence on time.

IV. SURFACE AND LEAKY WAVES

In this section, we study the asymptotic behaviour of the solution for large t. Since we assume that $w_0(z)$ is a function with finite support, it follows that $A(\omega)$ is defined on the whole complex ω-plane. It is a meromorphic function that has two poles coinciding with the zeros of $D(\omega)$. These zeros are equal to $\omega_\pm = \pm\omega_r - i\gamma$, where

$$\omega_r = \frac{\rho^{1/2}kV_A}{\rho_1+\rho_2}\sqrt{\left(\rho_1^{1/2}-\rho_2^{1/2}\right)^2 + \left(\rho_1^{1/2}+\rho_2^{1/2}\right)^2\cos^2\theta}, \tag{38}$$

$$\gamma = \frac{k(\rho_1V_{A1}+\rho_2V_{A2})\sin\theta}{\rho_1+\rho_2}. \tag{39}$$

Hence, we must take $\varsigma > -\gamma$, for example, $\varsigma = -\frac{1}{2}\gamma$.

It is shown in Appendix A that

$$Q(t) = e^{-\gamma t}\left(S_+e^{-i\omega_r t} - S_-e^{i\omega_r t}\right) \text{ when } t \ge t_m, \tag{40}$$

where

$$S_\pm = -\frac{iH(\omega_\pm)G(\omega_\pm)}{2k\omega_r(\rho_1+\rho_2)}, \tag{41}$$

$$t_m = \frac{z_m}{\sin\theta}\max\left(\frac{1}{V_{A1}}, \frac{1}{V_{A2}}\right). \tag{42}$$

Recalling that perturbations of all quantities are proportional to e^{ikx}, we see that the first and second terms in the brackets on the right-hand side of Eq. (40) describe the waves propagating with the phase speed ω_r/k in the positive and negative x-directions, respectively. We also see that the perturbations damp with the decrement γ.

The expression for w is obtained in Appendix B. It reads

$$w = w_s + w_l, \tag{43}$$

where

$$w_s = e^{-\gamma t}\begin{cases} e^{kz}(e^{-i\omega_r t}U_{1+} - e^{i\omega_r t}U_{1-}), & z<0, \\ e^{-kz}(e^{-i\omega_r t}U_{2+} - e^{i\omega_r t}U_{2-}), & z>0, \end{cases} \tag{44}$$

$$w_l(t,z) = e^{-\gamma t}$$
$$\times\left[e^{-i\omega_r t}W_{1+}\exp\left(-\frac{[\gamma + i(kV_{A1}\cos\theta + \omega_r)]z}{V_{A1}\sin\theta}\right)\right.$$
$$\left. - e^{i\omega_r t}W_{1-}\exp\left(-\frac{[\gamma + i(kV_{A1}\cos\theta - \omega_r)]z}{V_{A1}\sin\theta}\right)\right] \tag{45}$$

for $z<0$, and

$$w_l(t,z) = e^{-\gamma t}$$
$$\times\left[e^{-i\omega_r t}W_{2+}\exp\left(\frac{[\gamma - i(kV_{A2}\cos\theta - \omega_r)]z}{V_{A2}\sin\theta}\right)\right.$$
$$\left. - e^{i\omega_r t}W_{2-}\exp\left(\frac{[\gamma - i(kV_{A2}\cos\theta + \omega_r)]z}{V_{A2}\sin\theta}\right)\right] \tag{46}$$

for $z>0$, where $U_{1\pm}$, $W_{1\pm}$, $U_{2\pm}$, and $W_{2\pm}$ are given by Eqs. (B17), (B18), (B20), and (B21), respectively.

We note that w_s decays exponentially as $|z| \to \infty$, which implies that it corresponds to surface waves. If we restore the x-dependence, we can see that the terms proportional to U_{1+} and U_{2+} describe the wave propagating in the positive x-direction, while the terms proportional to U_{1-} and U_{2-} describe the wave propagating in the negative x-direction.

Equation (45) is only valid when

$$t \ge t_m \quad \text{and} \quad z \ge -tV_{A1}\sin\theta, \tag{47}$$

while Eq. (46) is only valid when

$$t \ge t_m \quad \text{and} \quad z \le tV_{A2}\sin\theta. \tag{48}$$

We note that the terms proportional to W_{1+} and W_{1-} grow exponentially as $|z|$ increases. This is a typical behaviour for leaky modes.

Recalling that the perturbations of all quantities are proportional to e^{ikx}, we conclude that the terms proportional to

U_{1+} and U_{1-} in Eq. (45) describe the waves propagating in the positive and negative x-directions, respectively. Restoring the x-dependence, we can write the term proportional to W_{1+} as

$$W_{1+} \exp\left[-\gamma\left(t + \frac{z}{V_{A1}\sin\theta}\right) - i(\omega_r - k_x x - k_z z)\right], \quad (49)$$

where

$$k_x = k, \quad k_z = -\frac{kV_{A1}\cos\theta + \omega_r}{V_{A1}\sin\theta}. \quad (50)$$

Hence, the expression given by Eq. (49) describes a wave propagating in both the x and z-directions. It follows from Eq. (50) that the dispersion equation for this wave is

$$\omega_r = -V_{A1}(k_x\cos\theta + k_z\sin\theta). \quad (51)$$

Then, the group velocity of this wave is $\mathbf{v}_g = -V_{A1}\mathbf{B}/B$ that is anti-parallel to the equilibrium magnetic field. Since the wave energy propagates with the group velocity, we conclude that it propagates in the direction anti-parallel to the equilibrium magnetic field, which is away from the interface as it should be expected. In a similar way, we can show that the term proportional to W_{1-} also describes the wave with the energy propagating away from the interface. Since $\gamma z < 0$, it follows that the absolute values of the terms proportional to W_{1+} and W_{1-} grow exponentially as $|z| \to \infty$.

Repeating this analysis, we show that the terms proportional to U_{2+} and U_{2-} describe the waves propagating in the positive and negative x-directions, respectively. The absolute values of the terms proportional to W_{2+} and W_{2-} grow exponentially when z increases. They describe two waves propagating in both the x and z-directions. Their group velocity is parallel to $\mathbf{B}$ meaning that the wave energy propagates away from the interface as it should be.

Let us take $t \gg \gamma^{-1}$. The increment of w_l with respect to z is $\gamma/V_A\sin\theta$. Since w_s is exponentially small for $t \gg \gamma^{-1}$, practically the whole wave energy is stored in the leaky waves, and, in turn, in the leaky waves it is concentrated in the regions defined by $V_A(t-\gamma^{-1})\sin\theta \lesssim |z| \le tV_A\sin\theta$. We do not write the indices 1 and 2 because this analysis is valid for both $z<0$ and $z>0$. It follows from Eqs. (45) and (46) that the wave amplitude does not change in these regions with time. Since the wave energy is proportional to the wave amplitude squared, we conclude that the wave energy propagates without damping along the magnetic field lines away from the interface.

V. THE LIMIT OF SMALL θ

We now assume that $\theta \ll 1$. Then, it follows from Eqs. (38) and (39) that

$$\omega_r = kC_k + \mathcal{O}(\theta^2), \quad \gamma = k\Gamma\theta + \mathcal{O}(\theta^3), \quad (52)$$

where

$$C_k^2 = \frac{2\rho V_A^2}{\rho_1 + \rho_2}, \quad \Gamma = \frac{\rho_1 V_{A1} + \rho_2 V_{A2}}{\rho_1 + \rho_2}. \quad (53)$$

It is shown in Appendix C that

$$G(\omega) = -\frac{k^2 w_0(0)(V_{A1}+V_{A2})}{(\omega - kV_{A1})(\omega + kV_{A2})} + \mathcal{O}(\theta). \quad (54)$$

Using Eqs. (52), (53), and (54), we reduce Eqs. (B17), (B18), (B20), and (B21) to

$$U_{1\pm} = U_{2\pm} = \pm\frac{1}{2}w_0(0) + \mathcal{O}(\theta), \quad (55)$$

$$W_{2\mp} = W_{1\pm} = \frac{i\theta w_0(0)C_k(V_{A2}-V_{A1})}{2(C_k \pm V_{A1})(C_k \pm V_{A2})} + \mathcal{O}(\theta^2). \quad (56)$$

Substituting Eqs. (55) and (56) in Eqs. (45) and (46), we obtain in the leading order approximation with respect to θ

$$w = e^{-\gamma t} w_0(0)\cos(kC_k t)\begin{cases} e^{kz}, & z<0, \\ e^{-kz}, & z>0. \end{cases} \quad (57)$$

Next, we obtain the expression for u. Using Eqs. (6), (45), (46), (55), and (56) yields

$$u = \tilde{u} + \bar{u}, \quad (58)$$

where

$$\tilde{u} = ie^{-\gamma t} w_0(0)\cos(kC_k t)\begin{cases} e^{kz}, & z<0, \\ -e^{-kz}, & z>0, \end{cases} \quad (59)$$

while $\bar{u}$ is given by

$$\bar{u} = \frac{iw_0(0)C_k(V_{A2}-V_{A1})\exp[-\gamma t - kz(\Gamma/V_{A1} + i/\theta)]}{2V_{A1}} \times \left(\frac{\exp[-ikC_k(t + z/\theta V_{A1})]}{C_k + V_{A2}} + \frac{\exp[ikC_k(t + z/\theta V_{A1})]}{C_k - V_{A2}}\right) \quad (60)$$

for $z<0$, and by

$$\bar{u} = \frac{iw_0(0)C_k(V_{A2}-V_{A1})\exp[-\gamma t + kz(\Gamma/V_{A2} - i/\theta)]}{2V_{A2}} \times \left(\frac{\exp[-ikC_k(t - z/\theta V_{A2})]}{C_k - V_{A1}} + \frac{\exp[ikC_k(t - z/\theta V_{A2})]}{C_k + V_{A1}}\right) \quad (61)$$

for $z>0$. Now, using Eqs. (6), (7), and (57) we obtain the similar expressions for the components of the magnetic field. They are given by Eqs. (C3)–(C7).

We now compare the expressions for the leaky modes and those for surface waves propagating on a tangential discontinuity. We obtain this tangential discontinuity by taking $\theta \to 0$. Below we use the subscript "t" to indicate quantities corresponding to the surface wave on the tangential discontinuity. It is straightforward to show that

$$w_t = \lim_{\theta\to 0} w, \; b_{zt} = \lim_{\theta\to 0} b_z, \; u_t = \lim_{\theta\to 0}\tilde{u}, \; b_{xt} = \lim_{\theta\to 0}\tilde{b}_x. \quad (62)$$

Since $\bar{u} \not\to 0$ and $\bar{b}_x \not\to 0$ as $\theta \to 0$, we conclude that $u \not\to u_t$ and $b_x \not\to b_{xt}$ as $\theta \to 0$. Hence, only w and b_z

tend to the corresponding quantities in a tangential discontinuity, while u and b_x do not. This implies that there is no continuous transition from the leaky mode on the contact discontinuity to the surface wave on the tangential discontinuity.

We now introduce a different definition of continuous transition. When $\theta \ll 1$, the z-dependence of u and b_z is highly oscillatory with the oscillation periods equal to

$$L_{1+} = \frac{2\pi\theta V_{A1}}{k(C_k + V_{A1})}, \quad L_{1-} = \frac{2\pi\theta V_{A1}}{k|C_k - V_{A1}|} \tag{63}$$

for $z < 0$, and

$$L_{2+} = \frac{2\pi\theta V_{A2}}{k(C_k + V_{A2})}, \quad L_{2-} = \frac{2\pi\theta V_{A2}}{k|C_k - V_{A2}|} \tag{64}$$

for $z > 0$. We introduce the average value of function $f(z)$ as

$$\langle f \rangle = \frac{k}{2\theta^{1/2}} \int_{z-k^{-1}\theta^{1/2}}^{z+k^{-1}\theta^{1/2}} f(z')\,dz'. \tag{65}$$

The choice of the averaging interval equal to $k^{-1}\theta^{1/2}$ is somewhat arbitrary. Instead of $\theta^{1/2}$, we can choose any quantity that is much smaller than unity and much larger than θ when $\theta \ll 1$. It is straightforward to obtain in the leading order approximation

$$\langle \tilde{u} \rangle = ie^{-\gamma t} w_0(0)\cos(kC_k t) \begin{cases} e^{kz}, & kz < -\theta^{1/2}, \\ -\theta^{-1/2}kz, & k|z| \le \theta^{1/2}, \\ -e^{-kz}, & kz > \theta^{1/2}. \end{cases} \tag{66}$$

After long but straightforward calculation we also obtain again in the leading order approximation

$$\langle \bar{u} \rangle = \frac{1}{4}\theta^{1/2} e^{-\gamma t} w_0(0) C_k (V_{A2} - V_{A1}) \begin{cases} \Upsilon_1, & kz < -\theta^{1/2}, \\ \Upsilon_t, & k|z| \le \theta^{1/2}, \\ \Upsilon_2, & kz > \theta^{1/2}. \end{cases} \tag{67}$$

The quantities Υ_1, Υ_2, and Υ_t are given in Appendix C. It follows from this equation that, for any value of z, $\langle \bar{u} \rangle \to 0$ as $\theta \to 0$. However, it also follows from the expressions for Υ_1 and Υ_2 that $\max_z|\langle \bar{u} \rangle| \to \infty$ as $|z| \to \infty$ while θ is fixed. Hence, the convergence of $\langle \bar{u} \rangle$ to zero is non-uniform with respect to z. In the same way, it can be shown that $\langle b_x \rangle \to b_{xt}$ as $\theta \to 0$, and again the convergence is non-uniform with respect to z.

It follows from Eq. (66) that $\langle \tilde{u} \rangle = \tilde{u} = u_t$ for $kz \ge \theta^{1/2}$. Hence, $\langle u \rangle = \langle \tilde{u} \rangle + \langle \bar{u} \rangle \to u_t$ as $\theta \to 0$ and $z \ne 0$. Summarising, we can state that the difference between $\langle u \rangle$ and u_t is on the order of $\theta^{1/2}$ except for a transitional layer of thickness on the order of $\theta^{1/2}$ when $\theta \ll 1$ and z is sufficiently small. It follows from the expressions for Υ_1 and Υ_2 that the latter condition is equivalent to $k|z| \ll 1$. Hence, $\langle u \rangle \approx u_t$ for $\theta^{1/2} \lesssim k|z| \ll 1$.

In Fig. 1, the real and imaginary parts of u (solid lines) and $\langle u \rangle$ (dashed lines) are shown for $\theta = 0.001$, $\rho_1/\rho_2 = 0.5$, and $kC_k t = 50\pi$. For this moment of time, we have $\cos(kC_k t) = 1$. As we have already pointed out, u involves two oscillation periods given by Eqs. (63) and (64). For the particular parameters chosen to calculate u, we obtain $kL_{1+} \approx 0.00346$, $kL_{1-} \approx 0.0342$, $kL_{2+} \approx 0.00290$, and $kL_{2-} \approx 0.0378$. Since $L_{1+} \ll L_{1-}$ and $L_{2+} \ll L_{2-}$, it follows that the graphs of the real and imaginary parts of u contain short and long period oscillations. The dashed curves for $z < 0$ do not show strong oscillations with the short period, while such oscillations have relatively large amplitude for $z > 0$. This behaviour is related to the fact that, for the particular

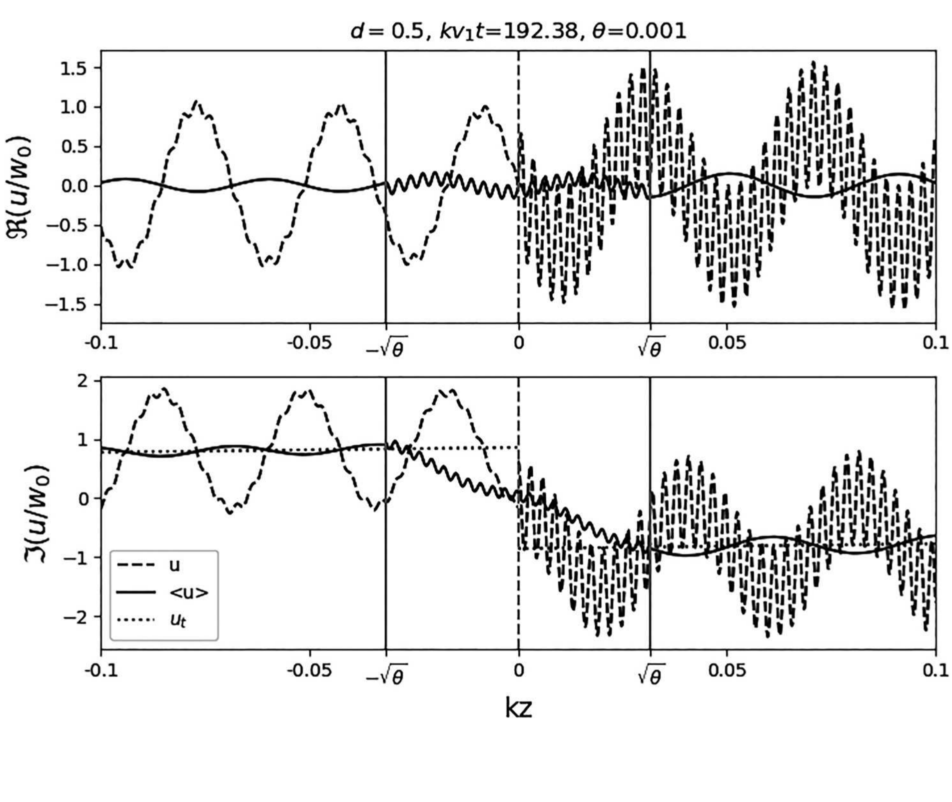

FIG. 1. The real and imaginary parts of u and $\langle u \rangle$ for $\theta = 0.001$, $\rho_1/\rho_2 = 0.5$, and $kC_t = 50\pi$. The solid lines show $\langle u \rangle$ and the dashed lines u. The dotted lines show u_t. The vertical solid lines show the boundaries of the transitional layer defined by $kz = \pm\theta^{1/2}$.

parameters chosen to calculate the dependences shown in Fig. 1, the amplitude of short-period oscillations is much smaller than that of the long-period oscillations for $z<0$, while the two amplitudes are on the same order for $z>0$. The dotted lines in the lower panel show the dependence of $\Im(u_t)$ on z. There are no dotted lines in the upper panel because $\Re(u_t)=0$. We can see that $\langle u\rangle$ is very close to u_t outside the transitional layer, that is, for $k|z|>\theta^{1/2}$.

VI. TIME EVOLUTION OF INITIAL PERTURBATIONS

In order to find how solutions vary in the direction of the wave propagation, we must perform an inverse Fourier transform of the velocities. Since $w(x, z, t)$ is real, we know that $w(-k, z, t) = w^*(k, z, t)$, where the asterisk denotes the complex conjugate. By splitting the integral into positive and negative ranges for k, we may rewrite the inverse Fourier transform as

$$w(x,z,t)=\frac{1}{2\pi}\int_0^\infty\left[e^{ikx}w(k,z,t)+e^{-ikx}w^*(k,z,t)\right]dk. \quad (68)$$

We consider the initial kink in the form of Lorentz function and take at $z=0$

$$w_0(x)=\frac{al}{x^2+l^2}, \quad (69)$$

where $a>0$ and $l>0$. The Fourier transform of this function is

$$w_0(k)=\pi a e^{-lk}. \quad (70)$$

Then, it follows from Eq. (57) that in the leading order with respect to θ

$$w(k,z,t)=\pi a\cos(kC_k t)\exp\left[-k(|z|+l+\theta\Gamma t)\right]. \quad (71)$$

After the straightforward calculation, we obtain

$$w(x,z,t)=\frac{a}{2}(\Gamma\theta t+|z|+l)\times\left[\frac{1}{(\Gamma\theta t+|z|+l)^2+(C_k t-x)^2}+\frac{1}{(\Gamma\theta t+|z|+l)^2+(C_k t+x)^2}\right]. \quad (72)$$

We see that, in contrast to the leaky modes, the solution to the initial value problem decays as $|z|\to\infty$, as it should be. It is a superposition of two perturbations propagating in the opposite directions with the phase speed C_k. Finally, the solution decays with time as t^{-1}. When $l\to 0$, we obtain $l/(x^2+l^2)\to\pi\delta(x)$, that is the initial condition in the form of a concentrated pulse. In this case, the solution to the initial value problem is given by

$$w(x,z,t)=\frac{a}{2}(\Gamma\theta t+|z|)\left[\frac{1}{(\Gamma\theta t+|z|)^2+(C_k t-x)^2}+\frac{1}{(\Gamma\theta t+|z|)^2+(C_k t+x)^2}\right]. \quad (73)$$

Finally, we point out that to derive Eqs. (72) and (73), we used Eq. (57) that is only valid when the conditions given by Eqs. (47) and (48) are satisfied. Taking into account that Eq. (57) is derived for $\theta\ll 1$, we conclude that Eqs. (72) and (73) are only valid for

$$t\ge\frac{z_m}{\theta}\max\left(\frac{1}{V_{A1}},\frac{1}{V_{A2}}\right), \quad -t\theta V_{A1}\le z\le t\theta V_{A2}. \quad (74)$$

We recall that z_m is determined by the condition that $w_0(z)=0$ for $|z|\ge z_m$, while there is such $z\in(-z_m, z_m)$ that $w_0(z)\ne 0$.

VII. SUMMARY AND CONCLUSIONS

We studied the propagation of surface waves on a magnetohydrodynamic contact discontinuity in an incompressible plasma. We assumed that at the initial moment of time the surface is perturbed and then we solved the initial value problem describing the evolution of this perturbation in time. The solution was obtained using the Laplace transform with respect to time and expressed in terms of the Bromwich integral.

We calculated the asymptotics of the solution valid for large time. In the case of tangential MHD discontinuity, the asymptotics of an initial perturbation consists of two surface waves with constant amplitudes propagating in the opposite directions. These waves are eigenmodes of ideal MHD. In contrast, in the case of contact discontinuity, the asymptotics of the initial perturbation consists of two leaky modes. The amplitudes of these modes exponentially decay with time and exponentially increase with the distance from the contact discontinuity, so they are not eigenmodes of ideal MHD. Moreover, these modes only determine the asymptotics of the solution to the initial value problem on a bounded intervals $z\in(-tv_{A1}, 0)$ below the discontinuity and $z\in(0, tv_{A2})$ above the discontinuity. The properties of leaky modes are similar to those of leaky modes related to kink oscillations of a magnetic tube with the internal plasma density smaller than that in the surrounding plasma.[47]

We obtained relatively simple approximate expressions for the leaky modes in the case of small inclination angle θ. Using these expressions, we studied the limit $\theta\to 0$. We found that the z-components of the velocity and magnetic field perturbation tend to the corresponding expressions for surface waves on tangential discontinuity. However, the x-components of the velocity and magnetic field perturbation do not tend to the corresponding expressions for surface waves on tangential discontinuity. Hence, there is no continuous transition from leaky waves on a contact discontinuity to surface waves on a tangential discontinuity.

The leaky modes are characterised by highly oscillatory behaviour in the z-direction that is orthogonal to the discontinuity. The characteristic scale of this oscillation is θL, where L is the wavelength in the direction parallel to the discontinuity. We introduced quantities averaged with respect to z over an interval of length $2k^{-1}\theta^{1/2}$, where $k=2\pi/L$. We showed that the average quantities tend to corresponding quantities in surface waves on a tangential discontinuity as $\theta\to 0$.

The solutions obtained in Secs. IV and V correspond to the harmonic initial perturbation. We also found the solution describing the time evolution of an initial perturbation described by the Lorentz function. It constitutes two counter-propagating wave pulses. Their amplitudes decay both in time and with the distance from the interface.

Finally we make a comment. We studied the one-dimensional problem and assumed that the perturbations are independent of the y-coordinate in Cartesian coordinates x, y, z, and the y-components of the velocity and magnetic field perturbation are zero. If we relax the latter assumption then, in addition to the system of Eqs. (5)–(9) we obtain two equations for the y-components of the velocity and magnetic field perturbation. These two equations are separated from the system of Eqs. (5)–(9). Hence, the solution to this system of equations remains the same, while the equations describing the y-components of the velocity and magnetic field perturbation can be solved separately. The solution to these equations is very simple. It describes the propagation of Alfvén waves along the magnetic field lines.

The situation is more involved when the perturbations depend on y. In this case, the system of Eqs. (5)–(9) and the equations describing the y-components of the velocity and magnetic field perturbation are related through the solenoidality conditions for the velocity and magnetic field, respectively. We plan to study this problem in the future.

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APPENDIX A: DERIVATION OF EXPRESSION FOR *Q*(*T*)

In this appendix, we derive the expression for $Q(t)$ valid for sufficiently large t. Using the integration by parts, we transform Eq. (36) to

$$G(\omega) = G_1(\omega) + \frac{G_2(\omega)}{\sin\theta}, \tag{A1}$$

where

$$G_1(\omega) = \frac{kw_0(0)\cos\theta}{\omega + kV_{A2}\cos\theta} - \frac{kw_0(0)\cos\theta}{\omega - kV_{A1}\cos\theta} + \frac{i\omega}{\sin\theta}\left(\frac{1}{V_{A1}^2\lambda_{1+}^2} - \frac{1}{V_{A2}^2\lambda_{2-}^2}\right)\frac{dw_0}{dz}\bigg|_{z=0}, \tag{A2}$$

$$G_2(\omega) = \omega\left(\frac{1}{V_{A1}^2\lambda_{1+}^3} - \frac{1}{V_{A2}^2\lambda_{2-}^3}\right)\frac{d^2w_0}{dz^2}\bigg|_{z=0} - e^{i\omega t}\left(\frac{\omega}{V_{A1}^2\lambda_{1+}^3}\int_{-\infty}^0 \frac{d^3w_0}{dz^3}e^{i\kappa_1 z}\,dz - \frac{\omega}{V_{A2}^2\lambda_{2-}^3}\int_0^\infty \frac{d^3w_0}{dz^3}e^{i\kappa_2 z}\,dz\right), \tag{A3}$$

$$\kappa_1 = -(\omega t + \lambda_{1+}z) = kz\cot\theta - \omega\left(t + \frac{z}{V_{A1}\sin\theta}\right), \tag{A4}$$

$$\kappa_2 = -(\omega t + \lambda_{2-}z) = kz\cot\theta - \omega\left(t - \frac{z}{V_{A2}\sin\theta}\right). \tag{A5}$$

We now consider a closed contour in the complex ω-plane shown in Fig. 2. We choose the radius R of the half-circle so large that the zeros of $D(\omega)$ are inside the contour. Using the residual theorem, we obtain that the integral of the integrant in Eq. (37) over this contour is equal to the sum of residuals at the zeros of D times $-2\pi i$

$$\left(\int_{\mathcal{C}} + \int_{i\varsigma-R}^{i\varsigma+R}\right)\frac{H(\omega)G(\omega)}{kD(\omega)}e^{-i\omega t}d\omega = -2\pi i\left[\text{res}_{\omega_-}\left(\frac{H(\omega)G(\omega)}{kD(\omega)}e^{-i\omega t}\right) + \left(\text{res}_{\omega_+}\frac{H(\omega)G(\omega)}{kD(\omega)}e^{-i\omega t}\right)\right], \tag{A6}$$

where $\mathcal{C}$ stays for the half-circle. The residues in Eq. (A6) are given by

$$\text{res}_{\omega_\pm}\left(\frac{H(\omega)G(\omega)}{kD(\omega)}e^{-i\omega t}\right) = \lim_{\omega\to\omega_\pm}\frac{(\omega-\omega_\pm)H(\omega)G(\omega)}{kD(\omega)}e^{-i\omega t} = \pm iS_\pm e^{-(\gamma\pm i\omega_r)t}, \tag{A7}$$

where

$$S_\pm = -\frac{iH(\omega_\pm)G(\omega_\pm)}{2k\omega_r(\rho_1+\rho_2)}.$$

Now, we calculate the limit of the integral over $\mathcal{C}$ in Eq. (A6) as $R\to\infty$. Using the integration by parts yields

$$\int_{\mathcal{C}}\frac{H(\omega)G_1(\omega)}{D(\omega)}e^{-i\omega t}d\omega = \frac{iH(\omega)G_1(\omega)e^{-i\omega t}}{tD(\omega)}\bigg|_{i\varsigma-R}^{i\varsigma+R} - \frac{i}{t}\int_{\mathcal{C}}\frac{d}{d\omega}\left(\frac{H(\omega)G_1(\omega)}{D(\omega)}\right)e^{-i\omega t}d\omega. \tag{A8}$$

It is straightforward to see that $G_1(\omega) = \mathcal{O}(R^{-1})$ and the integrand in the integral on the right-hand side of this equation is on the order of R^{-2} for $R \gg 1$ and $\omega\in\mathcal{C}$. In addition,

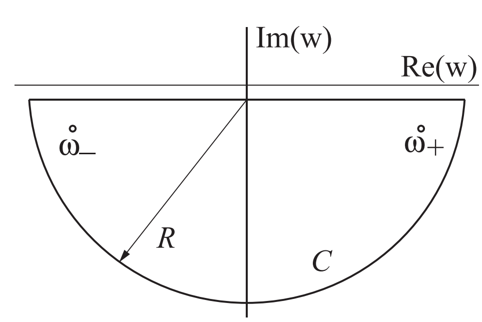

FIG. 2. Sketch of contour in the complex ω-plane used to derive Eq. (A6).

we have the estimates $D(\omega) = \mathcal{O}(R^2)$ and $D(\omega) = \mathcal{O}(R^2)$ for $R \gg 1$ and $\omega \in \mathcal{C}$. Then, since $|e^{-i\omega t}| \le 1$ it follows that the left-hand side of Eq. (A8) tends to zero as $R \to \infty$.

Next, we consider the quantities κ_1 and κ_2. We recall that $w_0(z)$ is assumed to be a function with the finite support and $w_0(z) = 0$ when $|z| \ge z_m$. On the other hand, $w_0(z) \ne 0$ at least at some points in the interval $(-z_m, z_m)$. Since $\mathcal{C}$ is in the lower half of the complex ω-plane, $\Re(i\kappa_1) \le 0$ for $-z_m \le z < 0$ and $\Re(i\kappa_2) \le 0$ for $0 < z \le z_m$ simultaneously if and only if

$$t \ge t_m \equiv \frac{z_m}{\sin\theta}\max\left(\frac{1}{V_{A1}}, \frac{1}{V_{A2}}\right).$$

When this inequality is satisfied, the two integrals on the right-hand side of Eq. (A3) are bounded, $G_2(\omega)e^{-i\omega t} = \mathcal{O}(R^{-2})$ for $\omega \in \mathcal{C}$, and

$$\int_{\mathcal{C}} \frac{H(\omega)G_2(\omega)}{D(\omega)} e^{-i\omega t} d\omega \to 0 \quad \text{as} \quad R \to \infty.$$

Hence, summarising we obtain that

$$\int_{\mathcal{C}} \frac{H(\omega)G_2(\omega)}{kD(\omega)} e^{-i\omega t} d\omega \to 0 \quad \text{as} \quad R \to \infty \text{ and } t \ge t_m. \tag{A9}$$

Then, taking $R \to \infty$ in Eq. (A6), using Eq. (A8), and noticing that

$$\int_{i\varsigma - R}^{i\varsigma + R} \frac{H(\omega)G(\omega)}{kD(\omega)} e^{-i\omega t} d\omega \to Q(t) \quad \text{as} \quad R \to \infty,$$

we eventually obtain Eq. (40)

APPENDIX B: DERIVATION OF EXPRESSION FOR *w*

In this appendix, we derive the expression for w valid for large t. Using Eqs. (24), (25), (28), (29), and (33), we obtain that

$$\hat{w}(\omega) = \frac{X_j(\omega)}{D(\omega)} + \frac{Y_j(\omega)}{\sin\theta}, \tag{B1}$$

where $j = 1$ and $j = 2$ refer to quantities in $z < 0$ and $z > 0$, respectively, and

$$X_1(\omega) = iG(\omega)(V_{A2} - V_{A1})\left[\frac{\rho_1 V_{A1} e^{i\lambda_{1-}z}(\omega - kV_{A1}e^{-i\theta})}{V_{A1} + V_{A2}} + \rho_2(\omega + kV_{A2}e^{i\theta})\left(\frac{\omega(e^{i\lambda_{1-}z} - e^{kz})}{\omega + kV_{A1}e^{-i\theta}} - \frac{V_{A2}e^{i\lambda_{1-}z}}{V_{A1} + V_{A2}}\right)\right], \tag{B2}$$

$$\begin{aligned} Y_1(\omega) = {} & \frac{e^{i\lambda_{1-}z}}{V_{A1} + V_{A2}}\left(\frac{V_{A1}}{V_{A2}}\int_0^\infty w_0(z)e^{-i\lambda_{2-}z}dz + \frac{V_{A2} - V_{A1}}{2V_{A1}}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{1+}z}dz\right) \\ & + \frac{e^{i\lambda_{1-}z}}{2V_{A1}}\int_z^0 w_0(z')e^{-i\lambda_{1-}z'}dz' + \frac{e^{i\lambda_{1+}z}}{2V_{A1}}\int_{-\infty}^z w_0(z')e^{-i\lambda_{1+}z'}dz', \end{aligned} \tag{B3}$$

$$X_2(\omega) = iG(\omega)(V_{A1} - V_{A2})\left[\frac{\rho_2 V_{A2} e^{i\lambda_{2+}z}(\omega + kV_{A2}e^{i\theta})}{V_{A1} + V_{A2}} + \rho_1(\omega - kV_{A1}e^{-i\theta})\left(\frac{\omega(e^{i\lambda_{2+}z} - e^{-kz})}{\omega - kV_{A2}e^{i\theta}} - \frac{V_{A1}e^{i\lambda_{2+}z}}{V_{A1} + V_{A2}}\right)\right], \tag{B4}$$

$$\begin{aligned} Y_2(\omega) = {} & \frac{e^{i\lambda_{2+}z}}{V_{A1} + V_{A2}}\left(\frac{V_{A2}}{V_{A1}}\int_{-\infty}^0 w_0(z)e^{-i\lambda_{2+}z}dz + \frac{V_{A1} - V_{A2}}{2V_{A2}}\int_0^\infty w_0(z)e^{-i\lambda_{1-}z}dz\right) \\ & + \frac{e^{i\lambda_{2+}z}}{2V_{A2}}\int_0^z w_0(z')e^{-i\lambda_{2+}z'}dz' + \frac{e^{i\lambda_{2-}z}}{2V_{A2}}\int_z^\infty w_0(z')e^{-i\lambda_{2-}z'}dz'. \end{aligned} \tag{B5}$$

It is obvious that $Y_1(\omega)$ and $Y_2(\omega)$ are holomorphic functions in the whole complex plane. Taking into account that $i\lambda_{1-} = k$ when $\omega = -kV_{A1}e^{-i\theta}$ and $i\lambda_{2+} = -k$ when $\omega = kV_{A2}e^{i\theta}$, we conclude that $X_1(\omega)$ and $X_2(\omega)$ are also holomorphic functions in the whole complex plane. Then it follows that $\hat{w}(\omega)$ is a meromorphic function in the whole complex plane with the simple poles at ω_+ and ω_-.

Now, we use the same closed contour shown in Fig. 2 as before and obtain

$$\left(\int_{\mathcal{C}} + \int_{i\varsigma - R}^{i\varsigma + R}\right)\hat{w}(\omega)e^{-i\omega t}d\omega = -2\pi i\left[\operatorname{res}_{\omega_-}\left(\frac{X_j(\omega)}{D(\omega)}e^{-i\omega t}\right) + \left(\operatorname{res}_{\omega_+}\frac{X_j(\omega)}{D(\omega)}e^{-i\omega t}\right)\right]. \tag{B6}$$

We again take $t \ge t_m$. Then, in accordance with Eqs. (A1)–(A3), $e^{-i\omega t}G(\omega) = \mathcal{O}(R^{-1})$ and, consequently, $e^{-i\omega t}X_j(\omega) = \mathcal{O}(1)$ when $R \gg 1$ and $\omega \in \mathcal{C}$. In addition, when $\Im(\omega) < 0$, $\Re(i\lambda_{1-}z - i\omega t) \le 0$ for $z \ge -V_{A1}\sin\theta$ and $\Re(i\lambda_{2+}z - i\omega t) \le 0$ for $z \le V_{A2}\sin\theta$. Now, taking into account that $D(\omega) = \mathcal{O}(R^2)$, we obtain that

$$\int_{\mathcal{C}} \frac{X_j(\omega)}{D(\omega)} e^{-i\omega t} d\omega \to 0 \quad \text{as} \quad R \to \infty \text{ and } t \ge t_j(z), \tag{B7}$$

where

$$t_j(z) = \frac{|z|}{V_{Aj}\sin\theta}. \tag{B8}$$

Using the integration by parts after some algebra, we transform Eq. (B3) to

$$Y_1(\omega) = Y_{11}(\omega) + Y_{12}(\omega) + Y_{13}(\omega), \tag{B9}$$

where

$$Y_{11}(\omega) = \frac{i\omega w_0(z)\sin\theta}{\omega^2 - k^2V_{A1}^2\cos^2\theta}, \tag{B10}$$

$$Y_{12}(\omega)l = \frac{2k\omega V_{A1}^2 \sin^2\theta\cos\theta}{(\omega^2 - k^2V_{A1}^2\cos^2\theta)^2}\frac{dw_0}{dz} + \frac{iV_{A1}(V_{A2}-V_{A1})w_0(0)k^2 e^{i\lambda_{1-}z}\sin\theta\cos^2\theta}{(\omega + kV_{A2}\cos\theta)(\omega^2 - k^2V_{A1}^2\cos^2\theta)} - \frac{e^{i\lambda_{1+}z}}{2V_{A1}\lambda_{1+}^2}\int_{-\infty}^{z}\frac{d^2w_0}{dz'^2}e^{-i\lambda_{1+}z'}dz', \tag{B11}$$

$$Y_{13}(\omega) = \frac{e^{i\lambda_{1-}z}}{2V_{A1}}\left[\frac{V_{A1}(V_{A2}-V_{A1})(\omega^2 + k^2V_{A1}^2\cos^2\theta)}{(V_{A1}+V_{A2})(\omega^2 - k^2V_{A1}^2\cos^2\theta)^2}\times\sin^2\theta\frac{dw_0}{dz}\bigg|_{z=0} - \frac{1}{\lambda_{1-}^2}\int_0^\infty\frac{d^2w_0}{dz^2}e^{-i\lambda_{1-}z}dz - \frac{2V_{A1}^2}{\lambda_{2-}^2V_{A2}(V_{A1}+V_{A2})}\int_0^\infty\frac{d^2w_0}{dz^2}e^{-i\lambda_{2-}z}dz - \frac{V_{A2}-V_{A1}}{\lambda_{1+}^2(V_{A1}+V_{A2})}\int_0^\infty\frac{d^2w_0}{dz^2}e^{-i\lambda_{1+}z}dz\right]. \tag{B12}$$

We again take $t \geq t_m$, which guaranties that the product of every integral in Eqs. (B11) and (B12) and $e^{-i\omega t}$ is bounded when $\omega \in \mathcal{C}$ and $R \to \infty$. Using the integration by parts, we obtain

$$\int_{\mathcal{C}} Y_{11}(\omega)\,e^{-i\omega t}d\omega = \frac{i}{t}Y_{11}(\omega).\,e^{-i\omega t}\Big|_{i\varsigma-R}^{i\varsigma+R} - \frac{\sin\theta}{t}\times\int_{\mathcal{C}}\frac{w_0(z)(\omega^2 + k^2V_{A1}^2\cos^2\theta)}{(\omega^2 - k^2V_{A1}^2\cos^2\theta)^2}\times e^{-i\omega t}d\omega \to 0 \tag{B13}$$

as $R \to \infty$. Since $\Re(i\lambda_{1+}) > 0$ and $\Re(-i\lambda_{1-}) > 0$, it follows that $Y_{12}(\omega) = \mathcal{O}(R^{-2})$ for $\omega \in \mathcal{C}$ and $R \gg 1$. Hence, we conclude that

$$\int_{\mathcal{C}} Y_{12}(\omega)\,e^{-i\omega t}d\omega \to 0 \quad \text{as} \quad R \to \infty. \tag{B14}$$

Finally, it follows from Eq. (B12) and the condition $t \geq t_m$ that $Y_{13}(\omega)e^{-i(\lambda_{1-}z+\omega t)} = \mathcal{O}(R^{-2})$ for $\omega \in \mathcal{C}$ and $R \gg 1$. Then it is obvious that $Y_{13}(\omega)$ decays as $R \to \infty$ only if the condition $t \geq t_1(z)$ is satisfied. Hence, we obtain that

$$\int_{\mathcal{C}} Y_{13}(\omega)\,e^{-i\omega t}d\omega \to 0 \tag{B15}$$

as $R \to \infty$ and $t \geq \max[t_m, t_1(z)]$.

Let us calculate the residues in Eq. (B6)

$$\text{res}_{\omega_\pm}\left(\frac{X_1(\omega)}{D(\omega)}e^{-i\omega t}\right) = \lim_{\omega\to\omega_\pm}\frac{(\omega-\omega_\pm)X_1(\omega)}{D(\omega)}e^{-i\omega t} = \pm ie^{-(\gamma\pm i\omega_r)t}\left[U_{1\pm}e^{kz} + W_{1\pm}\times\exp\left(-\frac{[\gamma + i(kV_{A1}\cos\theta\pm\omega_r)]z}{V_{A1}\sin\theta}\right)\right], \tag{B16}$$

where

$$U_{1\pm} = \frac{\rho_2\omega_\pm G(\omega_\pm)(V_{A1}-V_{A2})(\omega_\pm + kV_{A2}e^{i\theta})}{2\omega_r(\rho_1+\rho_2)(\omega_\pm + kV_{A1}e^{-i\theta})}, \tag{B17}$$

$$W_{1\pm} = \frac{G(\omega_\pm)V_{A1}(V_{A2}-V_{A1})}{2\omega_r(\rho_1+\rho_2)(V_{A1}+V_{A2})(\omega_\pm + kV_{A1}e^{-i\theta})}\times[(\rho_1+\rho_2)\omega_\pm^2 + 2ik\omega_\pm\rho_2V_{A2}\sin\theta - 2k^2\rho V_A^2e^{-i\theta}\cos\theta]. \tag{B18}$$

Then, taking $R \to \infty$, we obtain from Eq. (B6) the expression for w valid for $z < 0$ that is given by Eqs. (43)–(45). Recall that this expression is only valid for $z \geq -tV_{A1}\sin\theta$.

In the same way as it was done for $z < 0$, we prove that

$$\int_{\mathcal{C}} \hat{w}(\omega)\,e^{-i\omega t}d\omega \to 0 \tag{B19}$$

as $R \to \infty$, $z > 0$, and $t \geq \max[t_m, t_2(z)]$. Continuing, we obtain the expression for w valid for $t \geq t_m$ and $0 < z \leq tV_{A2}\sin\theta$ that is given by Eqs. (43), (44), and (46), where

$$U_{2\pm} = \frac{\rho_1\omega_\pm G(\omega_\pm)(V_{A2}-V_{A1})(\omega_\pm - kV_{A1}e^{-i\theta})}{2\omega_r(\rho_1+\rho_2)(\omega_\pm - kV_{A2}e^{i\theta})}, \tag{B20}$$

$$W_{2\pm} = \frac{G(\omega_\pm)V_{A2}(V_{A2}-V_{A1})}{2\omega_r(\rho_1+\rho_2)(V_{A1}+V_{A2})(\omega_\pm - kV_{A2}e^{i\theta})}\times[(\rho_1+\rho_2)\omega_\pm^2 + 2ik\omega_\pm\rho_1V_{A1}\sin\theta - 2k^2\rho V_A^2e^{i\theta}\cos\theta]. \tag{B21}$$

APPENDIX C: STUDYING THE LIMIT OF SMALL θ

We start from deriving the asymptotic expression for $G(\omega)$. Using the integration by parts, we obtain

$$\int_{-\infty}^0\frac{w_0(z)}{V_{A1}^2}e^{-i\lambda_{1+}z}dz = \frac{iw_0(0)}{\lambda_{1+}V_{A1}^2} + \frac{1}{\lambda_{1+}^2V_{A1}^2}\frac{dw_0}{dz}\bigg|_{z=0} - \frac{1}{\lambda_{1+}^2V_{A1}^2}\int_{-\infty}^0\frac{d^2w_0}{dz^2}e^{-i\lambda_{1+}z}dz, \tag{C1}$$

$$\int_0^\infty\frac{w_0(z)}{V_{A2}^2}e^{-i\lambda_{2-}z}dz = -\frac{iw_0(0)}{\lambda_{2-}V_{A2}^2} - \frac{1}{\lambda_{2-}^2V_{A2}^2}\frac{dw_0}{dz}\bigg|_{z=0} - \frac{1}{\lambda_{2-}^2V_{A2}^2}\int_0^\infty\frac{d^2w_0}{dz^2}e^{-i\lambda_{2-}z}dz. \tag{C2}$$

It follows from Eq. (20) that $\lambda_{1+} = \mathcal{O}(\theta^{-1})$ and $\lambda_{2-} = \mathcal{O}(\theta^{-1})$. Then, using Eqs. (C1) and (C2), we reduce Eqs. (36) to Eq. (54).

The asymptotic expressions for the magnetic field similar to the expressions for the velocity are given by

$$b_z = \frac{iBw_0(0)}{C_k}e^{-\gamma t}\sin(kC_kt)\begin{cases} e^{kz}, & z < 0, \\ e^{-kz}, & z > 0. \end{cases} \tag{C3}$$

When deriving this expression, we took into account that $b_z = 0$ at $t = 0$.

Finally, we obtain the expression for b_x. Using Eqs. (6)–(8), (45), (46), (52), (53), (55), (56), (60), (61), and (C3) yields

$$b_x = \tilde{b}_x + \bar{b}_x, \tag{C4}$$

where

$$\tilde{b}_x = \frac{Bw_0(0)}{C_k} e^{-\gamma t} \sin(kC_k t) \begin{cases} -e^{kz}, & z < 0, \\ e^{-kz}, & z > 0, \end{cases} \tag{C5}$$

while $\bar{b}_x$ is given by

$$\bar{b}_x = \frac{iw_0(0)BC_k(\rho_1 - \rho_2)\exp[-\gamma t - kz(\Gamma/V_{A1} + i/\theta)]}{2\rho_2(V_{A1} + V_{A2})} \left[\frac{\exp(-ikC_k t) - 1}{C_k + V_{A2}} \exp\left(-\frac{ikC_k z}{\theta V_{A1}}\right) + \frac{\exp(ikC_k t) - 1}{C_k - V_{A2}} \exp\left(\frac{ikC_k z}{\theta V_{A1}}\right) \right] \tag{C6}$$

for $z < 0$, and by

$$\bar{b}_x = \frac{iw_0(0)BC_k(\rho_2 - \rho_1)\exp[-\gamma t + kz(\Gamma/V_{A2} - i/\theta)]}{2\rho_1(V_{A1} + V_{A2})} \left[\frac{\exp(-ikC_k t) - 1}{C_k - V_{A1}} \exp\left(\frac{ikC_k z}{\theta V_{A2}}\right) + \frac{\exp(ikC_k t) - 1}{C_k + V_{A1}} \exp\left(-\frac{ikC_k z}{\theta V_{A2}}\right) \right] \tag{C7}$$

for $z > 0$.

The expressions for Υ_1, Υ_2, and Υ_t are given by

$$\Upsilon_1 = \exp\left[-kz\left(\frac{\Gamma}{v_{A1}} + \frac{i}{\theta}\right)\right] \left\{ \frac{\exp[-ikC_k(t + z/\theta v_{A1})]}{(C_k + v_1)(C_k + v_{A2})} \left[\exp\left(\frac{\Gamma\sqrt{\theta}}{v_{A1}} + \frac{i(1 + C_k/v_{A1})}{\sqrt{\theta}}\right) - \exp\left(-\frac{\Gamma\sqrt{\theta}}{v_{A1}} - \frac{i(1 + C_k/v_{A1})}{\sqrt{\theta}}\right) \right] \right.$$
$$\left. - \frac{\exp[ikC_k(t + z/\theta v_{A1})]}{(C_k - v_{A1})(C_k - v_{A2})} \left[\exp\left(\frac{\Gamma\sqrt{\theta}}{v_{A1}} + \frac{i(1 - C_k/v_{A1})}{\sqrt{\theta}}\right) - \exp\left(-\frac{\Gamma\sqrt{\theta}}{v_{A1}} - \frac{i(1 - C_k/v_{A1})}{\sqrt{\theta}}\right) \right] \right\}, \tag{C8}$$

$$\Upsilon_2 = \exp\left[kz\left(\frac{\Gamma}{v_{A2}} - \frac{i}{\theta}\right)\right] \left\{ \frac{\exp[-ikC_k(t - z/\theta v_{A2})]}{(C_k - v_{A1})(C_k - v_{A2})} \left[\exp\left(\frac{\Gamma\sqrt{\theta}}{v_{A2}} - \frac{i(1 - C_k/v_{A2})}{\sqrt{\theta}}\right) - \exp\left(-\frac{\Gamma\sqrt{\theta}}{v_{A2}} + \frac{i(1 - C_k/v_{A2})}{\sqrt{\theta}}\right) \right] \right.$$
$$\left. - \frac{\exp[ikC_k(t - z/\theta v_{A2})]}{(C_k + v_{A1})(C_k + v_{A2})} \left[\exp\left(\frac{\Gamma\sqrt{\theta}}{v_{A2}} - \frac{i(1 + C_k/v_{A2})}{\sqrt{\theta}}\right) - \exp\left(-\frac{\Gamma\sqrt{\theta}}{v_{A2}} + \frac{i(1 + C_k/v_{A2})}{\sqrt{\theta}}\right) \right] \right\}, \tag{C9}$$

$$\Upsilon_t = \frac{4i(C_k^2 + v_{A1}v_{A2})\sin(kC_k t)}{(C_k^2 - v1^2)(C_k^2 - v_{A2}^2)} + e^{-ikz/\theta} \left(\frac{\exp[\sigma_{1+} - ikC_k t] - \exp[\sigma_{2-} + ikC_k t]}{(C_k + v_{A1})(C_k + v_{A2})} - \frac{\exp[\sigma_{1-} + ikC_k t] - \exp[\sigma_{2+} - ikC_k t]}{(C_k - v_{A1})(C_k - v_{A2})} \right), \tag{C10}$$

where

$$\sigma_{1\pm} = \frac{\sqrt{\theta} - kz}{v_{A1}} \left(\Gamma \pm \frac{iC_k}{\theta}\right) + \frac{i}{\sqrt{\theta}}, \quad \sigma_{2\pm} = \frac{\sqrt{\theta} + kz}{v_{A2}} \left(\Gamma \mp \frac{iC_k}{\theta}\right) - \frac{i}{\sqrt{\theta}}. \tag{C11}$$

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