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Covariance Forecasting in Equity Markets*

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Abstract

We compare the performance of popular covariance forecasting models in the context of a portfolio of major European equity indices. We find that models based on high-frequency data offer a clear advantage in terms of statistical accuracy. They also yield more theoretically consistent predictions from an empirical asset pricing perspective, and, lead to superior out-of-sample portfolio performance. Overall, a parsimonious Vector Heterogeneous Autoregressive (VHAR) model that involves lagged daily, weekly and monthly realised covariances achieves the best performance out of the competing models. A promising new simple hybrid covariance estimator is developed that exploits option–implied information and high–frequency data while adjusting for the volatility risk premium. Relative model performance does not change during the global financial crisis, or, if a different forecast horizon, or, intraday sampling frequency is employed. Finally, our evidence remains robust when we consider an alternative sample of U.S. stocks.

JEL classification: C50, C58, G11, G12.

Keywords: Covariance forecasting, High-frequency data, Implied volatility, Asset allocation, Risk-return trade-off.

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1 Introduction

Little has changed since Robert Engle concluded in his Nobel Prize memorial lecture in 2003 that the best covariance forecasting method has not yet been discovered (for a review of this literature see Alexander, 2008; Andersen et al., 2013). Standard practice of modelling and forecasting covariance relies on multivariate GARCH models, whereas, more recent approaches advocate the use of high-frequency data. However, a synthesis of conclusions from different empirical studies is difficult due to the diversity in terms of econometric methods, asset classes, sampling frequencies, time periods, market regimes, and performance evaluation criteria. Besides that, the extant literature largely focuses on statistical evidence neglecting the rather important concepts of economic value of covariance forecasts for asset pricing and portfolio allocation.

We shed further light on this literature by undertaking a comprehensive empirical comparison of several alternatives in order to discover the best covariance forecasting model on the basis of statistical and economic criteria. Two broad families of models are considered: multivariate GARCH models that employ daily data, and, models that use high-frequency and options data. Our main dataset spans the period from January 2000 to April 2016, and, includes five major European equity markets (Germany, France, Netherlands, Switzerland, UK). In line with standard practice and theoretical results (Andersen et al., 2003; Barndorff-Nielsen and Shephard, 2004), we employ realised covariance computed from intraday returns as a proxy for the unobserved covariance. We consider daily, weekly and monthly forecasting horizons and compare the models during different market phases including the 2008-2009 global financial crisis.

Our econometric analysis demonstrates that high-frequency data are particularly valuable for covariance forecasting. Specifically, a Vector Heterogeneous Autoregressive (VHAR) model is shown to have the best performance from both a statistical and an economic perspective. Although GARCH models have relatively fewer data requirements, they have inferior predictive accuracy and entail substantial computational costs.

While option-implied information is useful for volatility modelling, employing it for covariance forecasting is neither technically straightforward nor clearly justified. Our paper addresses this issue by proposing a new simple approach for exploiting option implied information for covariance forecasting. Specifically, inspired by the option-implied betas literature (Buss and Vilkov, 2012), we apply a novel hybrid covariance estimator which combines option and high-frequency data. In addition to the benefit of being more forward-looking due to the use of implied volatilities, our hybrid estimator adjusts for the volatility risk premium. The proposed approach is non-parametric and achieves better forecasting performance relative to the multivariate GARCH models examined.

In the empirical asset pricing literature, there is contrasting evidence about the risk-return trade-off and whether this relationship is positive or negative (see the review by Bali and Engle 2010). We consider that the lack of consensus is partly due to the use of different models to estimate the conditional variances and covariances involved. We explore this possibility by comparing the covariance models studied in the context of the intertemporal capital asset pricing model (ICAPM) of Merton (1973). We use a system of seemingly unrelated regressions (SUR), which accounts for both time series and cross-sectional variation in excess stock returns (see Bali, 2008; Bali and Engle, 2010). The results show that conditional covariance estimates based on high-frequency models for our European equity data yield a positive and statistically significant risk-return trade-off. The common risk aversion coefficient has an economically meaningful size between 2 and 3, which is consistent with previous studies that employ covariance estimates from multivariate GARCH models (e.g., Bali and Engle, 2010). To the best of our knowledge, this is the first study that uses the VHAR model in an empirical asset pricing setting. Multivariate GARCH models provide somewhat mixed evidence on the significance of the common risk-return coefficient with the asymmetric DCC being the only model that shows evidence of a positive and significant risk-return trade-off across all test assets.

Finally, we evaluate the practical economic significance of the covariance forecasting models under study in a global minimum-variance portfolio framework. Specifically, we seek the model that achieves the best performance in terms of out-of-sample portfolio risk and turnover. Our results again confirm the superiority of high-frequency models, which, in the majority of scenarios we assume, yield the minimum risk along with low levels of turnover.

We perform a battery of additional tests to assess the robustness of our findings and discuss broader implications. First, we extend our analysis to a set of 10 liquid US stocks and obtain consistent evidence on the superiority of the three simple models that employ high-frequency and option-implied information. In line with the evidence from the European equity markets, these models yield a positive and significant risk-return trade-off. Second, we seek to address potential concerns about microstructure noise by providing evidence from lower intraday sampling frequencies of 10 and 30 minutes, respectively. Our conclusions are comparable with those obtained from the main analysis of 5-minute returns. Third, to account for a setting where intraday data is not available, we repeat our analysis estimating the high-frequency data-based models using daily data. The results are generally in line with those from the main analysis, which extends the applicability of models, such as the VHAR, to situations where only daily data is available. Fourth, we confirm the robustness of our results with respect to non-overlapping weekly and monthly forecasts and different breakpoints for the financial crisis. The results reveal that the relative ranking of the models changes little during turmoil periods, although forecast errors are generally higher.

2 Methodology

Let r_t be an $N \times 1$ vector of asset returns at time t, where: $t = 1, 2, ..., T$. Assuming a constant conditional mean, returns r_t can be expressed as follows:

$$r_t = \mu + e_t, \tag{1}$$

where μ is the unconditional mean of the return series and e_t is a vector of innovations satisfying:

$$e_t = H_t^{1/2} z_t, \tag{2}$$

where H_t is the $N \times N$ positive definite conditional covariance matrix of the innovations, i.e., $H_t = E_{t-1}(e_t e_t')$. z_t is the vector of i.i.d. standardized innovations that follow a multivariate standard normal distribution: $z_t \sim N(0, I_N)$, where I_N is an $N \times N$ identity matrix. We consider several alternative ways of modelling H_t in order to obtain conditional covariance matrix forecasts.

As the true covariance matrix is unobservable, we approximate it using the realised covariance matrix computed from intraday returns sampled at equally-spaced intervals. Suppose that on day t a grid of $M + 1$ equidistant intraday prices are observed at times

$t_0, t_1, ..., t_M$, with p_{t_j} being the logarithmic price at time t_j. The corresponding asset return, r_{t_j}, in the j^{th} intraday interval of day t is computed as $r_{t_j} = p_{t_j} - p_{t_{j-1}}$. We denote the $N \times 1$ vector of demeaned asset returns for the j^{th} interval of day t, by $r_{j,t}$. The realised daily covariance matrix, Σ_t, is then given by:

$$\Sigma_t = \sum_{j=1}^{M} r_{j,t}\, r'_{j,t} \tag{3}$$

The estimator of Equation (3) converges to the true unobserved covariance as the sampling frequency goes to infinity (Andersen et al., 2003). Following standard practice, we rely on 5-minute returns for the empirical estimation of Σ_t (see, for example, Andersen et al., 2001). Then, covariance over longer horizons of k days is computed by the sum of daily realised covariances.

We use the realised covariance computed from Equation (3) to compare the predictive performance of several parametric and nonparametric models. The first nine models belong to the GARCH family and are typically estimated using daily data. The remaining three models are estimated using intraday and option price data. A brief description of each model follows.

2.1 Diagonal BEKK (D-BEKK)

The BEKK model proposed by Engle and Kroner (1995) is extensively used in the empirical finance literature. It is particularly popular for modelling volatility spillovers, as it can allow conditional volatilities to depend on their own past values as well as past volatilities of other markets. The BEKK model has the desirable property that the conditional covariance matrix is ensured to be positive definite. The conditional covarince matrix, H_t, under the BEKK(1,1) model is parameterised as follows:

$$H_t = C'C + A'e_{t-1}e'_{t-1}A + B'H_{t-1}B, \qquad (4)$$

where C is a $N \times N$ positive definite upper triangular matrix of constant terms, and A and B are $N \times N$ matrices of parameters. The full version of the model requires the estimation of a comparatively large number of parameters and thus is less suitable for forecasting applications. Here, similar to Laurent et al. (2012), we estimate a reduced and more parsimonious version of the BEKK model, which assumes that the square matrices A and B are diagonal (D-BEKK).

2.2 Asymmetric Diagonal BEKK (A-D-BEKK)

The D-BEKK model assumes no difference in the impact of positive and negative shocks of the same magnitude on conditional covariance. However, there is evidence of asymmetric comovement in equity markets, which suggests that covariances tend to be higher following negative return shocks (Ang and Bekaert, 2002; Cappiello et al., 2006). To allow for such patterns, we also estimate the asymmetric version of the diagonal BEKK model (A-D-BEKK) specified as follows:

$$H_t = C'C + A'e_{t-1}e'_{t-1}A + \Gamma'u_{t-1}u'_{t-1}\Gamma + B'H_{t-1}B, \qquad (5)$$

where u_t corresponds to the $N \times 1$ vector of negative shocks, given by $u_t = min(e_t, 0)$, and Γ is an $N \times N$ matrix of parameters.

2.3 Constant Conditional Correlations (CCC)

The Constant Conditional Correlation model (CCC) of Bollerslev (1990) assumes that conditional correlations are constant, while conditional covariances vary over time and are proportional to conditional volatilities. The model is defined as follows:

$$H_t = D_t R D_t, \tag{6}$$

where $D_t = diag\{\sqrt{h_{11,t}}, \sqrt{h_{22,t}}, ..., \sqrt{h_{NN,t}}\}$ is a diagonal matrix whose elements are the conditional volatilities (i.e., square root of conditional variances) of the N return series. The $h_{ii,t}$ series are modelled through univariate GARCH(1,1) processes. R is the $N \times N$ unconditional correlation matrix of the standardized residuals from Equation (1), given by $z_{it} = e_{it}/\sqrt{h_{ii,t}}$.

The CCC model offers the advantage of easier estimation compared to BEKK, as it only requires estimation of N univariate GARCH(1,1) models. Moreover, the inverse covariance matrix required for the optimisation of the multivariate quasi-likelihood function is easily computed as a simple function of the univariate volatilities and of the unconditional correlation matrix, R.

2.4 Asymmetric Constant Conditional Correlations (A-CCC)

The asymmetric extension of the CCC model (A-CCC) results from estimating the GJR-GARCH(1,1) process of Glosten et al. (1993) for each diagonal element of D_t as follows:

$$h_{ii,t} = \omega_i + a_i e_{i,t-1}^2 + b_i h_{ii,t-1} + \gamma_i u_{i,t-1}^2, \tag{7}$$

where ω_i, a_i, b_i and γ_i are parameters for estimation.

2.5 Dynamic Conditional Correlations (DCC)

The Dynamic Conditional Correlation (DCC) model of Engle (2002), allows for time-varying correlations in Equation (6), i.e., $H_t = D_t R_t D_t$. The DCC model characterizes dynamic correlations through a two-stage process:

$$R_t = V_t^{-1} Q_t V_t^{-1}, \tag{8}$$

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha z_{t-1} z'_{t-1} + \beta Q_{t-1}, \tag{9}$$

where $V_t = diag\{\sqrt{q_{11,t}}, \sqrt{q_{22,t}}, ..., \sqrt{q_{NN,t}}\}$ and z_t is the vector of standardized innovations defined above. The scalar parameters α and β in Equation (9) should satisfy the restriction $\alpha + \beta < 1$ to ensure mean-reverting correlations. $\bar{Q}$ is the unconditional covariance matrix of the standardized residuals. The $q_{ij,t}$ elements of matrix Q_t represent quasi–correlations, which are re-scaled to obtain conditional correlations based on Equation (8) so that $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}$.

As with the CCC model, the DCC model is easily estimated through a two-step maximum likelihood procedure involving estimation of univariate GARCH processes followed by estimation of the correlation parameters through Equation (9). This makes the approach feasible in large systems. A limitation of the standard DCC model is that in order to reduce estimation complexity, parameters α and β in Equation (9) are scalars which means that all correlations are assumed to follow the same dynamics.

2.6 Asymmetric DCC (A-DCC)

The asymmetric extension of the standard DCC model (A–DCC) that allows for leverage effects on dynamic conditional correlations is the following:

$$Q_t = [(1-\alpha-\beta)\bar{Q} - \gamma\bar{N}] + \alpha z_{t-1}z'_{t-1} + \beta Q_{t-1} + \gamma u_{t-1}u'_{t-1}, \quad (10)$$

where $\bar{N}$ is the unconditional covariance matrix of the negative innovations (i.e., the $u'_t s$).

2.7 Orthogonal GARCH (O-GARCH)

The orthogonal GARCH model (O-GARCH) of Alexander and Chibumba (1997) and Alexander (2001) provides a parsimonious way of modelling and forecasting covariance matrices. This model belongs to the class of factor models, which assume that the observed return series can be expressed as a linear transformation of uncorrelated factors. In the O-GARCH model these factors are obtained through estimation of univariate GARCH(1,1) models on a few principal components of the full covariance matrix.

Let $z_t = U^{-1/2}e_t$ be the vector of standardized innovations at time t, where U is the $N \times N$ diagonal matrix of the unconditional variances of the innovations (i.e., of e_t). Then, the $p \times 1$ vector of principal components (factors) of the correlation matrix of the standardized innovations at time t is given by $f_t = \Lambda^{-1/2}P'z_t$, where Λ is the $p \times p$ diagonal matrix of the eigenvalues of the unconditional correlation matrix of the $z'_t s$ (ranked in decreasing order) and P is the $N \times p$ matrix of the corresponding eigenvectors. The (diagonal) conditional covariance matrix of e_t can then be approximated as follows:

$$H_t = A_t S_t A'_t + \Omega_e, \quad (11)$$

where $A_t = U^{1/2} P \Lambda^{1/2}$ is an $N \times p$ matrix of normalized factor loadings corresponding to the p principal components (for further details on the procedure see Alexander, 2001). S_t is an $p \times p$ diagonal matrix of the conditional variances of the p principal components obtained by estimating univariate GARCH(1,1) models (for the standard O-GARCH) or GJR-GARCH(1,1) models (for the asymmetric O-GARCH) on each of the p factors. Ω_e is the unconditional covariance matrix of the approximation error resulting from employing p ($p < N$) instead of the full number of principal components (see Alexander, 2001). Given the small dimension of our problem, we use the full set of the five principal components, i.e., $p = N$, and therefore, $\Omega_e = 0$ in Equation (11).

The main strength of the O-GARCH model family lies in its parsimony and estimation simplicity as it relies on estimation of univariate GARCH models on few principal components of the full covariance matrix. This leads to substantial advantages in modelling large covariance matrices as the dimension of the problem and thus the computational time are substantially reduced. The benefits from the dimension reduction are more tangible in highly correlated systems, as few principal components can explain most of the variation in the data. Moreover, the principal component analysis allows one to quantify the amount of risk associated with each factor (see Alexander, 2008, pp. 171–180, for more details about this method).

2.8 Exponentially Weigthed Moving Average (EWMA)

The Exponentially Weighted Moving Average (EWMA), widely known as the *RiskMetrics* estimator, employs exponentially decaying weights for the covariance matrix. The covariance matrix in the EWMA model is recursively computed as follows:

$$H_t = (1 - \lambda)\, e_t\, e_t' + \lambda\, H_{t-1}, \tag{12}$$

where the parameter λ determines the rate of decay. Following the standard approach, we adopt $\lambda = 0.94$.

The main feature of this method is that it is very simple to implement for large dimensions as it requires no optimisation and only the parameter λ needs to be specified. Yet, the single decay parameter λ does not have a solid theoretical basis and governs the dynamics of all conditional covariances.

2.9 Random Walk Estimator (RWE)

A naive covariance forecasting approach is based on the lagged realised covariance. This model assumes that covariance is a Markov process so that the covariance of the previous period is highly informative about the future covariances. This forecasting method is as simple as possible as it requires no optimisation.

2.10 Vector Heterogeneous Autoregressive Model (VHAR)

Corsi (2009) proposes the Heterogeneous Autoregressive Model (HAR) as a simple way to approximate the long-memory behaviour of volatility documented, for example, by Andersen et al. (2001, 2003). Chiriac and Voev (2011) implement the Vector HAR model (VHAR) as a multivariate extension in which, realised covariance is expressed as a linear combination of past daily, weekly and monthly realised covariances. The upper triangular elements of a factor, Y_t, obtained from the Cholesky decomposition of the realised covariance matrix are modelled as follows:

$$Y_{t+1} = c + \beta_d Y_t + \beta_w Y_{t-4:t} + \beta_m Y_{t-21:t} + u_{t+1}, \tag{13}$$

If H_t is the matrix of realised covariances, its Cholesky decomposition gives a matrix $H_t = X_t X_t'$ and then $Y_t = vech(X_t)$. The past k-day values of Y (where k=5 or 22) are computed as: $Y_{t-k:t} = \frac{1}{k}\sum_{j=0}^{k-1} Y_{t-j}$. c is a constant term and β_d, β_w, β_m are the parameters of the daily, weekly and monthly components of the model, respectively.

We obtain covariance forecasts, H_t, by a reverse transformation of the $Y_t's$. As pointed out in Chiriac and Voev (2011), modelling the Cholesky factors rather than covariances directly is done in order to avoid unnecessary restrictions that ensure positive definite covariance matrices. We iteratively produce k–step ahead covariance forecasts ($H_{t:t+k}$) based on 1-day ahead forecasts obtained from Equation (13).

Similar to the univariate HAR model of Corsi (2009), the VHAR model approximates long memory in a parsimonious way. The model involves a fixed number of parameters regardless of the number of assets. Furthermore, it is extremely easy to estimate through panel OLS in contrast to formal long memory models that require more complex optimisation and lack clear economic interpretation.

The above specification of the VHAR model assumes that all covariances obey the same dynamics. This assumption may appear restrictive as it does not allow for richer covariance dynamics. Nevertheless, a fully generalized model allowing each unique variance-covariance term to follow its own dynamics substantially increases the number of estimated parameters from 4 to $4 \times [N \times (N+1)/2]$. As a result, such a model may lead to worse forecasting performance, especially when the number of assets is large. To further illustrate this argument, we also estimate a generalized version of the VHAR model (GVHAR). Specifically, we model the dynamics of the time series of each unique element of the Cholesky factorization of the conditional covariance matrix through a univariate HAR model and then estimate the system using seemingly unrelated regressions (see Čech and Baruník, 2017). Comparing the in-sample fitting ability of the VHAR model with that of the GVHAR model, we find that there is very little improvement from using the more complex alternative. Furthermore, we compare the out-of-sample performance of the two models and find that the average loss of the GVHAR model is in many cases slightly higher than that of the VHAR model (for results see Table A8 of the appendix). Therefore, we decide to only include in our analysis the more parsimonious and easier-to-compute VHAR model.

2.11 Hybrid Implied Covariance (HYBICOV)

We also propose a hybrid implied covariance specification (HYBICOV) which combines realised correlations (from intraday returns) with model-free option implied volatilities. Our setting does not allow us to compute fully option-implied covariances using existing approaches. For example, the approach of Driessen et al. (2009) relies on implied volatilities of a market index or portfolio of assets and its constituents, which is not applicable to our case. Furthermore, the methodology of Chang et al. (2012) for estimating option-implied betas based on risk-neutral volatility, skewness or kurtosis assumes a linear asset pricing model consisting of an asset and the market which does not apply to our context either.

We start by deriving an estimate of implied volatility, which is robust to volatility risk. Simple estimates of implied volatility are biased forecasts of future realised volatility unless the market price of volatility risk is zero (Chernov, 2007). As this assumption is rejected by several studies (e.g., Carr and Wu, 2009; Driessen et al., 2009) we implement the non-parametric correction used by DeMiguel et al. (2013) to adjust the model-free implied volatility for the volatility risk premium.

Specifically, the variance risk premium of each asset is computed as follows:

$$VRP_{t:t+k} = \frac{IV^2_{t:t+k}}{E(RV^2_{t:t+k})}, \tag{14}$$

where $VRP_{t:t+k}$ is the variance risk premium between t and t+k. $IV_{t:t+k}$ is the model-free implied volatility and $E(RV^2_{t,t+k})$ is the expected realised variance for the period between t and t+k. We obtain realised variance forecasts using the HAR model of Corsi (2009). Then, following DeMiguel et al. (2013) we average the variance risk premium over a period of 252 - k days:

$$\overline{VRP}_t = \frac{1}{252-k} \sum_{j=t-251}^{t-k} VRP_{j:j+k} \tag{15}$$

Then, the adjusted model-free implied volatility of each asset is computed as follows:

$$\widetilde{IV}_t = \sqrt{\frac{IV^2_{t:t+k}}{\overline{VRP}_t}} \tag{16}$$

Finally, k-day ahead covariance forecasts are obtained as follows:

$$H_{t:t+k} = CIV_t \cdot Corr_{t-k:t} \cdot CIV_t, \tag{17}$$

where $H_{t:t+h}$ is the covariance matrix forecast for the period from day *t+1* to t+k, CIV_t is an $N \times N$ diagonal matrix containing the annualised k-day variance premium adjusted model-free implied volatilities (i.e., the $\widetilde{IV}_t'$s defined above) in its main diagonal, and $Corr_{t-k:t}$ is the realised correlation from day t-k to day t. There is ample evidence that forward-looking information from option prices help improve volatility forecasting (e.g., Kourtis et al., 2016) and accuracy of equity beta estimation (e.g., Buss and Vilkov, 2012). Therefore, our analysis offers an empirical test for the hypothesis that forward-looking information can help improve covariance forecasts.

3 Empirical Analysis

We obtain intraday data on five major European equity indices, namely: AEX (Netherlands), CAC 40 (France), DAX (Germany), FTSE 100 (UK), and SMI (Switzerland) from TickData Market (www.tickdatamarket.com) for the period from January 1, 2000 to April 19, 2016. These represent some of the largest economies and markets in Europe for which implied volatility indices and a long enough history of quality intraday data are available. The location and operating times of these markets allow us to avoid issues related to asynchronous trading times that would complicate the calculation of covariances using intraday data. These markets have recently been affected by the global financial crisis and the subsequent European sovereign debt crisis, which makes the analysis of their covariance dynamics particularly interesting.

Since the UK is located in a different time zone compared to the other four countries (one hour difference), we synchronize all markets using Coordinated Universal Time (UTC). To reduce the impact of known microstructure effects (e.g., bid-ask bounce), we create a grid of equally spaced 5-minute prices as is the standard approach in the literature (e.g., see Andersen et al., 2001). In order to avoid potential distortions resulting from opening and closing jumps, we discard the first and last 15 minutes from each trading day. We then calculate 5-minute returns for each market and match them across markets to create a balanced panel of intraday returns. We also drop from our dataset days with fewer than 70% of the maximum number of 5-minute intraday returns. Finally, we obtain daily dividend-adjusted closing prices for the above five equity indices and end-of-day values of model-free implied volatility indices from Datastream. In our sample we include 3,904 observation days.

Table 1 reports average realised correlations, as well as correlations computed from daily, weekly and monthly returns, respectively. We present correlations for the entire sample as well as for three sub-periods, namely: January 1, 2000–July 31, 2008 (pre-crisis), August 1, 2008–December 31, 2009 (crisis), and January 1, 2010–April 19, 2016 (post-crisis). Not surprisingly, all correlation coefficients in Panels B–D are quite high, indicating the close integration between major European equity markets. The average daily pairwise correlations computed from high-frequency data (Panel A) are generally lower than the corresponding correlations from daily returns. In line with the published literature, we observe that correlations are higher during the 2008–2009 global financial crisis period. The finding of increased comovement between equity markets during bad times is well-documented (see Ang and Bekaert, 2002; Aloui et al., 2011; Garcia and Tsafack, 2011). Some suggested explanations for this phenomenon include: commonality in liquidity during periods of market declines (Hameed et al., 2010), trade linkages between countries (Forbes, 2002), and co-movement in risk premiums across markets during periods of low liquidity (Vayanos, 2004).

Table 2 reports summary statistics for the time series of average daily realised correlations across the five markets. Specifically, every day we calculate the realised correlations between a market and all other markets and then take an average across these pairwise correlations. This yields a time series of daily average realised cross-correlations. The table shows the mean, median, range, standard deviation, skewness and kurtosis for the series of average daily cross-correlations. All average daily cross-correlations are positive and range between 0.58 and 0.71. Furthermore, their standard deviations indicate a substantial amount of variation, which is in line with the statistics in Panel A of Table 1. The distribution of average correlations is negatively skewed and leptokurtic for almost all markets. This non-normal feature of the empirical distribution of average cross-correlations is likely the outcome of sudden downward shifts during turmoil periods, such as the 2008–2009 global financial crisis.

Table 3 provides a summary description of the forecasting models that will be compared. The table also reports the number of parameters per model for 5, 10 and 100 assets, respectively, as an indication of estimation complexity. In general, more heavily parameterised models require more computational time and their estimation may even be infeasible in systems of large dimension. It can be seen from the table that the GARCH models are the most heavily parameterised ones, whereas HYBICOV and RWE are model-free. Furthermore, VHAR and EWMA involve a fixed number of parameters regardless of the dimension of the problem (four and one parameters, respectively).

3.1 Predictive Accuracy

3.1.1 Forecast Evaluation Approach

We evaluate the forecasting ability of the models based on three multivariate loss functions. We use the Euclidean distance (e.g., Laurent et al., 2012) along with two additional loss functions also used in Bollerslev et al. (2018). These loss functions are computed as follows:

$$\mathcal{L}_E = vech(\Sigma_t - H_t)' vech(\Sigma_t - H_t) \tag{18}$$

$$\mathcal{L}_F = Tr[(\Sigma_t - H_t)'(\Sigma_t - H_t)] \tag{19}$$

$$\mathcal{L}_Q = log|H_t| + Tr[H_t^{-1}\Sigma_t] \tag{20}$$

where $vech$ is the operator that stacks the lower portion of the covariance matrix (including the main diagonal) to a vector and Tr denotes the trace of a square matrix, defined as the sum of its diagonal elements. Σ_t denotes the realised covariance matrix at time t, defined in Equation (3), and H_t is the time t matrix of conditional covariance forecasts from a specific model. $\mathcal{L}_E$ is the Euclidean loss function computed by equally-weighting all the unique elements of the forecast error matrix. $\mathcal{L}_F$ is the Frobenius distance, which extends the mean squared error loss function to the multivariate space. $\mathcal{L}_Q$ is the multivariate quasi-likelihood loss function, which is scale invariant. Using a wide range of simulations, Laurent et al. (2013) show that rankings produced by the above three functions based on covariance proxies are consistent with those based on the true latent covariance matrix. The authors also provide theoretical conditions that ensure consistency.

Equipped with the values of the above three loss functions, we compare the forecasting accuracy of the models, using both parametric and non-parametric statistical tests. In particular, we employ the parametric unconditional predictive ability test (GW hereafter) (Giacomini and White (2006); for applications see Shephard and Sheppard (2010), Patton and Sheppard (2015)) and the non-parametric Model Confidence Set (MCS) (Hansen et al. (2011); for applications see Laurent et al. (2012); Liu et al. (2015); Duong and Swanson (2015)).

The GW test allows for comparisons of nested models and accounts for parameter uncertainty. The null hypothesis of the test is specified as follows:

$$H_0 : \overline{\Delta L}_{ij} = 0 \tag{21}$$

where $\overline{\Delta L}_{ij} = \frac{1}{P}\sum_{t=1}^{P} \Delta L_{ij,t}$ is the average loss difference between models i and j over the out-of-sample period. The Giacomini and White (2006) unconditional test follows a chi-squared distribution with 1 degree of freedom. To account for serial dependence in multi step-ahead forecasts we use a Newey-West estimator for the asymptotic variance of the out-of-sample loss differences with k lags (where k indicates the forecast horizon). Giacomini and White (2006) also proposed a more general conditional predictive ability test. However, as we test the average predictive accuracy across models, the unconditional test is deemed a more appropriate choice.

The MCS identifies a set of models that are superior to all other models at a given level of confidence. Specifically, given an initial set of models, $\mathcal{M}_0$, the test sequentially discards models with inferior predictive ability until a subset of superior models, $\mathcal{M}$, is reached. The elimination is based on sequentially testing the following hypothesis:

$$H_0 : E(\Delta L_{ij,t}) = 0, \qquad \text{for all } i, j \in \mathcal{M}. \tag{22}$$

The above null hypothesis is tested at each step, using the following two statistics:

$$T_{SQ} = \sum_{i<j} \frac{(\overline{\Delta L}_{ij})^2}{\widehat{var}(\overline{\Delta L}_{ij})} \tag{23}$$

$$T_R = \max_{i,j\in\mathcal{M}} \frac{|\overline{\Delta L}_{ij}|}{\sqrt{\widehat{var}(\overline{\Delta L}_{ij})}}, \tag{24}$$

where T_{SQ} is the semi-quadratic statistic, and T_R is the range statistic, respectively.[1] Let $\overline{\Delta L}_i = \frac{1}{m}\sum_{j=1}^{m}\overline{\Delta L}_{ij}$ be the average sample loss of model i relative to the average loss across all other m models that are currently in the set, $\mathcal{M}$. If the null hypothesis of Equation (22) is rejected, then the model with the highest value of the statistic $t_i = \frac{\overline{\Delta L}_i}{\sqrt{\widehat{var}(\overline{\Delta L}_i)}}$ is eliminated and the procedure is repeated until the MCS is constructed at the given confidence level (for more technical details refer to Hansen et al., 2011). $\widehat{var}(\overline{\Delta L}_{ij})$ and $\widehat{var}(\overline{\Delta L}_i)$ are estimates of the asymptotic variance of $\overline{\Delta L}_{ij}$ and $\overline{\Delta L}_i$, respectively, computed using a block bootstrap procedure with 10,000 replications and a block length of 2 observations.[2]

3.1.2 In-Sample Analysis

We begin by comparing the information content of the twelve models under consideration. In particular, we obtain in-sample estimates of the conditional covariance matrix from each model. We then compute the average values of the three multivariate loss functions, described in section 3.1.1, using the above covariance estimates and realised covariance as a proxy for the latent covariance. Multivariate GARCH models are estimated via quasi maximum likelihood (QML), the VHAR model is estimated with panel OLS, whereas EWMA, HYBICOV and RWE require no estimation.

Table 4 reports the average in-sample predictive losses of the models at each forecast horizon (i.e., 1, 5, and 22 trading days, respectively). To formally test for statistically significant loss differences we employ the GW test. The results show that the VHAR model achieves the best in-sample fit across all loss functions and forecast horizons. For

[1]To save space we only report results for the T_{SQ} statistic of Equation (23) and present the results for the T_R statistic in Table A10 of the appendix.

[2]Experimentation with different block lengths (e.g., 4 and 12) gave very similar results.

instance, looking at the value of the L_E loss function for the monthly forecast horizon, we observe that the average loss of VHAR is about 89% lower than the second best model (HYBICOV) and around 96% lower than the best performing GARCH model (A-D-BEKK). The difference in the in-sample forecast errors of VHAR relative to all other models is statistically significant at the 5% level across all loss functions and forecast horizons.

RWE and HYBICOV are the next best performing models. These two models in most cases demonstrate a superior in-sample fit compared to GARCH models. For instance, the average L_F (L_E) forecast loss of the HYBICOV model for the monthly forecast horizon, is approximately 69.5% (67.8%) lower than that of the best performing GARCH model. Finally, we cannot clearly distinguish the best among multivariate GARCH models as ranking differs across loss functions. Focusing, for example, on the weekly forecast horizon, the L_F loss function suggests that A-D-BEKK has the best in-sample predictive ability, whereas according to the L_Q loss function the DCC and A-DCC models are superior among the GARCH models considered. In sum, the results presented above show that less heavily parameterised models that rely on information from high-frequency and options data achieve a substantially better in-sample fit compared to models that employ daily data, such as the GARCH models we consider.

3.1.3 Out-of-Sample Analysis

We now focus on the out-of-sample predictive performance of the competing models. We produce rolling out-of-sample forecasts for each model and forecast horizon. We estimate the parameters of each model using the most recent 1,000 observations (approximately 4 years of daily data) and obtain k–day ahead forecasts, for $k = 1, 5$ and 22. We then move one day forward and repeat this procedure until we reach the end of our sample. In this manner, we obtain $M - k + 1$ out-of-sample forecasts where M is the length of the out-of-sample period. Following Bollerslev et al. (2018), we apply an "insanity filter" to replace negative-definite covariance forecasts with the average realised covariance over the respective in-sample estimation period. Multi-step ahead forecasts (i.e., 5–day and 22–day) for multivariate GARCH, EWMA and the VHAR are produced recursively from day-ahead forecasts. For HYBICOV and RWE we obtain multi-step ahead forecasts directly, using the estimators described in section 2.

Table 5 presents the average out-of-sample forecast losses for the 12 models under consideration.[3] The results show that the VHAR model yields the most accurate out-of-sample forecasts across all horizons and loss functions. Specifically, at the daily (monthly) forecast horizon and based on the L_F loss function the VHAR model has about 16.7% (24.6%) lower average forecast error compared to the second best model, which is the RWE. The average L_F loss for the VHAR model is approximately 43.1% (51.5%) lower than that of the best performing GARCH model at the weekly (monthly) forecast horizon. Similar conclusions are drawn from the other two loss functions.

Furthermore, the GW test indicates that the VHAR model produces significantly more accurate out-of-sample forecasts relative to all other models at the 5% significance level. The only exception is the RWE in some cases. The two non-parametric models based on intraday and options data, namely the RWE and HYBICOV, are consistently ranked either second or third in terms of their average forecast loss. Multivariate GARCH models generally underperform the more parsimonious VHAR, HYBICOV and RWE models, with

[3]We do not report the tables containing the pairwise comparisons of statistical significance between competing models in order to save space. These results are presented in Tables A11–A13 of the appendix.

very few exceptions mainly related to the L_Q loss function. Finally, based on the L_E and L_F loss functions, A-D-BEKK has the best performance among GARCH models.

Table 6 presents results for the Model Confidence Set (MCS hereafter).[4] The table reports the ranking of the models based on the semi-quadratic statistic of Equation (23). Models that are included in the MCS are marked with an asterisk. A few observations are in order. First, looking across forecast horizons and loss functions, we see that the MCS only includes the VHAR model and in very few cases the RWE. Second, even though the HYBICOV is not part of the MCS, it is consistently ranked in the top four models. Third, the ranking of multivariate GARCH models appears to vary across loss functions with the A-D-BEKK and A-CCC models exhibiting the lowest average forecast errors in slightly more cases. Tables A11–A13 of the appendix, which present pairwise GW tests, show that in many cases the latter two models yield significantly lower forecast errors relative to the other multivariate GARCH models. In sum, the above analysis clearly suggests that more parsimonious models that rely on intraday and option-implied information perform much better than parametric alternatives which employ daily data.

3.2 Consistency with Asset Pricing Theory

In line with Bali and Engle (2010) we now evaluate the ability of the covariance models considered to produce theoretically consistent results under the Intertemporal Capital Asset Pricing (ICAPM) model of Merton (1973). The ICAPM states that the expected return of an asset i varies linearly with its conditional covariance with the market and with economic state variables that represent shifts to the investment opportunity set. The

[4]Table A14 of the appendix presents results for the 25% MCS also employed in Laurent et al. (2012). Our results are not considerably affected by this alternative consideration.

equilibrium risk–return relation under the ICAPM can be expressed as follows:

$$E_t\left(R_{j,t+1}\right) - R_{f,t} = \beta Cov_t\left(R_{j,t+1}, R_{m,t+1}\right) + \gamma Cov_t\left(R_{j,t+1}, X_{t+1}\right), \tag{25}$$

where $E_t\left(R_{j,t+1}\right)$ is the time t expected return of asset j (for $j = 1, 2, ..., N + 1$, i.e. N individual assets and the market portfolio) conditional on the information set available at time t, $R_{f,t}$ is the risk-free rate at time t, $Cov_t\left(R_{j,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{j,t+1}, X_{t+1}\right)$ are the expected conditional covariances of asset j with the market m and with the economic state variables based on the information set available at time t. Therefore, there are two sources of risk compensation (premia), one associated with the conditional covariance of the asset with the market and another related to the conditional covariance of the asset with economic state variables (shifts to the investment opportunity set).

Empirical research on the risk-return trade-off within the ICAPM has produced conflicting results. For example, Campbell (1987), Whitelaw (1994), Harvey (2001) and Brandt and Kang (2004) find a negative risk-return relationship. In contrast, using daily data and GARCH-in-mean models Baillie and DeGennaro (1990), Campbell and Hentschel (1992), and Glosten et al. (1993) identify a positive but insignificant risk–return relationship in a time series setting. Alternatively, Harrison and Zhang (1999) find a positive risk-return trade-off only at longer horizons of one and two years but not at shorter horizons. Recent studies employing new methodologies provide evidence of a significant positive risk-return relationship (e.g., Ghysels et al., 2005; Bali and Peng, 2006; Guo and Whitelaw, 2006; Ludvigson and Ng, 2007; Lundblad, 2007; Bali, 2008; Bali and Engle, 2010).

We shall use the alternative covariance models within the ICAPM framework to investigate if a significantly positive risk-return trade-off exists in the five European markets under study. In addition to adding empirical evidence to this literature, this allows us to assess the theoretical consistency of alterative covariance estimators.

In the absence of readily available intraday data for computing economic state variables (e.g., default spread and term spread), we rely on a reduced form model which assumes zero intertemporal hedging demand.[5] We first obtain expected covariances between each European equity index and the European market portfolio from the lagged realised covariance (RWE) and the VHAR models. We employ the STOXX 50 index as a proxy for the European equity market portfolio. For the risk-free rate we employ the Euribor (the LIBOR rate produced similar results). Then, following Bali and Engle (2010) we estimate the following system of $N+1$ equations using the seemingly unrelated regression (SUR) methodology.

$$\begin{aligned} XR_{i,t+1} &= \alpha_i + \beta\sigma_{i,m,t+1} + e_{i,t+1} \\ XR_{m,t+1} &= \alpha_m + \beta\sigma_{m,m,t+1} + e_{m,t+1}, \end{aligned} \tag{26}$$

where $XR_{i,t+1} = R_{i,t+1} - R_{f,t}$ and $XR_{m,t+1} = R_{m,t+1} - R_{f,t}$ are the excess returns of asset i ($i = 1, 2, ..., N$) and market index m at time *t+1*. $\sigma_{i,m,t+1}$ is the time t prediction of next period's conditional covariance between asset i and the market portfolio and $\sigma_{m,m,t}$ is the time t predicted variance of the market portfolio. We approximate $\sigma_{i,m,t}$ and $\sigma_{m,m,t}$ using one-day ahead predicted covariances from the 12 covariance forecasting models, respectively. The common slope coefficient (β) represents the average relative risk aversion

[5]We hope to address this limitation in the future, if we can access intraday data for relevant economic variables.

in the ICAPM. Similar to Bali and Engle (2010) we assume a common slope but allow the intercept to vary across markets. For robustness purposes, we re-estimated the above system using weighted least squares (see Table A1 of the appendix for these results) similar to Bali and Engle (2010) and also as a panel with robust standard errors. The results obtained were very similar.

The results from estimating the above system are reported in Table 7. Each panel corresponds to a different intraday sampling frequency used for the estimation of the VHAR, HYBICOV and RWE models (i.e., 5, 10 and 30 minutes, respectively). All alphas are insignificant. In line with the theory, the common slope of the two best performing models in our preceding statistical analysis, namely the VHAR and the RWE, is positive and statistically significant with t-statistics ranging between 2.54 and 5.63. For example, the VHAR model yields a common slope coefficient of 2.24 (2.83) at the 5-minute (30-minute) frequency which is economically meaningful. In contrast, the multivariate GARCH models provide mixed evidence regarding the sign and statistical significance of the risk-return coefficient. In particular, among the multivariate GARCH models only the A–BEKK and A–DCC yield a positive and statistically significant common slope estimate at the 5% level, whereas the slope of the DCC model is significant at the 10% level. The O-GARCH model and its asymmetric extension suggest a negative and insignificant risk-return relation. Finally, the average relative risk aversion coefficient of the CCC and A-CCC models is too low to be economically meaningful and is statistically insignificant.

Overall, the analysis provides additional support for the two high-frequency models, namely the VHAR and the RWE, that are superior in terms of statistical accuracy. Specifically, these two models provide the most theoretically consistent results in that they yield a positive and strongly significant risk-return relation with economically plausible risk aversion coefficients.[6] These findings also suggest that empirical asset pricing studies in this area need to consider that conclusions may depend on the choice of the covariance model.

3.3 Portfolio Performance

We further assess the economic value of covariance forecasts in an asset allocation context. We assume an investor who allocates her wealth across the five European equity markets under study based on the covariance matrix forecasts obtained from each model. We employ daily, weekly and monthly rebalancing frequencies, respectively. In each period the investor solves the following minimisation problem:

$$min\, w_t' H_t w_t \quad \text{s.t.} \quad w_t' \iota = 1, \tag{27}$$

where w_t is an $N \times 1$ vector of global minimum variance (GMV) portfolio weights, H_t is the $N \times N$ matrix of conditional covariance forecasts from a particular model, and ι is an $N \times 1$ vector of ones. The optimal weights of the GMV portfolio are given by:

$$\widetilde{w}_t = \frac{H_t^{-1}\iota}{\iota' H_t^{-1}\iota} \tag{28}$$

From the last equation it becomes obvious that the optimal portfolio weights are only a function of conditional variances and covariances. In our simple asset allocation framework, we do not consider expected stock returns for two reasons. First, it is well known that estimation errors in sample means are large and the corresponding portfolios

[6] Even though the common slope estimate of the HYBICOV model is insignificant in the context of the six European equity indices, it appears strongly significant when we consider US stocks as test assets in our subsequent analysis (see section 4.2).

perform worse compared to the GMV portfolio (e.g., see DeMiguel et al., 2009; Kourtis et al., 2012). Second, since our focus in the current study is to forecast the covariance matrix, ignoring expected returns makes our results insensitive to errors in estimated expected returns.

Using the covariance forecasts from each model, we end up with 12 different portfolio strategies. We evaluate the out-of-sample performance of each strategy by following a rolling estimation approach, which is standard in the portfolio choice literature (DeMiguel et al., 2009). Specifically, we first estimate the optimal portfolio weights for each model m, using Equation (28), and then compute the ex-post average portfolio return for model m as $r_{t+1}^{(m)} = \left(\widetilde{w}_t^{(m)}\right)' r_{t+1}$, where r_t is an $N \times 1$ vector of asset returns. Following this approach, we obtain 12 time series of out-of-sample portfolio returns, one for each covariance forecasting model.

We then use the following two metrics to evaluate the portfolio performance of each forecasting model m:

- Out-of-sample portfolio variance:

$$\hat{\sigma}_m^2 = \frac{1}{P} \sum_{t=1}^{P} \left(r_t^{(m)} - \bar{r}_t^{(m)} \right) \tag{29}$$

- Average Portfolio Turnover:

$$\hat{\tau}_m = \frac{1}{P-1} \sum_{t=1}^{P-1} \| \widetilde{w}_{t+1}^{(m)} - \widetilde{w}_{t+}^{(m)} \|_1, \tag{30}$$

where P is the number of out-of-sample returns, $\bar{r}_t^{(m)}$ is the average return of portfolio m over the out-of-sample period, $\widetilde{w}_{t+}^{(m)}$ are the portfolio weights before rebalancing at the beginning of the period $t+1$ and $\|\cdot\|_1$ is the 1-norm. Portfolio turnover is a measure of the stability of a strategy. Strategies based on inaccurate estimates of the covariance matrix tend to lead to high turnover and trading costs (see Kourtis, 2014, for more details).

We compare the variance of the strategies with the variance of the equally-weighted portfolio (1/N). The 1/N portfolio is a typical benchmark in the portfolio choice literature because of its superiority over many sample-based portfolios (DeMiguel et al., 2009). We test the following hypothesis: $H_0 : \hat{\sigma}_m^2 - \hat{\sigma}_{1/N}^2 = 0$ (where $\hat{\sigma}_{1/N}$ is the estimated out-of-sample variance of the 1/N benchmark). P-values are estimated using the robust non–parametric bootstrap method of Ledoit and Wolf (2011), assuming an average block size of 10 and 10,000 trials.

The results in Table 8 show that employing high-frequency and option price data can lead to substantial gains in portfolio performance in comparison to using daily data. Predictions from the VHAR and RWE models lead to significantly lower out-of-sample portfolio variance compared to all other models for all considered forecast horizons. For example, for the monthly rebalancing frequency, we observe that the annualised out-of-sample variance of the VHAR, RWE and HYBICOV-based portfolios are equal to 0.0180, 0.0184 and 0.0185, respectively, compared to 0.0238 for the 1/N benchmark and these differences are significant at the 5% level. Furthermore, when rebalancing is performed weekly, the portfolios based on the VHAR, RWE and HYBICOV models have an annualised variance equal to 0.0308, 0.0304, and 0.0311 compared to 0.0405 for the 1/N benchmark and the differences are significant at the 1% level. We also observe that in the case of daily rebalancing, all models offer significant diversification benefits compared to the 1/N benchmark while only the portfolios constructed using intraday and options data are significantly less risky than the 1/N portfolio at the weekly and monthly horizons.

With regards to portfolio stability, for the weekly and monthly rebalancing frequencies, the portfolios constructed from models that employ high-frequency and options data yield the lowest average turnover among all the twelve models. This means that these portfolios are more attractive in the presence of transaction costs. One exception is the O-GARCH-based portfolio which has lower turnover relative to the portfolios based on the RWE and HYBICOV models when rebalancing is performed weekly. When the horizon is daily the O-GARCH-based portfolio is the least sensitive to transaction costs over time.

4 Robustness Tests

4.1 Stability across Market Regimes

We first investigate whether model performance varies across different market regimes. Our central question in this section is whether the predictive accuracy and the ranking of the various forecasting models under consideration change during periods of economic unrest, such as the 2008–2009 global financial crisis. To this end, we perform our out-of-sample analysis over three sub-periods of the full sample, namely: a relatively calm period from January 1, 2000 to July 31, 2008, the period of the global financial crisis between August 1, 2008 and December 31, 2009 and, finally, the January 1, 2010–April 19, 2016 period which includes the eurozone debt crisis.

Table 9 presents the MCS results for each forecast horizon and sub-period of the full sample. VHAR remains the best performing model. Moreover, in line with the results from the full sample, we observe that the VHAR, followed the HYBICOV and RWE are the models with the lowest average forecast errors across the three sub-samples. These three models are superior to all other models during the global financial crisis. None of the GARCH models is included in the MCS, and their ranking appears to be sensitive to the specific sub-period considered.

It is worth pointing out that asymmetric GARCH specifications (e.g., A-D-BEKK, A-CCC, A-O-GARCH) achieve better predictive performance relative to their symmetric counterparts during the 2008–2009 global financial crisis period. This finding highlights the importance of accounting for leverage effects during periods of market stress, when negative shocks are not only more frequent but also more sizeable. Moreover, this finding is in line with Martens and Poon (2001) and Laurent et al. (2012), who find that during turmoil periods, such as the 2001 .com bubble, GARCH models that incorporate asymmetries are superior to their symmetric counterparts.

The results reported in Table A15 of the appendix indicate that the average losses of most models are substantially higher during the 2008–2009 global financial crisis period. This extends the documented findings of inferior performance of volatility forecasting models during periods of market turmoil (Brownlees et al., 2012; Kourtis et al., 2016), to the multivariate case.

Finally, to check if these results are sensitive to the definition of the global financial crisis, we repeat the above analysis by defining the financial crisis to be between August 1, 2007 to December 31, 2009 as in Laurent et al. (2012). The results look qualitatively similar (Table A18 of the appendix).

4.2 Evidence from Individual Stocks

To further assess the robustness of our findings and ascertain that they are not specific to the equity markets under consideration, we repeat our out-of-sample analysis using ten highly liquid US stocks which match the ones used in Bollerslev et al. (2018). These stocks are: American Express (AXP), Boeing (BA), Chevron (CVX), DuPont (DD), General Electric (GE), IBM , JPMorgan Chase (JPM), Coca-Cola (KO), Microsoft (MSFT), and Exxon Mobil (XOM). The data are obtained from PiTrading and cover the period from January 2, 2003 to April 19, 2016. Options data for the calculation of the model-free implied volatility (MFIV) of each stock are collected from HistoricalOptionData. MFIV is computed using the methodology of Britten-Jones and Neuberger (2000). Details about this methodology and its practical implementation are provided in Section A of the appendix (see also Trolle and Schwartz, 2010; Prokopczuk et al., 2017).

Table 10 contains the average out-of-sample losses of the models and Table 11 reports the corresponding MCS results. The parsimonious and less parameterised models that rely on high-frequency and option-implied information, namely the RWE, VHAR and HYBICOV, consistently produce the lowest average forecast errors. The Giacomini and White (2006) test further suggests that, in most cases, the above three models yield equally accurate forecasts. Moreover, Table 11 indicates that these models are almost always the only ones included in the MCS. A potentially interesting finding is the improvement in the predictive performance of the HYBICOV model compared to the analysis involving the five European equity markets.

We also explore the risk-return relationship using the above 10 stocks and the S&P 500 index as a proxy for the market portfolio. We repeat the procedure illustrated in section 3.2. The estimation results from the SUR system, presented in Table 12,[7] reveal positive and strongly significant risk-return coefficients for the three parsimonious models that use intraday and option price data, namely the RWE, VHAR and HYBICOV with

[7]Table A9 of the appendix shows the corresponding results from a weighted least squares estimation.

t-statistics ranging between 4.52 and 6.94, depending on the intraday sampling frequency. The size of the relative risk aversion coefficient from the above three models is found to be between 2.07 and 5.69, which is consistent with previous research using different test assets (Bali, 2008). Also, the significance of the positive risk-return trade-off seems to strengthen as one moves to lower intraday sampling frequencies. This seems reasonable as individual stocks are more likely to be affected by microstructure noise as opposed to stock indices. Even though conditional covariances from multivariate GARCH models lead to positive common slope estimates, these are statistically significant only for the BEKK, A-CCC, A-DCC, and EWMA and insignificant for the rest of the models.

Finally, we compare the out-of-sample performance of the US stock global minimum variance portfolios constructed using the covariance forecasting models under consideration. The results presented in Table 13 are largely consistent with those for the European equity markets. In particular, the VHAR model again produces the portfolios with the minimum variance at the daily and weekly rebalancing frequencies. When rebalancing is performed monthly the HYBICOV model has slightly lower out-of-sample variance equal to 0.0152 compared to 0.0156 for the VHAR model. A slight difference compared to the analysis for the European equity markets is that even though the turnover of the VHAR-based portfolio is comparable to that of the other models, it is not the lowest overall.

4.3 Alternative Sampling Frequencies

A potential concern in our analysis may be that the superior forecasting ability of the high-frequency models could be driven to some extent by microstructure noise in intraday data. To investigate this possibility, we re-perform our analysis using prices sampled at lower frequencies, specifically every 10 and 30 minutes. The results reported in Tables A2–A5 of the appendix are largely consistent with those based on 5-minute intraday returns. Specifically, VHAR is the best performing model across loss functions and forecast horizons, followed by the HYBICOV and RWE models. Interestingly, the performance of the HYBICOV model is improved compared to the 5-minute sampling frequency. Furthermore, from Tables A3 and A5 we see that the MCS almost always includes the VHAR model and in a few cases the two non-parametric alternatives, namely RWE and HYBICOV. For instance, at the 10-minute sampling frequency, HYBICOV is consistently included in the MCS. The above analysis clearly indicates that microstructure noise is not a major driver of our findings.

Access to intraday data has become easier nowadays, but we appreciate that in some cases intraday data may not be readily available. Motivated by this consideration, we repeat our out-of-sample analysis by replacing the intraday data with daily data for estimating the VHAR, RWE and HYBICOV models. Given the requirement of the VHAR model for positive semi-definite covariances at the daily, weekly and monthly frequencies, we propose a specification that relies on EWMA covariances in order to ensure positive definite covariances in the model. We then run our main tests for a monthly forecast horizon. We exclude a daily or weekly analysis here as daily and weekly realised covariance matrices are in many cases negative semi-definite. Moreover, potential biases might occur as a consequence of using small samples for their estimation.

Table A6 of the appendix contains the out-of-sample forecast losses of the models. The results are generally consistent with those from our main analysis which is based on intraday data. Specifically, RWE, VHAR and HYBICOV are overall the best performing models. This finding is also supported by the results of the MCS tests contained in

Table A7. Not surprisingly, the relative performance of multivariate GARCH models improves when intraday data are not considered. However, their performance is still inferior to that of the three parsimonious and less parameterised models. The above findings have important and more general implications as they clearly suggest that the superior forecasting performance of the nonparametric models, such as VHAR and RWE, is valid in the absence of intraday data, extending the applicability of these models.

4.4 Alternative Forecasting Considerations

In line with Bollerslev et al. (2018), our main analysis is based on rolling samples of 1,000 observations. Using rolling forecasts from 1,250 observations does not significantly alter our results (Tables A16–A17 of the appendix). Finally, repeating our out-of-sample analysis employing non-overlapping weekly and monthly forecasts leads to very similar conclusions (Table A19 of the appendix).

5 Conclusions

We compare the ability of several popular models to predict the covariance matrix of equity returns. The models considered employ daily, intraday and, option-implied information and range from fully parametric to model-free. A comprehensive evaluation is performed across various forecast horizons, market regimes, intraday sampling frequencies and assets. In addition to the statistical significance, we explore the economic value of covariance forecasts based on their ability to generate theoretically consistent results and produce superior portfolio allocations.

Our analysis suggests that a Vector Heterogeneous Autoregressive model is the best performing model, both in statistical and economic terms. Lagged realised covariances and a novel hybrid estimator combining realised correlations with variance risk premium adjusted option-implied volatilities are good alternatives. Multivariate GARCH models are inferior both in statistical and economic terms in almost all cases. The ranking of the best performing models remains roughly the same, although forecast errors increase during periods of turmoil, such as global financial crisis. We further show that forecasts from models employing high-frequency data yield a positive and significant risk-return trade-off with economically meaningful relative risk aversion estimates. These models also lead to portfolios with lower risk relative to all GARCH-based models and the 1/N benchmark. Overall, our conclusions hold for a sample of ten liquid US stocks and are robust to a battery of additional robustness checks.

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Table 1 Realised and Historical Correlations

This table shows average daily realised correlations from 5-minute returns (Panel A) as well as pairwise correlations using daily, weekly and monthly returns (Panels B–D). The left panel contains correlations for the full sample (1/1/2000–19/4/2016), while the remaining panels show the same statistics for three sub-periods of the full sample, namely: 1/1/2000–31/7/2008 (pre-crisis sub-sample), 1/8/2008–31/12/2009 (crisis sub-sample), and 1/1/2010–19/4/2016 (post-crisis sub-sample).

	Full Sample: 1/1/2000-19/4/2016				*Pre-Crisis Sub-Sample: 1/1/2000-31/7/2008*				*Crisis Sub-Sample: 1/8/2008-31/12/2009*				*Post-Crisis Sub-Sample: 1/1/2010-19/4/2016*			
	AEX	**CAC**	**DAX**	**FTSE**	**AEX**	**CAC**	**DAX**	**FTSE**	**AEX**	**CAC**	**DAX**	**FTSE**	**AEX**	**CAC**	**DAX**	**FTSE**
Panel A: 5-minute Returns																
CAC	0.8457				0.7777				0.8986				0.8969			
DAX	0.7960	0.8238			0.7651	0.7797			0.8330	0.8785			0.8234	0.8582		
FTSE	0.6886	0.6892	0.6605		0.5091	0.5146	0.4708		0.8525	0.8678	0.8095		0.8091	0.8021	0.7974	
SMI	0.6334	0.6466	0.6530	0.5559	0.5644	0.5739	0.5893	0.3835	0.7286	0.7543	0.7591	0.7279	0.6906	0.7035	0.7011	0.6927
Panel B: Daily Returns																
CAC	0.9282				0.9152				0.9570				0.9421			
DAX	0.8652	0.8984			0.8396	0.8741			0.8825	0.9173			0.9129	0.9335		
FTSE	0.8804	0.8922	0.8226		0.8442	0.8694	0.7830		0.9357	0.9537	0.8867		0.8987	0.8878	0.8559	
SMI	0.8339	0.8339	0.7908	0.8218	0.8353	0.8312	0.7797	0.8076	0.8755	0.9075	0.8363	0.9071	0.7970	0.7910	0.7800	0.7786
Panel C: Weekly Returns																
CAC	0.9272				0.9052				0.9593				0.9428			
DAX	0.8821	0.9138			0.8581	0.8907			0.9125	0.9596			0.9003	0.9163		
FTSE	0.8781	0.8917	0.8449		0.8282	0.8421	0.7963		0.9397	0.9735	0.9510		0.8969	0.8863	0.8321	
SMI	0.8075	0.8111	0.7899	0.8193	0.8132	0.7999	0.7854	0.7855	0.8419	0.8901	0.8752	0.9040	0.7609	0.7591	0.7223	0.7793
Panel D: Monthly Returns																
AEX																
CAC	0.9121				0.9276				0.9141				0.9087			
DAX	0.8641	0.9075			0.8938	0.9174			0.8778	0.9680			0.8226	0.8623		
FTSE	0.8522	0.8668	0.7969		0.8244	0.8512	0.7893		0.9459	0.9448	0.9337		0.8242	0.8493	0.7339	
SMI	0.8007	0.8156	0.7915	0.7992	0.8340	0.8429	0.8388	0.8250	0.9073	0.9413	0.9381	0.9156	0.6705	0.7032	0.6259	0.6834

Table 2 Summary Statistics for Daily Realised Cross-Correlations

This table reports summary statistics for the average daily realised cross-correlations of the five equity markets under consideration. Each day we calculate the realised correlation matrix of stock returns and then take an average across all pairwise correlations. This yields a time series of average cross-correlations. The table shows the mean, median, range, standard deviation, skewness and kurtosis statistics. The sample period is from January 1, 2000 to April 19, 2016.

	Mean	**Median**	**Range**	**St.Dev**	**Skewness**	**Kurtosis**
AEX	0.6999	0.7567	1.1856	0.2052	-1.5987	6.0950
CAC	0.7106	0.7665	0.9654	0.2025	-1.5868	6.1215
DAX	0.6980	0.7494	0.9566	0.1944	-1.5476	6.1583
FTSE	0.5796	0.6936	0.9591	0.3071	-1.1508	3.0938
SMI	0.5881	0.6321	0.9394	0.2067	-0.9020	3.5392

Table 3 Description of Models

This table provides a description of the covariance forecasting models. Column 1 shows model names, while column 2 contains the number of estimated parameters for N assets. Columns 3 to 5 present the number of parameters for 5, 10, and 100 assets, respectively.

Model	N	$N = 5$	$N = 10$	$N = 100$
Diagonal BEKK (D-BEKK)	$2N + N(N+1)/2$	25	75	5,250
Asymmetric Diagonal BEKK (A-D-BEKK)	$3N + N(N+1)/2$	30	85	5,230
Constant Conditional Correlation (CCC)	$N(N+5)/2$	25	75	5,250
Asymmetric Constant Conditional Correlation (A-CCC)	$N(N+5)/2 + N$	30	85	5,350
Dynamic Conditional Correlation (DCC)	$N(N+5)/2 + 2$	27	77	5,252
Asymmetric Dynamic Conditional Correlation (A-DCC)	$N(N+5)/2 + 3N + 3$	43	108	5,553
Exponentially Weighted Moving Average (EWMA)	1	1	1	1
Orthogonal GARCH (O-GARCH)	$3N$	15	30	300
Asymmetric Orthogonal GARCH (A-O-GARCH)	$4N$	20	40	400
Random Walk Estimator (RWE)	0	0	0	0
Vector Heterogeneous Autoregressive (VHAR)	4	4	4	4
Hybrid Implied Covariance (HYBICOV)	0	0	0	0

Table 4 In-Sample Model Fit

This table reports the average in-sample losses of the twelve models under consideration for the 1–, 5–, and 22–day forecast horizons, respectively. L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The L_E and L_F values are multiplied by 10^4 to facilitate readability. The model with the best in-sample fit is marked with an asterisk (*).

	1–Day Horizon			5–Day Horizon			22–Day Horizon		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
D-BEKK	0.0473	0.0759	−47.3343	0.8746	1.4206	−39.3607	13.7819	22.5365	−31.9993
A-D-BEKK	0.0453	0.0730	−47.4693	0.8368	1.3656	−39.4851	13.4676	22.1214	−32.0889
CCC	0.0508	0.0798	−47.5971	0.9234	1.4579	−39.6003	13.9322	22.0597	−32.1441
A-CCC	0.0516	0.0803	−47.5883	0.9688	1.5124	−39.5948	15.3321	24.0135	−32.1111
DCC	0.0554	0.0889	−47.6315	1.0326	1.6766	−39.6493	15.7994	25.7996	−32.2162
A-DCC	0.0554	0.0889	−47.6315	1.0326	1.6766	−39.6493	15.7994	25.7996	−32.2162
EWMA	0.0554	0.0895	−46.8565	1.0737	1.7513	−38.9640	17.3593	28.4840	−31.7412
O-GARCH	0.0543	0.0876	−47.4545	1.0105	1.6458	−39.4561	15.4867	25.3375	−32.0403
A-O-GARCH	0.0613	0.0995	−47.4917	1.2053	1.9752	−39.4906	19.3896	31.8236	−32.0609
RWE	0.0236	0.0366	−47.9031	0.2939	0.4574	−39.9811	5.2074	8.1023	−32.1768
VHAR	0.0084*	0.0130*	−48.8868*	0.0695*	0.1077*	−40.5407*	0.4824*	0.7422*	−32.9133*
HYBICOV	0.0257	0.0399	−47.2979	0.3276	0.5076	−39.4018	4.3368	6.7232	−31.9224

Table 5 Out-of-Sample Forecast Losses

This table reports the average out-of-sample forecast losses for the 1–, 5–, and 22–day horizons, respectively. We employ a rolling window of 1,000 observations to produce forecasts from parametric models. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). A dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.

	1–Day Horizon			**5–Day Horizon**			**22–Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
D-BEKK	0.0279	0.0447	−19.5926	0.0221	0.0357	−19.4935	0.0215	0.0347	−19.1854
A-D-BEKK	0.0206	0.0329	−19.6601	0.0144	0.0232	−19.5400	0.0123	0.0196	−19.2106
CCC	0.0346	0.0550	−19.9575	0.0295	0.0470	−19.8107	0.0302	0.0481	−19.3823
A-CCC	0.0327	0.0514	−19.9266	0.0281	0.0444	−19.7770	0.0263	0.0414	−19.3651
DCC	0.0362	0.0581	−19.9827	0.0310	0.0501	−19.8392	0.0314	0.0505	−19.3962
A-DCC	0.0362	0.0581	−19.9815	0.0310	0.0501	−19.8390	0.0314	0.0506	−19.3973
EWMA	0.0357	0.0576	−19.2043	0.0309	0.0503	−19.0575	0.0342	0.0556	−18.5729
O-GARCH	0.0365	0.0592	−19.6562	0.0314	0.0515	−19.5511	0.0324	0.0530	−19.2155
A-O-GARCH	0.0516	0.0848	−19.6626	0.0454	0.0750	−19.5608	0.0386	0.0635	−19.2194
RWE	0.0152†	0.0236†	−20.2782	0.0076†	0.0118†	−20.3814†	0.0073	0.0114	−19.9184†
VHAR	0.0114*	0.0178*	−20.6043*	0.0064*	0.0100*	−20.4024$*$	0.0061*	0.0095*	−19.9277*
HYBICOV	0.0165	0.0256	−19.6659	0.0116	0.0182	−19.6377	0.0144	0.0225	−19.1767

Table 6 Model Confidence Set Results

This table shows the results of the 5% Model Confidence Set (MCS). We employ three statistical loss functions and 1–, 5–, and 22–day forecast horizons, respectively. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						**5–Day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	rank	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**
D-BEKK	6	0.00	6	0.00	7	0.00	6	0.00	6	0.00	9	0.00	6	0.00	6	0.00	9	0.00
A-D-BEKK	4	0.00	4	0.00	8	0.00	4	0.00	4	0.00	7	0.00	3	0.00	3	0.00	3	0.00
CCC	8	0.00	8	0.00	4	0.00	8	0.00	8	0.00	6	0.00	11	0.00	11	0.00	5	0.00
A-CCC	5	0.00	5	0.00	6	0.00	5	0.00	5	0.00	5	0.00	5	0.00	5	0.00	6	0.00
DCC	11	0.00	10	0.00	5	0.00	11	0.00	9	0.00	4	0.00	10	0.00	10	0.00	8	0.00
A-DCC	12	0.00	11	0.00	3	0.00	12	0.00	11	0.00	3	0.00	9	0.00	9	0.00	7	0.00
EWMA	9	0.00	9	0.00	12	0.00	9	0.00	10	0.00	12	0.00	12	0.00	12	0.00	12	0.00
O-GARCH	10	0.00	12	0.00	10	0.00	10	0.00	12	0.00	10	0.00	8	0.00	8	0.00	11	0.00
A-O-GARCH	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	10	0.00
RWE	2	0.05	2*	0.06	2	0.00	2	0.03	2	0.04	2*	0.06	2	0.00	2	0.00	2*	0.55
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	9	0.00	3	0.00	3	0.00	8	0.00	4	0.00	4	0.00	4	0.00

Table 7 Risk–Return Trade-off

This table reports the intercepts and the common slope estimates along with their t-statistics (in parentheses) from the SUR system estimation of Equation (26). The dependent variables correspond to the daily excess returns on the five European equity markets and the STOXX 50 index which serves as a proxy for the market portfolio. Each column contains results from using a different model to obtain conditional covariance estimates between the returns of each equity index and the STOXX 50 index. Three alternative intraday sampling frequencies are considered for the estimation of RWE, VHAR and HYBICOV (Panels A–C, respectively).

Panel A: 5-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AEX}	-0.0619 (-0.95)	-0.0717 (-1.02)	-0.0263 (-0.36)	-0.0598 (-0.79)	-0.0940 (-1.26)	-0.0055 (-0.08)	-0.0350 (-0.50)	-0.0524 (-0.73)	-0.0922 (-1.32)	0.0693 (0.91)	0.0388 (0.53)	-0.0389 (-0.54)
α_{CAC}	-0.0786 (-1.13)	-0.0892 (-1.18)	-0.0385 (-0.49)	-0.0733 (-0.90)	-0.1126 (-1.40)	-0.0145 (-0.18)	-0.0471 (-0.63)	-0.0667 (-0.86)	-0.1101 (-1.47)	0.0654 (0.81)	0.0332 (0.43)	-0.0515 (-0.67)
α_{DAX}	-0.0088 (-0.13)	-0.0187 (-0.25)	0.0291 (0.38)	-0.0067 (-0.08)	-0.0442 (-0.56)	0.0518 (0.67)	0.0203 (0.27)	0.0009 (0.01)	-0.0409 (-0.55)	0.1320 (1.63)	0.0994 (1.28)	0.0159 (0.21)
α_{SMI}	-0.0386 (-0.70)	-0.0460 (-0.79)	-0.0112 (-0.19)	-0.0371 (-0.60)	-0.0646 (-1.05)	0.0034 (0.06)	-0.0179 (-0.31)	-0.0328 (-0.55)	-0.0627 (-1.07)	0.0583 (0.94)	0.0360 (0.60)	-0.0225 (-0.38)
α_{FTSE}	-0.0439 (-0.78)	-0.0524 (-0.87)	-0.0141 (-0.23)	-0.0411 (-0.64)	-0.0706 (-1.10)	0.0032 (0.05)	-0.0193 (-0.32)	-0.0355 (-0.58)	-0.0665 (-1.11)	0.0601 (0.94)	0.0371 (0.60)	-0.0253 (-0.41)
α_{Market}	-0.0892 (-1.27)	-0.1000 (-1.31)	-0.0484 (-0.60)	-0.0847 (-1.02)	-0.1245 (-1.52)	-0.0231 (-0.29)	-0.0573 (-0.75)	-0.0770 (-0.98)	-0.1226 (-1.61)	0.0600 (0.73)	0.0265 (0.33)	-0.0610 (-0.78)
β_{Pooled}	1.6923 (3.80)	2.2436 (2.54)	0.9654 (0.97)	1.5297 (1.60)	2.5167 (2.51)	0.3252 (0.40)	0.9650 (1.38)	1.3493 (1.70)	2.1907 (3.17)	-1.2590 (-1.35)	-0.6116 (-0.76)	1.0725 (1.35)

Panel B: 10-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AEX}	-0.0748 (-1.15)	-0.0810 (-1.15)	-0.0335 (-0.46)	-0.0598 (-0.79)	-0.0940 (-1.26)	-0.0055 (-0.08)	-0.0350 (-0.50)	-0.0524 (-0.73)	-0.0922 (-1.32)	0.0693 (0.91)	0.0388 (0.53)	-0.0389 (-0.54)
α_{CAC}	-0.0928 (-1.34)	-0.0993 (-1.32)	-0.0464 (-0.59)	-0.0733 (-0.90)	-0.1126 (-1.40)	-0.0145 (-0.18)	-0.0471 (-0.63)	-0.0667 (-0.86)	-0.1101 (-1.47)	0.0654 (0.81)	0.0332 (0.43)	-0.0515 (-0.67)
α_{DAX}	-0.0225 (-0.33)	-0.0285 (-0.38)	0.0213 (0.27)	-0.0067 (-0.08)	-0.0442 (-0.56)	0.0518 (0.67)	0.0203 (0.27)	0.0009 (0.01)	-0.0409 (-0.55)	0.1320 (1.63)	0.0994 (1.28)	0.0159 (0.21)
α_{SMI}	-0.0483 (-0.88)	-0.0531 (-0.91)	-0.0165 (-0.28)	-0.0371 (-0.60)	-0.0646 (-1.05)	0.0034 (0.06)	-0.0179 (-0.31)	-0.0328 (-0.55)	-0.0627 (-1.07)	0.0583 (0.94)	0.0360 (0.60)	-0.0225 (-0.38)
α_{FTSE}	-0.0550 (-0.98)	-0.0608 (-1.01)	-0.0201 (-0.32)	-0.0411 (-0.64)	-0.0706 (-1.10)	0.0032 (0.05)	-0.0193 (-0.32)	-0.0355 (-0.58)	-0.0665 (-1.11)	0.0601 (0.94)	0.0371 (0.60)	-0.0253 (-0.41)
α_{Market}	-0.1034 (-1.47)	-0.1097 (-1.44)	-0.0563 (-0.70)	-0.0847 (-1.02)	-0.1245 (-1.52)	-0.0231 (-0.29)	-0.0573 (-0.75)	-0.0770 (-0.98)	-0.1226 (-1.61)	0.0600 (0.73)	0.0265 (0.33)	-0.0610 (-0.78)
β_{Pooled}	1.9817 (4.50)	2.4923 (2.83)	1.1411 (1.14)	1.5297 (1.60)	2.5167 (2.51)	0.3252 (0.40)	0.9650 (1.38)	1.3493 (1.70)	2.1907 (3.17)	-1.2590 (-1.35)	-0.6116 (-0.76)	1.0725 (1.35)

Panel C: 30-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AEX}	-0.0899 (-1.39)	-0.0922 (-1.33)	-0.0326 (-0.45)	-0.0598 (-0.79)	-0.0940 (-1.26)	-0.0055 (-0.08)	-0.0350 (-0.50)	-0.0524 (-0.73)	-0.0922 (-1.32)	0.0693 (0.91)	0.0388 (0.53)	-0.0389 (-0.54)
α_{CAC}	-0.1094 (-1.59)	-0.1109 (-1.49)	-0.0454 (-0.58)	-0.0733 (-0.90)	-0.1126 (-1.40)	-0.0145 (-0.18)	-0.0471 (-0.63)	-0.0667 (-0.86)	-0.1101 (-1.47)	0.0654 (0.81)	0.0332 (0.43)	-0.0515 (-0.67)
α_{DAX}	-0.0392 (-0.58)	-0.0404 (-0.55)	0.0219 (0.28)	-0.0067 (-0.08)	-0.0442 (-0.56)	0.0518 (0.67)	0.0203 (0.27)	0.0009 (0.01)	-0.0409 (-0.55)	0.1320 (1.63)	0.0994 (1.28)	0.0159 (0.21)
α_{SMI}	-0.0609 (-1.12)	-0.0627 (-1.08)	-0.0166 (-0.28)	-0.0371 (-0.60)	-0.0646 (-1.05)	0.0034 (0.06)	-0.0179 (-0.31)	-0.0328 (-0.55)	-0.0627 (-1.07)	0.0583 (0.94)	0.0360 (0.60)	-0.0225 (-0.38)
α_{FTSE}	-0.0686 (-1.23)	-0.0710 (-1.19)	-0.0198 (-0.32)	-0.0411 (-0.64)	-0.0706 (-1.10)	0.0032 (0.05)	-0.0193 (-0.32)	-0.0355 (-0.58)	-0.0665 (-1.11)	0.0601 (0.94)	0.0371 (0.60)	-0.0253 (-0.41)
α_{Market}	-0.1198 (-1.72)	-0.1206 (-1.60)	-0.0550 (-0.69)	-0.0847 (-1.02)	-0.1245 (-1.52)	-0.0231 (-0.29)	-0.0573 (-0.75)	-0.0770 (-0.98)	-0.1226 (-1.61)	0.0600 (0.73)	0.0265 (0.33)	-0.0610 (-0.78)
β_{Pooled}	2.3297 (5.63)	2.8345 (3.34)	1.1254 (1.13)	1.5297 (1.60)	2.5167 (2.51)	0.3252 (0.40)	0.9650 (1.38)	1.3493 (1.70)	2.1907 (3.17)	-1.2590 (-1.35)	-0.6116 (-0.76)	1.0725 (1.35)

Table 8 Out-of-Sample Portfolio Performance

*This table summarizes the performance of the global minimum variance portfolios constructed using the covariance forecasts from the 12 models under consideration. The portfolios are compared on the basis of their annualised out-of-sample variance ($\hat{\sigma}^2_m$) and average out-of-sample turnover ($\hat{\tau}_m$), respectively. Results for three different rebalancing frequencies are presented. Each portfolio is compared with the 1/N benchmark (last row). *, and ** indicate rejections of the hypothesis of equal out-of-sample variances of a given portfolio and the 1/N benchmark at the 10% and 5% levels, respectively. The non-parametric methodology of Ledoit and Wolf (2011) is adopted for the above test in order to calculate the corresponding p-values.*

	Daily Rebalancing		**Weekly Rebalancing**		**Monthly Rebalancing**	
	$\hat{\sigma}^2_m$	$\hat{\tau}_m$	$\hat{\sigma}^2_m$	$\hat{\tau}_m$	$\hat{\sigma}^2_m$	$\hat{\tau}_m$
D-BEKK	0.0299**	0.3351	0.0475	0.7435	0.0195	1.1482
A-D-BEKK	0.0296**	0.5052	0.0436	0.8635	0.0207	1.2014
CCC	0.0311**	0.3875	0.0349	0.8466	0.0201	1.2592
A-CCC	0.0299**	0.3379	0.0337*	0.9109	0.0204	1.4296
DCC	0.0307**	0.4111	0.0367	0.9106	0.0203	1.3501
A-DCC	0.0307**	0.4117	0.0368	0.9122	0.0203	1.3494
EWMA	0.0304**	0.4024	0.0477	1.0240	0.0263	1.9513
O-GARCH	0.0309**	0.2690	0.0426	0.6016	0.0187	0.8627
A-O-GARCH	0.0319**	0.4031	0.0454	0.7518	0.0193	0.9998
RWE	0.0297**	1.2313	0.0304**	0.7364	0.0184*	0.6838
VHAR	0.0282**	0.5524	0.0308**	0.5490	0.0180*	0.7535
HYBICOV	0.0293**	0.7951	0.0311**	0.6462	0.0185*	0.6919
1/N	0.0397	0.0033	0.0405	0.0085	0.0238	0.0208

Table 9 MCS Results for Different Market Regimes

This table shows the results of the 5% Model Confidence Set (MCS) for three sub-periods of the full sample. We employ three statistical loss functions and 1–, 5–, and 22–day forecast horizons, respectively. Each panel shows the results for a different sub-period of the full sample (i.e. pre-crisis, crisis and post-crisis). L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						5–Day Horizon						22–Day Horizon					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	Rank	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**
Panel A: 1/1/2000 - 31/7/2008 (pre-crisis sub-sample)																		
D-BEKK	5	0.03	5	0.03	7	0.00	5	0.00	5	0.00	11	0.00	5	0.00	5	0.00	9	0.00
A-D-BEKK	6	0.03	6	0.03	10	0.00	4	0.00	4	0.00	9	0.00	3	0.00	4	0.00	4	0.00
CCC	8	0.03	8	0.03	6	0.00	7	0.00	7	0.00	8	0.00	8	0.00	8	0.00	11	0.00
A-CCC	4*	0.15	4*	0.17	5	0.00	6	0.00	6	0.00	3	0.00	6	0.00	6	0.00	3	0.00
DCC	11	0.03	11	0.03	3	0.00	8	0.00	8	0.00	5	0.00	10	0.00	10	0.00	8	0.00
A-DCC	10	0.03	10	0.03	4	0.00	9	0.00	9	0.00	4	0.00	11	0.00	11	0.00	6	0.00
EWMA	12	0.02	12	0.02	12	0.00	11	0.00	11	0.00	12	0.00	7	0.00	7	0.00	12	0.00
O-GARCH	9	0.03	9	0.03	8	0.00	12	0.00	12	0.00	10	0.00	12	0.00	12	0.00	7	0.00
A-O-GARCH	7	0.03	7	0.03	9	0.00	10	0.00	10	0.00	7	0.00	9	0.00	9	0.00	5	0.00
RWE	2*	0.89	2*	0.89	2	0.00	3	0.02	3	0.03	2*	0.36	2*	0.32	2*	0.32	1*	1.00
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	2	0.02
HYBICOV	3*	0.15	3*	0.18	11	0.00	2	0.02	2	0.03	6	0.00	4	0.00	3	0.00	10	0.00
Panel B: 1/8/2008- 31/12/2009 (crisis sub-sample)																		
D-BEKK	6	0.00	6	0.00	8	0.00	6	0.00	6	0.00	8	0.00	6	0.00	6	0.00	8	0.00
A-D-BEKK	4	0.01	4	0.01	9	0.00	4	0.00	4	0.00	9	0.00	4	0.00	4	0.00	9	0.00
CCC	9	0.00	8	0.00	5	0.00	9	0.00	8	0.00	6	0.00	11	0.00	11	0.00	6	0.00
A-CCC	5	0.01	5	0.01	3	0.00	5	0.00	5	0.00	4	0.00	5	0.00	5	0.00	4	0.00
DCC	12	0.00	12	0.00	6	0.00	12	0.00	12	0.00	5	0.00	10	0.00	10	0.00	5	0.00
A-DCC	11	0.00	11	0.00	4	0.00	11	0.00	11	0.00	7	0.00	9	0.00	9	0.00	7	0.00
EWMA	8	0.00	9	0.00	10	0.00	10	0.00	10	0.00	11	0.00	12	0.00	12	0.00	12	0.00
O-GARCH	10	0.00	10	0.00	11	0.00	8	0.00	9	0.00	10	0.00	8	0.00	8	0.00	11	0.00
A-O-GARCH	7	0.00	7	0.00	12	0.00	7	0.00	7	0.00	12	0.00	7	0.00	7	0.00	10	0.00
RWE	2*	0.11	2*	0.12	2	0.00	2*	0.61	2*	0.67	1*	1.00	2	0.00	2	0.01	2*	0.08
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	2*	0.50	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.02	3	0.02	7	0.00	3	0.00	3	0.00	3	0.00	3	0.00	3	0.00	3	0.03
Panel C: 1/1/2010- 19/4/2016 (post-crisis sub-sample)																		
D-BEKK	5	0.00	6	0.00	8	0.00	5	0.00	5	0.00	8	0.00	5	0.00	5	0.00	4	0.00
A-D-BEKK	4	0.00	4	0.00	9	0.00	4	0.00	4	0.00	9	0.00	3	0.00	3	0.00	3	0.00
CCC	10	0.00	10	0.00	4	0.00	10	0.00	10	0.00	4	0.00	10	0.00	10	0.00	5	0.00
A-CCC	6	0.00	5	0.00	7	0.00	6	0.00	6	0.00	7	0.00	7	0.00	7	0.00	9	0.00
DCC	8	0.00	8	0.00	5	0.00	8	0.00	8	0.00	5	0.00	9	0.00	9	0.00	7	0.00
A-DCC	9	0.00	9	0.00	3	0.00	9	0.00	9	0.00	3	0.00	8	0.00	8	0.00	6	0.00
EWMA	7	0.00	7	0.00	10	0.00	7	0.00	7	0.00	10	0.00	6	0.00	6	0.00	10	0.00
O-GARCH	11	0.00	11	0.00	11	0.00	12	0.00	12	0.00	11	0.00	12	0.00	12	0.00	11	0.00
A-O-GARCH	12	0.00	12	0.00	12	0.00	11	0.00	11	0.00	12	0.00	11	0.00	11	0.00	12	0.00
RWE	2	0.05	2*	0.05	2	0.00	2	0.01	2	0.02	2	0.02	2	0.00	2	0.00	2*	0.14
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	6	0.00	3	0.00	3	0.00	6	0.00	4	0.00	4	0.00	8	0.00

Table 10 Out–of–Sample Forecast Losses (US Stocks)

This table reports the average out-of-sample forecast losses for the 1–, 5–, and 22–day horizons, respectively. The sample consists of ten US stocks (details presented in section 4.2). We employ a rolling window of 1,000 observations to produce forecasts from parametric models. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). A dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.

	1–Day Horizon			**5–Day Horizon**			**22–Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
D-BEKK	0.5722	0.9014	−26.3993	0.3887	0.6102	−26.2553	0.3380	0.5326	−25.8502
A-D-BEKK	0.5745	0.9094	−26.4812	0.3932	0.6217	−26.3236	0.3455	0.5490	−25.8857
CCC	0.5342	0.7950	−27.0146	0.3725	0.5330	−26.7738	0.3540	0.4963	−26.1790
A-CCC	0.4776	0.7134	−26.9941	0.3282	0.4688	−26.7493	0.3157	0.4413	−26.1594
DCC	0.5444	0.8154	−27.0667	0.3843	0.5566	−26.8177	0.3666	0.5214	−26.2055
A-DCC	0.5442	0.8151	−27.0508	0.3844	0.5567	−26.7952	0.3660	0.5203	−26.1536
EWMA	0.6420	1.0150	−25.5509	0.4857	0.7665	−25.2413	0.4898	0.7774	−24.5879
O-GARCH	0.6061	0.9738	−26.6555	0.4452	0.7188	−26.4499	0.4423	0.7202	−25.9722
A-O-GARCH	0.6464	1.0493	−26.6282	0.5152	0.8438	−26.4333	0.4934	0.8120	−25.9609
RWE	$0.4443^{\dagger}$	$0.7081^{\dagger}$	−26.7815	$0.1967^{\dagger}$	$0.3072^{\dagger}$	-27.9492^{*}	$0.1538^{\dagger}$	$0.2417^{\dagger}$	-27.3109^{*}
VHAR	$0.3054^{\dagger}$	$0.4820^{\dagger}$	-28.3505^{*}	$0.1569*$	0.2441^{*}	−27.8470	0.1345^{*}	$0.2103*$	−26.8206
HYBICOV	0.2983^{*}	0.4714^{*}	−26.9947	$0.1582^{\dagger}$	$0.2474^{\dagger}$	$-27.8507^{\dagger}$	$0.1559^{\dagger}$	$0.2431^{\dagger}$	$-27.1039^{\dagger}$

Table 11 Model Confidence Set Results (US Stocks)

This table shows the results of the 5% Model Confidence Set (MCS). The sample consists of ten US stocks (details presented in section 4.2). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 trading days, respectively. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						**5–Day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	rank	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**
D-BEKK	10	0.028	10	0.017	9	0.000	6	0.024	8	0.013	9	0.000	5	0.027	7	0.025	9	0.000
A-D-BEKK	11	0.028	11	0.017	8	0.000	9	0.024	10	0.013	6	0.000	6	0.027	9	0.025	8	0.000
CCC	7	0.028	6	0.020	6	0.000	8	0.024	7	0.013	5	0.000	8	0.027	6	0.025	5	0.000
A-CCC	4	0.028	4	0.023	5	0.000	5	0.024	4	0.013	7	0.000	4	0.027	5	0.025	7	0.000
DCC	8	0.028	8	0.020	2	0.000	10	0.024	6	0.013	4	0.000	10	0.027	8	0.025	4	0.000
A-DCC	6	0.028	5	0.020	4	0.000	7	0.024	5	0.013	8	0.000	11	0.027	4	0.025	6	0.000
EWMA	12	0.028	12	0.017	12	0.000	12	0.024	12	0.013	12	0.000	12	0.027	12	0.025	12	0.000
O-GARCH	9	0.028	9	0.020	10	0.000	4	0.024	11	0.013	10	0.000	9	0.027	11	0.025	10	0.000
A-O-GARCH	5	0.028	7	0.020	11	0.000	11	0.024	9	0.013	11	0.000	7	0.027	10	0.025	11	0.000
RWE	3*	0.093	3*	0.103	7	0.000	3*	0.156	3*	0.141	1*	1.000	2*	0.170	2*	0.180	1*	1.000
VHAR	2*	0.456	2*	0.428	1*	1.000	1*	1.000	1*	1.000	3*	0.166	1*	1.000	1*	1.000	3	0.000
HYBICOV	1*	1.000	1*	1.000	3	0.000	2*	0.846	2*	0.765	2*	0.295	3*	0.170	3*	0.180	2*	0.197

Table 12 Risk–Return Trade-off (US Stocks)

This table reports the intercepts and the common slope coefficient estimates along with their t-statistics (in parentheses) from the SUR system estimation of Equation (26). The dependent variables correspond to daily excess returns on the ten US stocks presented in section 4.2 and the S&P 500 index, which serves as a proxy for the market portfolio. Each column contains the results from using a different model to obtain estimates of the expected conditional covariance between the returns of each stock and the return on S&P 500 index. Three alternative intraday sampling frequencies are considered for the estimation of RWE, VHAR and HYBICOV (Panels A–C).

Panel A: 5-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AXP}	-0.1085 (-0.79)	-0.1906 (-1.35)	-0.2550 (-1.77)	-0.1565 (-1.01)	-0.1123 (-0.76)	-0.0280 (-0.20)	-0.0670 (-0.48)	-0.0440 (-0.31)	-0.1062 (-0.75)	-0.0139 (-0.09)	-0.0559 (-0.38)	-0.1117 (-0.78)
α_{BA}	-0.0470 (-0.48)	-0.1068 (-1.05)	-0.1430 (-1.39)	-0.0718 (-0.65)	-0.0474 (-0.44)	0.0188 (0.19)	-0.0049 (-0.05)	0.0086 (0.09)	-0.0291 (-0.29)	0.0269 (0.25)	-0.0021 (-0.02)	-0.0370 (-0.36)
α_{CVX}	-0.0577 (-0.61)	-0.1189 (-1.21)	-0.1575 (-1.57)	-0.0840 (-0.78)	-0.0431 (-0.43)	0.0116 (0.12)	-0.0128 (-0.13)	0.0008 (0.01)	-0.0388 (-0.40)	0.0202 (0.19)	-0.0098 (-0.10)	-0.0492 (-0.49)
α_{DD}	-0.0575 (-0.57)	-0.1204 (-1.16)	-0.1660 (-1.57)	-0.0920 (-0.80)	-0.0693 (-0.62)	0.0127 (0.13)	-0.0137 (-0.14)	0.0013 (0.01)	-0.0409 (-0.40)	0.0220 (0.20)	-0.0100 (-0.09)	-0.0507 (-0.48)
α_{GE}	-0.1226 (-1.12)	-0.1902 (-1.69)	-0.2527 (-2.18)	-0.1553 (-1.26)	-0.1217 (-1.03)	-0.0501 (-0.45)	-0.0809 (-0.73)	-0.0623 (-0.56)	-0.1100 (-0.99)	-0.0380 (-0.32)	-0.0712 (-0.61)	-0.1132 (-1.00)
α_{IBM}	-0.0329 (-0.42)	-0.0801 (-1.01)	-0.1061 (-1.32)	-0.0487 (-0.57)	-0.0291 (-0.35)	0.0201 (0.26)	0.0022 (0.03)	0.0124 (0.16)	-0.0161 (-0.21)	0.0265 (0.32)	0.0041 (0.05)	-0.0234 (-0.29)
α_{JPM}	-0.0836 (-0.55)	-0.1636 (-1.06)	-0.2280 (-1.46)	-0.1499 (-0.88)	-0.0934 (-0.58)	-0.0095 (-0.06)	-0.0527 (-0.34)	-0.0262 (-0.17)	-0.0927 (-0.60)	0.0083 (0.05)	-0.0372 (-0.23)	-0.0961 (-0.61)
α_{KO}	0.0171 (0.28)	-0.0166 (-0.26)	-0.0334 (-0.52)	-0.0002 (0.00)	0.0127 (0.19)	0.0539 (0.87)	0.0412 (0.67)	0.0486 (0.78)	0.0286 (0.46)	0.0587 (0.90)	0.0434 (0.67)	0.0233 (0.37)
α_{MSFT}	-0.0158 (-0.17)	-0.0708 (-0.73)	-0.1019 (-1.03)	-0.0435 (-0.41)	-0.0044 (-0.04)	0.0457 (0.48)	0.0250 (0.26)	0.0364 (0.38)	0.0032 (0.03)	0.0526 (0.51)	0.0249 (0.25)	-0.0077 (-0.08)
α_{XOM}	-0.0706 (-0.81)	-0.1259 (-1.41)	-0.1609 (-1.77)	-0.0946 (-0.96)	-0.0717 (-0.75)	-0.0057 (-0.06)	-0.0282 (-0.32)	-0.0153 (-0.17)	-0.0510 (-0.58)	0.0026 (0.03)	-0.0249 (-0.27)	-0.0598 (-0.66)
α_{Market}	-0.0460 (-0.63)	-0.1027 (-1.34)	-0.1430 (-1.82)	-0.0746 (-0.85)	-0.0519 (-0.61)	0.0149 (0.20)	-0.0151 (-0.20)	0.0046 (0.06)	-0.0398 (-0.53)	0.0266 (0.32)	-0.0021 (-0.03)	-0.0390 (-0.50)
β_{Pooled}	2.0695 (4.52)	4.0231 (4.88)	5.1888 (5.25)	3.6290 (2.11)	2.6009 (1.89)	0.4918 (0.82)	1.1416 (2.24)	0.7312 (1.11)	1.6922 (2.93)	0.2139 (0.22)	0.8372 (0.96)	1.6161 (2.38)

Panel B: 10-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AXP}	-0.1761 (-1.28)	-0.2148 (-1.51)	-0.2839 (-1.97)	-0.1565 (-1.01)	-0.1123 (-0.76)	-0.0280 (-0.20)	-0.0670 (-0.48)	-0.0440 (-0.31)	-0.1062 (-0.75)	-0.0139 (-0.09)	-0.0559 (-0.38)	-0.1117 (-0.78)
α_{BA}	-0.0946 (-0.96)	-0.1206 (-1.18)	-0.1577 (-1.54)	-0.0718 (-0.65)	-0.0474 (-0.44)	0.0188 (0.19)	-0.0049 (-0.05)	0.0086 (0.09)	-0.0291 (-0.29)	0.0269 (0.25)	-0.0021 (-0.02)	-0.0370 (-0.36)
α_{CVX}	-0.1080 (-1.13)	-0.1310 (-1.33)	-0.1734 (-1.74)	-0.0840 (-0.78)	-0.0431 (-0.43)	0.0116 (0.12)	-0.0128 (-0.13)	0.0008 (0.01)	-0.0388 (-0.40)	0.0202 (0.19)	-0.0098 (-0.10)	-0.0492 (-0.49)
α_{DD}	-0.1096 (-1.09)	-0.1350 (-1.30)	-0.1831 (-1.74)	-0.0920 (-0.80)	-0.0693 (-0.62)	0.0127 (0.13)	-0.0137 (-0.14)	0.0013 (0.01)	-0.0409 (-0.40)	0.0220 (0.20)	-0.0100 (-0.09)	-0.0507 (-0.48)
α_{GE}	-0.1767 (-1.61)	-0.2034 (-1.80)	-0.2646 (-2.31)	-0.1553 (-1.26)	-0.1217 (-1.03)	-0.0501 (-0.45)	-0.0809 (-0.73)	-0.0623 (-0.56)	-0.1100 (-0.99)	-0.0380 (-0.32)	-0.0712 (-0.61)	-0.1132 (-1.00)
α_{IBM}	-0.0712 (-0.92)	-0.0899 (-1.13)	-0.1173 (-1.46)	-0.0487 (-0.57)	-0.0291 (-0.35)	0.0201 (0.26)	0.0022 (0.03)	0.0124 (0.16)	-0.0161 (-0.21)	0.0265 (0.32)	0.0041 (0.05)	-0.0234 (-0.29)
α_{JPM}	-0.1492 (-0.99)	-0.1801 (-1.16)	-0.2517 (-1.61)	-0.1499 (-0.88)	-0.0934 (-0.58)	-0.0095 (-0.06)	-0.0527 (-0.34)	-0.0262 (-0.17)	-0.0927 (-0.60)	0.0083 (0.05)	-0.0372 (-0.23)	-0.0961 (-0.61)
α_{KO}	-0.0107 (-0.17)	-0.0238 (-0.37)	-0.0416 (-0.65)	-0.0002 (0.00)	0.0127 (0.19)	0.0539 (0.87)	0.0412 (0.67)	0.0486 (0.78)	0.0286 (0.46)	0.0587 (0.90)	0.0434 (0.67)	0.0233 (0.37)
α_{MSFT}	-0.0597 (-0.63)	-0.0799 (-0.82)	-0.1139 (-1.16)	-0.0435 (-0.41)	-0.0044 (-0.04)	0.0457 (0.48)	0.0250 (0.26)	0.0364 (0.38)	0.0032 (0.03)	0.0526 (0.51)	0.0249 (0.25)	-0.0077 (-0.08)
α_{XOM}	-0.1152 (-1.33)	-0.1357 (-1.51)	-0.1751 (-1.93)	-0.0946 (-0.96)	-0.0717 (-0.75)	-0.0057 (-0.06)	-0.0282 (-0.32)	-0.0153 (-0.17)	-0.0510 (-0.58)	0.0026 (0.03)	-0.0249 (-0.27)	-0.0598 (-0.66)
α_{Market}	-0.0917 (-1.25)	-0.1116 (-1.45)	-0.1556 (-1.99)	-0.0746 (-0.85)	-0.0519 (-0.61)	0.0149 (0.20)	-0.0151 (-0.20)	0.0046 (0.06)	-0.0398 (-0.53)	0.0266 (0.32)	-0.0021 (-0.03)	-0.0390 (-0.50)
β_{Pooled}	3.3028 (6.80)	4.5272 (5.05)	5.6885 (5.90)	3.6290 (2.11)	2.6009 (1.89)	0.4918 (0.82)	1.1416 (2.24)	0.7312 (1.11)	1.6922 (2.93)	0.2139 (0.22)	0.8372 (0.96)	1.6161 (2.38)

Panel C: 30-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AXP}	-0.1300 (-0.95)	-0.1302 (-0.92)	-0.2004 (-1.41)	-0.1565 (-1.01)	-0.1123 (-0.76)	-0.0280 (-0.20)	-0.0670 (-0.48)	-0.0440 (-0.31)	-0.1062 (-0.75)	-0.0139 (-0.09)	-0.0559 (-0.38)	-0.1117 (-0.78)
α_{BA}	-0.0620 (-0.63)	-0.0580 (-0.57)	-0.1033 (-1.02)	-0.0718 (-0.65)	-0.0474 (-0.44)	0.0188 (0.19)	-0.0049 (-0.05)	0.0086 (0.09)	-0.0291 (-0.29)	0.0269 (0.25)	-0.0021 (-0.02)	-0.0370 (-0.36)
α_{CVX}	-0.0739 (-0.79)	-0.0590 (-0.60)	-0.1165 (-1.19)	-0.0840 (-0.78)	-0.0431 (-0.43)	0.0116 (0.12)	-0.0128 (-0.13)	0.0008 (0.01)	-0.0388 (-0.40)	0.0202 (0.19)	-0.0098 (-0.10)	-0.0492 (-0.49)
α_{DD}	-0.0746 (-0.75)	-0.0682 (-0.66)	-0.1230 (-1.19)	-0.0920 (-0.80)	-0.0693 (-0.62)	0.0127 (0.13)	-0.0137 (-0.14)	0.0013 (0.01)	-0.0409 (-0.40)	0.0220 (0.20)	-0.0100 (-0.09)	-0.0507 (-0.48)
α_{GE}	-0.1408 (-1.30)	-0.1328 (-1.18)	-0.2007 (-1.77)	-0.1553 (-1.26)	-0.1217 (-1.03)	-0.0501 (-0.45)	-0.0809 (-0.73)	-0.0623 (-0.56)	-0.1100 (-0.99)	-0.0380 (-0.32)	-0.0712 (-0.61)	-0.1132 (-1.00)
α_{IBM}	-0.0442 (-0.58)	-0.0412 (-0.52)	-0.0756 (-0.95)	-0.0487 (-0.57)	-0.0291 (-0.35)	0.0201 (0.26)	0.0022 (0.03)	0.0124 (0.16)	-0.0161 (-0.21)	0.0265 (0.32)	0.0041 (0.05)	-0.0234 (-0.29)
α_{JPM}	-0.1062 (-0.71)	-0.0973 (-0.63)	-0.1755 (-1.13)	-0.1499 (-0.88)	-0.0934 (-0.58)	-0.0095 (-0.06)	-0.0527 (-0.34)	-0.0262 (-0.17)	-0.0927 (-0.60)	0.0083 (0.05)	-0.0372 (-0.23)	-0.0961 (-0.61)
α_{KO}	0.0074 (0.12)	0.0168 (0.27)	-0.0110 (-0.18)	-0.0002 (0.00)	0.0127 (0.19)	0.0539 (0.87)	0.0412 (0.67)	0.0486 (0.78)	0.0286 (0.46)	0.0587 (0.90)	0.0434 (0.67)	0.0233 (0.37)
α_{MSFT}	-0.0291 (-0.31)	-0.0242 (-0.25)	-0.0660 (-0.68)	-0.0435 (-0.41)	-0.0044 (-0.04)	0.0457 (0.48)	0.0250 (0.26)	0.0364 (0.38)	0.0032 (0.03)	0.0526 (0.51)	0.0249 (0.25)	-0.0077 (-0.08)
α_{XOM}	-0.0831 (-0.97)	-0.0656 (-0.74)	-0.1234 (-1.38)	-0.0946 (-0.96)	-0.0717 (-0.75)	-0.0057 (-0.06)	-0.0282 (-0.32)	-0.0153 (-0.17)	-0.0510 (-0.58)	0.0026 (0.03)	-0.0249 (-0.27)	-0.0598 (-0.66)
α_{Market}	-0.0587 (-0.81)	-0.0501 (-0.66)	-0.1039 (-1.35)	-0.0746 (-0.85)	-0.0519 (-0.61)	0.0149 (0.20)	-0.0151 (-0.20)	0.0046 (0.06)	-0.0398 (-0.53)	0.0266 (0.32)	-0.0021 (-0.03)	-0.0390 (-0.50)
β_{Pooled}	2.4526 (6.94)	3.8922 (3.22)	4.0646 (4.80)	3.6290 (2.11)	2.6009 (1.89)	0.4918 (0.82)	1.1416 (2.24)	0.7312 (1.11)	1.6922 (2.93)	0.2139 (0.22)	0.8372 (0.96)	1.6161 (2.38)

Table 13 Out-of-Sample Portfolio Performance (US Stocks)

*This table summarizes the performance of the global minimum variance portfolios of the 10 US stocks constructed using the covariance forecasts from the 12 models under consideration. The portfolios are compared on the basis of their annualised out-of-sample variance ($\hat{\sigma}^2_m$) and average out-of-sample turnover ($\hat{\tau}_m$), respectively. Results for three different rebalancing frequencies are presented. Each portfolio is compared with the 1/N benchmark (last row). *, and ** indicate rejections of the hypothesis of equal out-of-sample variances of a given portfolio and the 1/N benchmark at the 10% and 5% levels, respectively. The non-parametric methodology of Ledoit and Wolf (2011) is adopted for the above test in order to calculate the corresponding p-values.*

	Daily Rebalancing		**Weekly Rebalancing**		**Monthly Rebalancing**	
	$\hat{\sigma}^2_m$	$\hat{\tau}_m$	$\hat{\sigma}^2_m$	$\hat{\tau}_m$	$\hat{\sigma}^2_m$	$\hat{\tau}_m$
D-BEKK	0.0281**	0.0877	0.0249**	0.2363	0.0198**	0.5474
A-D-BEKK	0.0279**	0.0833	0.0245**	0.2449	0.0195**	0.5696
CCC	0.0307**	0.2647	0.0273**	0.6312	0.0182*	1.0016
A-CCC	0.0307**	0.2376	0.0267**	0.6035	0.0169**	1.0029
DCC	0.0301**	0.2613	0.0268**	0.6198	0.0181*	0.9832
A-DCC	0.0300**	0.2622	0.0266**	0.6206	0.0181*	0.9832
EWMA	0.0342**	0.3320	0.0329*	0.8882	0.0241	1.6505
O-GARCH	0.0287**	0.1433	0.0270**	0.3215	0.0206*	0.5921
A-O-GARCH	0.0287**	0.1916	0.0262**	0.3894	0.0215*	0.6518
RWE	0.0293**	1.4539	0.0308**	1.5577	0.0187*	1.5677
VHAR	0.0261**	0.4953	0.0239**	0.6777	0.0156**	0.8348
HYBICOV	0.0286**	1.0916	0.0253**	1.1814	0.0152**	1.2369
1/N	0.0543	0.0087	0.0525	0.0210	0.0306	0.0531

Appendix to:

"Covariance Forecasting in Equity Markets"

Not intended for publication

A. Calculation of the Model-Free Implied Volatility of US Stocks

The calculation of the model-free implied volatility (MFIV) for each stock is based on the formula of Britten-Jones and Neuberger (2000):

$$MFIV_{t,t+\tau} = \sqrt{\frac{2e^{R_{f,t}\tau}}{\tau}\left[\int_{0}^{S_t}\frac{P(K,\tau)}{K^2}dK + \int_{S_t}^{\infty}\frac{C(K,\tau)}{K^2}dK\right]}, \tag{31}$$

where $MFIV_{t,t+\tau}$ is the τ-day ahead *MFIV*, $R_{f,t}$ is the risk-free rate at time t with an horizon of τ days, S_t is the underlying spot price at time t, and $C(K,\tau)$ and $P(K,\tau)$ are the prices of the out-of-the money call and put options with strike K and maturity τ. To obtain the τ-day risk-free rate we interpolate between the LIBOR rates of all available maturities. The options on individual stocks are American. However, the above *MFIV* formula relies on European options. To this end, we convert the American option prices to European using the Barone-Adesi and Whaley (1987) approach. Specifically, we first calculate the Barone-Adesi-Whaley implied volatility and we then use this implied volatility in order to compute the price of the European option using the Black and Scholes (1973) formula (see Trolle and Schwartz, 2010, for a similar task).

To empirically evaluate the above formula and obtain daily *MFIV* estimates we closely follow the approach of Carr and Wu (2009) and Prokopczuk et al. (2017). Specifically, every day we sort all out-of-the money call and put options based on their time to maturity. We only retain maturities with more than three out-of-the money options, since the integral of the *MFIV* formula of Equation (31) requires a continuum of strike prices for a good approximation. Nevertheless, in practice there is only a limited number of strikes per maturity. To overcome this problem, similar to Jiang and Tian (2005), we average the Black and Scholes (1973) implied volatilities of the cross-section of out-of-the money options of each specific maturity and then define a lower and an upper bound for the strikes as follows (see also Prokopczuk et al., 2017):

$$K_{u,t} = S_t e^{8 \cdot \sigma_t}$$
$$K_{l,t} = S_t e^{-8 \cdot \sigma_t}, \tag{32}$$

where $K_{u,t}$ and $K_{l,t}$ are the upper and lower bounds of the range of strikes as described above, and σ_t is the average Black-Scholes implied volatility on day t of all out-of-the money options with the same maturity.

We then interpolate between the Black and Scholes (1973) implied volatilities of the 2,000 equidistant strikes. To deal with truncation errors, for any observed strike price greater (lower) than $K_{u,t}$ ($K_{l,t}$) we assume that its implied volatility is equal to the upper (lower) bound. Finally, we map the 2,000 equidistant implied volatilities to call ($C(K, \tau)$) and put ($P(K, \tau)$) option prices and evaluate the integral of Equation (31) using the trapezoidal rule (see Jiang and Tian, 2005, for details). We repeat this procedure for each maturity and finally interpolate between implied volatilities of the closest maturities in order to obtain $MFIV_{t,t+\tau}$ (in the case of the VIX index, τ=30 calendar days).

Table A1 Risk–Return Trade-off (Weighted Least Squares Estimation)

This table reports the intercepts and the common slope coefficient estimates along with their t-statistics (in parentheses) from the SUR system estimation of Equation (26). The system is estimated using weighted least squares (WLS) similar to Bali and Engle (2010). The dependent variables correspond to daily excess returns on the five European equity markets and the STOXX 50 index which serves as a proxy for the European market portfolio. Each column presents results from using a different model to obtain conditional covariance estimates between the returns of each equity index and the STOXX 50 index. Three alternative intraday sampling frequencies are considered for the estimation of the RWE, VHAR and HYBICOV (Panels A–C).

Panel A: 5-Minute Frequency

	RWE	**VHAR**	**HYBICOV**	**D-BEKK**	**A-D-BEKK**	**CCC**	**A-CCC**	**DCC**	**A-DCC**	**O-GARCH**	**A-O-GARCH**	**EWMA**
α_{AEX}	-0.0617 (-0.95)	-0.0719 (-1.02)	-0.0253 (-0.35)	-0.0604 (-0.80)	-0.0950 (-1.27)	-0.0057 (-0.08)	-0.0353 (-0.50)	-0.0531 (-0.74)	-0.0934 (-1.33)	0.0694 (0.91)	0.0392 (0.53)	-0.0393 (-0.55)
α_{CAC}	-0.0784 (-1.13)	-0.0894 (-1.19)	-0.0373 (-0.48)	-0.0740 (-0.91)	-0.1136 (-1.41)	-0.0147 (-0.19)	-0.0474 (-0.63)	-0.0675 (-0.87)	-0.1115 (-1.49)	0.0655 (0.81)	0.0336 (0.43)	-0.0519 (-0.67)
α_{DAX}	-0.0087 (-0.13)	-0.0189 (-0.26)	0.0302 (0.39)	-0.0074 (-0.09)	-0.0452 (-0.57)	0.0515 (0.67)	0.0200 (0.27)	0.0001 (0.00)	-0.0422 (-0.57)	0.1321 (1.63)	0.0998 (1.28)	0.0156 (0.20)
α_{SMI}	-0.0385 (-0.70)	-0.0462 (-0.79)	-0.0105 (-0.18)	-0.0376 (-0.60)	-0.0654 (-1.06)	0.0032 (0.05)	-0.0182 (-0.31)	-0.0333 (-0.56)	-0.0636 (-1.09)	0.0584 (0.94)	0.0362 (0.60)	-0.0228 (-0.38)
α_{FTSE}	-0.0438 (-0.78)	-0.0526 (-0.87)	-0.0132 (-0.21)	-0.0416 (-0.65)	-0.0713 (-1.12)	0.0030 (0.05)	-0.0195 (-0.33)	-0.0360 (-0.58)	-0.0674 (-1.13)	0.0602 (0.95)	0.0373 (0.61)	-0.0256 (-0.41)
α_{Market}	-0.0890 (-1.27)	-0.1003 (-1.31)	-0.0472 (-0.59)	-0.0854 (-1.03)	-0.1256 (-1.53)	-0.0234 (-0.29)	-0.0577 (-0.76)	-0.0778 (-0.99)	-0.1239 (-1.62)	0.0601 (0.73)	0.0269 (0.34)	-0.0614 (-0.78)
β_{Pooled}	1.6892 (3.79)	2.2493 (2.55)	0.9383 (0.94)	1.5437 (1.61)	2.5400 (2.53)	0.3310 (0.40)	0.9721 (1.39)	1.3643 (1.72)	2.2166 (3.21)	-1.2612 (-1.36)	-0.6189 (-0.77)	1.0804 (1.36)

Panel B: 10-Minute Frequency

	RWE	**VHAR**	**HYBICOV**	**D-BEKK**	**A-D-BEKK**	**CCC**	**A-CCC**	**DCC**	**A-DCC**	**O-GARCH**	**A-O-GARCH**	**EWMA**
α_{AEX}	-0.0748 (-1.15)	-0.0812 (-1.16)	-0.0325 (-0.44)	-0.0604 (-0.80)	-0.0950 (-1.27)	-0.0057 (-0.08)	-0.0353 (-0.50)	-0.0531 (-0.74)	-0.0934 (-1.33)	0.0694 (0.91)	0.0392 (0.53)	-0.0393 (-0.55)
α_{CAC}	-0.0928 (-1.34)	-0.0996 (-1.32)	-0.0454 (-0.57)	-0.0740 (-0.91)	-0.1136 (-1.41)	-0.0147 (-0.19)	-0.0474 (-0.63)	-0.0675 (-0.87)	-0.1115 (-1.49)	0.0655 (0.81)	0.0336 (0.43)	-0.0519 (-0.67)
α_{DAX}	-0.0224 (-0.33)	-0.0287 (-0.39)	0.0223 (0.29)	-0.0074 (-0.09)	-0.0452 (-0.57)	0.0515 (0.67)	0.0200 (0.27)	0.0001 (0.00)	-0.0422 (-0.57)	0.1321 (1.63)	0.0998 (1.28)	0.0156 (0.20)
α_{SMI}	-0.0483 (-0.88)	-0.0533 (-0.91)	-0.0159 (-0.26)	-0.0376 (-0.60)	-0.0654 (-1.06)	0.0032 (0.05)	-0.0182 (-0.31)	-0.0333 (-0.56)	-0.0636 (-1.09)	0.0584 (0.94)	0.0362 (0.60)	-0.0228 (-0.38)
α_{FTSE}	-0.0550 (-0.98)	-0.0610 (-1.01)	-0.0194 (-0.31)	-0.0416 (-0.65)	-0.0713 (-1.12)	0.0030 (0.05)	-0.0195 (-0.33)	-0.0360 (-0.58)	-0.0674 (-1.13)	0.0602 (0.95)	0.0373 (0.61)	-0.0256 (-0.41)
α_{Market}	-0.1034 (-1.47)	-0.1100 (-1.44)	-0.0552 (-0.69)	-0.0854 (-1.03)	-0.1256 (-1.53)	-0.0234 (-0.29)	-0.0577 (-0.76)	-0.0778 (-0.99)	-0.1239 (-1.62)	0.0601 (0.73)	0.0269 (0.34)	-0.0614 (-0.78)
β_{Pooled}	1.9815 (4.50)	2.4989 (2.84)	1.1173 (1.11)	1.5437 (1.61)	2.5400 (2.53)	0.3310 (0.40)	0.9721 (1.39)	1.3643 (1.72)	2.2166 (3.21)	-1.2612 (-1.36)	-0.6189 (-0.77)	1.0804 (1.36)

Panel C: 30-Minute Frequency

	RWE	**VHAR**	**HYBICOV**	**D-BEKK**	**A-D-BEKK**	**CCC**	**A-CCC**	**DCC**	**A-DCC**	**O-GARCH**	**A-O-GARCH**	**EWMA**
α_{AEX}	-0.0898 (-1.39)	-0.0925 (-1.33)	-0.0317 (-0.43)	-0.0604 (-0.80)	-0.0950 (-1.27)	-0.0057 (-0.08)	-0.0353 (-0.50)	-0.0531 (-0.74)	-0.0934 (-1.33)	0.0694 (0.91)	0.0392 (0.53)	-0.0393 (-0.55)
α_{CAC}	-0.1092 (-1.59)	-0.1113 (-1.50)	-0.0444 (-0.56)	-0.0740 (-0.91)	-0.1136 (-1.41)	-0.0147 (-0.19)	-0.0474 (-0.63)	-0.0675 (-0.87)	-0.1115 (-1.49)	0.0655 (0.81)	0.0336 (0.43)	-0.0519 (-0.67)
α_{DAX}	-0.0391 (-0.57)	-0.0407 (-0.55)	0.0229 (0.29)	-0.0074 (-0.09)	-0.0452 (-0.57)	0.0515 (0.67)	0.0200 (0.27)	0.0001 (0.00)	-0.0422 (-0.57)	0.1321 (1.63)	0.0998 (1.28)	0.0156 (0.20)
α_{SMI}	-0.0608 (-1.11)	-0.0630 (-1.09)	-0.0160 (-0.27)	-0.0376 (-0.60)	-0.0654 (-1.06)	0.0032 (0.05)	-0.0182 (-0.31)	-0.0333 (-0.56)	-0.0636 (-1.09)	0.0584 (0.94)	0.0362 (0.60)	-0.0228 (-0.38)
α_{FTSE}	-0.0685 (-1.23)	-0.0713 (-1.19)	-0.0190 (-0.31)	-0.0416 (-0.65)	-0.0713 (-1.12)	0.0030 (0.05)	-0.0195 (-0.33)	-0.0360 (-0.58)	-0.0674 (-1.13)	0.0602 (0.95)	0.0373 (0.61)	-0.0256 (-0.41)
α_{Market}	-0.1196 (-1.72)	-0.1209 (-1.61)	-0.0539 (-0.67)	-0.0854 (-1.03)	-0.1256 (-1.53)	-0.0234 (-0.29)	-0.0577 (-0.76)	-0.0778 (-0.99)	-0.1239 (-1.62)	0.0601 (0.73)	0.0269 (0.34)	-0.0614 (-0.78)
β_{Pooled}	2.3267 (5.62)	2.8426 (3.34)	1.1017 (1.11)	1.5437 (1.61)	2.5400 (2.53)	0.3310 (0.40)	0.9721 (1.39)	1.3643 (1.72)	2.2166 (3.21)	-1.2612 (-1.36)	-0.6189 (-0.77)	1.0804 (1.36)

Table A2 Out-of-Sample Forecast Losses (10-min. Sampling Frequency)

This table reports the average out-of-sample forecast losses for the 1–, 5–, and 22–day horizons, respectively. We employ a rolling window of 1,000 observations to produce forecasts from parametric models. VHAR, RWE and HYBICOV model forecasts are based on intraday data sampled at the 10-minute frequency. L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). A dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.

	1–Day Horizon			**5–Day Horizon**			**22–Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
D-BEKK	0.0288	0.0463	−20.2210	0.0229	0.0368	−20.0576	0.0220	0.0354	−19.7743
A-D-BEKK	0.0215	0.0344	−20.2909	0.0152	0.0244	−20.0908	0.0128	0.0203	−19.7861
CCC	0.0355	0.0564	−20.4750	0.0304	0.0482	−20.3188	0.0308	0.0489	−19.9065
A-CCC	0.0334	0.0527	−20.4556	0.0289	0.0454	−20.3095	0.0268	0.0421	−19.9062
DCC	0.0370	0.0595	−20.5266	0.0319	0.0513	−20.3586	0.0320	0.0514	−19.9304
A-DCC	0.0370	0.0595	−20.5254	0.0319	0.0513	−20.3581	0.0321	0.0514	−19.9308
EWMA	0.0366	0.0592	−19.9778	0.0317	0.0514	−19.7713	0.0347	0.0563	−19.3458
O-GARCH	0.0370	0.0601	−20.2589	0.0321	0.0523	−20.0811	0.0327	0.0534	−19.7484
A-O-GARCH	0.0518	0.0850	−20.2688	0.0458	0.0753	−20.0932	0.0389	0.0639	−19.7560
RWE	0.0173†	0.0272†	−20.7172	0.0080	0.0124†	−21.0406†	0.0074	0.0115	−20.6030*
VHAR	0.0122*	0.0195*	−21.3501*	0.0067*	0.0105*	−21.0632*	0.0062*	0.0097*	−20.5594†
HYBICOV	0.0135†	0.0214†	−20.5585	0.0073†	0.0114†	−20.7984	0.0066†	0.0103†	−19.6591

Table A3 Model Confidence Set Results (10-minute Sampling Frequency)

This table shows the results of the 5% Model Confidence Set (MCS). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 trading days, respectively. VHAR, RWE and HYBICOV model forecasts are based on intraday data sampled at the 10-minute frequency. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						**5–Day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	rank	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**
D-BEKK	6	0.001	6	0.001	9	0.000	6	0.001	6	0.001	9	0.000	6	0.000	6	0.001	9	0.000
A-D-BEKK	4	0.005	4	0.005	8	0.000	4	0.001	4	0.001	8	0.000	4	0.000	4	0.001	8	0.000
CCC	10	0.001	8	0.001	7	0.000	12	0.001	9	0.001	5	0.000	11	0.000	11	0.001	7	0.000
A-CCC	5	0.002	5	0.002	5	0.000	5	0.001	5	0.001	4	0.000	5	0.000	5	0.001	5	0.000
DCC	11	0.001	11	0.001	4	0.000	10	0.001	11	0.001	6	0.000	10	0.000	10	0.001	6	0.000
A-DCC	12	0.001	12	0.001	6	0.000	11	0.001	12	0.001	7	0.000	9	0.000	9	0.001	3	0.000
EWMA	8	0.001	9	0.001	10	0.000	9	0.001	10	0.001	11	0.000	12	0.000	12	0.001	12	0.000
O-GARCH	9	0.001	10	0.001	11	0.000	8	0.001	8	0.001	12	0.000	8	0.000	8	0.001	11	0.000
A-O-GARCH	7	0.001	7	0.001	12	0.000	7	0.001	7	0.001	10	0.000	7	0.000	7	0.001	10	0.000
RWE	3*	0.085	3*	0.098	2	0.000	3*	0.078	3*	0.097	2*	0.068	3	0.004	3	0.004	1*	1.000
VHAR	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2	0.014
HYBICOV	2*	0.115	2*	0.138	3	0.000	2*	0.158	2*	0.184	3	0.000	2*	0.153	2*	0.180	4	0.000

Table A4 Out-of-Sample Forecast Losses (30-minute Sampling Frequency)

This table reports average out-of-sample forecast losses for the 1–, 5–, and 22–day horizons, respectively. We employ a rolling window of 1,000 observations to produce forecasts from parametric models. VHAR, RWE and HYBICOV model forecasts are based on intraday data sampled at the 30-minute frequency. L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). A dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.

	1–Day Horizon			**5–Day Horizon**			**22–Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
D-BEKK	0.0340	0.0548	−21.0445	0.0248	0.0398	−20.8445	0.0231	0.0372	−20.5353
A-D-BEKK	0.0267	0.0430	−21.1184	0.0169	0.0270	−20.8747	0.0135	0.0215	−20.5351
CCC	0.0408	0.0651	−21.1736	0.0326	0.0516	−21.0063	0.0321	0.0509	−20.5696
A-CCC	0.0387	0.0612	−21.1725	0.0314	0.0492	−21.0124	0.0281	0.0440	−20.5871
DCC	0.0424	0.0682	−21.2591	0.0341	0.0547	−21.0697	0.0334	0.0534	−20.6096
A-DCC	0.0424	0.0682	−21.2590	0.0341	0.0547	−21.0700	0.0334	0.0534	−20.6100
EWMA	0.0420	0.0680	−21.0208	0.0339	0.0547	−20.7898	0.0362	0.0585	−20.3922
O-GARCH	0.0424	0.0689	−21.0166	0.0343	0.0558	−20.8067	0.0341	0.0555	−20.4625
A-O-GARCH	0.0575	0.0942	−21.0272	0.0488	0.0798	−20.8185	0.0405	0.0662	−20.4678
RWE	$0.0258^{\dagger}$	$0.0409^{\dagger}$	−18.5153	0.0098	0.0155	-21.5565^{*}	0.0072	0.0114	-21.3128^{*}
VHAR	0.0171^{*}	0.0276^{*}	-21.7544^{*}	0.0079^{*}	0.0125^{*}	−21.3997	0.0060^{*}	0.0096^{*}	−20.7592
HYBICOV	$0.0179^{\dagger}$	$0.0289^{\dagger}$	−19.2128	$0.0083^{\dagger}$	$0.0132^{\dagger}$	$-16.3704^{\dagger}$	$0.0066^{\dagger}$	$0.0105^{\dagger}$	−21.0811

Table A5 Model Confidence Set Results (30-minute Sampling Frequency)

This table shows the results of the 5% Model Confidence Set (MCS). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 trading days, respectively. VHAR, RWE and HYBICOV model forecasts are based on intraday data sampled at the 30-minute frequency. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						5–Day Horizon						22–Day Horizon					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	rank	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**
D-BEKK	6	0.002	6	0.001	8	0.000	6	0.000	6	0.000	9	0.000	6	0.000	6	0.000	9	0.000
A-D-BEKK	4	0.008	4	0.008	5	0.000	4	0.000	4	0.000	7	0.000	4	0.000	4	0.000	8	0.000
CCC	8	0.002	8	0.001	6	0.000	12	0.000	10	0.000	6	0.000	11	0.000	11	0.000	7	0.000
A-CCC	5	0.003	5	0.004	4	0.000	5	0.000	5	0.000	5	0.000	5	0.000	5	0.000	4	0.000
DCC	11	0.002	11	0.001	2	0.000	10	0.000	11	0.000	4	0.000	10	0.000	10	0.000	6	0.000
A-DCC	12	0.002	12	0.001	3	0.000	11	0.000	12	0.000	3	0.000	9	0.000	9	0.000	5	0.000
EWMA	9	0.002	9	0.001	7	0.000	9	0.000	9	0.000	8	0.000	12	0.000	12	0.000	10	0.000
O-GARCH	10	0.002	10	0.001	10	0.000	8	0.000	8	0.000	11	0.000	8	0.000	8	0.000	12	0.000
A-O-GARCH	7	0.002	7	0.001	9	0.000	7	0.000	7	0.000	10	0.000	7	0.000	7	0.000	11	0.000
RWE	3*	0.099	3*	0.113	12	0.000	3	0.030	3	0.031	1*	1.000	3	0.003	3	0.005	1*	1.000
VHAR	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2	0.001	1*	1.000	1*	1.000	3	0.000
HYBICOV	2*	0.260	2*	0.279	11	0.000	2*	0.255	2*	0.278	12	0.000	2	0.034	2	0.037	2	0.000

Table A6 Out-of-Sample Forecast Losses (Daily Sampling Frequency)

This table reports average out-of-sample forecast losses for the forecast horizon of 22 trading days based on the 12 models under consideration. Daily data are employed for the VHAR, RWE and HYBICOV estimation as well as for the latent covariance proxy. We employ a rolling window of 1,000 observations to produce forecasts from parametric models. L_E *is the Euclidean distance,* L_F *is the Frobenius distance, and* L_Q *is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). A dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.*

	22-Day Horizon		
	L_E	L_F	L_Q
D-BEKK	0.0488	0.0784	-24.2213
A-D-BEKK	0.0310	0.0493	-24.2387
CCC	0.0659	0.1045	-23.8267
A-CCC	0.0585	0.0922	-23.9117
DCC	0.0678	0.1083	-23.9326
A-DCC	0.0678	0.1083	-23.9315
EWMA	0.0720	0.1161	-25.3788
O-GARCH	0.0692	0.1120	-24.0905
A-O-GARCH	0.0773	0.1255	-24.0868
RWE	0.0001*	0.0002*	-32.1800*
VHAR	0.0384†	0.0619†	-25.7395†
HYBICOV	0.0423	0.0682	-26.6651

Table A7 Model Confidence Set Results (Daily Sampling Frequency)

This table shows the results of the 5% Model Confidence Set (MCS) using daily data for all the 12 models under consideration and employing three statistical loss functions at 22-day horizon. L_E *denotes the Euclidean distance,* L_F *is the Frobenius distance, and* L_Q *is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.*

	22-Day Horizon					
	L_E		L_F		L_Q	
	Rank	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**
D-BEKK	4	0.000	4	0.000	7	0.000
A-D-BEKK	3	0.000	3	0.000	5	0.000
CCC	9	0.000	9	0.000	8	0.000
A-CCC	6	0.000	6	0.000	6	0.000
DCC	12	0.000	12	0.000	11	0.000
A-DCC	11	0.000	11	0.000	12	0.000
EWMA	8	0.000	8	0.000	4	0.000
O-GARCH	10	0.000	10	0.000	9	0.000
A-O-GARCH	7	0.000	7	0.000	10	0.000
RWE	1*	1.000	1*	1.000	1*	1.000
VHAR	5	0.000	5	0.000	3	0.000
HYBICOV	2	0.000	2	0.000	2	0.000

Table A8 Out-of-Sample Forecast Comparison of VHAR vs. GVHAR

This table shows the average out-of-sample losses of the VHAR model and a fully generalized alternative (GVHAR). Three forecast horizons are considered, namely: 1, 5, and 22 trading days, respectively. Panels A–C display the results for three alternative intraday sampling frequencies. The VHAR model assumes that all pairwise conditional covariances are determined by the same parameters. The GVHAR model relaxes the previous assumption by allowing the time series of each unique conditional covariance element to follow its own HAR dynamics and their system is estimated through seemingly unrelated regressions (SUR). We employ a rolling window of 1,000 observations to produce out-of-sample forecasts. L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). A dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.

	1-Day Horizon			**5-Day Horizon**			**22-Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
Panel A: 5-Minute Returns									
VHAR	0.0114*	0.0178*	-20.6043*	0.0064*	0.0100*	-20.4024*	0.0061*	0.0095*	-19.9277*
GVHAR	0.0127†	0.0198†	-19.8945	0.0072†	0.0112†	-19.7218	0.0062†	0.0097†	-19.3322
Panel B: 10-Minute Returns									
VHAR	0.0131*	0.0205*	-20.8973	0.0069*	0.0109*	-20.7175	0.0063†	0.0099†	-20.2994†
GVHAR	0.0139†	0.0217†	-20.9985*	0.0074†	0.0116†	-20.8147*	0.0062*	0.0097*	-20.3987*
Panel C: 30-Minute Returns									
VHAR	0.0188*	0.0295*	-21.661*	0.0079*	0.0125*	-21.4832*	0.0064†	0.0101†	-21.1034*
GVHAR	0.0191†	0.0301†	-21.5954†	0.0081†	0.0128†	-21.4150	0.0061*	0.0096*	-21.0135†

Table A9 Risk–Return Trade-off for US Stocks (Weighted Least Squares Estimation)

This table reports the intercepts and the common slope coefficient estimates along with their t-statistics (in parentheses) from the SUR system estimation of Equation (26). The system is estimated using weighted least squares (WLS) similar to Bali and Engle (2010). The dependent variables correspond to daily excess returns on the ten US stocks presented in section 4.2 and the S&P 500 index, which serves as a proxy for the market portfolio. Each column presents results from using a different model to obtain estimates of the expected conditional covariance between the excess returns of each stock and the excess return on S&P 500 index. Three alternative intraday sampling frequencies are considered for the estimation of RWE, VHAR and HYBICOV models (Panels A–C).

Panel A: 5-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AXP}	-0.1089 (-0.79)	-0.1926 (-1.36)	-0.2561 (-1.77)	-0.1584 (-1.02)	-0.1138 (-0.77)	-0.0283 (-0.20)	-0.0671 (-0.48)	-0.0446 (-0.31)	-0.1071 (-0.76)	-0.0139 (-0.09)	-0.0559 (-0.38)	-0.1130 (-0.79)
α_{BA}	-0.0472 (-0.48)	-0.1082 (-1.07)	-0.1438 (-1.40)	-0.0731 (-0.67)	-0.0485 (-0.45)	0.0186 (0.19)	-0.0050 (-0.05)	0.0083 (0.08)	-0.0296 (-0.30)	0.0269 (0.25)	-0.0021 (-0.02)	-0.0378 (-0.37)
α_{CVX}	-0.0580 (-0.61)	-0.1204 (-1.23)	-0.1584 (-1.58)	-0.0854 (-0.79)	-0.0441 (-0.43)	0.0114 (0.12)	-0.0129 (-0.13)	0.0004 (0.00)	-0.0394 (-0.41)	0.0202 (0.19)	-0.0098 (-0.10)	-0.0500 (-0.50)
α_{DD}	-0.0578 (-0.57)	-0.1220 (-1.18)	-0.1669 (-1.58)	-0.0935 (-0.81)	-0.0706 (-0.63)	0.0125 (0.12)	-0.0138 (-0.14)	0.0009 (0.01)	-0.0415 (-0.41)	0.0220 (0.20)	-0.0100 (-0.09)	-0.0516 (-0.49)
α_{GE}	-0.1229 (-1.12)	-0.1919 (-1.70)	-0.2537 (-2.19)	-0.1568 (-1.27)	-0.1230 (-1.04)	-0.0504 (-0.45)	-0.0810 (-0.74)	-0.0627 (-0.56)	-0.1107 (-1.00)	-0.0380 (-0.32)	-0.0713 (-0.61)	-0.1142 (-1.01)
α_{IBM}	-0.0331 (-0.43)	-0.0812 (-1.02)	-0.1067 (-1.33)	-0.0497 (-0.58)	-0.0300 (-0.36)	0.0200 (0.26)	0.0021 (0.03)	0.0121 (0.15)	-0.0165 (-0.21)	0.0265 (0.32)	0.0041 (0.05)	-0.0241 (-0.30)
α_{JPM}	-0.0840 (-0.56)	-0.1656 (-1.07)	-0.2291 (-1.46)	-0.1519 (-0.89)	-0.0950 (-0.59)	-0.0099 (-0.06)	-0.0529 (-0.35)	-0.0269 (-0.17)	-0.0937 (-0.61)	0.0083 (0.05)	-0.0372 (-0.23)	-0.0974 (-0.62)
α_{KO}	0.0169 (0.27)	-0.0175 (-0.28)	-0.0338 (-0.53)	-0.0010 (-0.01)	0.0120 (0.18)	0.0538 (0.87)	0.0412 (0.67)	0.0484 (0.77)	0.0283 (0.45)	0.0587 (0.90)	0.0434 (0.67)	0.0227 (0.36)
α_{MSFT}	-0.0160 (-0.17)	-0.0722 (-0.74)	-0.1026 (-1.04)	-0.0448 (-0.42)	-0.0053 (-0.05)	0.0455 (0.48)	0.0249 (0.26)	0.0361 (0.37)	0.0027 (0.03)	0.0526 (0.51)	0.0249 (0.25)	-0.0085 (-0.09)
α_{XOM}	-0.0708 (-0.82)	-0.1274 (-1.42)	-0.1617 (-1.78)	-0.0959 (-0.97)	-0.0728 (-0.76)	-0.0059 (-0.07)	-0.0283 (-0.33)	-0.0157 (-0.18)	-0.0515 (-0.59)	0.0026 (0.03)	-0.0250 (-0.27)	-0.0606 (-0.67)
α_{Market}	-0.0462 (-0.63)	-0.1042 (-1.36)	-0.1438 (-1.83)	-0.0760 (-0.86)	-0.0530 (-0.62)	0.0147 (0.19)	-0.0152 (-0.20)	0.0042 (0.05)	-0.0404 (-0.53)	0.0266 (0.32)	-0.0021 (-0.03)	-0.0399 (-0.51)
β_{Pooled}	2.0761 (4.54)	4.0647 (4.93)	5.2124 (5.28)	3.6729 (2.14)	2.6360 (1.91)	0.4974 (0.82)	1.1440 (2.24)	0.7405 (1.12)	1.7068 (2.95)	0.2138 (0.22)	0.8374 (0.96)	1.6337 (2.40)

Panel B: 10-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AXP}	-0.1780 (-1.29)	-0.2172 (-1.52)	-0.2855 (-1.98)	-0.1584 (-1.02)	-0.1138 (-0.77)	-0.0283 (-0.20)	-0.0671 (-0.48)	-0.0446 (-0.31)	-0.1071 (-0.76)	-0.0139 (-0.09)	-0.0559 (-0.38)	-0.1130 (-0.79)
α_{BA}	-0.0960 (-0.97)	-0.1224 (-1.20)	-0.1588 (-1.55)	-0.0731 (-0.67)	-0.0485 (-0.45)	0.0186 (0.19)	-0.0050 (-0.05)	0.0083 (0.08)	-0.0296 (-0.30)	0.0269 (0.25)	-0.0021 (-0.02)	-0.0378 (-0.37)
α_{CVX}	-0.1094 (-1.15)	-0.1329 (-1.35)	-0.1745 (-1.75)	-0.0854 (-0.79)	-0.0441 (-0.43)	0.0114 (0.12)	-0.0129 (-0.13)	0.0004 (0.00)	-0.0394 (-0.41)	0.0202 (0.19)	-0.0098 (-0.10)	-0.0500 (-0.50)
α_{DD}	-0.1111 (-1.10)	-0.1369 (-1.31)	-0.1843 (-1.75)	-0.0935 (-0.81)	-0.0706 (-0.63)	0.0125 (0.12)	-0.0138 (-0.14)	0.0009 (0.01)	-0.0415 (-0.41)	0.0220 (0.20)	-0.0100 (-0.09)	-0.0516 (-0.49)
α_{GE}	-0.1783 (-1.62)	-0.2055 (-1.81)	-0.2660 (-2.32)	-0.1568 (-1.27)	-0.1230 (-1.04)	-0.0504 (-0.45)	-0.0810 (-0.74)	-0.0627 (-0.56)	-0.1107 (-1.00)	-0.0380 (-0.32)	-0.0713 (-0.61)	-0.1142 (-1.01)
α_{IBM}	-0.0723 (-0.93)	-0.0914 (-1.14)	-0.1181 (-1.47)	-0.0497 (-0.58)	-0.0300 (-0.36)	0.0200 (0.26)	0.0021 (0.03)	0.0121 (0.15)	-0.0165 (-0.21)	0.0265 (0.32)	0.0041 (0.05)	-0.0241 (-0.30)
α_{JPM}	-0.1511 (-1.00)	-0.1825 (-1.18)	-0.2532 (-1.62)	-0.1519 (-0.89)	-0.0950 (-0.59)	-0.0099 (-0.06)	-0.0529 (-0.35)	-0.0269 (-0.17)	-0.0937 (-0.61)	0.0083 (0.05)	-0.0372 (-0.23)	-0.0974 (-0.62)
α_{KO}	-0.0115 (-0.19)	-0.0248 (-0.39)	-0.0422 (-0.66)	-0.0010 (-0.01)	0.0120 (0.18)	0.0538 (0.87)	0.0412 (0.67)	0.0484 (0.77)	0.0283 (0.45)	0.0587 (0.90)	0.0434 (0.67)	0.0227 (0.36)
α_{MSFT}	-0.0609 (-0.64)	-0.0815 (-0.83)	-0.1149 (-1.17)	-0.0448 (-0.42)	-0.0053 (-0.05)	0.0455 (0.48)	0.0249 (0.26)	0.0361 (0.37)	0.0027 (0.03)	0.0526 (0.51)	0.0249 (0.25)	-0.0085 (-0.09)
α_{XOM}	-0.1165 (-1.35)	-0.1374 (-1.53)	-0.1761 (-1.95)	-0.0959 (-0.97)	-0.0728 (-0.76)	-0.0059 (-0.07)	-0.0283 (-0.33)	-0.0157 (-0.18)	-0.0515 (-0.59)	0.0026 (0.03)	-0.0250 (-0.27)	-0.0606 (-0.67)
α_{Market}	-0.0930 (-1.27)	-0.1133 (-1.47)	-0.1567 (-2.01)	-0.0760 (-0.86)	-0.0530 (-0.62)	0.0147 (0.19)	-0.0152 (-0.20)	0.0042 (0.05)	-0.0404 (-0.53)	0.0266 (0.32)	-0.0021 (-0.03)	-0.0399 (-0.51)
β_{Pooled}	3.3372 (6.87)	4.5790 (5.10)	5.7201 (5.93)	3.6729 (2.14)	2.6360 (1.91)	0.4974 (0.82)	1.1440 (2.24)	0.7405 (1.12)	1.7068 (2.95)	0.2138 (0.22)	0.8374 (0.96)	1.6337 (2.40)

Panel C: 30-Minute Frequency

	RWE	VHAR	HYBICOV	D-BEKK	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	O-GARCH	A-O-GARCH	EWMA
α_{AXP}	-0.1303 (-0.95)	-0.1305 (-0.92)	-0.2006 (-1.41)	-0.1584 (-1.02)	-0.1138 (-0.77)	-0.0283 (-0.20)	-0.0671 (-0.48)	-0.0446 (-0.31)	-0.1071 (-0.76)	-0.0139 (-0.09)	-0.0559 (-0.38)	-0.1130 (-0.79)
α_{BA}	-0.0623 (-0.63)	-0.0583 (-0.57)	-0.1035 (-1.02)	-0.0731 (-0.67)	-0.0485 (-0.45)	0.0186 (0.19)	-0.0050 (-0.05)	0.0083 (0.08)	-0.0296 (-0.30)	0.0269 (0.25)	-0.0021 (-0.02)	-0.0378 (-0.37)
α_{CVX}	-0.0741 (-0.79)	-0.0592 (-0.61)	-0.1167 (-1.19)	-0.0854 (-0.79)	-0.0441 (-0.43)	0.0114 (0.12)	-0.0129 (-0.13)	0.0004 (0.00)	-0.0394 (-0.41)	0.0202 (0.19)	-0.0098 (-0.10)	-0.0500 (-0.50)
α_{DD}	-0.0749 (-0.75)	-0.0685 (-0.66)	-0.1232 (-1.19)	-0.0935 (-0.81)	-0.0706 (-0.63)	0.0125 (0.12)	-0.0138 (-0.14)	0.0009 (0.01)	-0.0415 (-0.41)	0.0220 (0.20)	-0.0100 (-0.09)	-0.0516 (-0.49)
α_{GE}	-0.1411 (-1.30)	-0.1331 (-1.18)	-0.2009 (-1.77)	-0.1568 (-1.27)	-0.1230 (-1.04)	-0.0504 (-0.45)	-0.0810 (-0.74)	-0.0627 (-0.56)	-0.1107 (-1.00)	-0.0380 (-0.32)	-0.0713 (-0.61)	-0.1142 (-1.01)
α_{IBM}	-0.0445 (-0.58)	-0.0414 (-0.52)	-0.0757 (-0.96)	-0.0497 (-0.58)	-0.0300 (-0.36)	0.0200 (0.26)	0.0021 (0.03)	0.0121 (0.15)	-0.0165 (-0.21)	0.0265 (0.32)	0.0041 (0.05)	-0.0241 (-0.30)
α_{JPM}	-0.1065 (-0.71)	-0.0977 (-0.63)	-0.1757 (-1.14)	-0.1519 (-0.89)	-0.0950 (-0.59)	-0.0099 (-0.06)	-0.0529 (-0.35)	-0.0269 (-0.17)	-0.0937 (-0.61)	0.0083 (0.05)	-0.0372 (-0.23)	-0.0974 (-0.62)
α_{KO}	0.0073 (0.12)	0.0167 (0.27)	-0.0111 (-0.18)	-0.0010 (-0.01)	0.0120 (0.18)	0.0538 (0.87)	0.0412 (0.67)	0.0484 (0.77)	0.0283 (0.45)	0.0587 (0.90)	0.0434 (0.67)	0.0227 (0.36)
α_{MSFT}	-0.0294 (-0.31)	-0.0244 (-0.25)	-0.0662 (-0.68)	-0.0448 (-0.42)	-0.0053 (-0.05)	0.0455 (0.48)	0.0249 (0.26)	0.0361 (0.37)	0.0027 (0.03)	0.0526 (0.51)	0.0249 (0.25)	-0.0085 (-0.09)
α_{XOM}	-0.0834 (-0.98)	-0.0658 (-0.75)	-0.1236 (-1.38)	-0.0959 (-0.97)	-0.0728 (-0.76)	-0.0059 (-0.07)	-0.0283 (-0.33)	-0.0157 (-0.18)	-0.0515 (-0.59)	0.0026 (0.03)	-0.0250 (-0.27)	-0.0606 (-0.67)
α_{Market}	-0.0590 (-0.82)	-0.0503 (-0.66)	-0.1041 (-1.36)	-0.0760 (-0.86)	-0.0530 (-0.62)	0.0147 (0.19)	-0.0152 (-0.20)	0.0042 (0.05)	-0.0404 (-0.53)	0.0266 (0.32)	-0.0021 (-0.03)	-0.0399 (-0.51)
β_{Pooled}	2.4593 (6.96)	3.9032 (3.23)	4.0692 (4.81)	3.6729 (2.14)	2.6360 (1.91)	0.4974 (0.82)	1.1440 (2.24)	0.7405 (1.12)	1.7068 (2.95)	0.2138 (0.22)	0.8374 (0.96)	1.6337 (2.40)

Table A10 MCS Results using the Range Statistic

This table shows the results of the 5% Model Confidence Set (MCS). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 trading days, respectively. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The range statistic of Equation (24) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						**5–Day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	Rank	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**
D-BEKK	6	0.00	6	0.00	7	0.00	6	0.00	6	0.00	9	0.00	6	0.00	6	0.00	9	0.00
A-D-BEKK	4	0.00	4	0.00	8	0.00	4	0.00	4	0.00	7	0.00	3	0.00	3	0.00	3	0.00
CCC	8	0.00	8	0.00	4	0.00	8	0.00	8	0.00	6	0.00	11	0.00	11	0.00	5	0.00
A-CCC	5	0.00	5	0.00	6	0.00	5	0.00	5	0.00	5	0.00	5	0.00	5	0.00	6	0.00
DCC	11	0.00	10	0.00	5	0.00	11	0.00	9	0.00	4	0.00	10	0.00	10	0.00	8	0.00
A-DCC	12	0.00	11	0.00	3	0.00	12	0.00	11	0.00	3	0.00	9	0.00	9	0.00	7	0.00
EWMA	9	0.00	9	0.00	12	0.00	9	0.00	10	0.00	12	0.00	12	0.00	12	0.00	12	0.00
O-GARCH	10	0.00	12	0.00	10	0.00	10	0.00	12	0.00	10	0.00	8	0.00	8	0.00	11	0.00
A-O-GARCH	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	10	0.00
RWE	2	0.05	2*	0.06	2	0.00	2	0.03	2	0.04	2*	0.06	2	0.00	2	0.00	2*	0.55
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	9	0.00	3	0.00	3	0.00	8	0.00	4	0.00	4	0.00	4	0.00

Table A11 Pairwise Comparisons of Forecasts (L_E Loss Function)

*This table reports the signed value of the χ^2 statistic of the Giacomini-White test related to the null hypothesis that the difference in the average L_E loss of the model in the column and that of the row model is equal to zero. Negative (positive) values indicate superiority of the column (row) model over the row (column) model. L_E is the Euclidean loss function. Each panel (A–C) shows results for a different forecast horizon. We use a rolling window of 1,000 observations to obtain forecasts from parametric models. * and ** indicate significance at 5% and 1% levels, respectively.*

	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	EWMA	O-GARCH	A-O-GARCH	RWE	HYBICOV	VHAR
					Panel A: 1–Day Horizon						
D-BEKK	−59.79**	8.06**	1.72	9.47**	9.49**	64.69**	9.33**	10.43**	−16.35**	−41.21**	−57.20**
A-D-BEKK		31.82**	12.57**	30.97**	30.99**	82.98**	30.18**	18.16**	−3.86*	−10.53**	−31.85**
CCC			−0.75	20.05**	20.16**	0.32	13.04**	10.03**	−22.51**	−26.37**	−37.61**
A-CCC				2.57	2.58	0.83	3.01	16.58**	−18.93**	−14.59**	−24.28**
DCC					1.66	−0.06	1.28	9.11**	−23.59**	−26.50**	−37.04**
A-DCC						−0.06	1.22	9.10**	−23.60**	−26.51**	−37.05**
EWMA							0.11	5.38*	−32.12**	−53.89**	−65.56**
O-GARCH								9.09**	−23.53**	−26.23**	−36.57**
A-O-GARCH									−22.13**	−19.14**	−24.57**
RWE										0.23	−3.17
HYBICOV											−40.32**
					Panel B: 5–Day Horizon						
D-BEKK	−15.07**	3.24	1.16	3.57	3.57	18.81**	3.59	4.65*	−17.23**	−11.80**	−18.48**
A-D-BEKK		10.12**	6.16*	9.72**	9.72**	19.04**	9.59**	7.80**	−14.72**	−3.94*	−17.58**
CCC			−0.29	5.64*	5.66*	0.17	4.26*	5.02*	−12.51**	−9.07**	−13.07**
A-CCC				1.46	1.47	0.28	1.95	8.60**	−9.51**	−6.50*	−10.22**
DCC					2.04	0.00	1.00	4.81*	−12.05**	−8.92**	−12.61**
A-DCC						0.00	0.95	4.81*	−12.05**	−8.92**	−12.61**
EWMA							0.02	2.13	−18.96**	−15.57**	−19.66**
O-GARCH								4.76*	−11.95**	−8.92**	−12.51**
A-O-GARCH									−9.68**	−7.92**	−10.02**
RWE										15.44**	−3.32
HYBICOV											−29.71**
					Panel C: 22–Day Horizon						
D-BEKK	−4.01*	1.94	0.53	2.01	2.01	4.56*	2.21	2.16	−5.44*	−1.58	−5.69*
A-D-BEKK		3.76	3.15	3.67	3.67	4.45*	3.75	3.63	−9.02**	1.31	−9.75**
CCC			−1.37	2.46	2.45	0.63	3.13	2.13	−4.59*	−2.32	−4.79*
A-CCC				1.96	1.97	1.04	2.30	4.12*	−4.34*	−1.77	−4.63*
DCC					1.58	0.27	2.72	1.96	−4.46*	−2.35	−4.66*
A-DCC						0.27	2.69	1.96	−4.46*	−2.34	−4.66*
EWMA							−0.12	0.21	−5.13*	−3.03	−5.27*
O-GARCH								1.58	−4.50*	−2.45	−4.70*
A-O-GARCH									−4.33*	−2.67	−4.5*
RWE										12.56**	−6.41*
HYBICOV											−14.80**

Table A12 Pairwise Comparisons of Forecasts (L_F Loss Function)

*This table reports the signed value of the χ^2 statistic of the Giacomini-White test related to the null hypothesis that the difference in the average L_F loss of the model in the column and that of the row model is equal to zero. Negative (positive) values indicate superiority of the column (row) model over the row (column) model. L_F denotes the Frobenius distance. Each panel (A–C) contains results for a different forecast horizon. We use a rolling window of 1,000 observations to obtain forecasts from parametric models. * and ** indicate significance at 5% and 1% levels, respectively.*

	A-D-BEKK	**CCC**	**A-CCC**	**DCC**	**A-DCC**	**EWMA**	**O-GARCH**	**A-O-GARCH**	**RWE**	**HYBICOV**	**VHAR**
						Panel A: 1–Day Horizon					
D-BEKK	−60.85**	7.48**	1.37	9.26**	9.28**	65.57**	9.62**	10.61**	−17.48**	−42.36**	−56.90**
A-D-BEKK		31.41**	11.97**	30.40**	30.43**	84.14**	29.98**	18.11**	−4.42*	−11.57**	−31.35**
CCC			−0.96	20.05**	20.16**	0.80	15.45**	10.59**	−22.63**	−26.35**	−37.02**
A-CCC				3.62	3.63	1.40	5.02*	17.29**	−18.63**	−14.46**	−23.72**
DCC					1.66	−0.02	5.23*	9.52**	−23.92**	−26.49**	−36.36**
A-DCC						−0.02	5.11*	9.51**	−23.93**	−26.51**	−36.37**
EWMA							0.17	5.57*	−33.72**	−54.80**	−65.46**
O-GARCH								9.22**	−24.40**	−26.62**	−36.27**
A-O-GARCH									−22.18**	−19.26**	−24.28**
RWE										0.25	−2.90
HYBICOV											−37.56**
						Panel B: 5–Day Horizon					
D-BEKK	−15.38**	3.02	0.97	3.45	3.46	18.81**	3.63	4.66*	−17.22**	−12.29**	−18.56**
A-D-BEKK		10.01**	5.97*	9.52**	9.53**	19.22**	9.47**	7.71**	−14.58**	−4.55*	−17.43**
CCC			−0.41	5.64*	5.66*	0.38	4.85*	5.16*	−12.41**	−9.11**	−12.99**
A-CCC				2.21	2.23	0.50	3.49	8.58**	−9.44**	−6.49*	−10.12**
DCC					2.04	0.00	2.42	4.93*	−11.87**	−8.92**	−12.42**
A-DCC						0.00	2.36	4.93*	−11.87**	−8.92**	−12.42**
EWMA							0.03	2.19	−18.95**	−15.87**	−19.71**
O-GARCH								4.76*	−11.82**	−9.01**	−12.38**
A-O-GARCH									−9.57**	−7.93**	−9.89**
RWE										15.00**	−3.20
HYBICOV											−28.67**
						Panel C: 22–Day Horizon					
D-BEKK	−4.06*	1.84	0.42	1.94	1.94	4.57*	2.21	2.16	−5.44*	−1.75	−5.69*
A-D-BEKK		3.73	3.11	3.63	3.63	4.48*	3.73	3.59	−8.88**	0.97	−9.68**
CCC			−1.54	2.46	2.45	0.85	3.21	2.26	−4.58*	−2.37	−4.78*
A-CCC				2.22	2.22	1.28	2.70	4.01*	−4.35*	−1.81	−4.65*
DCC					1.58	0.34	3.45	2.10	−4.42*	−2.40	−4.62*
A-DCC						0.34	3.44	2.10	−4.42*	−2.40	−4.61*
EWMA							−0.08	0.24	−5.14*	−3.14	−5.28*
O-GARCH								1.59	−4.47*	−2.54	−4.65*
A-O-GARCH									−4.27*	−2.73	−4.44*
RWE										12.31**	−6.66**
HYBICOV											−14.48**

Table A13 Pairwise Comparisons of Forecasts (L_Q Loss Function)

*This table reports the signed value of the χ^2 statistic of the Giacomini-White test related to the null hypothesis that the difference in the average L_Q loss of the model in the column and that of the row model is equal to zero. Negative (positive) values indicate superiority of the column (row) model over the row (column) model. L_Q denotes the multivariate quasi-likelihood loss function. Each panel (A–C) shows the results for a different forecast horizon. We use a rolling window of 1,000 observations to obtain forecasts from parametric models. * and ** indicate significance at 5% and 1% levels, respectively.*

	A-D-BEKK	CCC	A-CCC	DCC	A-DCC	EWMA	O-GARCH	A-O-GARCH	RWE	HYBICOV	VHAR
					Panel A: 1–Day Horizon						
D-BEKK	−26.23**	−67.77**	−40.86**	−107.94**	−107.92**	178.82**	−8.21**	−9.20**	−135.71**	−1.72	−336.10**
A-D-BEKK		−56.26**	−35.52**	−97.50**	−97.65**	210.98**	0.03	−0.02	−140.76**	−0.01	−383.75**
CCC			1.62	−6.12*	−5.41*	168.24**	103.16**	107.62**	−57.91**	38.47**	−247.14**
A-CCC				−4.52*	−4.28*	130.90**	49.13**	52.49**	−134.23**	55.84**	−556.44**
DCC					4.07*	235.29**	175.03**	186.76**	−51.20**	48.14**	−251.04**
A-DCC						235.36**	175.12**	186.39**	−51.33**	47.64**	−251.06**
EWMA							−112.98**	−116.76**	−274.97**	−48.26**	−505.72**
O-GARCH								−1.60	−162.18**	−0.04	−423.06**
A-O-GARCH									−171.01**	0.00	−447.77**
RWE										212.54**	−351.08**
HYBICOV											−620.01**
					Panel B: 5–Day Horizon						
D-BEKK	−6.37*	−20.5**	−12.15**	−34.53**	−34.63**	97.46**	−2.13	−2.54	−134.41**	−2.76	−153.15**
A-D-BEKK		−15.43**	−9.39**	−26.44**	−26.60**	88.66**	−0.08	−0.26	−127.42**	−1.32	−144.91**
CCC			1.17	−3.23	−3.13	74.39**	38.34**	38.96**	−206.99**	16.94**	−176.51**
A-CCC				−2.87	−2.83	55.47**	18.35**	19.44**	−333.91**	9.21**	−261.65**
DCC					0.04	103.60**	78.13**	79.00**	−194.58**	19.69**	−181.12**
A-DCC						103.59**	78.90**	79.54**	−193.87**	19.63**	−180.73**
EWMA							−56.84**	−57.08**	−236.49**	−28.74**	−272.35**
O-GARCH								−1.29	−259.53**	−2.07	−271.14**
A-O-GARCH									−276.02**	−1.75	−284.07**
RWE										179.83**	−2.63
HYBICOV											−155.58**
					Panel C: 22–Day Horizon						
D-BEKK	−1.29	−2.01	−1.84	−3.49	−3.52	22.26**	−0.15	−0.20	−22.32**	0.00	−22.59**
A-D-BEKK		−1.53	−1.37	−2.65	−2.68	22.49**	0.00	−0.01	−20.68**	0.03	−20.78**
CCC			0.45	−0.21	−0.25	18.46**	5.54*	4.90*	−47.11**	3.81	−50.23**
A-CCC				−0.96	−1.02	18.19**	4.94*	4.62*	−52.40**	3.09	−54.46**
DCC					−0.73	24.01**	14.88**	13.32**	−46.26**	3.43	−48.78**
A-DCC						24.01**	14.95**	13.40**	−46.04**	3.47	−48.47**
EWMA							−18.61**	−19.23**	−63.27**	−5.09*	−63.71**
O-GARCH								−0.10	−54.37**	0.07	−53.73**
A-O-GARCH									−53.50**	0.09	−52.80**
RWE										24.15**	−0.14
HYBICOV											−24.71**

Table A14 MCS Results for the 25% Confidence Level1

This table shows the results of the 25% Model Confidence Set (MCS). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 days, respectively. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The range statistic of Equation (24) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 25% MCS.

	1–Day Horizon						**5–day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	Rank	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**
D-BEKK	6	0.00	6	0.00	7	0.00	6	0.00	6	0.00	9	0.00	6	0.00	6	0.00	9	0.00
A-D-BEKK	4	0.00	4	0.00	8	0.00	4	0.00	4	0.00	7	0.00	3	0.00	3	0.00	3	0.00
CCC	8	0.00	8	0.00	4	0.00	8	0.00	8	0.00	6	0.00	11	0.00	11	0.00	5	0.00
A-CCC	5	0.00	5	0.00	6	0.00	5	0.00	5	0.00	5	0.00	5	0.00	5	0.00	6	0.00
DCC	11	0.00	10	0.00	5	0.00	11	0.00	9	0.00	4	0.00	10	0.00	10	0.00	8	0.00
A-DCC	12	0.00	11	0.00	3	0.00	12	0.00	11	0.00	3	0.00	9	0.00	9	0.00	7	0.00
EWMA	9	0.00	9	0.00	12	0.00	9	0.00	10	0.00	12	0.00	12	0.00	12	0.00	12	0.00
O-GARCH	10	0.00	12	0.00	10	0.00	10	0.00	12	0.00	10	0.00	8	0.00	8	0.00	11	0.00
A-O-GARCH	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	10	0.00
RWE	2	0.05	2	0.06	2	0.00	2	0.03	2	0.04	2	0.06	2	0.00	2	0.00	2*	0.55
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	9	0.00	3	0.00	3	0.00	8	0.00	4	0.00	4	0.00	4	0.00

Table A15 Out-of-Sample Forecast Losses for Different Market Regimes

This table reports average out-of-sample losses for the 1–, 5–, and 22–day forecast horizons, respectively. We employ a rolling window of 1,000 observations to produce forecasts from parametric models. Each panel reports results for a different sub-period of the full sample (i.e. pre-crisis, crisis and post-crisis). L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the 5% MCS based on a particular loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). Also, a dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.

	1–Day Horizon			**5–Day Horizon**			**22–Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
Panel A: 1/1/2000 - 31/7/2008									
D-BEKK	0.0077	0.0124	−20.5252	0.0054	0.0088	−20.4627	0.0041	0.0067	−20.3359
A-D-BEKK	0.0074	0.0119	−20.6422	0.0054	0.0087	−20.5638	0.0042	0.0067	−20.4047
CCC	0.0078	0.0125	−20.9554	0.0065	0.0104	−20.7614	0.0053	0.0085	−20.3640
A-CCC	0.0071†	0.0114†	−20.9759	0.0066	0.0106	−20.8094	0.0053	0.0084	−20.4240
DCC	0.0081	0.0130	−21.0029	0.0068	0.0109	−20.8255	0.0056	0.0090	−20.4438
A-DCC	0.0081	0.0130	−21.0004	0.0068	0.0109	−20.8247	0.0056	0.0090	−20.4472
EWMA	0.0096	0.0157	−19.9905	0.0079	0.0129	−19.8680	0.0070	0.0115	−19.6659
O-GARCH	0.0082	0.0133	−20.6944	0.0070	0.0114	−20.6136	0.0059	0.0096	−20.4242
A-O-GARCH	0.0097†	0.0158	−20.7097	0.0095	0.0155	−20.6358	0.0074	0.0120	−20.4470
RWE	0.0050†	0.0080†	−21.3877	0.0051†	0.0082†	−21.4681†	0.0024†	0.0039†	−21.2394*
VHAR	0.0048*	0.0077*	−21.6891*	0.0035*	0.0057*	−21.4837*	0.0021*	0.0034*	−21.1843†
HYBICOV	0.0065	0.0104	−20.5342	0.0044	0.0069	−20.4490	0.0041	0.0062	−19.9704
Panel B: 1/8/2008- 31/12/2009									
D-BEKK	0.1560	0.2518	−14.2032	0.1268	0.2058	−13.8043	0.1253†	0.2027†	−12.6199
A-D-BEKK	0.1050	0.1687	−14.1417	0.0712	0.1149	−13.7008	0.0593†	0.0939†	−12.5108
CCC	0.2034	0.3245	−15.2557	0.1747	0.2791	−15.0773	0.1771†	0.2821†	−14.1743
A-CCC	0.1865	0.2953	−15.3165	0.1562	0.2472	−15.1067	0.1403†	0.2204†	−14.1437
DCC	0.2153	0.3482	−15.1931	0.1860	0.3018	−14.9710	0.1863†	0.3006†	−13.9702
A-DCC	0.2153	0.3482	−15.1845	0.1861	0.3019	−14.9638	0.1864†	0.3008†	−13.9642
EWMA	0.2092	0.3397	−13.8235	0.1860	0.3037	−13.4248	0.2123†	0.3459†	−12.2009
O-GARCH	0.2157	0.3536	−14.5248	0.1874	0.3095	−14.2690	0.1913†	0.3159†	−13.4279
A-O-GARCH	0.3138	0.5190	−14.5430	0.2692	0.4471	−14.2907	0.2171†	0.3594†	−13.4092
RWE	0.0780†	0.1221†	−15.6721	0.0304†	0.0469†	−15.6910*	0.0393†	0.0607†	−14.7210†
VHAR	0.0563*	0.0884*	−15.9351*	0.0289*	0.0451*	−15.6645†	0.0326*	0.0508*	−14.8127*
HYBICOV	0.0830	0.1299	−15.1376	0.0540	0.0843	−15.1175	0.0524	0.0816	−14.5050†
Panel C: 1/1/2010- 19/4/2016									
D-BEKK	0.0125	0.0195	−20.1858	0.0095	0.0152	−20.1325	0.0094	0.0151	−19.9143
A-D-BEKK	0.0104	0.0162	−20.2494	0.0076	0.0121	−20.1766	0.0070	0.0112	−19.9414
CCC	0.0145	0.0225	−20.3522	0.0120	0.0190	−20.2466	0.0133	0.0211	−19.9155
A-CCC	0.0150	0.0232	−20.2664	0.0135	0.0211	−20.1444	0.0143	0.0225	−19.8508
DCC	0.0146	0.0228	−20.3824	0.0122	0.0193	−20.2818	0.0134	0.0213	−19.9355
A-DCC	0.0146	0.0228	−20.3838	0.0122	0.0193	−20.2836	0.0134	0.0213	−19.9366
EWMA	0.0140	0.0220	−19.8933	0.0112	0.0180	−19.7892	0.0117	0.0189	−19.2957
O-GARCH	0.0149	0.0234	−20.1210	0.0125	0.0200	−20.0363	0.0138	0.0221	−19.7310
A-O-GARCH	0.0205	0.0329	−20.1188	0.0188	0.0305	−20.0350	0.0188	0.0305	−19.7277
RWE	0.0079†	0.0118†	−20.5771	0.0041	0.0063	−20.7172	0.0033	0.0052	−20.2276†
VHAR	0.0057*	0.0086*	−20.9339*	0.0032*	0.0049*	−20.7526*	0.0027*	0.0042*	−20.2604*
HYBICOV	0.0081	0.0123	−20.1080	0.0069	0.0108	−20.1180	0.0126	0.0198	−19.7116

Table A16 Out-of-Sample Forecast Losses (Rolling Window of 1,250 Obs.)

This table reports average out-of-sample losses for the daily, weekly, and monthly forecast horizons, respectively. We employ a rolling window of 1,250 observations to produce forecasts from parametric models. L_E *denotes the Euclidean distance,* L_F *is the Frobenius distance, and* L_Q *is the multivariate quasi-likelihood loss function. The model with the lowest average out-of-sample loss is marked with an asterisk (*). Also, a dagger (†) indicates models that yield as accurate forecasts as the best model at the 5% significance level based on the Giacomini-White test.*

	1–Day Horizon			**5–Day Horizon**			**22–Day Horizon**		
	L_E	L_F	L_Q	L_E	L_F	L_Q	L_E	L_F	L_Q
D-BEKK	0.0278	0.0445	−19.3332	0.0210	0.0340	−19.2226	0.0193	0.0312	−18.8920
A-D-BEKK	0.0221	0.0352	−19.4388	0.0154	0.0248	−19.3048	0.0131	0.0210	−18.9686
CCC	0.0352	0.0557	−19.7275	0.0291	0.0462	−19.5844	0.0282	0.0449	−19.1732
A-CCC	0.0327	0.0514	−19.6941	0.0274	0.0431	−19.5433	0.0247	0.0389	−19.1603
DCC	0.0368	0.0590	−19.7366	0.0307	0.0494	−19.5930	0.0295	0.0474	−19.1602
A-DCC	0.0368	0.0590	−19.7356	0.0307	0.0494	−19.5924	0.0295	0.0474	−19.1601
EWMA	0.0390	0.0629	−18.9449	0.0337	0.0548	−18.7815	0.0373	0.0607	−18.2442
O-GARCH	0.0366	0.0593	−19.4115	0.0304	0.0497	−19.3033	0.0292	0.0478	−18.9683
A-O-GARCH	0.0500	0.0820	−19.4051	0.0430	0.0709	−19.2978	0.0354	0.0582	−18.9572
RWE	0.0167†	0.0258†	−20.0219	0.0083†	0.0129†	−20.1105†	0.0080	0.0124	−19.5932†
VHAR	0.0124*	0.0194*	−20.3502*	0.0069*	0.0108*	−20.1324*	0.0066*	0.0103*	−19.6097*
HYBICOV	0.0179	0.0278	−19.4934	0.0125	0.0195	−19.4963	0.0153	0.0239	−19.0733

Table A17 MCS Results (Rolling Window of 1,250 Obs.)

This table shows the results of the 5% Model Confidence Set (MCS). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 trading days, respectively. A rolling window of 1,250 observations is used to produce forecasts from parametric models. L_E is the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function while the p-val column shows the associated p-value of the test. The semi-quadratic statistic of Equation (24) is used to test the null hypothesis of the MCS. An asterisk (*) indicates models that are part of the 5% MCS.

	1–Day Horizon						**5–Day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	Rank	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**	**Rank**	**p-val**
D-BEKK	6	0.00	6	0.00	9	0.00	6	0.00	6	0.00	9	0.00	6	0.00	6	0.00	9	0.00
A-D-BEKK	4	0.00	4	0.00	7	0.00	4	0.00	4	0.00	8	0.00	3	0.00	3	0.00	6	0.00
CCC	8	0.00	8	0.00	3	0.00	9	0.00	8	0.00	7	0.00	11	0.00	11	0.00	5	0.00
A-CCC	5	0.00	5	0.00	6	0.00	5	0.00	5	0.00	6	0.00	5	0.00	5	0.00	4	0.00
DCC	12	0.00	11	0.00	4	0.00	11	0.00	11	0.00	4	0.00	10	0.00	10	0.00	7	0.00
A-DCC	10	0.00	10	0.00	5	0.00	10	0.00	10	0.00	5	0.00	8	0.00	8	0.00	8	0.00
EWMA	11	0.00	12	0.00	12	0.00	12	0.00	12	0.00	11	0.00	12	0.00	12	0.00	12	0.00
O-GARCH	9	0.00	9	0.00	10	0.00	8	0.00	9	0.00	10	0.00	7	0.00	7	0.00	10	0.00
A-O-GARCH	7	0.00	7	0.00	11	0.00	7	0.00	7	0.00	12	0.00	9	0.00	9	0.00	11	0.00
RWE	2	0.05	2*	0.05	2	0.00	2	0.03	2	0.03	2*	0.06	2	0.00	2	0.00	2*	0.34
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	8	0.00	3	0.00	3	0.00	3	0.00	4	0.00	4	0.00	3	0.00

Table A18 MCS Results for Alternative Sub-samples

*This table shows the results of the 5% Model Confidence Set (MCS). Each panel shows results for a different sub-period of the full sample (i.e. pre-crisis, crisis and post-crisis). The crisis period is defined to be from 1 August, 2007 to 31 December, 2009. We employ three statistical loss functions and forecast horizons of 1, 5 and 22 days, respectively. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function, while the p-val column contains the p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. * indicates models that are part of the 5% MCS.*

	1–Day Horizon						**5–Day Horizon**						**22–Day Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	rank	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**
Panel A: 1/1/2000-31/7/2007 (pre-crisis sub-sample)																		
D-BEKK	7	0.00	7	0.00	10	0.00	7	0.00	7	0.00	10	0.00	7	0.00	7	0.00	8	0.00
A-D-BEKK	10	0.00	10	0.00	9	0.00	8	0.00	8	0.00	9	0.00	6	0.00	6	0.00	4	0.00
CCC	9	0.00	8	0.00	5	0.00	10	0.00	9	0.00	8	0.00	12	0.00	12	0.00	10	0.00
A-CCC	5	0.00	4	0.00	7	0.00	5	0.00	5	0.00	5	0.00	4	0.00	5	0.00	9	0.00
DCC	11	0.00	11	0.00	4	0.00	11	0.00	11	0.00	4	0.00	10	0.00	10	0.00	6	0.00
A-DCC	12	0.00	12	0.00	3	0.00	12	0.00	12	0.00	3	0.00	11	0.00	11	0.00	3	0.00
EWMA	4	0.00	5	0.00	12	0.00	3	0.00	3	0.00	12	0.00	3	0.00	3	0.00	11	0.00
O-GARCH	8	0.00	9	0.00	8	0.00	9	0.00	10	0.00	7	0.00	9	0.00	9	0.00	7	0.00
A-O-GARCH	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	8	0.00	8	0.00	5	0.00
RWE	2*	0.11	2*	0.11	2	0.00	2	0.02	2	0.02	2	0.00	2	0.01	2	0.01	1*	1.00
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	2	0.02
HYBICOV	3	0.00	3	0.00	11	0.00	4	0.00	4	0.00	11	0.00	5	0.00	4	0.00	12	0.00
Panel B: 1/8/2007 - 31/12/2009 (crisis sub-sample)																		
D-BEKK	6	0.00	6	0.00	8	0.00	6	0.00	6	0.00	9	0.00	6	0.01	6	0.01	6	0.00
A-D-BEKK	4	0.01	4	0.01	9	0.00	4	0.00	4	0.00	6	0.00	4	0.01	4	0.01	9	0.00
CCC	8	0.00	8	0.00	5	0.00	9	0.00	8	0.00	5	0.00	11	0.01	11	0.01	5	0.00
A-CCC	5	0.01	5	0.01	4	0.00	5	0.00	5	0.00	4	0.00	5	0.01	5	0.01	4	0.00
DCC	12	0.00	12	0.00	7	0.00	12	0.00	12	0.00	7	0.00	10	0.01	10	0.01	7	0.00
A-DCC	11	0.00	11	0.00	6	0.00	11	0.00	10	0.00	8	0.00	9	0.01	9	0.01	8	0.00
EWMA	9	0.00	9	0.00	11	0.00	10	0.00	11	0.00	10	0.00	12	0.00	12	0.00	12	0.00
O-GARCH	10	0.00	10	0.00	10	0.00	8	0.00	9	0.00	12	0.00	8	0.01	8	0.01	11	0.00
A-O-GARCH	7	0.00	7	0.00	12	0.00	7	0.00	7	0.00	11	0.00	7	0.01	7	0.01	10	0.00
RWE	2*	0.13	2*	0.14	2	0.00	2*	0.18	2*	0.19	1*	1.00	2	0.01	2	0.01	3*	0.80
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	2*	0.16	1*	1.00	1*	1.00	2*	0.80
HYBICOV	3	0.01	3	0.02	3	0.00	3	0.00	3	0.00	3	0.00	3	0.01	3	0.01	1*	1.00
Panel C: 1/1/2010- 19/4/2016 (post-crisis sub-sample)																		
D-BEKK	5	0.00	6	0.00	8	0.00	5	0.00	5	0.00	8	0.00	5	0.00	5	0.00	4	0.00
A-D-BEKK	4	0.00	4	0.00	9	0.00	4	0.00	4	0.00	9	0.00	3	0.00	3	0.00	3	0.00
CCC	10	0.00	10	0.00	4	0.00	10	0.00	10	0.00	4	0.00	10	0.00	10	0.00	5	0.00
A-CCC	6	0.00	5	0.00	7	0.00	6	0.00	6	0.00	7	0.00	7	0.00	7	0.00	9	0.00
DCC	8	0.00	8	0.00	5	0.00	8	0.00	8	0.00	5	0.00	9	0.00	9	0.00	7	0.00
A-DCC	9	0.00	9	0.00	3	0.00	9	0.00	9	0.00	3	0.00	8	0.00	8	0.00	6	0.00
EWMA	7	0.00	7	0.00	10	0.00	7	0.00	7	0.00	10	0.00	6	0.00	6	0.00	10	0.00
O-GARCH	11	0.00	11	0.00	11	0.00	12	0.00	12	0.00	11	0.00	12	0.00	12	0.00	11	0.00
A-O-GARCH	12	0.00	12	0.00	12	0.00	11	0.00	11	0.00	12	0.00	11	0.00	11	0.00	12	0.00
RWE	2	0.05	2*	0.05	2	0.00	2	0.01	2	0.02	2	0.02	2	0.00	2	0.00	2*	0.14
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	6	0.00	3	0.00	3	0.00	6	0.00	4	0.00	4	0.00	8	0.00

Table A19 MCS Results: Non-Overlapping Forecasts

*This table shows the results of the Model Confidence Set (MCS) using non-overlapping forecasts (for weekly and monthly forecasts). We employ three statistical loss functions and forecast horizons of 1, 5 and 22 trading days, respectively. L_E denotes the Euclidean distance, L_F is the Frobenius distance, and L_Q is the multivariate quasi-likelihood loss function. The column labelled rank reports the relative ranking of a model in the MCS based on a specific loss function, while the p-val column contains the p-value of the test. The semi-quadratic statistic of Equation (23) is used to test the null hypothesis of the MCS. * indicates models that are part of the 5% MCS.*

	Daily Horizon						**Weekly Horizon**						**Monthly Horizon**					
	L_E		L_F		L_Q		L_E		L_F		L_Q		L_E		L_F		L_Q	
	rank	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**	**rank**	**p-val**
D-BEKK	6	0.00	6	0.00	7	0.00	5	0.04	5	0.04	9	0.00	9*	0.09	9*	0.09	5	0.00
A-D-BEKK	4	0.00	4	0.00	8	0.00	4	0.04	4	0.04	8	0.00	3*	0.09	3*	0.09	3	0.00
CCC	8	0.00	8	0.00	4	0.00	9	0.04	7	0.04	7	0.00	5*	0.09	5*	0.09	6	0.00
A-CCC	5	0.00	5	0.00	6	0.00	6	0.04	6	0.04	6	0.00	6*	0.09	6*	0.09	7	0.00
DCC	11	0.00	10	0.00	5	0.00	10	0.04	10	0.04	5	0.00	7*	0.09	7*	0.09	9	0.00
A-DCC	12	0.00	11	0.00	3	0.00	11	0.04	11	0.04	4	0.00	8*	0.09	8*	0.09	8	0.00
EWMA	9	0.00	9	0.00	12	0.00	7	0.04	8	0.04	12	0.00	12*	0.09	12*	0.08	12	0.00
O-GARCH	10	0.00	12	0.00	10	0.00	12	0.04	12	0.04	10	0.00	11*	0.09	11*	0.09	10	0.00
A-O-GARCH	7	0.00	7	0.00	11	0.00	8	0.04	9	0.04	11	0.00	10*	0.09	10*	0.09	11	0.00
RWE	2	0.05	2*	0.06	2	0.00	2*	0.12	2*	0.14	2*	0.25	2*	0.09	2*	0.09	2*	0.58
VHAR	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00	1*	1.00
HYBICOV	3	0.00	3	0.00	9	0.00	3	0.04	3	0.04	3	0.00	4*	0.09	4*	0.09	4	0.00