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1 **A review of numerical modelling of the dynamics of microstructural development in**  
2 **rocks and ice: Past, present and future**

3

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32

### 33 **Abstract**

34 This review provides an overview of the emergence and current status of numerical  
35 modelling of microstructures, a powerful tool for predicting the dynamic behaviour of rocks  
36 and ice at the microscale with consequence for the evolution of these materials at a larger  
37 scale. We emphasize the general philosophy behind such numerical models and their  
38 application to important geological phenomena such as dynamic recrystallization and strain  
39 localization. We focus in particular on the dynamics that emerge when multiple processes,  
40 which may either be enhancing or competing with each other, are simultaneously active.  
41 Here, the ability to track the evolving microstructure is a particular advantage of numerical  
42 modelling. We highlight advances through time and provide glimpses into future  
43 opportunities and challenges.

44

45 **Keywords:** review, evolution of microstructures, numerical modelling, dynamic  
46 recrystallization, process interaction

47

## 48 **1. Introduction**

49 Structural geology studies how rocks deform under applied stress or strain. These studies are  
50 applied from the sub-grain scale to that of mountain belts and tectonic plates. To know what  
51 has happened to a volume of rock millions of years ago, or to predict what would happen to a  
52 rock volume under certain (future) conditions, we need to know the link between (i) the  
53 boundary conditions (e.g. applied stress or strain, pressure-temperature conditions), (ii) the  
54 intrinsic material properties (e.g. Young's modulus, crystal symmetry, slip systems) and (iii)  
55 the processes that may be activated (e.g. dislocation creep, pressure solution, mineral phase  
56 changes). These three together determine the evolution of the state of the rock (e.g.  
57 developing cleavage, folds, lineation, stylolites) and its bulk material properties (e.g.  
58 viscosity). The microstructure is thus a state variable of a rock, describing the state it  
59 achieved as a result of the interplay between various processes and boundary conditions.  
60 Importantly, the microstructure is not a passive log of events, but plays an active and central  
61 role in the evolution of a rock throughout its history (Gottstein, 2004). In this context non-  
62 linear feedback between a specific microstructural configuration and local changes in  
63 intrinsic and extrinsic parameters may be important but difficult to predict. Hence, the  
64 microstructure is the link that couples the material properties, boundary conditions and  
65 processes that together control the behaviour and evolution of a rock. Consequently, the  
66 analysis and correct interpretation of microstructures is crucial in gaining an understanding of  
67 how rocks, including ice, deform on Earth and other planets. Here we define microstructure  
68 as the full spatial, compositional and orientational arrangement of all entities in a rock,  
69 typically on the scale of a thin section to hand specimen (roughly  $\mu\text{m}$  to cm) (Hobbs et al.,  
70 1976). These entities include minerals, grain and subgrain boundaries, crystal lattice  
71 orientations, and chemical composition from the nano- to micro-scale.

72 While the rock's temperature or elastic strain are ephemeral, the microstructure may be  
73 preserved for the geologist to interpret millions to billions of years later. Therefore,

74 microstructures are one of the prime forensic tools to unravel the history of a rock that allows  
75 us to deduce the succession of strain rates, stresses, diagenetic and metamorphic conditions  
76 and the rock's rheology during deformation (e.g. Hobbs et al., 1976; Vernon, 2004; Passchier  
77 and Trouw, 2005).

78

79 One major problem in microstructural studies is that we normally only have a static record as  
80 post-mortem "images" (grain arrangement in thin section, chemical and orientation  
81 characteristics derived using electron microscopy, 3D datasets from computer tomography,  
82 neutron diffraction and synchrotron beam analysis, etc.). To interpret microstructures, we  
83 need to know how they form and change. Means (1977) wrote: "a more valuable kind of  
84 kinematic investigation is that in which the time sequence of incremental strains or  
85 incremental displacements, the kinematic history, is correlated with progressive structural  
86 change. This promises to reveal more about the physics of rock deformation and to provide  
87 more sensitive structural methods for reconstructing tectonic history". Such investigations  
88 first became possible using in-situ analogue experiments that were introduced to geology by  
89 influential scientists such as Cloos (1955) and Ramberg (1981), and at the microstructural  
90 scale by Means (1977). In in-situ analogue experiments by Means (1977), inspired by those  
91 of McCrone and Chen (1949), a thin-section sized sample of crystalline analogue material is  
92 sandwiched and deformed between glass plates to observe the changing microstructure under  
93 a microscope. These experiments were followed by a large number of studies that  
94 investigated deformation related features such as dynamic recrystallization and crystal  
95 plasticity (e.g. Means, 1977; 1980; Means and Ree, 1988; Park and Means, 1996; Urai et al.,  
96 1980; Urai and Humphreys, 1981; Wilson, 1986; Jessell, 1986; Ree, 1991; Bons and Jessell,  
97 1999; Ree and Park, 1997; Herwegh and Handy, 1996, 1998; Nama et al., 1999; Wilson et  
98 al., 2014), melt or fluid-bearing microstructures (e.g. Urai, 1983; Rosenberg and Handy,

99 2000, 2001; Rosenberg, 2001; Schenk and Urai, 2004, 2005; Walte et al., 2005), the  
100 development of vein and fringe microstructures (e.g. Hilgers et al., 1997; Koehn et al., 2003)  
101 and the formation of  $\delta$ -clasts (ten Brink et al., 1995). Many of these experiments made clear  
102 that when a polycrystalline material deforms, multiple processes act upon the microstructure  
103 and govern its dynamic behaviour throughout its deformation and post-deformation history  
104 (Fig. 1a; see also movies at <http://www.tectonique.net/MeansCD/>). In addition, pre-existing  
105 heterogeneities and/or those developing during deformation can have a significant effect on  
106 the dynamics of the system since local variations in stress and strain play a significant role in  
107 the evolution of microstructures and material behaviour, with their effects on chemistry being  
108 particularly intriguing (Wheeler, 2018).

109 Although changes in microstructural characteristics (e.g. grain size) can be quantified in such  
110 experiments, the underlying principles of the active processes still need to be deduced.  
111 Therefore, an additional tool is needed. Here numerical models simulating microstructural  
112 development based on several concurrent processes become important. Such numerical  
113 simulations share the advantage of in-situ experiments that the full microstructural  
114 development can be traced in form of time-series, with the opportunity to systematically  
115 study the effect of different processes and/or pre-existing heterogeneities on the  
116 microstructural development. In addition, numerical simulations are neither constrained by  
117 time nor specific boundary conditions. Once calibrated against laboratory experiments (e.g.  
118 Piazzolo et al., 2004) or analytical solutions, they can be applied to a wide range of conditions  
119 or materials including those not attainable or suitable for laboratory experiments. In many  
120 cases, the developed microphysical behaviour may be applied to problems on the continental  
121 scale.

122 In this contribution, we aim to give an overview of the concepts behind numerical modelling  
123 of microstructures, i.e. microdynamic modelling, and the achievements that have been made

124 in this field. This is followed by a selection of examples of the current state of the art. Finally,  
125 future possibilities and directions are briefly discussed, including some work in progress. We  
126 focus on those studies that (i) are applied to geological materials, (ii) involve several  
127 processes and (iii) allow prediction and visualization of the development of the  
128 microstructure. Hence, we exclude studies where microstructures are not spatially mapped  
129 along with models in which grains are modelled as if embedded in a medium with averaged  
130 properties. Table 1 summarizes the microdynamic numerical models cited, the numerical  
131 method used and processes included in each model.

132

## 133 **2. Numerical simulation of microstructures**

### 134 **2. 1. Emergence and philosophy**

135 Soon after the first in-situ experiments, computers had advanced to a stage allowing the first  
136 numerical simulations of microstructural development, with a full "image" of the  
137 microstructure, to appear in geology (Etchecopar, 1977; Jessell, 1988a,b; Jessell and Lister,  
138 1990). When using such models we accept the fact that the scientific description of the  
139 phenomena studied does not fully capture reality. This is not a shortcoming but a strength, as  
140 general system behaviour and interrelationships can be studied systematically and  
141 transferred to geological questions. In a numerical model the behaviour of a system emerges  
142 from the piece-meal enumeration of the behaviour of each of its component parts. As a  
143 consequence, numerical models are powerful and necessary when the bulk behaviour is  
144 influenced by the local interaction of the components. In contrast, global, averaged or mean  
145 field solutions cannot fully encompass the effect of processes that interact with each other  
146 on the small scale. In nature, patterns emerge (e.g. orientation and arrangement of high-  
147 strain zones, foliation, fractures) which may be used to deduce conditions of their formation.

148 Numerical models are powerful tools to investigate this link between patterns observed in  
149 nature and processes that are responsible for their development.

150

## 151 **2.2. Multiprocess modelling – approaches**

152 As shown convincingly by the aforementioned experiments, several processes act  
153 concurrently on a microstructure and control the dynamics of the microstructural evolution  
154 and material properties (e.g. Means, 1977, 1980). While there are a large number of models  
155 that focus on one process alone, there are fewer systems that aim to model the effect of  
156 multiple interacting processes. Here, we focus on the latter. Solving all necessary equations  
157 simultaneously is only practical when the number of interacting processes is small.  
158 Alternatively, the set of equations, hence processes, needs to be reduced to a manageable  
159 number necessitating a user driven selection which may result in the omission of important  
160 feedbacks. Operator splitting provides an alternative approach, whereby each process acts  
161 sequentially on a microstructure that was incrementally changed by all operating processes in  
162 the previous step. In this case, the number of equations to be solved is not limited; hence  
163 there is no need to artificially reduce the number of modelled processes. Clearly, length scale  
164 and time steps have to be considered carefully for this approach to be valid. The operator  
165 splitting approach is fundamental to the numerical platform Elle (Jessell et al., 2001, Bons et  
166 al., 2008, Piazzolo et al., 2010; [www.elle.ws](http://www.elle.ws)) which is, within the realm of microdynamic  
167 modelling, the most used numerical platform in geology. However, operator splitting has also  
168 been employed in other numerical schemes (e.g. Cross et al., 2015).

169

## 170 **2.3. The numerical representation of a microstructure**

171 It is necessary to describe numerically the microstructure so that in the model the processes  
172 can act upon it spatially. Up to now, for rocks and ice, microstructural models have been



173 mostly restricted to two-dimensions. Two basic approaches can be taken: (i) a lattice data  
174 structure or (ii) an element data structure. In the lattice data structure, the microstructure is  
175 mapped onto an irregular or regular lattice, like the pixels in a digital image or gridded  
176 analysis points obtained during chemical mapping and orientation mapping of geological  
177 samples. Potts, cellular automata, lattice-spring, particle-in-cell, phase field and  
178 micromechanical Fast Fourier Transform (FFT) based models utilize the lattice data structure  
179 (Fig. 2; see also chapter 2 of Bons et al., 2008 for a detailed review of different numerical  
180 methods). Numerically the lattice data structure is easy to use and calculations are  
181 straightforward. In the element data structure, the microstructure is described by discrete  
182 elements which can be points, line segments, or polygons (Fig. 2). Finite-element models and  
183 boundary models typically use elements to describe a system. Element data structure is well  
184 suited for processes acting upon a surface, e.g. grain boundary migration, but is numerically  
185 more “expensive” due to necessary topology checks. Both elements and lattice points can  
186 have, in addition to their location in xy space, specific properties or store specific state  
187 variables, such as mineral phase, chemical composition, crystallographic orientation, and  
188 stress and strain (rate) values, respectively. The numerical platform Elle (Jessell et al., 2001)  
189 combines these two approaches: a microstructure is represented by boundary nodes  
190 connected by straight segments (Fig.2). A polygon enclosed by boundary nodes represents a  
191 grain or subgrain. Property variations within the individual polygons are defined at interior  
192 points where information is recorded and tracked. Processes acting on the microstructure can  
193 move boundaries and/or interior points and/or change the properties at those points, e.g.  
194 composition, crystallographic orientation or dislocation density. The system has now matured  
195 to a stage that it can robustly be used for a large variety of models relevant to geology.

196

### 197 **3. Examples of modelling several coupled processes**

198 **3.1. Dynamic recrystallization**

199 3.1.1. A historical perspective

200 Dynamic recrystallization is the response of a crystalline aggregate to lower its free energy by  
201 formation and movement of sub- and grain boundaries (Means, 1983). It occurs by a number  
202 of concurrent processes that act upon the polycrystalline material. For decades, geologists  
203 have used microstructural characteristics developed during dynamic recrystallization to infer  
204 dominance of processes and linked boundary conditions (e.g. Urai et al., 1986; Hirth and  
205 Tullis, 1992; Stipp et al., 2002). Following the first attempts of such a model (Etchecopar,  
206 1977), Jessell (1988a,b) and Jessell and Lister (1990) developed a numerical model  
207 incorporating rotation of the crystal lattice and dynamic recrystallization for quartz. The data  
208 structure used was a 100x100 hexagonal lattice structure where each lattice point represented  
209 a grain or subgrain (Fig. 3a inset). Calculation of the crystal lattice rotation in Jessell and  
210 Lister (1990) uses the Taylor-Bishop-Hill calculation method (Taylor, 1938; Bishop and Hill,  
211 1951a, 1951b; Lister and Paterson, 1979) and the critical resolved shear stress of the different  
212 slip systems in quartz. This work reproduced the general microstructures seen in a mylonite.  
213 A major step forward was that the simulations showed that dynamic recrystallization can  
214 significantly modify crystallographic preferred orientation (CPO) development. Piazzolo et al.  
215 (2002) advanced Jessell's model by utilizing the more robust Elle data structure to describe  
216 the local deformation and the dynamics of the grain boundary network (Jessell et al., 2001).  
217 The model now included a finite element solution for incompressible, linear or nonlinear  
218 viscous flow (BASIL; Barr and Houseman 1992, 1996) to compute the local velocity field,  
219 the front tracking method for the motion of grain boundaries by moving nodes and segments  
220 (e.g. Bons and Urai, 1992), the Taylor-Bishop-Hill formulation (Lister and Paterson, 1979)  
221 and additional features such as crystal lattice rotation, formation of subgrains,  
222 recrystallization by nucleation, grain boundary migration, recovery, work hardening and

223 tracking of dislocation densities. This list of additional features shows the advance made in  
224 approximately a decade in reproducing natural microstructures (Fig. 3b).

225 So far, the rheological anisotropy of minerals deforming by dislocation creep was not taken  
226 into account in the calculation of the stress-strain rate field and lattice rotation. About another  
227 decade after Piazzolo et al. (2002), the viscoplastic implementation of the full-field crystal  
228 plasticity micromechanical code based on Fast Fourier Transforms (VPFFT; Lebensohn,  
229 2001) was introduced in geological microstructural modelling (Griera et al., 2011, 2013,  
230 2015). The VPFFT is a spectral method that specifically assumes that deformation is  
231 achieved by dislocation glide on crystallographic slip planes (each with their own critical  
232 resolved shear stress) whose orientations are mapped on a regular grid (for details see Griera  
233 et al. (2013), Montagnat et al. (2011, 2014) and Llorens et al. (2016a)). The simulations of  
234 Griera et al. (2011, 2013) and Ran et al. (2018) illustrate that heterogeneous stress within and  
235 between grains that emerge from inhomogeneous slip has a significant effect on the rotation  
236 rate of porphyroclasts and -blasts and the strain (rate) field around these objects.

237 More recently, Llorens et al. (2016a; 2016b; 2017) and Steinbach et al. (2016; 2017) coupled  
238 dynamic recrystallization (DRX) by grain boundary migration (GBM), intracrystalline  
239 recovery and polygonisation with VPFFT crystal plasticity models, and applied them to study  
240 the deformation of polar ice. Llorens et al. (2016a) presents a comprehensive description of  
241 the method. They found that DRX produces large and equidimensional grains, but only  
242 marginally affects the development of the c-axes CPO. However, DRX can alter the activity  
243 of slip systems and does modify the distribution of a-axes. In simple shear, the strong  
244 intrinsic anisotropic of ice crystals is transferred from the crystal to the polycrystal scale,  
245 leading to strain localisation bands that can be masked by GBM (Llorens et al., 2016b).

246 Llorens et al. (2017) compared the dynamics of pure polar ice polycrystalline aggregates in  
247 pure and simple shear deformation. It was found that, due to the vorticity of deformation, it is

248 expected that ice is effectively weaker in the lower parts of ice sheets (where simple shear  
249 dominates) than in the upper parts (where ice is mostly deformed coaxially). The method was  
250 also applied to reproduce the development of tilted-lattice (kink) bands found in ice cores  
251 (Jansen et al., 2016) and crenulation cleavage during folding (Ran et al., 2018) as a result of  
252 mechanical anisotropy. Steinbach et al. (2016) included air inclusions as a second phase to  
253 simulate DRX in porous firn and found that DRX can occur despite the low strain and stress in  
254 firn.

255 Apart from polar ice, the VPFFT-Elle model has also been used to analyse subgrain rotation  
256 recrystallization of halite polycrystals in simple shear (Gomez-Rivas et al., 2017) by coupling  
257 VPFFT with the Elle routines that simulate intracrystalline recovery (Borthwick et al, 2013)  
258 and subgrain rotation. They found that recovery does not affect CPOs, but strongly decreases  
259 grain size reduction. These authors also evaluated the use of mean subgrain misorientations  
260 as a strain gauge.

### 261 3.1.2. Testing the effect of different combinations of recrystallization processes

262 We show results from three combinations of processes that occur during dynamic  
263 recrystallization. These processes act on the same initial 10x10 cm aggregate of ice Ih  
264 crystals (Fig. 4a). The numerical approach in these simulations is based on the VPFFT  
265 micromechanical model in combination with several Elle processes (Llorens et al., 2016a,  
266 2016b, 2017; Steinbach et al., 2016; 2017). The deformation-induced lattice rotation and the  
267 estimation of geometrically necessary dislocation densities calculated from the stress and  
268 velocity field provided by the VPFFT algorithm are used to simulate recrystallization by  
269 intra-crystalline recovery, grain boundary migration and nucleation. Grain boundary  
270 migration (GBM) is simulated based on the algorithm by Becker et al. (2008). Here, a front-  
271 tracking approach is used in which the movement of boundaries is mapped through time.  
272 Recovery reduces the intra-granular stored energy in a deformed crystal (Borthwick et al.,

273 2013). Nucleation creates new grain boundaries with areas of misorientation values above a  
274 pre-defined threshold (Piazolo et al., 2002; Llorens et al., 2017; Steinbach et al., 2017).  
275 Dextral simple shear deformation is applied in strain increments of  $\Delta\gamma = 0.04$ . Three different  
276 combinations of recrystallization processes are tested: (1) GBM only, (2) GBM and recovery  
277 or (3) GBM, recovery and nucleation (Fig. 4a). Simulations including VPFPT viscoplastic  
278 deformation show a similar evolution of c-axis orientation, regardless of the dynamic  
279 recrystallization processes included (Fig. 4a). The lack of influence of nucleation on the CPO  
280 is due to the fact that new grains are modelled to have lattice orientations close to those of  
281 their parent grains. However, inclusion of the nucleation process result in grain size reduction  
282 (Fig. 4b). Results illustrate how different combinations of microdynamic processes affect the  
283 microstructural characteristics (e.g. CPO, grain size, shape and orientation) in different ways  
284 where no single microstructural parameter can grasp the full dynamics of polycrystalline  
285 deformation.

### 286 **3.2. Diffusion creep**

287 Diffusion creep is a physico-chemical process that operates in large regions of the Earth – in  
288 the upper Earth as “pressure solution” where diffusion is enhanced along fluid films and  
289 elsewhere at low strain rates, maybe in large parts of the lower mantle (Karato, 1992). The  
290 mechanism involves diffusion driven by gradients in normal stress along boundaries. In  
291 simple models no other driving forces need to be included. It is implicit in our understanding  
292 of this mechanism that grain boundary sliding occurs in conjunction with diffusion. Ford et  
293 al. (2002) devised a numerical model to understand the evolution of grain shapes and CPO in  
294 grain boundary diffusion creep. The mathematical framework allows for the consideration of  
295 diffusion and sliding together. Operator splitting is not required and the evolution at each  
296 timestep is based on a single matrix inversion operation. The model couples the  
297 microstructure to an evolving system of local stresses (Fig. 5). The model predicts that grain

298 shapes become somewhat elongate, in accordance with experiments on calcite (Schmid et al.,  
299 1987) and observations on an albite-bearing mylonite (Jiang et al., 2000) (Fig. 5). It also  
300 predicts that grain rotations are not chaotic and that CPO may be present to high strains  
301 (Wheeler, 2009), as seen in, for example, experiments on olivine-orthopyroxene (Sundberg  
302 and Cooper, 2008) and predicted by numerical simulations of Bons and den Brok (1999). A  
303 second phase, if insoluble, can be included and such a model was used by Berton et al. (2006,  
304 2011) to prove the hypothesis that a different grain boundary diffusion coefficient along the  
305 two-phase boundaries could explain fibre growth in pressure shadows. Not all model  
306 predictions are in agreement with other investigations – for example some materials  
307 deformed by diffusion creep e.g. olivine show quite equant grains (Karato et al., 1986). In  
308 such experiments grain growth occurs but this process was not included in the numerical  
309 model, hence results differ. In a later section we address this issue, but this example  
310 illustrates that, in common with other numerical models, the applicability reflects the  
311 processes included.

### 312 **3.3. Stress driven dissolution, growth and dynamic roughening**

313 Another example where stress leads to micro- and macrostructures is the roughening of grain  
314 boundaries in the presence of a fluid. In this case dissolution and/or mineral growth are  
315 driven not just by normal stress gradients at the interface (c.f. section 3.2) but also by changes  
316 in elastic and surface energies. In addition to these driving forces, the detailed  
317 electrochemistry of interfaces has a kinetic effect on rates (Gratier et al., 2013). In the  
318 example presented here, the process is modelled in Elle by coupling a lattice spring code that  
319 calculates the strain with a background fluid that dissolves lattice elements. This process can  
320 produce transient patterns of interface geometry at the grain boundaries (Koehn et al., 2006).  
321 In addition, the process itself produces rough interfaces that can be seen on the larger scale as  
322 stylolites (Koehn et al., 2007; Ebner et al., 2009). The scaling behaviour of stylolites is

323 important for stress inversion and compaction on the basin scale (Koehn et al., 2012; 2016)  
324 and the shape of stylolites is partly rooted in the microstructure of the host-rock (Fig. 6).  
325 Slower dissolving material on various scales, from small grains to fossils and layers, can  
326 influence the dissolution process and thus the development of patterns, whereas deformation  
327 leads to changes in long-range interactions (Koehn et al., 2016). Similarly, pinning behaviour  
328 in the presence of a fluid is seen in grain growth where grain boundary pinning results in  
329 restriction of grain growth. Pinning particles can also be moved around so that growing  
330 grains become clean. Such a process can lead to layered rocks, for example, zebra dolomites  
331 (Kelka et al., 2015).

## 332 **4. Numerical simulations of microdynamic processes: Examples of ongoing work**

### 333 **4.1. Linking chemistry and microstructural evolution**

334 Even now, most of the numerical models that incorporated more than one process are  
335 restricted to deformation processes alone. Although in diffusion creep there is an intrinsic  
336 chemical aspect, the model described in 3.2 involves just a single phase of fixed composition.  
337 There are some models that incorporate the growth of new mineral phases and investigate the  
338 rheological effect of such growth (e.g. Groome et al., 2006; Smith et al., 2015), however the  
339 location of new phase growth and/or growth rate is predefined. While Park et al. (2004)  
340 showed convincingly the effect of microstructure on the diffusion pathways and chemical  
341 patterns in garnet and biotite, there are now many opportunities available in a numerical  
342 system such as Elle (Jessell et al., 2001) to couple local chemistry and microstructural  
343 evolution. Below we present preliminary results from two current projects linking chemical  
344 changes and a dynamically evolving grain network.

#### 345 4.1.1. Grain boundary diffusion creep and grain growth

346 Here we present the first results using a model combining, using operator splitting,  
347 deformation by diffusion creep (Ford et al., 2002; Wheeler, 2009) and surface energy driven

348 grain boundary migration (GBM), i.e. grain growth (modified after Becker et al., 2008). Four  
349 different scenarios are shown: diffusion creep only, grain boundary migration only and two  
350 combinations with medium and high grain boundary migration GBM rates (Fig. 7a). The  
351 starting material is an almost regular hexagonal grain mesh with equant grains in which triple  
352 junctions have had small random perturbations imposed (Fig. 7a). The perturbations are  
353 required to avoid mathematical problems that arise in perfectly regular hexagonal networks  
354 (Wheeler, 2010). Topology checks such as neighbour switching routines are performed after  
355 each step in all simulations, ensuring that no topology problems arise. The microstructural  
356 development is markedly different if grain growth is coupled with diffusion creep (Fig. 7a)  
357 where with increasing GBM rate the aspect ratios are generally reduced relative to that of  
358 comparable experiments modelling diffusion creep only. This is provisionally in accord with  
359 the hypothesis that there will be a ‘steady state’ grain elongation developed which is a  
360 function of the relative magnitudes of strain rate and grain growth kinetic parameters  
361 (Wheeler, 2009) but more simulations are required (with less regular starting  
362 microstructures). In contrast, for grain growth only, the microstructure does not change as the  
363 initial configuration consists of stable near 120° triple junctions (Fig. 7a). Grain numbers (i.e.  
364 grain size) do not change in any of the simulations.

#### 365 4.1.2. Trace element partitioning between fluid and solid coupled with recrystallization

366 Field and experimental studies illustrate the importance of the time and length scales on the  
367 evolution of grain networks during trace element diffusion (e.g. Ashley et al., 2014 and  
368 references therein). Therefore, understanding the influence of recrystallization on trace  
369 element distribution is essential for correctly interpreting patterns of trace element  
370 distribution (e.g. Wark and Watson, 2006). However, to our knowledge there are currently no  
371 numerical approaches able to simulate diffusion coupled with microstructure evolution.  
372 Recently, a new Elle finite difference process that solves Fick’s second law to simulate trace



373 element diffusion in a polycrystalline medium has been developed. The polycrystal is defined  
374 by n-phases where grain boundaries and grains are differentiated (Fig. 7b). The trace element  
375 partitioning coefficient between different phases is also considered. The approach uses an  
376 element data structure representing grain boundaries and a lattice data structure of a regular  
377 grid of unconnected lattice point to track diffusion along the grain boundaries and the  
378 physical and chemical properties within grains, respectively. To ensure elemental exchange  
379 between grain boundaries and grain interiors, a grain interior-grain boundary partition  
380 coefficient is implemented that takes the proximity to grain boundaries into account. This  
381 allows the simulation of coupled bulk and grain boundary diffusion. This approach can be  
382 fully coupled with other processes of the numerical platform Elle and therefore allows  
383 simulating simultaneous diffusion, deformation and static or dynamic recrystallization. The  
384 use of this approach is demonstrated through the modelling of diffusion of a tracer with  
385 fractionation during static grain growth of a single-phase aggregate (Fig. 7b) (after that  
386 presented by Jessell et al., 2001). Patterns of chemical concentration qualitatively resemble  
387 those of Ti-distribution observed in recrystallized quartz in shear zones and can help to  
388 understand the redistribution of Ti in quartz during dynamic recrystallization (e.g. Grujic et  
389 al., 2011), among many other geological problems.

#### 390 4.2. **Modelling in three dimensions**

391 We restricted this review to two-dimensional numerical approaches, as very few models have  
392 been published that are in three dimensions and also specific to geological microstructures.  
393 An exception is the phase field approach by Ankit et al. (2015) who investigate the growth of  
394 crystals in an open fracture. However, within the material science community numerical  
395 models exist that investigate the behaviour of a microstructure in response to single processes  
396 in three dimensions (e.g. three dimensional crystal plasticity modelling platform DAMASK  
397 (Roters et al., 2012; Eisenlohr et al., 2013), which includes the option of solving the

398 micromechanical problem using a 3D implementation of the FFT-based formulation  
399 (presently implemented in Elle in its 2-D version) and grain growth (e.g. Krill and Chen,  
400 2002; Kim et al., 2006). With the technical advances made within the geological and material  
401 science community it is now in principle possible to extend currently available 3D models to  
402 geological questions and/or multi-process scenarios.

#### 403 4.2.1. Dissolution of reactive surfaces in three dimensions

404 Mineral dissolution controls important processes in geoscience such as serpentinization (e.g.  
405 Seyfried et al., 2007), retrogression and replacement reactions (Putnis and Austrheim, 2012),  
406 deformation by dissolution-precipitation creep (e.g. Rutter, 1976) and also affects processes  
407 relevant to society such as the stability of spent nuclear fuel and mine waste. Recent work has  
408 shown that dissolution rates are linked to the evolving surface structure, and thus are time-  
409 dependent (Godinho et al., 2012). Different surfaces have different structures in three  
410 dimensions and the presence of etch pits play a major role in dissolution (Pluemper et al.,  
411 2012). Hence, for accurate representation of dissolution behaviour, modelling in three  
412 dimensions is important. Here, we present a numerical model that simulates the dissolution  
413 process as a potential tool to quantify the links between dissolution rates, reactive surface  
414 area and topography over periods of time beyond reasonable for a laboratory experiment. The  
415 program uses empirical equations that relate the dissolution rate of a point of the surface with  
416 its crystallographic orientation (Godinho et al., 2012) to simulate changes of topography  
417 during dissolution, which ultimately results in the variation of the overall dissolution rate  
418 (Godinho et al., 2012, 2014). The initial surface is composed of a group of nodes with a xy  
419 position and a set height (z). For each lattice point a local surface orientation is calculated  
420 from the inclination of the segment node and its neighbours. This orientation together with  
421 the crystallographic orientation of the grain the surface belongs to is then used to calculate a  
422 dissolution rate (surface reactivity). Based on this the displacement of the node using the

423 equations published in Godinho et al. (2012) is calculated. The model allows the graphical  
424 display of the three-dimensional topographic development of the surface, tracking of the  
425 variation of the surface area and calculation of the overall dissolution rate at each stage of the  
426 simulation. Results obtained with the 3D simulation at three consecutive stages of dissolution  
427 (150 hrs, 180 hrs, 276 hrs dissolution duration) show that numerical results are in accordance  
428 to experimental results (Fig. 7c).

429

## 430 **5. Numerical simulations of microstructures: Possibilities and challenges**

### 431 **5.1. Linking laboratory and natural data with numerical models**

432 Over the last two decades numerical capabilities have advanced markedly and models have  
433 come of age. Consequently, a large range of models and process combinations is now  
434 available that can be utilized to gain further insight into the link between processes, material  
435 properties and boundary conditions. This also includes polymineralic systems making the  
436 models more appropriate to use to investigate processes occurring within polyphase rocks  
437 (e.g. Roessiger et al., 2014; Steinbach et al, 2016). New avenues of effective verification  
438 against laboratory and natural data are now opening up, due to the development of analytical  
439 tools allowing rapid complete microstructural and microchemical analysis providing dataset  
440 similar to those possible in numerical models (e.g. Steinbach et al, 2017, Piazzolo et al., 2016);  
441 many of the analytical data sets have a lattice data structure.

### 442 **5.2. Linking chemistry and microstructural evolution**

443 Advances in numerical methods, theoretical treatment of thermodynamic data, analytical  
444 tools and theoretical and experimental insights into the coupling between chemical and  
445 physical process that emerged over the last decade, call for a major effort in this area of  
446 research. Examples of areas of opportunities include investigation of the (1) significance of  
447 local stress versus bulk stress on mineral reactions and reaction rates (e.g. Wheeler, 2014), (2)

448 characteristics of replacement microstructures and their potential significance for  
449 microstructural interpretation (e.g. Putnis, 2009, Spruzeniece et al., 2017), (3) mobility of  
450 trace elements enhanced by deformation that change significantly the local elemental  
451 distributions (e.g. Reddy et al., 2007, Piazzolo et al., 2012, 2016) and (4) coupling between  
452 reactive fluid-solid systems and hydrodynamics (Kelka et al., 2017). Increasing computer  
453 processing speeds will aid the running of such models. However, the technical challenges  
454 include the harmonisation of fundamentally different numerical approaches (crystal plasticity  
455 versus diffusion creep, for example) and the resolution of fundamental mathematical  
456 problems, e.g. the current lack of an internally consistent model for multiphase diffusion  
457 creep, highlighted by Ford and Wheeler (2004).

### 458 **5.3. Linking brittle and ductile deformation: elasto-viscoplastic behaviour**

459 When examining the rock record, it is clear that in many cases, brittle and ductile behaviour  
460 often occurs at the same time within a rock (e.g. Bell and Etheridge, 1973, Hobbs et al.,  
461 1976). With the potential significance of grain-scale brittle behaviour now measurable on  
462 seismic signals (e.g. Fagereng et al., 2014) there is an increased need in developing numerical  
463 techniques that allow us to model the dynamic link between brittle and ductile behaviour.  
464 Such elasto-viscoplastic behaviour combines the elastic reversible fast deformation with a  
465 viscous time dependent flow and a plastic behaviour. These behaviours can be included in  
466 continuous or discontinuous models. For example, in the numerical platform Elle, a lattice  
467 structure is used to deform the model elastically up to a critical stress where bonds fracture  
468 (plastic behaviour) and the particles themselves deform as a function of stress and time  
469 (viscous behaviour). In this case the viscous behaviour conserves the volume whereas shear  
470 forces and differential stresses converge to zero (Sachau and Koehn, 2010, 2012; Arslan et  
471 al., 2012; Koehn and Sachau, 2014). Alternatively, linking the elasto-viscoplastic FFT based  
472 (EVPFFT) (Lebensohn et al., 2012) with models such as Elle would allow calculation of the

473 Cauchy stresses that are the local driving force for grain scale damage processes.  
474 One of the major challenges is the large range of time-scales in these processes, with  
475 fracturing and fluid flow on fast to intermediate and viscous deformation and potentially  
476 reactions on very long time scales. This complexity either requires the assumption that some  
477 processes are instantaneous or it requires an “up-scaling in time” or non-linear time scales in  
478 models.

479

#### 480 **5.4. Expansion of capability to three dimensions**

481 With the advent of supercomputers and new numerical approaches now is the time to develop  
482 techniques to investigate microstructural development in three dimensions. This is of  
483 particular importance if material transport such as aqueous fluid and/or melt flow is to be  
484 considered. This would enable, for example, modelling of strain fringes, shear veins and en-  
485 echelon tension gashes. However, these require models that link brittle and ductile behaviour  
486 along with modelling in three dimensions.

487 One of the biggest problems faced with three-dimensional models of microstructures are the  
488 three-dimensional topology changes that are common in dynamic microstructural  
489 development (e.g. Fig. 1-7). This is a major problem if an element data structure with  
490 segments, i.e. grain boundaries is used. However, the phase field approach does not work  
491 with such distinct boundaries, and is therefore well suited for 3D problems (e.g. Ankit et al.  
492 2014). In addition, a three dimensional network of unconnected nodes, in which there is no  
493 physical movement of boundaries but only changes in the properties of the 3D nodes (voxels)  
494 (cf. Fig. 7c) may be a way forward (e.g. Sachau and Koehn, 2012).

495

#### 496 **5.5. Link between geophysical signals and microstructure**

497 At a time where there is an ever-increasing amount of geophysical data being collected,  
498 numerical simulations that are used to test the link between microstructural development and  
499 geophysical signal will become increasingly important. Cyprych et al. (2017) showed that not  
500 only crystallographic preferred orientation but also the spatial distribution of phases i.e. the  
501 microstructure, has a major impact on seismic anisotropy. Therefore, a direct link between  
502 the microdynamic models such as Elle allowing tracking of microstructural changes through  
503 time and space and geophysical signal generation offers a wealth of new opportunities  
504 including interpretation of strong reflectors in the lower crust and mantle. Current efforts by  
505 Johnson, Gerbi and co-workers (Naus-Thijssen et al., 2011; Cook et al., 2013; Vel et al., 2016)  
506 are in line with this direction.

507

## 508 **5.6. Application of numerical models to polymineralic rock deformation and ice-related** 509 **questions**

510 The dynamic behaviour of the Earth is strongly influenced by the deformation of  
511 polymineralic rocks. At the same time Earth's polar ice caps and glacial ice which often  
512 include ice and a second phase (e.g. air inclusions, dust, entrained bedrock) is of major  
513 importance to society, especially in view of changing climate (e.g. Petit et al., 1999; EPICA,  
514 2004). Application of the current numerical capabilities to polycrystalline ice and  
515 polymineralic rocks is therefore urgently needed. Over the last years, there has been an  
516 increased effort in this direction (Roessiger et al., 2011, 2014; Piazzolo et al., 2015; Llorens et  
517 al., 2016a, 2016b, 2017; Jansen et al., 2016; Steinbach et al. 2016, 2017), which promises to  
518 continue. Here, the development of the link between elemental mobility and microstructural  
519 development is of major importance, as only with such models the can chemical signals of,  
520 for example, ice cores be correctly interpreted.

521

522 **5.7. Upscaling: Utilizing operator splitting and utilities developed for microdynamic**  
523 **systems to larger-scale problems**

524 One of the strengths of the numerical approach taken by the microstructural community has  
525 been the close link between different processes and the ability of the models to take into  
526 account the local differences in properties such as stress, strain and chemistry. The technique  
527 of operator splitting has proven extremely powerful. Furthermore, the ability to model  
528 anisotropic material behaviour utilizing for example VPFFT viscoplastic deformation  
529 formulations (Lebensohn, 2001), has enabled realistic and dynamic models. Upscaling this  
530 approach to investigate problems at a large scale e.g. folding (Llorens et al., 2013a, 2013b;  
531 Bons et al., 2016; Ran et al., 2018) and shear deformation (Gardner et al., 2017) have shown  
532 to be very beneficial. There is great scope to expand further on this in view of fluid flow,  
533 mineralization and fault formation.

534

535 **6. Numerical simulations of microstructures: Lessons learnt and future challenges**

536 Numerical simulations of microstructural development have caught our imagination over the  
537 last three decades. They have markedly advanced our ability to explain phenomena and  
538 patterns we observe in nature and experiments by allowing us to test the link between  
539 boundary conditions, material properties, processes, and microstructural development.  
540 Importantly, models, especially those that couple several process and/or investigate pre-  
541 existing heterogeneities can train the geologist to think of the dynamics of the system rather  
542 than a linear development. For example, different patterns of strain localization observed in  
543 nature can be explained by differences in the relative rates of interacting processes (e.g.  
544 Jessell et al., 2005; Gardner et al., 2017). At the same time, specific indicative microstructural  
545 parameters can be developed to help interpret natural microstructures (e.g. Piazzolo et al.,  
546 2002; Gomez-Rivas et al., 2017, Llorens et al., 2017; Steinbach et al., 2017).

547 However, including chemistry coupled to other processes remains a particular challenge.  
548 Whilst, for example, trace-element diffusion can be enacted in parallel with other processes  
549 (section 4.1.2), chemical transport of major elements and diffusion creep cannot yet be fully  
550 integrated with many other processes. Indeed, when multiphase systems are considered, there  
551 are unsolved problems with diffusion creep modelling even in the absence of other processes  
552 (Ford and Wheeler, 2004). This challenge is closely linked to our current inability to  
553 confidently model grain boundary sliding.  
554 Nevertheless, the studies we describe have shown that numerical models are extremely  
555 powerful in providing benchmark results to investigate what kind of microstructure may  
556 develop under certain conditions. These models are sophisticated mind experiments that are  
557 firmly based on physical and chemical laws for which the theory is well known individually  
558 but their interaction is difficult to predict analytically.

559

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569

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915

916 **Table Captions**

917 **Table 1** List of numerical models of microstructural (mm to dm) development identifying  
918 processes modelled, numerical method used and providing relevant references. This list is  
919 restricted to geological applications and those referenced in the main text. Note processes are  
920 categorized as "S" for static (material points do not move) and "D" for dynamic (material  
921 points move). Furthermore, unless stated otherwise models are two-dimensional.

922 Abbreviations: TBH – Taylor Bishop Hill calculation method for crystal lattice rotation,  
923 VPFPT – Viscoplastic Fast Fourier Transform based model, EVPFPT - Elasto-viscoplastic  
924 Fast Fourier Transform based model, FEM Finite Element, reXX – recrystallization.

925

926 **Figure Captions**

927 **Figure 1** Microstructural development during in-situ deformation of the rock analogue  
928 octochloropropane within a circular shear zone (Bons and Jessell, 1999); dashed red lines  
929 indicate the shear distribution between the two steps shown. Experiments run with top to the  
930 right shear at an average strain rate of  $4.6 \cdot 10^{-4} \text{ s}^{-1}$  where the strain rate near edge of the shear  
931 zone ( $\sim 1.2 \cdot 10^{-3} \text{ s}^{-1}$ ) is 10 x higher than in the top half of the image ( $\sim 1.2 \cdot 10^{-4} \text{ s}^{-1}$ ). (a)  $t_1$  at a  
932 bulk shear strain of  $\sim 40$ ; (b)  $t_1 + 16 \text{ min.}$ . Note multiple concurrent processes: grain boundary  
933 migration, leading to dissection of grains (locations 1 & 2), subgrain rotation (black arrow),  
934 nucleation (white arrow). The different shear rates lead to a different balance of  
935 recrystallization processes and differences in microstructures. At the low shear rate grains are  
936 equant, have straight sub-grain boundaries and basal planes at an angle to the NS and EW  
937 polarisers. The high-strain rate zone shows serrate grain boundaries, an oblique grain-shape  
938 foliation, basal grains approximately parallel to the shear-zone boundary, as well as shear  
939 localisation on grain boundaries (location 3), indicative of grain boundary sliding/shearing.



940 Such "micro-shear zones" may now have been detected in polar ice sheets as well (Weikusat  
941 et al., 2009).

942 **Figure 2** Numerical representation of a microstructure. (a) Micrograph of quartzite; (b)  
943 numerical representation combining an element data structure with nodes (black circles),  
944 segments (black lines) and polygons (enclosed area) and a lattice data structure with  
945 unconnected lattice points (open circles). This structure is used in the numerical platform Elle  
946 (see text for details).

947 **Figure 3** Numerical modelling of dynamic recrystallization – a historical perspective;  
948 mineral modelled is quartz; simple shear (see text for details); (a) numerical microstructure  
949 after  $\gamma = 1$ ; different grey scales signify different crystallographic orientations; top inset shows  
950 data structure of hexagonal lattice points (modified after Jessell and Lister, 1990); (b)  
951 numerical microstructure after  $\gamma = 1$ ; colours show crystallographic orientation, grain  
952 boundaries are red, subgrain boundaries black; for data structure see inset (modified after  
953 Piazzolo et al., 2002).

954 **Figure 4** Numerical modelling of dynamic recrystallization – testing the effect of process  
955 combination on microstructural development; model parameters: mineral modelled - ice; time  
956 step - 20 years, simple shear;  $\Delta\gamma = 0.04$  per time step. FFT and GBM signify fast fourier  
957 transform formulation for crystal plasticity and grain boundary migration, respectively; (a)  
958 initial microstructure and results after  $\gamma = 2.4$  for different process combinations. Results are  
959 shown as grain network with orientation related colour coding according to crystal  
960 orientations relative to the shortening direction  $y$  (see legend) and in pole figures. In the latter  
961 the colour bar indicates the multiples of uniform distribution; (b) grain area distribution  
962 normalized to the initial average grain area for all models shown.

963 **Figure 5** Results from diffusion creep modelling in pure shear; 2D microstructure as in the  
964 starting frame of Fig 1b of Wheeler (2009). Shown is the oblique view of microstructure with

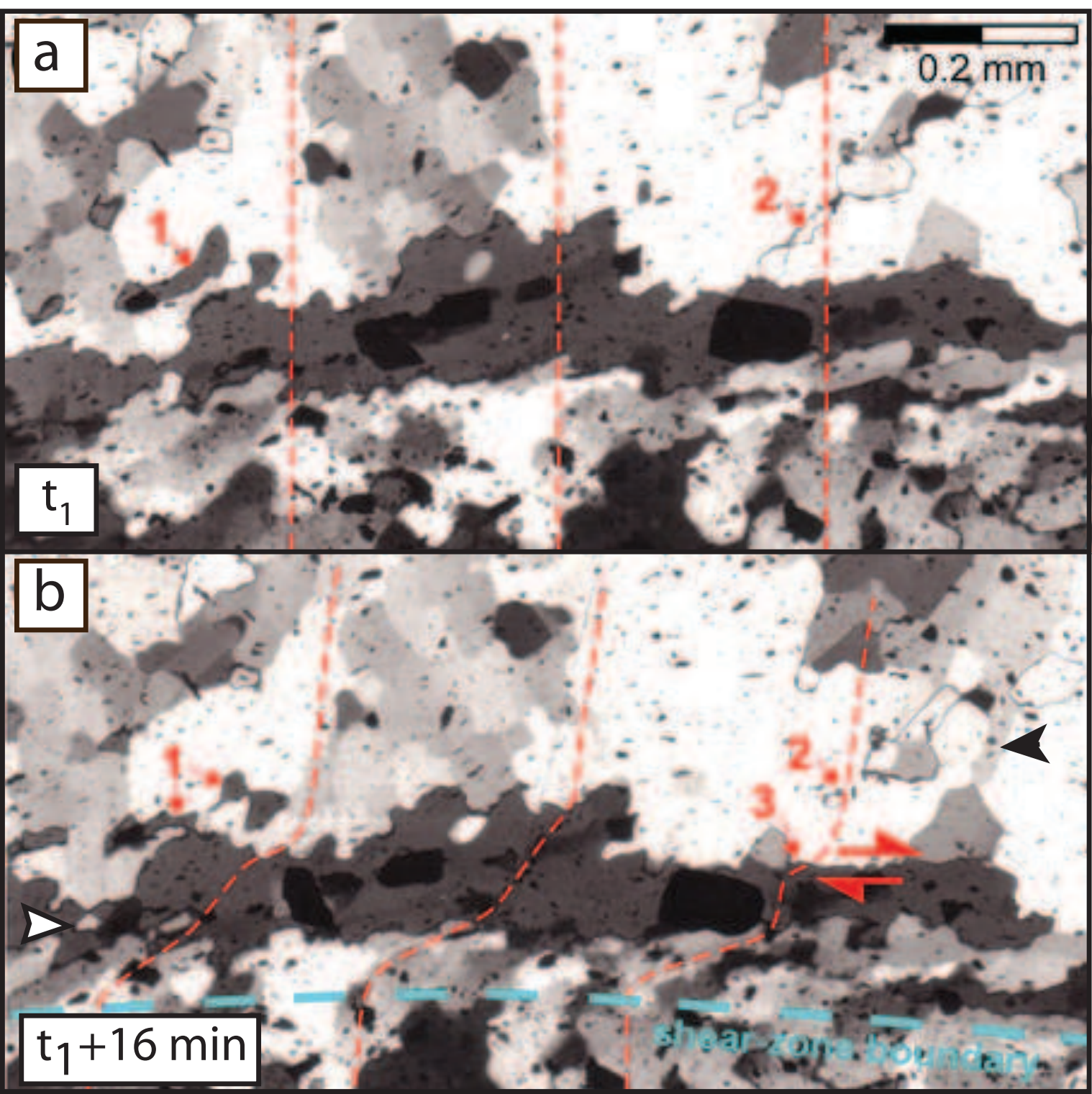
965 patterns of normal stress shown along grain boundaries in the 3<sup>rd</sup> dimension as “fences”. Red  
966 arrows show stretching direction, hence stresses are tensile (shown as negative) on  
967 boundaries at a high angle to stretching direction are tensile. Fences are colour coded  
968 according to dissolution rate with blue low and red high. When there is no relative grain  
969 rotation the fences have a single colour and the stress is parabolic. When there is relative  
970 grain rotation the fences vary in colour and the stress is a cubic function of position.

971 **Figure 6** Dynamic development of stylolite roughness in numerical simulations; (a) time  
972 series (left to right) with the stylolite nucleating in the middle of a slow dissolving layer  
973 (layer in green and stylolite in black colour). Once the stylolite has dissolve the layer on one  
974 side the layer starts to pin and teeth develop. (b) Variation of the pinning strength of the layer  
975 in three different simulations showing a strong dependency. The compaction (movement of  
976 upper and lower walls) is shown in quite arrows. L in the picture on the right hand side is the  
977 initial position of the layer and P shown as black arrows indicates the pinning of the layer  
978 upwards and downwards during dissolution.

979 **Figure 7** Example of new development in microdynamic numerical modelling; preliminary  
980 results (see text for details). (a) Coupling of diffusion creep and surface energy driven grain  
981 boundary migration (GBM); (left) starting microstructure; (middle) microstructure at stretch  
982 2; (right) graph showing average aspect ratio versus stretch; note that the microstructure after  
983 a significant period of exclusive grain boundary migration is the same as the starting  
984 microstructure as no movement occurs as all triple junctions are 120°. Number of grains  
985 stays constant for all simulations. (b) Evolution of chemical concentration of an arbitrary  
986 element during surface energy driven GBM. The material is a single-phase polycrystalline  
987 aggregate with different initial chemical content. This example assumes very low bulk  
988 diffusion ( $D_{\text{bulk}} = 1e^{-20} \text{ m}^2/\text{s}$ ) and fast grain boundary diffusion ( $D_{\text{boundary}} = 1e^{-8} \text{ m}^2/\text{s}$ ), hence  
989 grain boundary diffusion dominates. Colour code indicates chemical concentration and white

990 lines represent grain boundaries. (c) Dissolution of reactive surfaces in 3D using the example  
991 of fluorite dissolution. Microstructures after three dissolution periods are show (150hrs, 180  
992 hrs, 276 hrs); surfaces correspond to the {111} plane at the start of experiment/simulation;  
993 colours identify different depths where blue signifies low and red high; images are 60  $\mu\text{m}$   
994 width. Top panel row shows numerical results. Lower panel shows experimental results using  
995 confocal microscopy images of a grain of a sintered  $\text{CaF}_2$  pellet at the same three dissolution  
996 times as the numerical models. Note the formation of etch pits with similar triangular shape  
997 and the faster/enhanced dissolution of the grain boundaries in both experiment and numerical  
998 simulations.

Figure 1



← strain rate increase

Figure 2

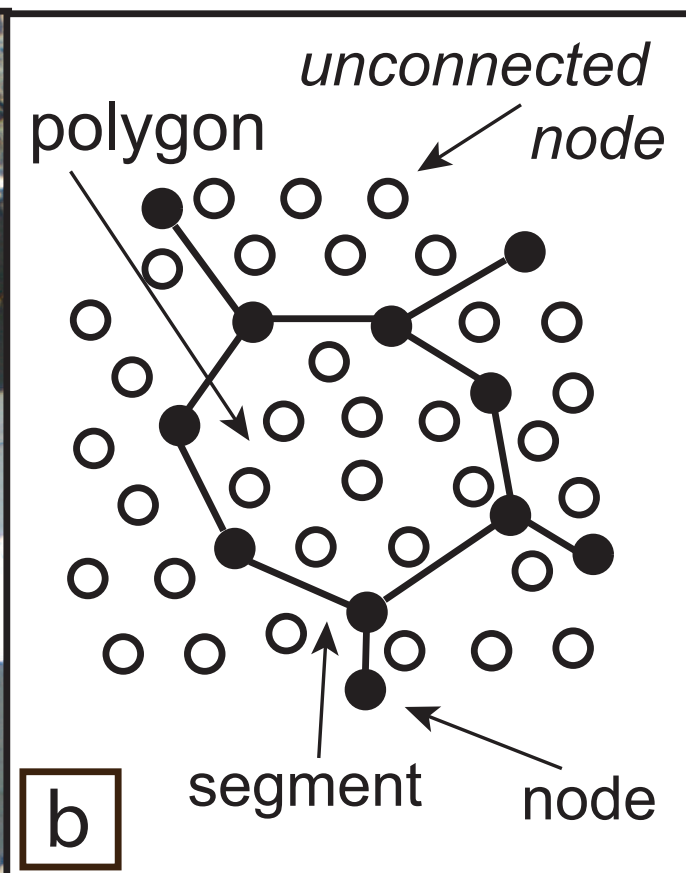
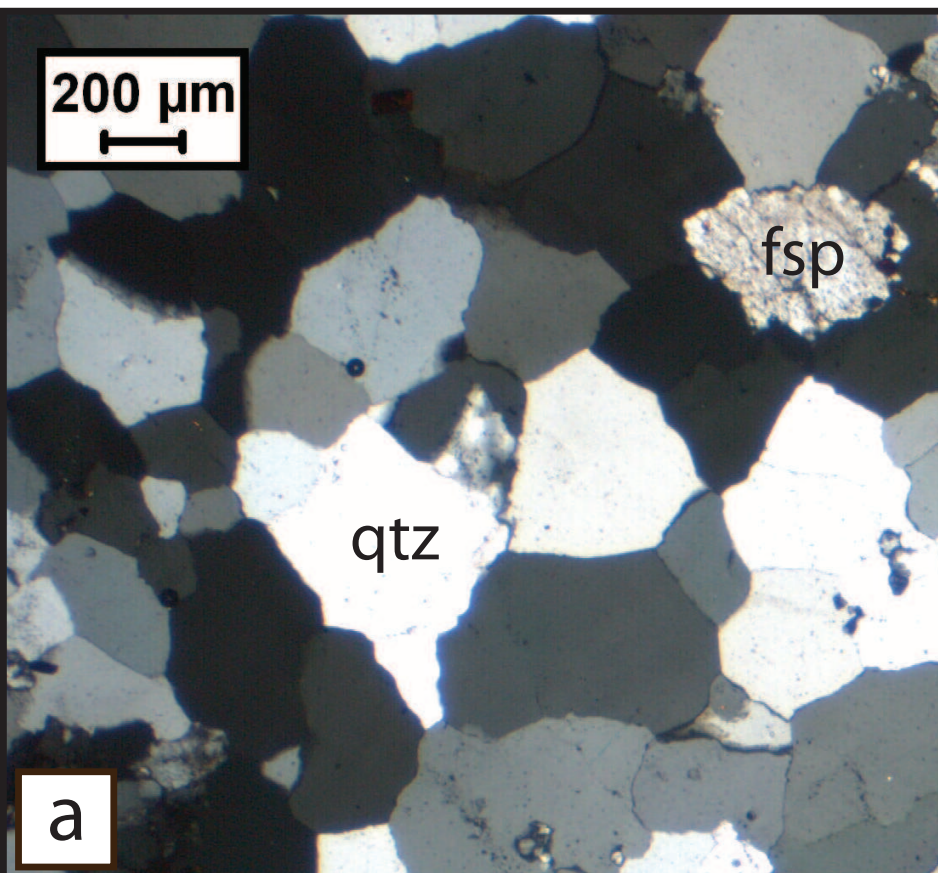




Figure 3

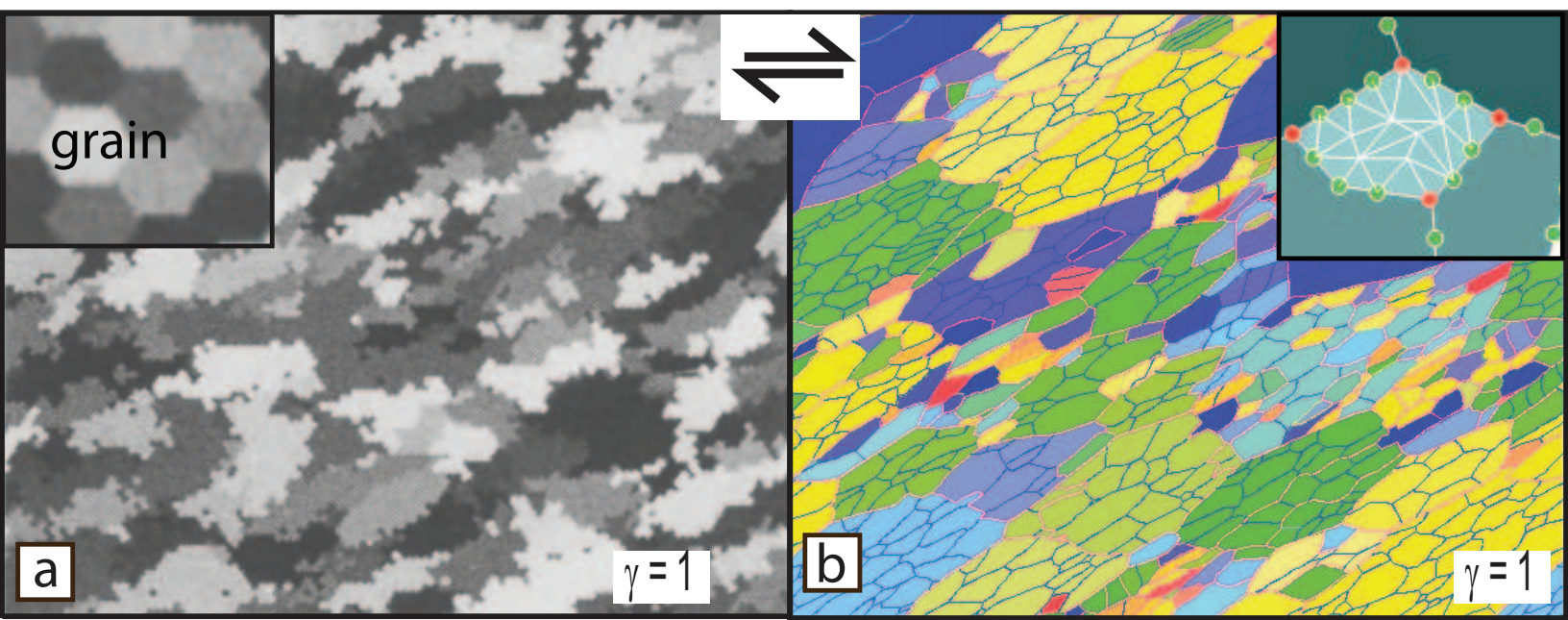


Figure 4

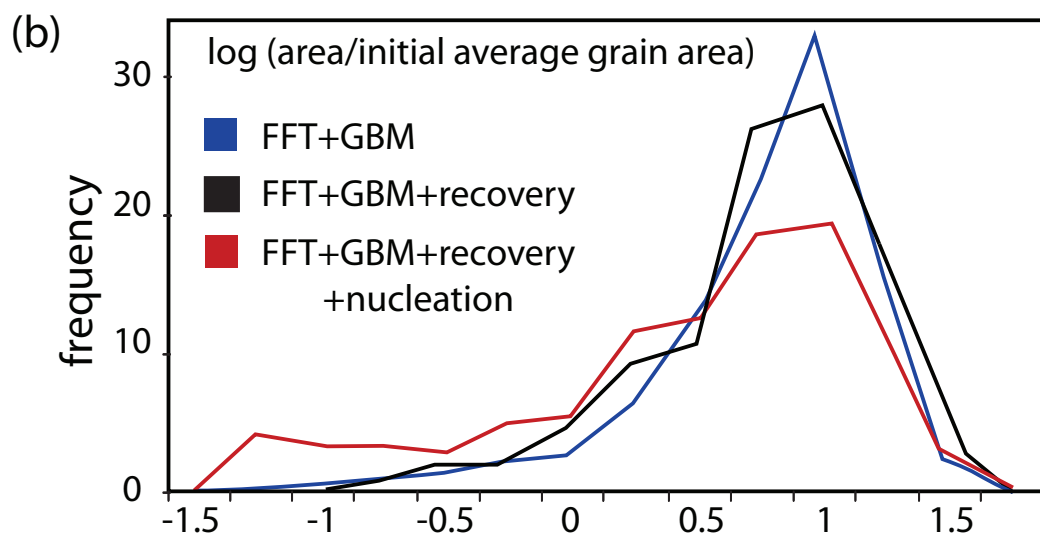
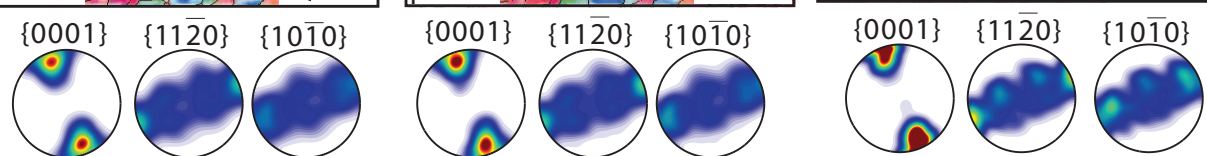
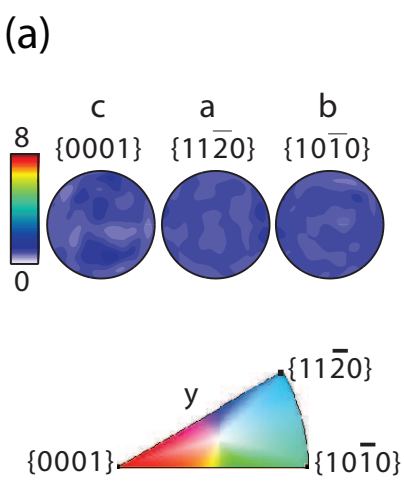
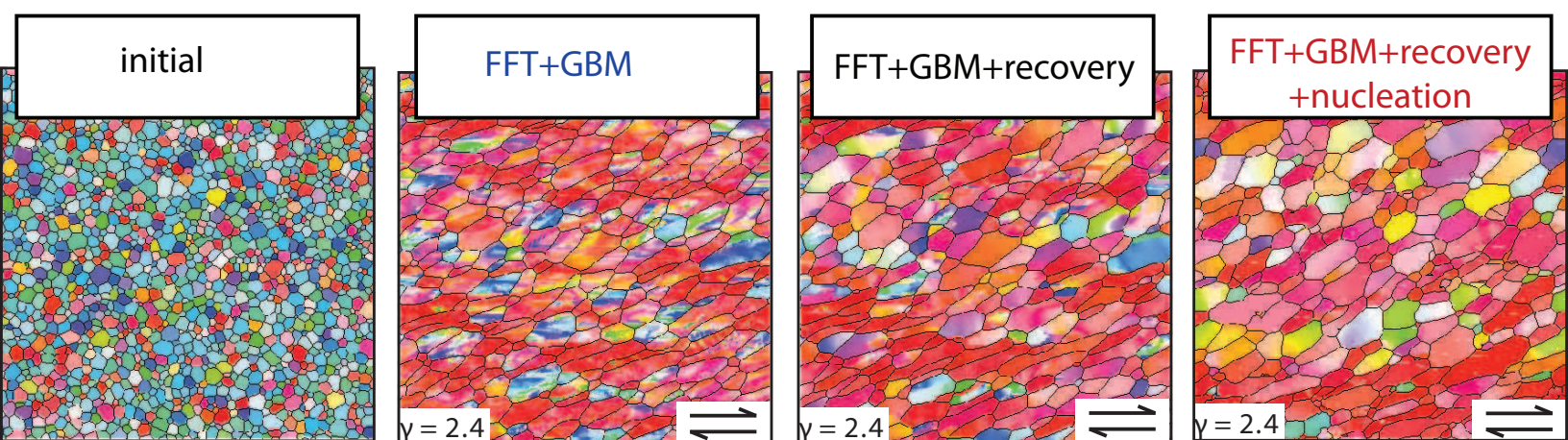


Figure 5

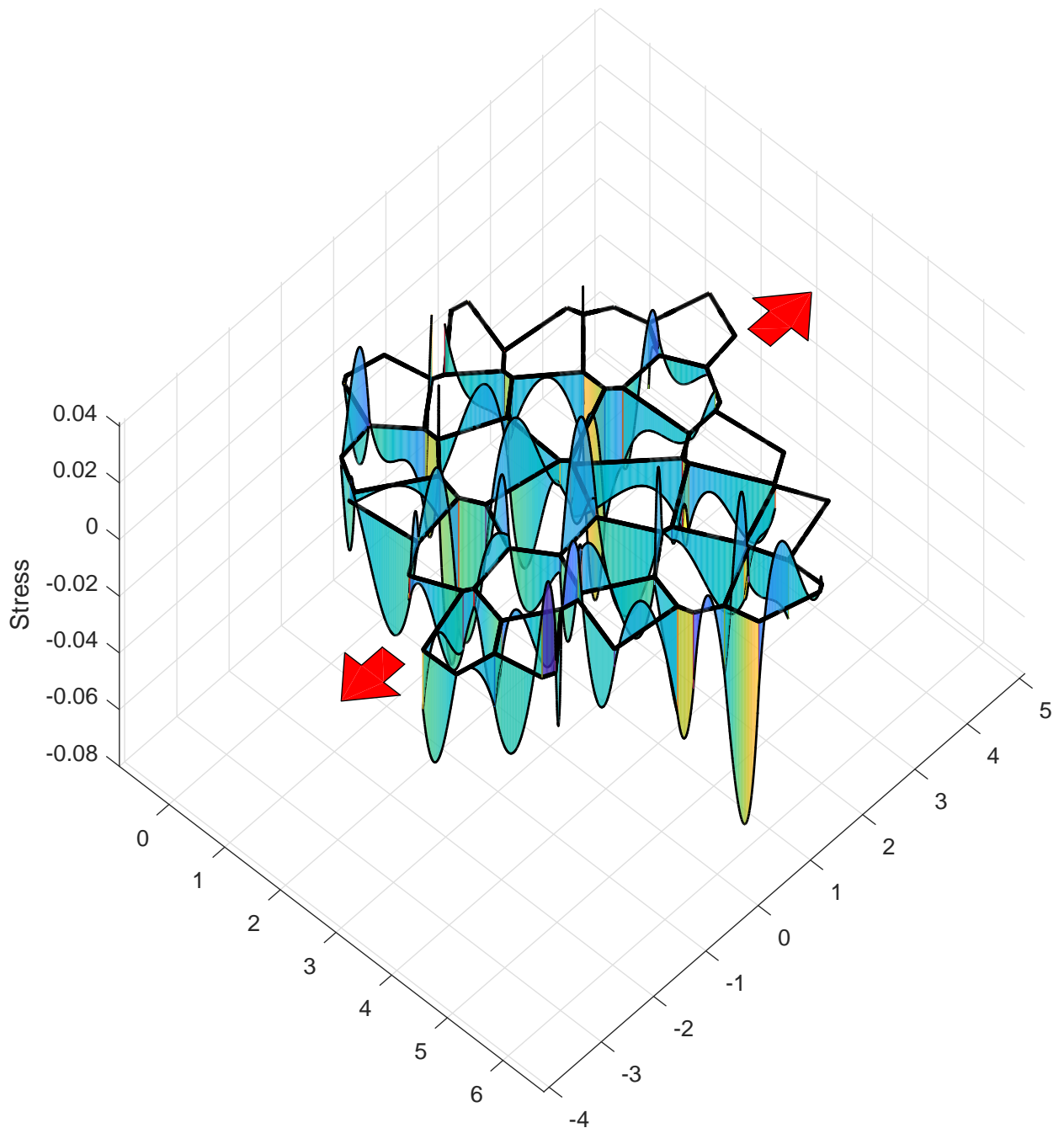




Figure 6

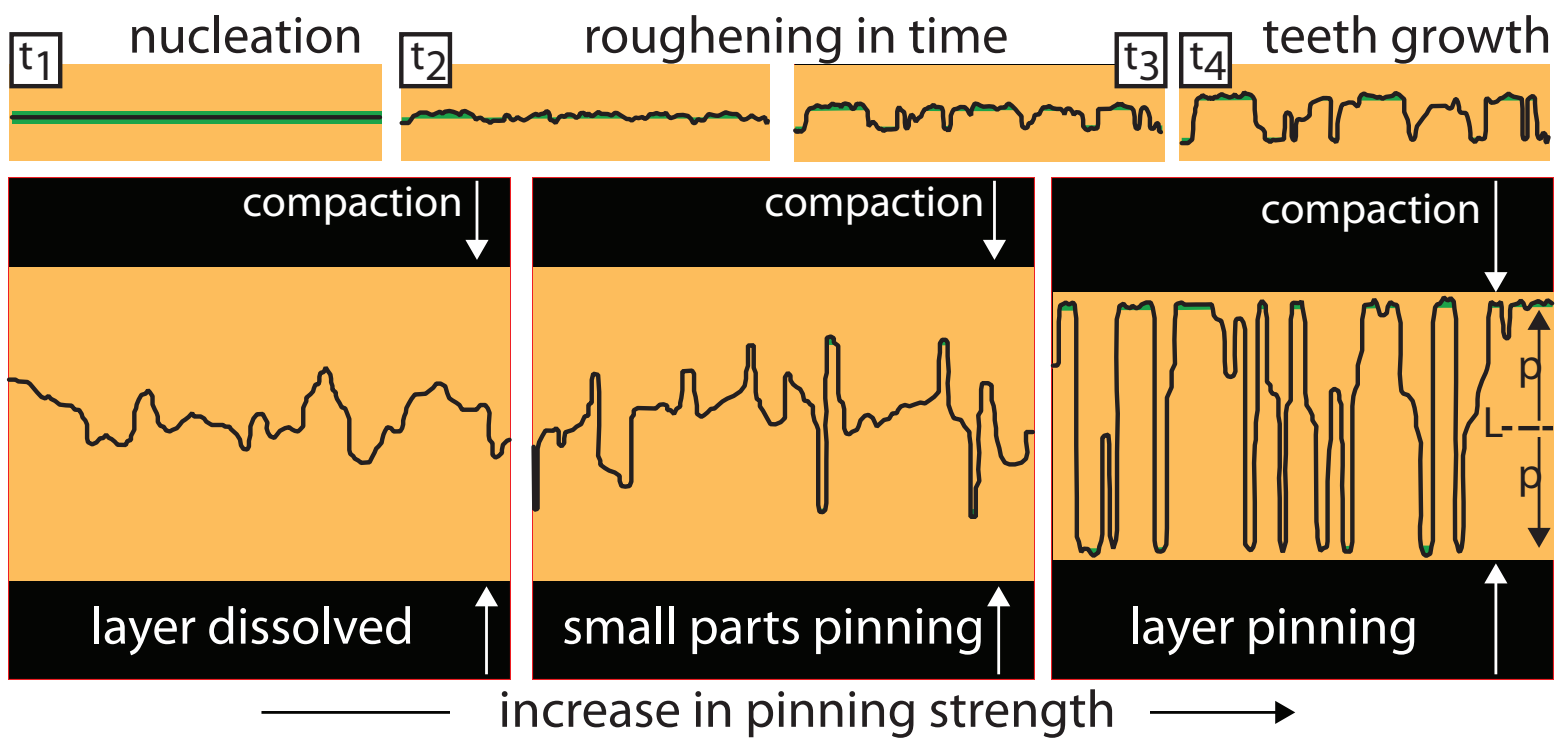


Figure 7

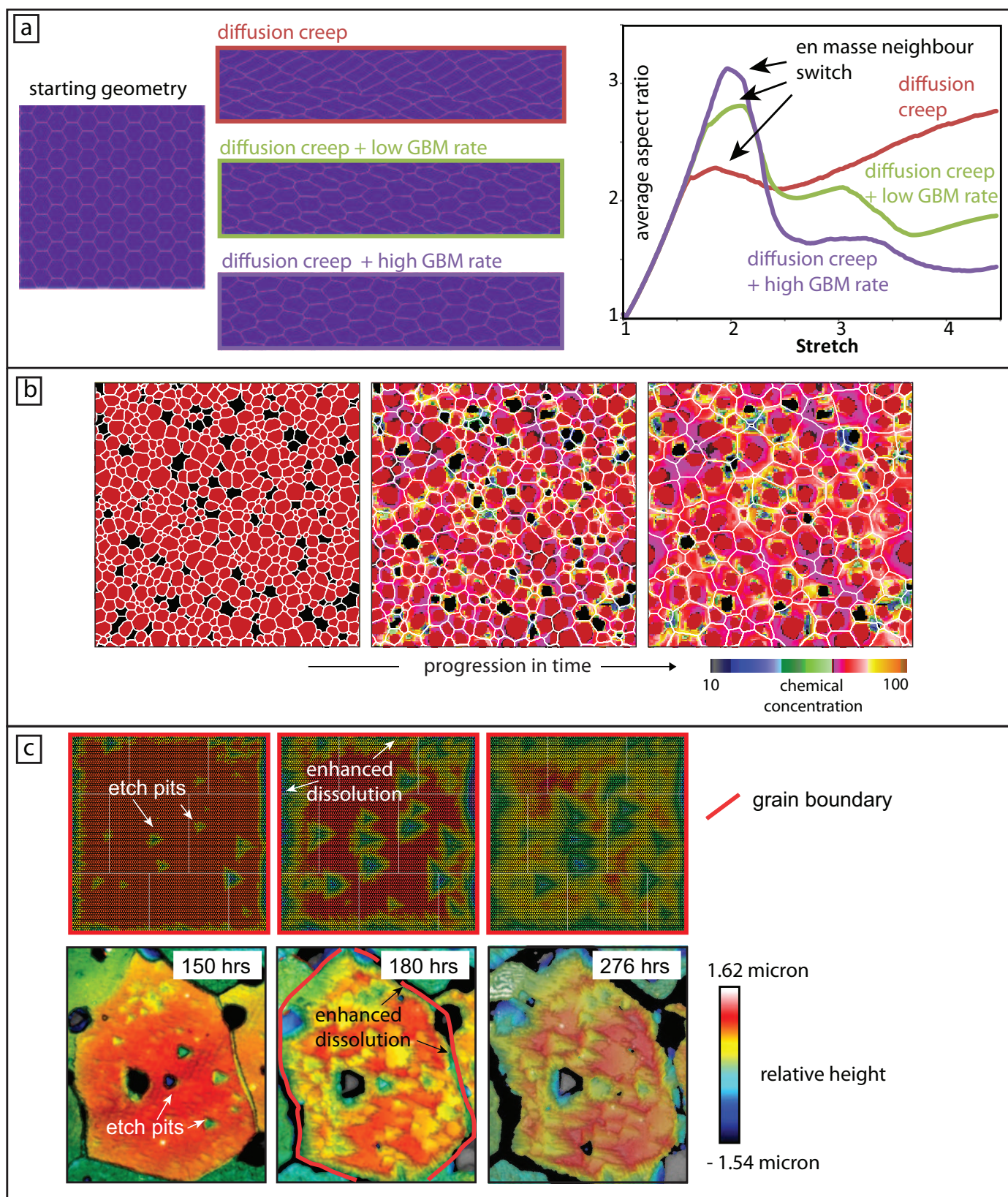


Table 1

| (micro) structure/process  |    | Numerical Method                           | References  |
|--|----|--|---|
| <b>General papers – review, overview, state of the art</b>   |    |  | Jessell et al., 2001; Jessell and Bons, 2002; Bons et al., 2008; Piazzolo et al., 2010  |
| <b>Single processes</b>  |    |  |   |
| Crystal lattice rotation for single phase  | D  | slip, rotation, translation                | Etchecopar, 1977  |
|  | D  | TBH  | Lister and Paterson, 1979   |
|  | D  | VPFFT                                      | Lebensohn et al., 2001; Montagnat et al., 2011, 2014  |
|  | D  | VPFFT – 3D                                 | Roters et al., 2012; Eisenlohr et al., 2013   |
| Grain rotations for two phases   | D  | VPFFT                                      | Griera et al., 2011, 2013, 2015; Ran et al., 2018   |
| Elasto-viscoplastic behaviour  | D  | EVPFFT                                     | Lebensohn et al., 2012  |
|  | D  | lattice spring model + viscous deformation | Sachau and Koehn, 2010, 2012; Arslan et al., 2012; Koehn and Sachau, 2014   |
| Diffusion creep for single phase   | D  | front tracking                             | Ford et al., 2002, 2004; Wheeler, 2009, 2010  |
| Diffusion creep for two phases   | D  | front tracking                             | Berton et al., 2006, 2011; Ford and Wheeler, 2004   |
| Dissolution and/or precipitation   | D  | lattice spring model                       | Koehn et al., 2006, 2007, 2012, 2016; Ebner et al., 2009,   |
| Dissolution at reactive surfaces   | D  | front tracking<br>front tracking - 3D      | Godinho et al., 2014<br>Piazzolo et al. this contribution   |
| Grain growth   |    |  |   |
| – isotropic surface energy   | S  | front tracking                             | Roessiger et al. 2011, 2014   |
| – isotropic surface energy   | SD | front tracking (triple points only)        | Bons & Urai, 1992   |
| – anisotropic surface energy   | S  | front tracking                             | Becker et al., 2008; Piazzolo et al., 2016  |
| – growth with pinning  | S  | front tracking                             | Kelka et al., 2015  |
| – isotropic grain growth   | SD | Potts – 2 & 3D                             | Kim et al. 2006; Krill and Chen, 2002   |
| – two phases   | S  | front tracking                             | Roessiger et al., 2014  |
| Growth of crystals into a crack or in strain fringes   | SD | phase field – 3D                           | Ankit et al., 2015  |
|  | D  | front tracking                             | Hilgers et al. ,1997; Koehn et al., 2003  |
| Strain-induced grain boundary migration  | S  | front tracking                             | Becker et al., 2008; Roessiger et al., 2011   |
| Single phase ductile deformation without reXX + crystallography  | D  | FEM – thin sheet model                     | Barr and Houseman, 1992, 1996   |
| Grain boundary diffusion   | S  | finite difference                          | Park et al., 2004   |
| Strain/stress localisation   | D  | VPFFT                                      | Griera et al. , 2011, 2015; Ran et al., 2018  |
| <b>Combination of Processes</b>  |    |  |   |
| Intracrystalline recovery and subgrain rotation  | S  | Potts-like                                 | Borthwick et al., 2013  |
| Ductile deformation and grain growth - polyphase   | D  | FEM + front tracking                       | Groome et al., 2006; Smith et al., 2015   |
| Dynamic reXX:  | D  | TBH+Potts-like                             | Jessell, 1988a, b; Jessell & Lister, 1990   |
| - crystal plastic deformation, grain boundary migration, rotation recrystallization  |    |  |   |
| - grain growth and polygonisation  | S  | front tracking                             | Roessiger et al., 2011  |
| - crystal plastic deformation, grain boundary migration, rotational recrystallization, nucleation, recovery                | D  | TBH+FEM+front tracking                     | Piazzolo et al., 2002   |
| - crystal plastic deformation, grain boundary migration, rotational recrystallization, recovery                            | D  | VPFFT+ front tracking                      | Jansen et al., 2016; Llorens et al., 2016a, b, 2017; Steinbach et al., 2016, 2017; Gomez-Rivas et al., 2017; Piazzolo et al., this contribution |
| Strain/stress localization<br>- linked to grain size variations & rheology<br>- linked to grain size variations & rheology | D  | FEM+front tracking                         | Jessell et al., 2005; Gardner et al., 2017  |
|  | D  | FEM  | Cross et al., 2015  |
| Diffusion and grain growth   | D  | front tracking                             | Piazzolo et al. this contribution   |
| Trace element diffusion and reXX   | S  | front tracking                             | Piazzolo et al. this contribution   |