
**Thermodynamic Efficiency Gains and their Role as a
Key 'Engine of Economic Growth'**

**Supplementary Materials: The UK
MAcroeconometric Resource CONsumption Model
(MARCO-UK)**

S1. Introduction

The UK MAcroeconomic Resource COnsumption (MARCO-UK) model is a macroeconomic representation of the UK economy with a particular emphasis on the demand for energy and materials. The model describes the behaviour of different economic agents and their interrelationships and relations with other elements of the economy.

The main objective of the MARCO-UK model is to provide a better understanding of the macroeconomic effects in the UK derived from policy changes aimed at reducing energy, resource use and emissions. The model is useful to conduct ex-post and ex-ante simulations.

Alike other macroeconomic models, MARCO-UK is a demand-driven model, following the tradition of other similar post-Keynesian-related models, such as E3ME[1], developed by Cambridge Econometrics, and the macroeconomic model used by the Office for Budget Responsibility (OBR)[2].

Macroeconomic models generally contain two types of equations. The first type involves definitional relationships, also known as ‘identities’, which represent definitions of given variables, which must hold true in all time periods. For example, gross domestic product (GDP) is defined as $GDP=C+I+G+X-M$; that is, the sum of private (C) and public (G) consumption, investment (I), plus net exports (X-M). This identity must hold each year covered by the model. The second type of equations are known as ‘behavioural’ or ‘stochastic’. We refer to them for simplicity in this paper as ‘econometric’ equations. Their main difference in relation to identities is that these equations are empirically validated to capture the structure of the economy. In simple terms, econometric equations contain parameters that are estimated using rigorous econometric approaches.

Note on the current version (v1.0): This version is highly aggregated, containing only two sectors: industry and the rest of the economy (agriculture and services). Moreover, material use is not included in this version.

S2. Additional model assumptions

Apart from the assumptions presented in the ‘methods’ section in the main body of the article, some other key assumptions are described here.

The model recognises the importance of supply-side factors on the economy in the long run. Factor productivity determines the level of investment in capital, which expands economic growth in the long term. Moreover, since the model solution does not entail any optimisation, non-optimal capital allocation is possible. Labour, on the other hand, is assumed to be substitutable with capital and energy. It is determined through a simple Cobb-Douglas production function, which is solved for labour rather than for output. Decisions on how much labour to use has consequences on the level of capital and long-term growth. Useful exergy is also similarly determined through a Cobb-Douglas production function, and its level likewise influences long-term growth.

Unemployment is incorporated in the model and is calculated as the difference between the labour force and the amount of people that are actually at work. However, there are no restrictions on the unemployment level, which can grow without compromising growth.

Furthermore, prices do not adjust automatically after a shock. Prices, as well as wages, are considered to be ‘sticky’ and adjust gradually. Due to the use of time lags in the econometric equations, prices tend to react after certain number of time periods.

S3. Model equations

The model contains 57 equations, of which 30 are identities and 27 are econometric. These are described next.

Gross domestic product (GDP)

From the expenditure side, GDP (Y) is defined as the sum of total consumption (C), investment (I), government expenditure (G), net exports (X-M), plus a statistical difference (STAT1) (as reported by the ONS).

$$Y_t = CT_t + I_t + G_t + X_t - M_t + STAT1_t$$

From the income side, GDP (Y) is defined as the sum of revenue by private firms (i.e. gross operating surplus of corporations and other income) (YF), total compensation of employees (W), government income (taxes minus subsidies) (YG), plus a statistical difference (STAT2). In the model, this equation is solved for YF, since Y is already defined as an endogenous variable.

$$YF_t = Y_t - W_t - YG_t - STAT2_t$$

In addition, GDP (Y) is also defined as the sum of gross value added (GVA), net taxes (NET_TAX) and a statistical difference (STAT3). This equation is solved for GVA, since Y is already defined as an endogenous variable.

$$GVA_t = Y_t - NET_TAX_t - STAT3_t$$

Sectoral GDP

In the model, it is assumed that all the income is spent. In this sense, from the expenditure side, it can be said that GDP (Y) is equal to total expenditure by industry (IND_T) and total expenditure by other sectors (i.e. agriculture and services) (OTH_T). The following equation is solved for OTH_T, since Y has already been defined as an endogenous variable.

$$OTH_T_t = Y_t - IND_T_t - STAT1_t$$

Total expenditure by industry (IND_T) is the sum of industry expenditure on non-energy (IND_NE) goods and energy (IND_E).

$$IND_T_t = IND_NE_t + IND_E_t$$

Industry non-energy (IND_NE) is a function of investment (I), total useful exergy (UEX_TOT) and the real interest rate (R_REAL). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$IND_NE_t = f(I_t, UEX_TOT_t, R_REAL_t)$$

Industry energy, in turn, is calculated by the physical amount of final energy used by industry (FEN_IND) multiplied by its price (P_EN_IND).

$$IND_E_t = ((P_EN_IND_t) / CPI_t) * FEN_IND_t$$

For the remaining sectors, their expenditure on energy is similarly calculated by the physical amount of final energy used by other sectors (FEN_OTH) multiplied by its price (P_EN_OTH).

$$OTH_E_t = ((P_EN_OTH_t) / CPI_t) * FEN_OTH_t$$

Expenditure on non-energy by other sectors (OTH_NE) is the difference between the total (OTH_T) and their expenditure on energy (OTH_E).

$$OTH_NE_t = OTH_T_t - OTH_E_t$$

Consumption

Total consumption (CT) is the sum of consumption expenditure on non-energy (CNE) and energy goods (CE).

$$CT_t = CNE_t + CE_t$$

Consumption of non-energy goods (CNE) is a function of disposable income (YD), wages (W) and total useful exergy (UEX_TOT). This function represents the long-term specification. The short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$CNE_t = f(YD_t, W_t, UEX_TOT_t)$$

Consumption of energy goods (CE) is given by the physical amount of final energy used by households (FEN_C) multiplied by its price (P_EN_C).

$$CE_t = ((P_EN_C_t) / (CPI_t / 100)) * FEN_C_t$$

Investment

Investment (I) by private firms is expressed as a function of profits made by firms (YF), capital productivity (Y/K_NET), the productivity of useful exergy (Y/UEX_TOT), and labour productivity (Y/L). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$I_t = f(YF_t, Y_t/K_NET_t, Y_t/UEX_TOT_t, Y_t/L_t)$$

Exports

Exports (X) are a function of GDP from the rest of the world (Y_RW), the price of exports (PX) and total useful exergy (UEX_TOT). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$X_t = f(Y_RW_t, PX_t, UEX_TOT_t)$$

Imports

Imports are a function of total consumption expenditure (CT), GDP from the rest of the world (Y_RW), and the real exchange rate (E_INDEX_REAL). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$M_t = f(CT_t, Y_RW_t, E_INDEX_REAL_t)$$

Trade balance

The trade balance is simply defined as exports (X) minus imports (M).

$$TB_t = X_t - M_t$$

Capital stock

Gross capital stock (K_GRS) is defined as the existing stock in time $t-1$ plus the flow of gross fixed capital formation (I) in period t , minus the amount of obsolete capital that is retired from use (i.e. assets at end of life and loss from scrapping) (K_RETIRE).

$$K_GRS_t = K_GRS_{t-1} + I_t - K_RETIRE_t$$

Net capital stock (K_NET), in turn, is the gross capital stock (K_GRS) minus the depreciation of fixed (DEP_FIX) and non-fixed assets (DEP_NFIX).

$$K_NET_t = K_GRS_t - DEP_FIX_t - DEP_NFIX_t$$

Capital services (K_SERV) is calculated by multiplying the net stock (K_NET) by an index (K_serv_index).

$$K_SERV_t = K_NET_t * K_serv_index_t$$

Depreciation of fixed assets is equal to the depreciation rate multiplied by the net capital stock.

$$DEP_FIX_t = DEP_RATE_t * K_NET_t$$

Labour

Labour force (LF) is defined as the amount of people in the UK economy that are available to work. It is a function of the labour force in time $t-1$, GDP (Y) and population (POP).

$$LF_t = f(LF_{t-1}, Y_t, POP_t)$$

Labour supply represents the amount of people that are actually employed. It is a function of GDP (Y) and the other two factors of production: capital services (K_SERV) and total useful exergy (UEX_TOT). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$L_t = f(Y_t, K_SERV_t, UEX_TOT_t)$$

Quality-corrected labour is calculated by multiplying labour (L) by two indices: the average annual hours worked by persons engaged (L_HRS_INDEX) and the human capital index, based on years of schooling and returns to education (L_HC_INDEX).

$$HL_t = L_t * L_HRS_INDEX_t * L_HC_INDEX_t$$

Labour productivity is simply the ratio between GDP (Y) and labour (L).

$$YL_t = Y_t / L_t$$

The unemployment rate (UR) is the percentage of people that are out of work, according to the following equation:

$$UR_t = ((LF_t - L_t) / LF_t) * 100$$

Disposable income and net wealth

Disposable income is a function of wages (W) and net wealth (NW). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$YD_t = f(W_t, NW_t)$$

Net wealth (NW) is a function of its value in time $t-1$, profits made by firms (YF), the unemployment rate (UR) and disposable income (YD).

$$NW_t = f(NW_{t-1}, UR_t, YD_t)$$

Savings, on the other hand, is defined as a ratio (S_RATIO) given as the percentage of disposable income (YD) that is not destined to total consumption expenditure (CT).

$$S_RATIO_t = ((YD_t - CT_t) / YD_t) * 100$$

Wages

Total remuneration to employees (W) is a function of profits received by firms (YF), average hourly wages (W_HOUR), the consumer price index (CPI) and quality adjusted labour (HL). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$W_t = f(YF_t, W_HOUR_t, CPI_t, HL_t)$$

Average hourly wages, in turn, is a function of its own level in the previous period ($t-1$), the consumer price index (CPI), labour productivity (Y/L) and the unemployment rate (UR).

$$W_HOUR_t = f(W_HOUR_{t-1}, CPI_t, Y_t/L_t, UR_t)$$

Energy

The model makes a distinction between primary energy (PEN), primary exergy (PEX), total final energy (FEN_T) and total useful exergy (UEX_TOT). Primary exergy (PEX) is calculated by dividing total final energy (FEN_T) by the efficiency to transform primary energy into final energy ($EXEFF_PF$).

$$PEX_t = FEN_T_t / EXEFF_PF_t$$

Primary energy, in turn, is calculated by dividing primary exergy (PEX) by the ratio between primary exergy and primary energy (PEX_PEN_RATIO).

$$PEN_t = PEX_t / PEX_PEN_RATIO_t$$

Total final energy is given by the sum of final energy used by households (FEN_C), industry (FEN_IND) and remaining sectors (i.e. agriculture and services) (FEN_OTH).

$$FEN_T_t = FEN_C_t + FEN_IND_t + FEN_OTH_t$$

Final energy used by households (FEN_C) is a function of energy prices faced by households (P_EN_C), total useful exergy (UEX_TOT), heating degree days (HDD) and average hourly wages (W_HOUR). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$FEN_{Ct} = f(P_{EN_Ct}, UEX_{TOTt}, HDD_t, W_HOUR_t)$$

Final energy used by industry (FEN_IND) is a function of prices faced by industry (P_EN_IND), total useful exergy (UEX_TOT) and the level of imports (M). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$FEN_{INDt} = f(P_{EN_INDt}, UEX_{TOTt}, M_t)$$

Final energy used by other sectors (FEN_OTH) is a function of its own level in time $t-1$, prices faced by other sectors (P_EN_OTH) and total useful exergy (UEX_TOT).

$$FEN_{OTHt} = f(FEN_{OTH_{t-1}}, P_{EN_OTHt}, UEX_{TOTt})$$

Finally, total useful exergy (UEX_TOT) is a function of its own level in time $t-1$, quality-adjusted labour, gross capital stock (K_GRS) and GDP (Y).

$$UEX_{TOTt} = f(UEX_{TOT_{t-1}}, HL_t, K_GRSt, Y_t)$$

Energy efficiency

Total energy efficiency (EN_EFF_TOT) is expressed as the combined efficiency of transforming primary energy to final (EXEFF_PF) and the efficiency to transform final energy to its useful state (EXEFF_FU).

$$EN_EFF_TOT_t = EXEFF_PF_t * EXEFF_FU_t$$

The efficiency to transform final energy to useful exergy (EXEFF_FU) is simply a ratio between total useful exergy (UEX_TOT) and total final energy (FEN_T).

$$EXEFF_FU_t = UEX_{TOTt} / FEN_T_t$$

Energy intensity (EY), which is often used as a proxy for energy efficiency, is calculated as the ratio between total final energy (FEN_T) and GDP (Y).

$$EY_t = FEN_T_t / Y_t$$

The energy-GDP ratio (EN_GDP_RATIO) is calculated by dividing total final expenditure (i.e. the sum of energy expenditure by households, industry and other sectors) by GDP (Y).

$$EN_GDP_RATIO_t = (CE_t + IND_E_t + OTH_E_t) / Y_t$$

Prices

The general level of prices is represented by the consumer price index (CPI), which is a function of general price of energy (CPI_E), the price of imports (PM), wage productivity (W/Y) and the real exchange rate (E_INDEX_REAL). A short-term specification is used with variables expressed in

differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$CPI_t = f(CPI_{E_t}, PM_t, W_t/Y_t, E_INDEX_REAL_t)$$

The general price of energy (CPI_E) is a function of its own levels in the previous period ($t-1$), and the energy prices for households (P_EN_C), industry (P_EN_IND) and others (P_EN_OTH).

$$CPI_{E_t} = f(CPI_{E_{t-1}}, P_EN_C_t, P_EN_IND_t, P_EN_OTH_t)$$

The relative price of energy is thus calculated as the ratio between the general price of energy (CPI_E) and the consumer price index (CPI).

$$CPI_REL_EN_t = CPI_{E_t} / CPI_t$$

Energy prices for households (P_EN_C) is expressed as a demand function, given by the amount of final energy consumed by households (FEN_C) and the consumer price index (CPI). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$P_EN_C_t = f(P_EN_C_{t-1}, FEN_C_t, CPI_t)$$

Similarly, energy prices for industry (P_EN_IND) is a function of the amount of final energy consumed by industry (FEN_IND) and the consumer price index (CPI). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$P_EN_IND_t = f(FEN_IND_t, CPI_t)$$

Finally, energy prices for other sectors (P_EN_OTH) is a function of the amount of final energy consumed by households (FEN_C) and the consumer price index (CPI). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$P_EN_OTH_t = f(FEN_OTH_t, CPI_t)$$

Price of exports is given by its own level in the previous period ($t-1$), the consumer price index (CPI), GDP (Y) and the real exchange rate (E_INDEX_REAL).

$$PX_t = f(PX_{t-1}, CPI_t, Y_t, E_INDEX_REAL_t)$$

The price of imports is a function of its own level in the previous period ($t-1$), GDP (Y) relative to that of the rest of the world (Y/Y_RW), and the real exchange rate (E_INDEX_REAL).

$$PM_t = f(PM_{t-1}, Y_t/Y_RW_t, E_INDEX_REAL_t)$$

Inflation is simply the change of the consumer price index from period $t-1$ to period t .

$$INF_t = (((CPI_t / CPI_{t-1}) - 1) * 100)$$

Money supply and interest rates

Money supply (MS) is a function of its own level in time $t-1$, GDP (Y), the real interest rate (R_REAL), inflation (INF) and the savings ratio (S_RATIO).

$$MS_t = f(MS_{t-1}, Y_t, R_REAL_t, INCf_t, S_RATIO_t)$$

The nominal interest rate (R_NOM) is a function of its own level in the previous period ($t-1$), money supply, GDP (Y) and the consumer price index (CPI).

$$R_NOM_t = f(R_NOM_{t-1}, MS_t, Y_t, CPI_t)$$

The real interest rate (R_REAL) is calculated by subtracting inflationary effects from the nominal interest rate (R_NOM).

$$R_REAL_t = R_NOM_t - INF_t$$

Exchange rate

The nominal exchange rate (E_INDEX_NOM) is a function of its own level in the previous period ($t-1$), the relative price of imports (CPI/PM), the ratio between domestic GDP (Y) and GDP from the rest of the world (Y_RW) and the nominal interest rate (R_NOM).

$$E_INDEX_NOM_t = f(E_INDEX_NOM_{t-1}, CPI_t/PM_t, Y_t/Y_RW_t, R_NOM_t)$$

The real exchange rate is calculated by multiplying the nominal exchange rate (E_INDEX_NOM) by the relative price of imports (CPI/PM).

$$E_INDEX_REAL_t = E_INDEX_NOM_t * (CPI_t / PM_t)$$

CO2 emissions

CO2 per capita (CO2_TERR/POP), from the territorial perspective, is expressed in the reduced form of the Kaya identity, being a function of GDP per capita (Y/POP) and (primary) energy intensity (PEN/Y). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$CO2_TERR_t/POP_t = f(Y_t/POP_t, PEN_t/Y_t)$$

From a consumption approach, CO2 per capita (CO2_CONS/POP) is a function of its own level in the previous period ($t-1$), GDP per capita (Y/POP), (primary) energy intensity (PEN/Y) and imports per capita (M/POP). A short-term specification is used with variables expressed in differenced logs, incorporating lags of the endogenous and exogenous variables and an error correction term.

$$CO2_CONS_t/POP_t = f(CO2_CONS_{t-1}/POP_{t-1}), Y_t/POP_t, PEN_t/Y_t, M_t/POP_t)$$

Exogenous variables

There are 17 exogenous variables in the model:

- G – Government (public) expenditure
- YG – Government income
- NET_TAX – Net taxes
- Y_RW – GDP from the rest of the world
- K_RETIRE – Capital retired from use

K_SERV_INDEX – Capital services index
DEP_NFIX – Depreciation of non-fixed assets
DEP_RATE – Depreciation rate
L_HC_INDEX – Average annual hours worked by persons engaged
L_HRS_INDEX – Human capital index, based on years of schooling and returns to education
EXEFF_PF – Energy efficiency to transform primary to final energy
PEX_PEN_RATIO
HDD – Heating degree days
POP – Population
STAT1 – GDP Statistical differences (as reported by the ONS in the national accounts)
STAT2 – GDP Statistical differences (as reported by the ONS in the national accounts)
STAT3 – GDP Statistical differences (as reported by the ONS in the national accounts)

S4. Estimated parameters

The econometric equations are estimated using Ordinary Least Squares (OLS). In some cases, where it is found that the involved variables share common long-term equilibrium relationships using the Johansen cointegration test, long-run and short-run specifications are estimated following the Engle-Granger approach[3]. A long-term specification is estimated using the variables in levels (i.e. without differencing) and without including time lags. Some of these specifications present autocorrelation (i.e. low Durbin-Watson stats) and heteroscedasticity in the residuals, but they are not used directly in the model. Only the residuals from these equations are used by introducing them into the short-term specifications. Short-term specifications are then estimated using the variables expressed in first differences including lags for all variables. The residuals from the long-term specifications are then incorporated, but lagged one time period, resulting in an error correction term (ECT). The estimated parameter of the ECT must be significant and negative, in which case the existence of a cointegrating relationship can be confirmed. Finally, the residuals from the short-term specification are tested for autocorrelation, heteroscedasticity, serial correlation and normality to ensure the robustness of the estimation. The residuals are also tested for the presence of unit roots and confirm that they are considered as ‘white noise’.

An important issue is that econometric simultaneous equation systems often violate the assumption of no correlation between the residuals of endogenous variables and the independent variables. Such violation occurs when endogenous variables are also treated as exogenous in other equations, giving rise to simultaneity bias. Correction can be achieved using two-stage least squares (TSLS) rather than OLS solution methods. However, TSLS methods can yield less accurate simulations due to data errors or sampling bias[4]. Moreover, due to the size of the model, TSLS is not feasible due to the large number of instrumental variables and the use of cointegration techniques. Therefore, we estimate the equations using OLS following the procedures suggested by Brillet^[5].

Dummy variables are used in some equations, acknowledging that their inclusion is restricted by the degrees of freedom in the data, in order to capture certain periods in the UK economy characterised by unusual movements in the time-series data. Several variables present signs of structural break. When estimating an equation, the Chow structural breakpoint test is applied to determine the year when the trends change. A dummy variable is then included in order to capture the break and avoid heteroscedasticity in the residuals. Structural breaks were found mainly for 1976 and 1982, years of economic crisis. Moreover, a dummy for the year 2009 was often used to capture the effects of the global financial crisis. To avoid repetition, dummy variables are not presented in the estimations below. However, they constitute an important element in the solution of the model.

The econometric equations are estimated following an iterative approach in order to ensure that the model is able to run and to replicate as much as possible the historical behaviour of the variables involved. Once an individual equation has been estimated, it is inserted into the model and tested. If the residuals derived from any of the estimated variables in the model present any issues, such as instability or heteroscedasticity (i.e. fail to track the historical behaviour), then the equation is revised

and estimated again in order to resolve the problem. This procedure is repeated until a statistically-robust basefit is produced.

The estimated parameters for each econometric equation are presented next. All the variables are expressed in logs, except when noted.

Consumption of non-energy goods

Long-term specification:

Dependent variable $\log(\text{CNE})$

| Variables | Estimated coefficients | t-stats |
|-------------------------|------------------------|---------|
| Constant | -3.8195*** | -4.3044 |
| $\log(\text{YD})$ | 0.8192*** | 11.6631 |
| $\log(\text{UEX_TOT})$ | 0.1868** | 2.0455 |
| $\log(\text{W})$ | 0.2644*** | 3.6330 |
| R ² | | 0.9966 |
| DW | | 0.5550 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{CNE})$

| Variables | Estimated coefficients | t-stats |
|-------------------------------|------------------------|---------|
| Error Correction Term (ECT) | -0.3889*** | -4.3860 |
| $\Delta\log(\text{CNE}(-1))$ | 0.4610*** | 4.5834 |
| $\Delta\log(\text{CNE}(-2))$ | -0.2878*** | -3.2377 |
| $\Delta\log(\text{YD})$ | 0.5494*** | 5.8016 |
| $\Delta\log(\text{UEX_TOT})$ | 0.1512** | 2.5014 |
| $\Delta\log(\text{W})$ | 0.3426*** | 3.0243 |
| R ² | | 0.8391 |
| DW | | 2.0325 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Investment

Long-term specification:

Dependent variable $\log(\text{I})$

| Variables | Estimated coefficients | t-stats |
|--|------------------------|---------|
| Constant | -6.6667*** | -3.9349 |
| $\log(\text{PROFIT})$ | 0.8180*** | 4.9451 |
| $\log(\text{Y})-\log(\text{K_NET})$ | 1.7091*** | 5.7128 |
| $\log(\text{Y})-\log(\text{UEX_TOT})$ | -0.2498 | -1.6765 |
| $\log(\text{Y})-\log(\text{L})$ | -0.4969* | -2.0053 |
| R ² | | 0.9630 |
| DW | | 1.1333 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(I)$

| Variables | Estimated coefficients | t-stats |
|---|------------------------|---------|
| ECT | -0.4093*** | -3.0185 |
| $\Delta\log(\text{PROFIT})$ | 0.3817** | 2.0803 |
| $\Delta(\log(Y)-\log(K_NET))$ | 2.7435*** | 5.5522 |
| $\Delta(\log(Y(-1))-\log(\text{UEX_TOT}(-1)))$ | 0.4232* | 1.8429 |
| $\Delta(\log(Y(-2))-\log(L(-2)))$ | -0.7138** | -2.4244 |
| R ² | | 0.8361 |
| DW | | 1.7681 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Exports

Long-term specification:

Dependent variable $\log(X)$

| Variables | Estimated coefficients | t-stats |
|-------------------------|------------------------|---------|
| $\log(Y_RW)$ | 0.2116* | 1.8375 |
| $\log(PX)$ | -0.0935*** | -3.2339 |
| $\log(\text{UEX_TOT})$ | 0.4348* | 1.8793 |
| time trend | 0.0352*** | 29.4273 |
| R ² | | 0.9927 |
| DW | | 0.7340 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(X)$

| Variables | Estimated coefficients | t-stats |
|-------------------------------|------------------------|---------|
| ECT | -0.4800*** | -5.6523 |
| $\Delta\log(X(-1))$ | 0.1467 | 1.6234 |
| $\Delta\log(Y_RW)$ | 0.3784** | 2.5134 |
| $\Delta\log(PX)$ | 0.1303** | 2.2218 |
| $\Delta\log(\text{UEX_TOT})$ | 0.2300 | 1.5721 |
| R ² | | 0.6365 |
| DW | | 1.7520 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Imports

Long-term specification:

Dependent variable $\log(M)$

| Variables | Estimated coefficients | t-stats |
|---------------|------------------------|---------|
| Constant | -14.7779*** | -6.6870 |
| $\log(C_T)$ | 1.4765*** | 14.0289 |
| $\log(Y_RW)$ | 0.2772** | 2.1734 |
| $\log(PM)$ | -0.1143*** | -4.0545 |

| | |
|----------------|--------|
| R ² | 0.9978 |
| DW | 1.2035 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(M)$

| Variables | Estimated coefficients | t-stats |
|----------------------------------|------------------------|---------|
| ECT | -0.7850*** | -7.0120 |
| $\Delta\log(M(-2))$ | -0.2162*** | -3.2239 |
| $\Delta\log(M(-3))$ | -0.0832 | -1.4058 |
| $\Delta\log(C_T)$ | 1.3461*** | 9.1013 |
| $\Delta\log(Y_RW)$ | 0.6683*** | 4.9923 |
| $\Delta\log(E_INDEX_REAL(-3))$ | -0.0738* | -1.7420 |
| R ² | | 0.8836 |
| DW | | 1.9441 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Industrial non-energy value added

Long-term specification:

Dependent variable $\log(IND_NE)$

| Variables | Estimated coefficients | t-stats |
|------------------|------------------------|---------|
| Constant | 6.2425*** | 5.5430 |
| $\log(I)$ | 0.1784*** | 3.2194 |
| $\log(UEX_TOT)$ | 0.2956*** | 3.1020 |
| R_REAL | 0.0066*** | 5.3448 |
| time trend | 0.0022** | 2.0978 |
| R ² | | 0.9532 |
| DW | | 1.0760 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(IND_NE)$

| Variables | Estimated coefficients | t-stats |
|----------------------------|------------------------|---------|
| ECT | -0.4663** | -2.6534 |
| $\Delta\log(IND_NE(-2))$ | 0.2493** | 2.2046 |
| $\Delta\log(I)$ | 0.2619*** | 6.1465 |
| $\Delta\log(UEX_TOT(-1))$ | 0.1924* | 1.9986 |
| $\Delta(R_REAL)$ | 0.0022 | 1.4154 |
| R ² | | 0.7094 |
| DW | | 1.9006 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Wages

Dependent variable $\log(W)$

| Variables | Estimated coefficients | t-stats |
|-----------|------------------------|---------|
| Constant | -4.8482*** | -5.6489 |

| | | |
|----------------|------------|---------|
| log(W(-1)) | 0.5627*** | 9.1347 |
| log(YF(-1)) | 0.0702*** | 2.8359 |
| log(W_HOUR) | 0.3455*** | 7.5483 |
| log(CPI) | -0.0436*** | -5.1839 |
| log(HL) | 0.6283*** | 7.0222 |
| log(HL(-1)) | -0.3045*** | -3.6446 |
| R ² | | 0.9995 |
| DW | | 1.9137 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Disposable income

Long-term specification:

Dependent variable log(YD)

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| Constant | 4.1731*** | 3.0289 |
| log(W) | 0.4419*** | 4.9118 |
| log(NW) | 0.2138*** | 4.5010 |
| Time trend | 0.0100*** | 3.4166 |
| R ² | | 0.9914 |
| DW | | 0.3380 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(YD)$

| Variables | Estimated coefficients | t-stats |
|----------------------|------------------------|---------|
| ECT | -0.1636* | -1.9870 |
| $\Delta\log(YD(-1))$ | 0.3086*** | 3.0816 |
| $\Delta\log(W)$ | 0.4360*** | 3.2382 |
| $\Delta\log(NW)$ | 0.1164** | 2.4487 |
| R ² | | 0.5634 |
| DW | | 1.7527 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Net wealth

Dependent variable log(NW)

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| log(NW(-1)) | 0.8903*** | 22.6930 |
| $\Delta(Ur)$ | -0.0196** | -2.4891 |
| log(YD) | 0.1269*** | 2.8744 |
| R ² | | 0.9938 |
| DW | | 1.7252 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Labour force

Dependent variable log(LF)

| Variables | Estimated coefficients | t-stats |
|-------------|------------------------|---------|
| log(LF(-1)) | 1.3720*** | 12.5147 |

| | | |
|----------------|------------|---------|
| log(LF(-2)) | -0.5117*** | -4.8660 |
| log(Y) | 0.0203*** | 3.7328 |
| log(POP) | 0.1050*** | 3.0545 |
| R ² | | 0.9962 |
| DW | | 1.8470 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Labour supply

Long-term specification:

Dependent variable log(L)

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| Constant | 5.4259*** | 8.3887 |
| LOG(Y) | 0.5694*** | 10.6527 |
| LOG(UEX_TOT) | -0.1177* | -1.9264 |
| LOG(K_SERV) | -0.1808*** | -6.1863 |
| R ² | | 0.9439 |
| DW | | 0.5616 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(L)$

| Variables | Estimated coefficients | t-stats |
|------------------|------------------------|---------|
| ECT | -0.2802*** | -4.9165 |
| DLOG(L(-1)) | 0.5263*** | 7.2403 |
| DLOG(Y) | 0.3190*** | 7.8693 |
| DLOG(UEX_TOT) | -0.0599* | -1.9645 |
| DLOG(K_SERV(-1)) | -0.1151*** | -4.1236 |
| R ² | | 0.8747 |
| DW | | 1.8900 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Consumer price index

Long-term specification:

Dependent variable log(CPI)

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|----------|
| Constant | -1.2706*** | -18.9318 |
| log(CPI_E) | 0.1307*** | 4.5023 |
| log(PM) | 1.1823*** | 26.6341 |
| log(W)-log(Y) | -0.3321** | -2.5997 |
| R ² | | 0.9981 |
| DW | | 1.2484 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{CPI})$

| Variables | Estimated coefficients | t-stats |
|---|------------------------|---------|
| ECT | -0.2970*** | -3.3175 |
| $\Delta\log(\text{CPI_E})$ | 0.0855* | 1.8709 |
| $\Delta\log(\text{PM})$ | 1.1891*** | 15.4800 |
| $\Delta\log(\text{W}(-4)) - \Delta\log(\text{Y}(-4))$ | 0.3004** | 2.6977 |
| $\Delta(\text{E_INDEX_REAL})$ | 0.0011*** | 4.5750 |
| R ² | | 0.9336 |
| DW | | 1.8968 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Consumer price index (energy)

Dependent variable $\log(\text{CPI_E})$

| Variables | Estimated coefficients | t-stats |
|-----------------------------|------------------------|---------|
| Constant | 1.1140*** | 11.4422 |
| $\log(\text{CPI_E}(-1))$ | 0.6842*** | 9.2486 |
| $\log(\text{CPI_E}(-2))$ | -0.2215*** | -2.8588 |
| $\log(\text{P_EN_C}(-2))$ | 0.0574 | 1.2811 |
| $\log(\text{P_EN_IND})$ | 0.2004*** | 8.8205 |
| $\log(\text{P_EN_OTH})$ | 0.2191*** | 7.9590 |
| R ² | | 0.9993 |
| DW | | 2.2372 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Export prices

Dependent variable $\log(\text{PX})$

| Variables | Estimated coefficients | t-stats |
|---------------------------------|------------------------|---------|
| $\log(\text{PX}(-1))$ | 0.9580*** | 84.3690 |
| $\Delta\log(\text{CPI})$ | 1.2809*** | 8.8076 |
| $\Delta\log(\text{CPI}(-1))$ | -0.4073*** | -3.7871 |
| $\log(\text{Y}(-2))$ | 0.0126*** | 3.4390 |
| $\Delta(\text{E_INDEX_REAL})$ | -0.0033*** | -8.9488 |
| R ² | | 0.9988 |
| DW | | 2.2290 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Import prices

Dependent variable $\log(\text{PM})$

| Variables | Estimated coefficients | t-stats |
|---|------------------------|----------|
| $\log(\text{PM}(-1))$ | 0.9360*** | 182.6190 |
| $\log(\text{Y}) - \log(\text{Y_RW})$ | -0.0194*** | -14.0782 |
| $\Delta\log(\text{E_INDEX_REAL}(-1))$ | 0.0785** | 2.3945 |
| R ² | | 0.9993 |
| DW | | 1.5919 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Average hourly wages

Dependent variable log(W_HOUR)

| Variables | Estimated coefficients | t-stats |
|-----------------|------------------------|---------|
| log(W_HOUR(-1)) | 0.9152*** | 11.1976 |
| log(W_HOUR(-3)) | -0.1879*** | -3.1618 |
| log(CPI) | 0.0401*** | 2.9947 |
| log(Y)-log(L) | 0.1471*** | 6.2213 |
| UR(-1) | -0.0073*** | -4.9766 |
| R ² | | 0.9967 |
| DW | | 2.2465 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Nominal interest rate

Dependent variable R_NOM

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| R_NOM(-1) | 0.6391*** | 7.9428 |
| log(MS) | -4.5027*** | -3.6791 |
| log(Y) | 4.5855*** | 3.6866 |
| Δlog(CPI) | 34.1850*** | 4.5826 |
| Δlog(CPI(-2)) | -28.8548*** | -4.5000 |
| R ² | | 0.9370 |
| DW | | 2.3065 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Nominal exchange rate

Dependent variable LOG(E_INDEX_NOM)

| Variables | Estimated coefficients | t-stats |
|--------------------------|------------------------|---------|
| LOG(E_INDEX_NOM(-1)) | 1.0932*** | 8.9948 |
| LOG(E_INDEX_NOM(-2)) | -0.2902** | -2.2562 |
| LOG(CPI(-1))-LOG(PM(-1)) | -0.1298** | -2.4406 |
| LOG(Y)-LOG(Y_RW) | -0.0668** | -2.0974 |
| D(R_NOM) | 0.0086** | 2.3420 |
| R ² | | 0.8848 |
| DW | | 2.3004 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Money supply

Dependent variable log(MS)

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| LOG(MS(-1)) | 1.3656*** | 17.0355 |
| LOG(MS(-2)) | -0.4752*** | -7.0005 |
| LOG(Y(-2)) | 0.1150*** | 5.8449 |
| D(R_REAL) | 0.0028** | 2.5265 |
| INF | -0.0059*** | -6.9468 |
| S_RATIO | -0.0046*** | -3.6365 |
| R ² | | 0.9995 |
| DW | | 2.1737 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Useful exergy (total)

Dependent variable log(UEX_TOT)

| Variables | Estimated coefficients | t-stats |
|------------------|------------------------|---------|
| log(UEX_TOT(-1)) | 0.4794*** | 3.7383 |
| log(UEX_TOT(-2)) | 0.3040** | 2.2302 |
| log(HL(-1)) | 0.5910* | 1.9685 |
| log(HL(-2)) | -0.4456* | -1.9114 |
| log(Y) | 0.8887*** | 5.2997 |
| log(Y(-1)) | -1.1924*** | -4.8784 |
| log(K_GRS(-1)) | 0.7069*** | 3.7112 |
| time trend | -0.0086*** | -3.0336 |
| R ² | | 0.9833 |
| DW | | 2.1132 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Final energy use (households)

Long-term specification:

Dependent variable log(FEN_C)

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| Constant | 2.6971** | 2.2081 |
| log(P_EN_C) | -0.0510*** | -3.1787 |
| log(UEX_TOT) | 0.5338*** | 5.7720 |
| log(HDD) | 0.4458*** | 7.8592 |
| log(W_HOUR) | 0.4231*** | 3.9831 |
| R ² | | 0.9447 |
| DW | | 1.3114 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable Δ log(FEN_C)

| Variables | Estimated coefficients | t-stats |
|-----------------------|------------------------|---------|
| ECT | -0.7035*** | -6.4174 |
| Δ log(P_EN_C) | -0.0791*** | -2.9893 |
| Δ log(UEX_TOT) | 0.4551*** | 4.8950 |
| Δ log(HDD) | 0.4906*** | 15.4421 |
| Δ log(W_HOUR) | 0.5579*** | 5.6818 |
| R ² | | 0.9344 |
| DW | | 1.8595 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Final energy use (industry)

Long-term specification:

Dependent variable log(FEN_IND)

| Variables | Estimated coefficients | t-stats |
|-----------|------------------------|---------|
| Constant | 8.0083*** | 5.6905 |

| | | |
|----------------|------------|---------|
| log(P_EN_IND) | -0.0643*** | -4.4075 |
| log(UEX_TOT) | 0.3860*** | 3.0610 |
| log(M) | 0.1252** | 2.4153 |
| R ² | | 0.9886 |
| DW | | 1.2410 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{FEN_IND})$

| Variables | Estimated coefficients | t-stats |
|---------------------------------|------------------------|---------|
| ECT | -0.6434*** | -8.1358 |
| $\Delta\log(\text{P_EN_IND})$ | -0.0696** | -2.7053 |
| $\Delta\log(\text{UEX_TOT})$ | 0.3999*** | 3.1416 |
| $\Delta\log(\text{M})$ | 0.0990* | 1.6940 |
| R ² | | 0.8456 |
| DW | | 1.7108 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Final energy use (other sectors)

Dependent variable $\log(\text{FEN_OTH})$

| Variables | Estimated coefficients | t-stats |
|---------------------------------|------------------------|---------|
| $\log(\text{FEN_OTH}(-1))$ | 0.8719*** | 16.7172 |
| $\Delta\log(\text{P_EN_OTH})$ | -0.1435*** | -5.6028 |
| $\log(\text{UEX_TOT})$ | 0.1383** | 2.4749 |
| time trend | -0.0005** | -2.4504 |
| R ² | | 0.9936 |
| DW | | 1.8639 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Energy prices (households)

Long-term specification:

Dependent variable $\log(\text{P_EN_C})$

| Variables | Estimated coefficients | t-stats |
|-----------------------|------------------------|---------|
| Constant | 25.4654*** | 5.2119 |
| $\log(\text{FEN_C})$ | -1.9708*** | -5.6597 |
| $\log(\text{CPI})$ | 1.2151*** | 25.6980 |
| R ² | | 0.9819 |
| DW | | 1.0680 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{P_EN_C})$

| Variables | Estimated coefficients | t-stats |
|-----------------------------------|------------------------|---------|
| ECT | -0.1641*** | -3.0014 |
| $\Delta\log(\text{P_EN_C}(-1))$ | 0.2289*** | 2.8559 |
| $\Delta\log(\text{FEN_C})$ | -0.6925*** | -7.1379 |

| | | |
|--------------------------|-----------|--------|
| $\Delta\log(\text{CPI})$ | 0.9193*** | 8.8672 |
| R ² | | 0.8577 |
| DW | | 2.1802 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Energy prices (industry)

Long-term specification:

Dependent variable $\log(\text{P_EN_IND})$

| Variables | Estimated coefficients | t-stats |
|-------------------------|------------------------|---------|
| Constant | 17.1322*** | 2.8149 |
| $\log(\text{FEN_IND})$ | -1.3603*** | -3.4374 |
| $\log(\text{CPI})$ | 1.0741*** | 8.3891 |
| R ² | | 0.9655 |
| DW | | 0.8825 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{P_EN_IND})$

| Variables | Estimated coefficients | t-stats |
|-------------------------------|------------------------|---------|
| ECT | -0.1228*** | -2.7734 |
| $\Delta\log(\text{FEN_IND})$ | -0.4003** | -2.6975 |
| $\Delta\log(\text{CPI})$ | 1.1585*** | 10.4675 |
| R ² | | 0.8982 |
| DW | | 2.1573 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Energy prices (other sectors)

Long-term specification:

Dependent variable $\log(\text{P_EN_OTH})$

| Variables | Estimated coefficients | t-stats |
|-------------------------|------------------------|---------|
| Constant | 20.9611*** | 6.5045 |
| $\log(\text{FEN_OTH})$ | -1.6329*** | -7.0685 |
| $\log(\text{CPI})$ | 1.3771*** | 24.3987 |
| R ² | | 0.9894 |
| DW | | 1.0270 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{P_EN_OTH})$

| Variables | Estimated coefficients | t-stats |
|-------------------------------|------------------------|---------|
| ECT | -0.2538** | -2.1823 |
| $\Delta\log(\text{FEN_OTH})$ | -1.7701*** | -4.7371 |
| $\Delta\log(\text{CPI})$ | 1.3635*** | 11.5710 |
| R ² | | 0.7230 |
| DW | | 2.0468 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

CO2 (territorial perspective)

Long-term specification:

Dependent variable $\log(\text{CO2_TERR/POP})$

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| Constant | -0.9782*** | -3.5973 |
| $\log(Y/POP)$ | 0.9434*** | 11.9547 |
| $\log(PEN/Y)$ | 0.4351*** | 4.7956 |
| Time trend | -0.0155*** | -5.3162 |
| R ² | | 0.9509 |
| DW | | 0.9183 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{CO2_TERR/POP})$

| Variables | Estimated coefficients | t-stats |
|-----------------------------|------------------------|---------|
| ECT | -0.4796*** | -4.9297 |
| $\Delta\log(Y/POP)$ | 0.8265*** | 9.0195 |
| $\Delta\log(Y(-1)/POP(-1))$ | -0.1442 | -1.6483 |
| $\Delta\log(PEN/Y)$ | 0.6957*** | 8.5801 |
| $\Delta\log(PEN(-1)/Y(-1))$ | -0.0990 | -1.5583 |
| R ² | | 0.9146 |
| DW | | 1.5931 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

CO2 (consumption perspective)

Long-term specification:

Dependent variable $\log(\text{CO2_CONS/POP})$

| Variables | Estimated coefficients | t-stats |
|----------------|------------------------|---------|
| $\log(Y/POP)$ | 0.6167*** | 5.8081 |
| $\log(PEN/Y)$ | 0.3006*** | 3.0986 |
| $\log(M/POP)$ | 0.4314*** | 7.8041 |
| Time trend | -0.0232*** | -6.8530 |
| R ² | | 0.9005 |
| DW | | 0.9639 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

Short-term specification:

Dependent variable $\Delta\log(\text{CO2_CONS/POP})$

| Variables | Estimated coefficients | t-stats |
|---|------------------------|---------|
| ECT | -0.5040*** | -5.0055 |
| $\Delta\log(\text{CO2_CONS}(-1)/\text{POP}(-1))$ | 0.1444** | 2.1819 |
| $\Delta\log(Y/POP)$ | 0.8098*** | 4.2345 |
| $\Delta\log(PEN/Y)$ | 0.6635*** | 6.5756 |

| | | |
|---------------------|----------|--------|
| $\Delta\log(M/POP)$ | 0.2140** | 2.4476 |
| R ² | | 0.8657 |
| DW | | 1.6582 |

***, **, * Significant at 0.01, 0.05, and 0.10 levels respectively.

S5. Energy-economy relationship

One of the characteristics of model is its incorporation of energy as an indispensable element in the economic system. Energy is represented in its three different stages (i.e. primary, final and useful). The conversion from one stage to another is determined by two types of thermodynamic efficiency, as can be seen in Figure S1. The first one is the efficiency to transform primary energy into its final stage, while the second one is the efficiency to transform final energy into its useful form. Given an increase in thermodynamic efficiency from the final to useful stages, keeping everything else constant, increases the amount of useful energy (i.e. energy services) in the system. In other words, the same amount of final energy is able to deliver a greater amount of energy services.

It must be noted that Figure S1 only shows a selected view of the relationships between key economic and energy variables, and it does not pretend to show a graphical representation of the entire model.

In our framework, investment responds to changes in the productivity of useful energy and the productivity of capital stock (i.e. GDP per unit of useful energy/capital). A higher productivity of energy and capital stimulate a higher level of investment which, in turn, translates into economic growth. Furthermore, the additional availability of energy services increases the demand for non-energy goods which, once again, generates growth. However, the added demand for energy services and the higher rates of economic activity eventually lead to an increase in final energy. Final energy demand depends, among other factors (see supplementary information for more details), on the demand for energy services (useful energy) and energy prices.

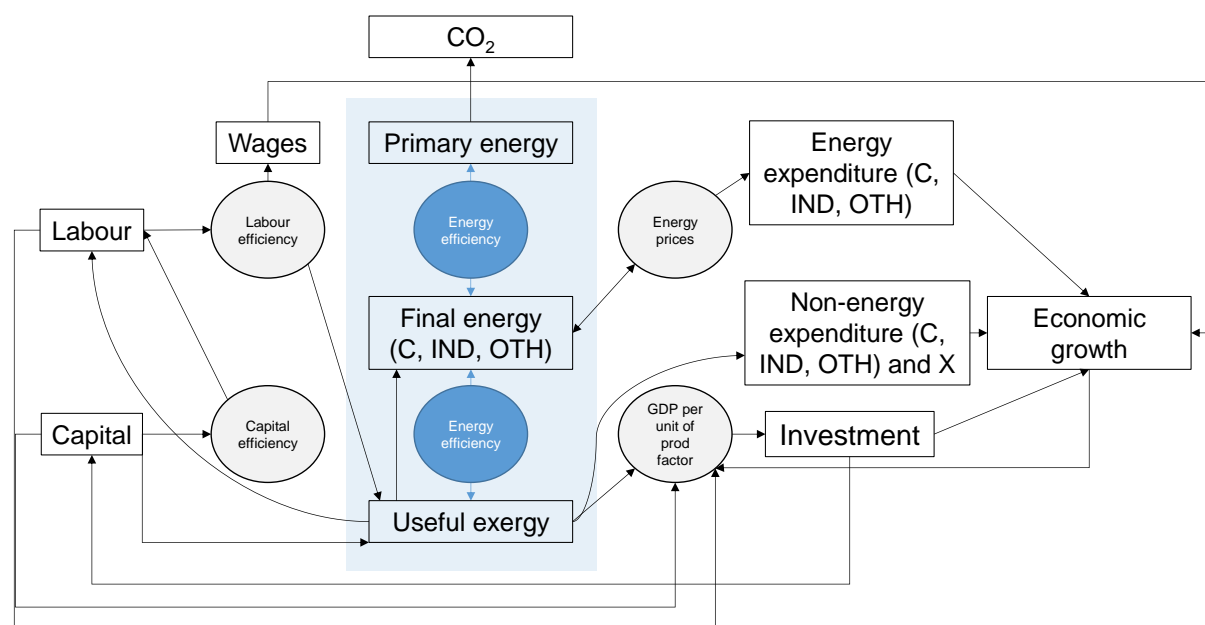


Figure S1 – Economy-energy framework

Capital and energy services are treated in the model as complements. This means that capital goods cannot be put into work without useful work. In other words, an expansion of the capital stock through investment of fixed capital requires an increase in useful energy and, consequently, on final energy as well (given that thermodynamic efficiency does not rise enough to satisfy this new energy requirement). Useful energy is also influenced by labour, which in the model has been quality-corrected

by an efficiency index, given by the amount of hours worked and level of human capital. An increase in quality-corrected labour demands more useful energy. This also changes the volume of wages which, in turn, drive GDP growth from the income side. In contrast, the model allows the substitutability of capital and labour. As the economy becomes more capital intensive, the amount of labour can decrease in relative terms. Labour is determined by the demand of energy services and capital services (in turn given by capital efficiency or the extent to which the capital stock can provide capital services).

S6. Econometric tests

Unit root tests were applied to all variables determine their order of integration. When the variables included in a regression share the same order of integration, cointegration tests were applied to determine the existence of a common long-term equilibrium relationship.

All estimated equations were checked to determine their goodness of fit (R squared), as well for the existence of normality, homoscedasticity and no serial correlation in the residuals.

S7. Solving the model

The model consists of a system of linear equations, which is dynamically solved for each time period (1971-2013) using the Gauss-Seidel iterative method[6] included in the EViews econometric software. This technique allows to determine the values of the endogenous variables, based on the known values of the exogenous variables. The method requires only the values of the endogenous variables for first time periods. Lagged endogenous variables in the model are calculated using the solutions calculated in previous periods, and not from actual historical values. As explained in section S4, an iterative procedure is followed to ensure that the model is able to produce a base-fit solution and that the estimated endogenous variables track their historical behaviour over the sampling period.

S8. Model limitations

One of the most obvious drawbacks of the model when compared to other existing ones is the absence of sectoral disaggregation. Economic sectors are limited to two large groups: industry and the rest. While the high aggregation makes the model easier to be solved, it does not allow a detailed analysis of sectoral impacts or the modelling of sector-specific policies. This limitation also constrains, among other issues, the proper treatment of crowding-out effects, as explained in a previous section. Investment is assumed to take place across the whole economy, rather than in particular sectors.

The model depicts an open economy. Nevertheless, the rest of the world is treated as exogenous, although the decision of how much to export and import is determined within the model. In addition, technology transfers are not considered, as technology is treated as an aggregate and there is no detail of specific technologies. This creates a series of further limitations. For example, it is not possible to model the role of foreign technological activities and policies. Moreover, it is not feasible to model the composition of trade flows.

The model does not contain a description of different energy technologies and it does not distinguish between different energy sources (i.e. coal, gas, solar, wind, etc.). Once again, this limits the ability of modelling specific policies and their impacts on carbon emissions and economic growth.

Our model does not contain Constant Elasticity of Substitution (CES) functions, meaning we cannot directly extract elasticities of substitution in the manner of Kander and Stern [7].

Fiscal pressures have also not been integrated into the model. Government spending and the fiscal budget (i.e. gap between government revenue and expenditure) are determined exogenously. However, there is no constraint regarding how much the government can increase the national debt before it starts having a negative effect on growth. Similarly, there is no constraint on how much the economy can spend on energy as a percentage of GDP. It has been suggested that if the ratio between energy expenditure and GDP surpasses a certain level, there is a risk of jeopardising economic growth [8].

More broadly, the model is not entirely stock-flow consistent, as it does not integrate all the flows and the stocks of the economy. This is due to the poor description of the monetary and fiscal sectors, and to an incomplete description of the rest of the world.

S9. Data sources

Table S1 – Data sources

| Variable | Description | Year coverage | Units | Source |
|----------|---|---------------|-------|-----------------|
| Y | Gross Domestic Product: chained volume measures | 1971-2013 | £m | ONS |
| CT | Household final consumption expenditure :National concept CVM and Final Consumption Expenditure of NPISHs CVM | 1971-2013 | £m | ONS |
| G | General Government: Final consumption expenditure CVM | 1971-2013 | £m | ONS |
| I | Total Gross Fixed Capital Formation CVM, plus changes in inventories including alignment adjustment, plus acquisitions less disposals of valuables, total economy CVM | 1971-2013 | £m | ONS |
| X | Balance of Payments: Trade in Goods & Services: Total exports CVM | 1971-2013 | £m | ONS |
| M | Balance of Payments: Imports: Total Trade in Goods & Services CVM | 1971-2013 | £m | ONS |
| NW | Total net worth: total non-financial assets plus net financial assets/liabilities | 1971-2013 | £m | ONS |
| YD | Real households' disposable income per head, CVM | 1971-2013 | £ | ONS |
| R_NOM | Annual average of official bank rate | 1971-2013 | % | Bank of England |
| Y_RW | World GDP (without UK) | 1971-2013 | \$USD | World Bank |
| PX | Export deflator | 1971-2013 | Index | ONS |
| PM | Import deflator | 1971-2013 | Index | ONS |

| Variable | Description | Year coverage | Units | Source |
|-------------|---|---------------|-----------|------------------------------------|
| E_INDEX_NOM | Exchange rate index 1990=100 new | 1971-2013 | Index | Bank of England |
| W | Total compensation of employees | 1971-2013 | £m | ONS |
| YF | Gross operating surplus of corporations | 1971-2013 | £m | ONS |
| NET_TAX | Taxes on products & production less subsidies | 1971-2013 | £m | ONS |
| K_GRS | Gross capital stock | 1971-2013 | £bn | ONS |
| K_NET | Net Capital Stock | 1971-2013 | £bn | ONS |
| DEP_FIX | Capital Consumption | 1970-2013 | £m | ONS |
| CPI | RPI All Items Index: 2011=100 | 1971-2013 | Index | ONS |
| W_HOUR | Average wage per hour | 1971-2013 | £ | ONS |
| UR | Unemployment rate: all aged 16 and over | 1971-2013 | % | ONS |
| L | In employment: all aged 16+ | 1971-2016 | 000's | ONS |
| LF | Economically Active: all aged 16-64. | 1971-2013 | 000's | ONS |
| PEN | Primary energy | 1971-2013 | TJ | DUKES |
| CO2T | UK CO2 production | 1970-2013 | tonnes | DEFRA |
| CO2C | UK CO2 footprint | 1970-2013 | tonnes | DEFRA |
| GHGT | UK GHG production | 1970-2013 | tonnes | DEFRA |
| GHGC | UK GHG footprint | 1970-2013 | tonnes | DEFRA |
| POP | Population | 1950-2013 | thousands | United Nations Population Division |
| MS | M4 (monetary financial institutions' sterling M4 liabilities to private sector) | 1971-2013 | £m | Bank of England |

| Variable | Description | Year coverage | Units | Source |
|----------|---|--------------------|-------|-----------------------|
| IND_T | Gross value added (industry) | 1971-2013 | £m | AMECO |
| OTH_T | Gross value added (agriculture and services) | 1971-2013 | £m | AMECO |
| IND_E | Expenditure on final energy by industry | 1971-2013 | £m | DUKES 1.1.6 |
| OTH_E | Expenditure on final energy by other sectors | 1971-2013 | £m | DUKES 1.1.6 |
| C_E | Expenditure on final energy by households | 1971-2013 | £m | DUKES 1.1.6 |
| C_NE | ABJR (Household final consumption expenditure :National concept CVM SA) | 1971 - 2013 (2016) | £m | ONS |
| GVA | Gross Value Added (Average) at basic prices | 1971-2013 | £m | ONS |
| NET_TAX | Total adjustment to basic prices | 1971-2013 | £ | ONS |
| CPI_E | CPI INDEX: Energy 2015=100 | 1971-2013 | Index | ONS |
| FEN_T | Final energy use (total) | 1971-2013 | TJ | DUKES |
| FEN_IND | Final energy use (industry) | 1971-2013 | TJ | DUKES |
| FEN_C | Final energy use (households) | 1971-2013 | TJ | DUKES |
| FEN_OTH | Final energy use (other sectors) | 1971-2013 | TJ | DUKES |
| P_EN_IND | Energy prices (industry) | 1971-2013 | £/MJ | DUKES |
| P_EN_C | Energy prices (households) | 1971-2013 | £/MJ | DUKES |
| P_EN_OTH | Energy prices (other sectors) | 1971-2013 | £/MJ | DUKES |
| UEX_TOT | Total exergy | 1971-2013 | TJ | Brockway, Barrett [9] |

| Variable | Description | Year coverage | Units | Source |
|--------------|---|---------------|--------------------------|---------------------------|
| L_HRS_INDEX | Average annual hours worked by persons engaged | 1971-2013 | Index | Penn World Tables 9.0[10] |
| L_HC_INDEX | Human capital index, based on years of schooling and returns to education | 1971-2013 | Index | Penn World Tables 9.0[10] |
| K_SERV_INDEX | Capital services index | 1971-2013 | Index | Penn World Tables 9.0[10] |
| HDD | Heating degree days | 1971-2013 | Number of days in a year | Palmer and Cooper [11] |

S10. Simulation responses to 6 key variables

In this section we plot the responses of the model to each of the 6 variables constrained in the counterfactual simulations (refer to main article).

First, there is the response of thermodynamic efficiency (final energy to useful exergy) in Figure S2 under the 6 simulations, which replicates an expanded version (covering all simulations) of Figure 2 in the main paper. Three cases stand out, beyond that of the constant efficiency simulation.

First, constraining capital investment to 1971 levels decreases thermodynamic efficiency (purple line) from c.1990. This is because actual capital investment only increased significantly after c. 1985, causing thermodynamic efficiency under constant investment simulation to fall after 1990.

Second, constrained energy prices increase final energy use as energy is relatively cheaper, but this weakens the incentive for energy efficiency gains, i.e. cheap energy leads to lower efficiency gains. Thus, thermodynamic efficiency gains (green line) are lower in this counterfactual case.

Third, constant energy services (useful exergy) also reduces counterfactual thermodynamic efficiency (blue line), as thermodynamic efficiency does not have to increase as much to supply the (reduced) counterfactual energy service demand. In fact, this simulation reveals that thermodynamic efficiency gains can be split into two components. One is a distinct demand-sided gain, stimulated by the increased demand for energy services. Figure S2 shows this as the difference between the constant energy services (blue) and basefit (black) lines, i.e. 17.5% versus 19.4% in 2013. The other is a 'natural' or 'technical' efficiency gain which occurs independently of any increased energy service demands. In Figure S2, this is seen as the difference between the constant efficiency (grey) and constant energy services (blue) lines, i.e. 14.0% versus 17.5% in 2013. These are parts of the efficiency-led 'growth engine' mechanism presented in the main paper.

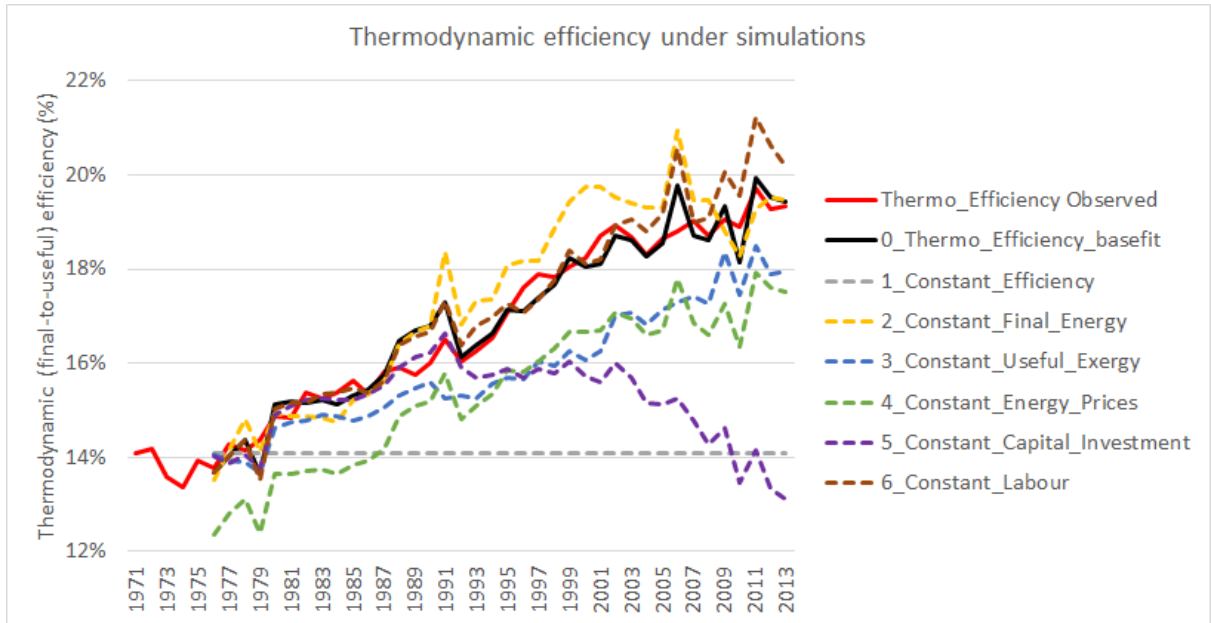


Figure S2 – Thermodynamic efficiency (exeff2_fu) under simulations

Figure S3 presents the effects on final energy from the simulations. Under most simulations, the slowdown in economic growth translates to a slowdown in final energy consumption.

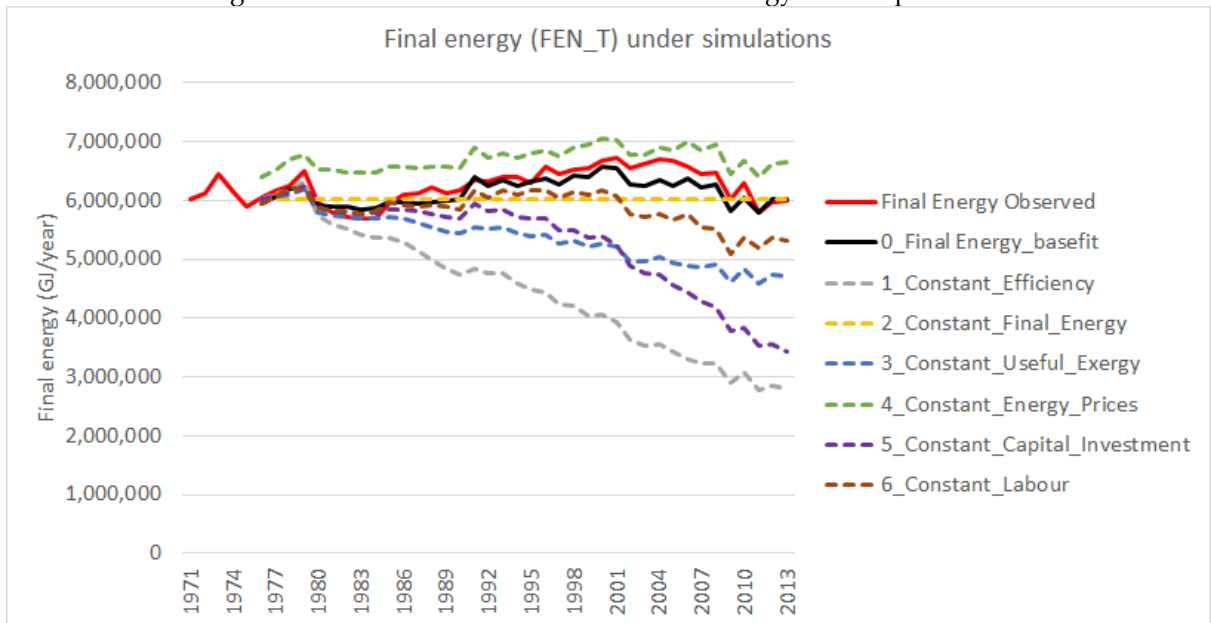


Figure S3 – Final energy (FEN_T) under simulations

Next, the response of energy services demand (useful exergy) to the simulations is given in Figure S4. The constant efficiency and capital investment simulations reduce end demand for energy services by over half.

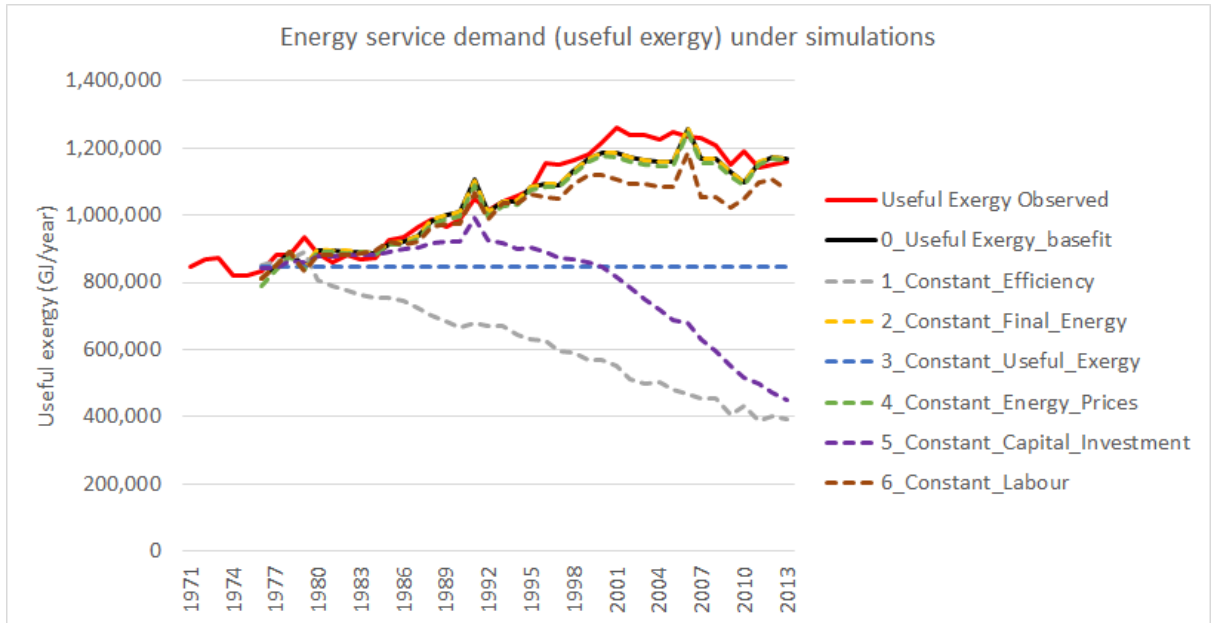


Figure S4 – Energy service demand (useful exergy) under simulations

The level of energy prices under simulations is given in Figure S5. This shows the effect of constraints to final energy (in Figure S3) is to increase the energy price.

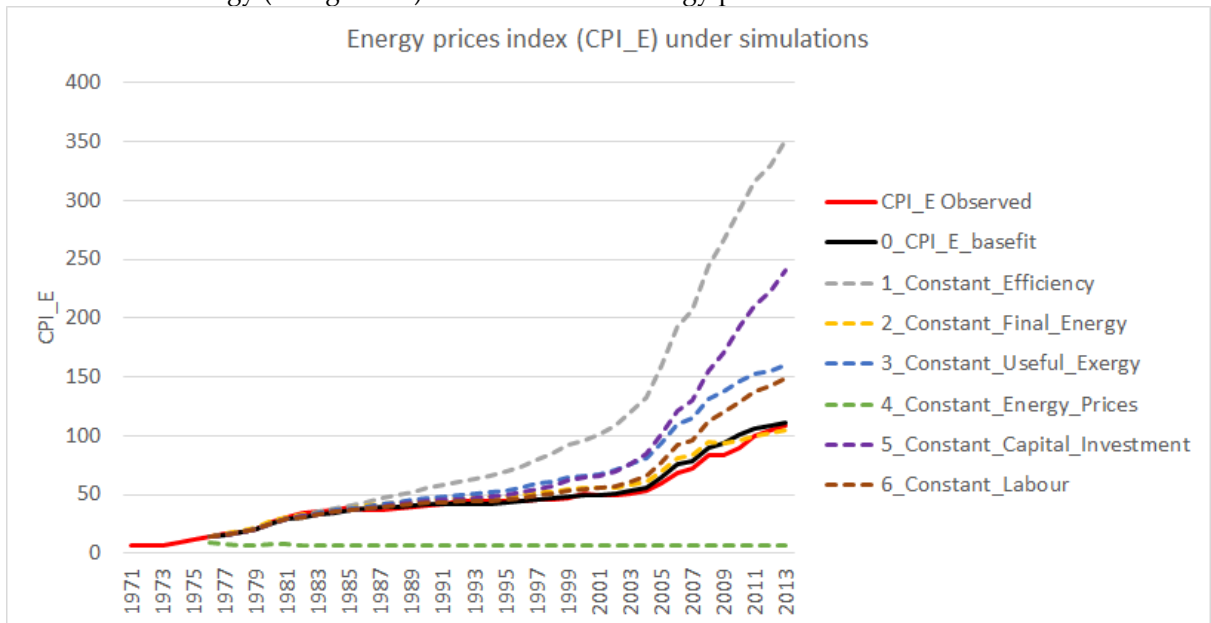


Figure S5 – Energy price index (CPI_E) under simulations

Figure S6 below shows the cost of energy relative to overall GDP. We see the ratio of energy/GDP costs to be 0.15-0.20 by 2013 in the cases of constant efficiency and capital investment, caused by the sharp increases to energy prices, due in turn to the constraint to available final energy.

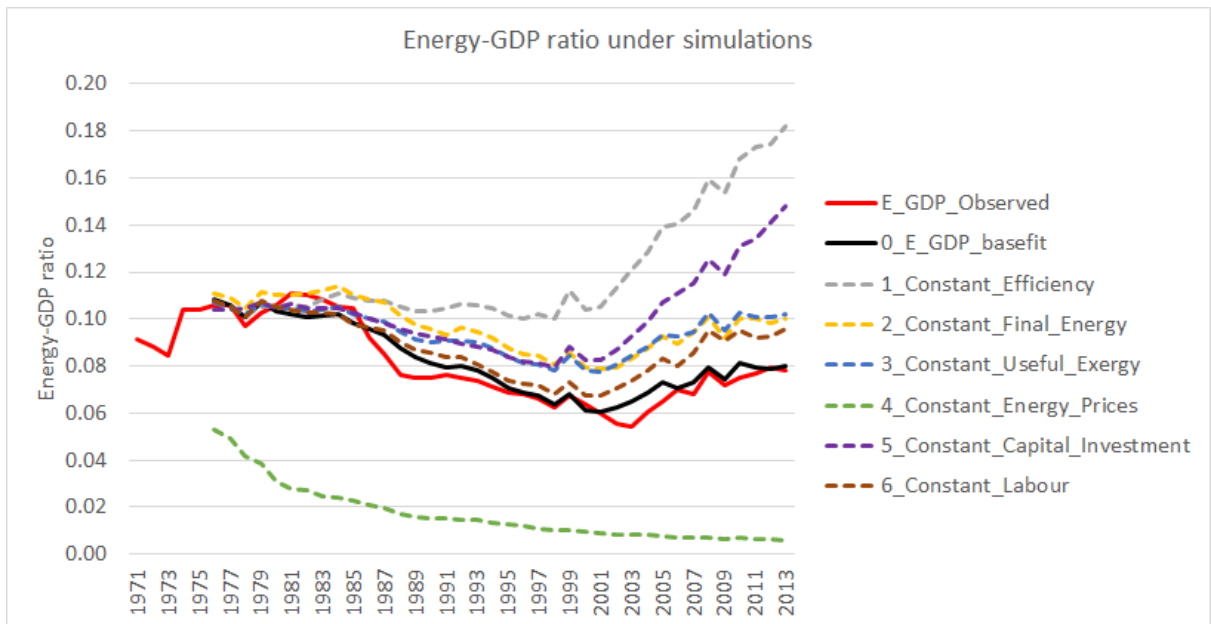


Figure S6 – Energy-GDP cost ratio under simulations

Next, we can see the level of capital stock under the simulations in Figure S7. We see the expected result that the constraint of capital investment simulation has the largest adverse effect on capital stock. However, the constraint to thermodynamic efficiency and demand for useful exergy also significantly constrain capital stock.

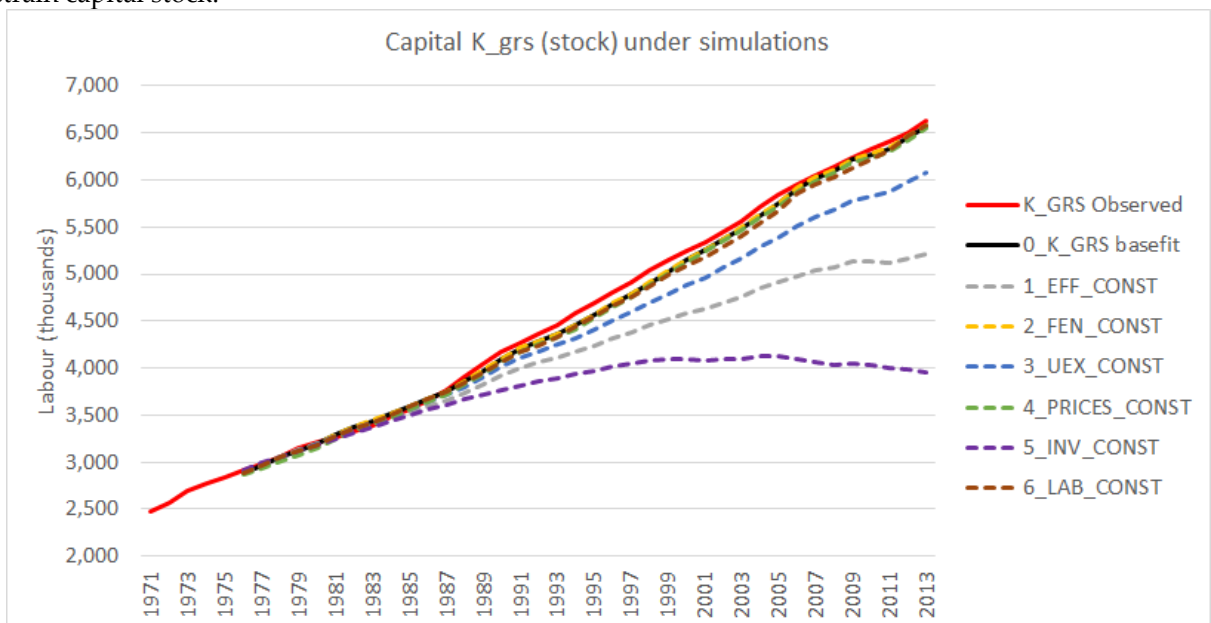


Figure S7 – Capital stock (K_GRS) under simulations

Last, we see the changes to employment (labour) in Figure S8. The constraint to capital (in 1. constant capital investment, 2. thermodynamic efficiency and 3. useful exergy simulations) causes a substitution effect whereby increased labour supply is required to contribute to economic output.

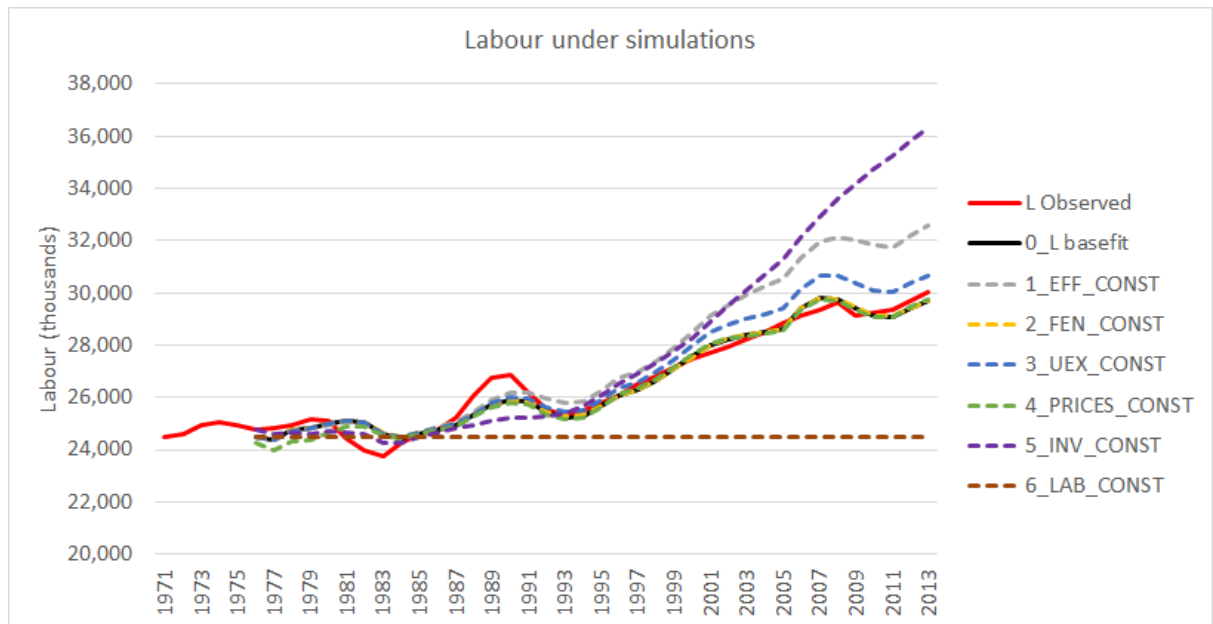


Figure S8 – Labour employment under simulations

References

- [1] CE. E3ME Technical Manual, Version 6.0. Cambridge, UK: Cambridge Econometrics; 2014.
- [2] OBR. The macroeconomic model. Briefing paper No 5: Office for Budget Responsibility; 2013.
- [3] Engle RF, Granger CWJ. Co-integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*. 1987;55:251-76.
- [4] Solomon BD, Rubin BM. Environmental Linkages in Regional Econometric Models: An Analysis of Coal Development in Western Kentucky. *Land Economics*. 1985;61:43-57.
- [5] Brilllet JL. Structural econometric modelling: Methodology and tools with applications under EViews. IHS Global Inc.; 2016.
- [6] Varga RS. Basic Iterative Methods and Comparison Theorems. In: Varga RS, editor. *Matrix Iterative Analysis*. Berlin, Heidelberg: Springer Berlin Heidelberg; 2000. p. 63-110.
- [7] Kander A, Stern DI. Economic growth and the transition from traditional to modern energy in Sweden. *Energy Economics*. 2014;46:56-65.
- [8] Bashmakov I. Three laws of energy transitions. *Energy Policy*. 2007;35:3583-94.
- [9] Brockway PE, Barrett JR, Foxon TJ, Steinberger JK. Divergence of Trends in US and UK Aggregate Exergy Efficiencies 1960–2010. *Environmental Science & Technology*. 2014;48:9874-81.
- [10] Feenstra RC, Inklaar R, Timmer MP. The Next Generation of the Penn World Table. *American Economic Review*. 2015;105:3150-82.
- [11] Palmer J, Cooper I. United Kingdom housing energy fact file. Department of Energy and Climate Change; 2013.