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A Pathway to the Athermal Impact Initiation of Energetic Azides

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S1. Crystal Structures and Structural Optimisation Criteria

All input structures were taken from the ICSD database, or published literature data, as indicated in Table S1. For calculations performed in CASTEP v16,¹ structures were converged using the GGA functional of Perdew-Burke-Ernzerhof (PBE)² with dispersion corrections as outlined in Table S1. Dispersion corrections were chosen so as to yield reasonable unit cell parameters on structural relaxation, Section S1.2. The OOPBE_OP norm-conserving pseudopotentials were used with a plane wave kinetic energy cut-off of 1800 eV. The optgg1 pseudopotential was used for the Zn atom. The electronic structure was sampled on a Monkhorst-Pack (MP) grid,³ with k -point spacing of no more than 0.05 Å⁻¹, Table S1. Geometry optimisation convergence criteria are given in Table S1. Note that tighter convergence was required to ensure no imaginary phonons in the Zn material. Fully optimised structures were used for subsequent phonon calculations using density-functional perturbation theory (DFPT). For structures studied by QuantumEspresso v6.1,⁴ geometry convergence was accepted with SCF convergence of 1.0x10⁻¹³ Ry, a total energy convergence of 1x10⁻¹⁰ Ry. Norm-conserving pseudopotentials were used in each case.

Table S1: Computational Parameters for the azides studied in this work. The ICSD code, exchange-correlation functional (XC), quantum chemistry code (CASTEP, C; Quantum Espresso, Q), convergence criteria and kinetic energy cut-off (ECut) are all given.

| Azide | ICSD Code | Code | XC | D(Energy) / eV.atom ⁻¹ | Max. Force / eV Å ⁻¹ | Max. Atomic Disp. / Å | Max. Stress GPa | ECut / eV |
|----------------------------------|-----------|------|----------|-----------------------------------|---------------------------------|-----------------------|--------------------|-----------|
| NaN ₃ | 29370 | C | PBE + D2 | 2x10 ⁻⁶ | 0.001 | 0.001 | 0.005 | 1800 |
| NH ₄ N ₃ | 2236 | C | PBE + TS | 2x10 ⁻⁶ | 0.001 | 0.001 | 0.005 | 1800 |
| TAGZ | Ref 5 | C | PBE + TS | 2x10 ⁻⁶ | 0.001 | 0.001 | 0.005 | 1800 |
| HN ₃ | 261955 | C | PBE + TS | 2x10 ⁻⁶ | 0.001 | 0.001 | 0.005 | 1800 |
| LiN ₃ | 34675 | Q | rVV10 | 1x10 ⁻⁹ | 1.0 x 10 ⁻⁸ Ry/Bohr | 0.0001 | 5x10 ⁻⁶ | 2312.9 |
| Zn(N ₃) ₂ | 430428 | C | PBE+D2 | 2x10 ⁻⁹ | 0.0005 | 0.0005 | 0.0005 | 1800 |
| Ba(N ₃) ₂ | 26202 | Q | rVV10 | 1x10 ⁻⁹ | 1.0 x 10 ⁻⁸ Ry/Bohr | 0.0001 | 5x10 ⁻⁶ | 2312.9 |
| AgN ₃ | 88335 | Q | rVV10 | 1x10 ⁻⁹ | 1.0 x 10 ⁻⁸ Ry/Bohr | 0.0001 | 5x10 ⁻⁶ | 1768.7 |
| Sn(N ₃) ₂ | 433812 | C | PBE + TS | 2x10 ⁻⁶ | 0.001 | 0.001 | 0.005 | 1800 |

S1.2 Comparison of Optimised Structures with Experimental Structures

As a means to benchmark the models used in this work, all computed structures are compared with experimental crystal structures. In all cases the optimised unit cell was found to adhere very closely to experimentally determined structures.

Table S1.2.1: Comparison of experimental (exp) and computed (calc) unit cell geometries.

| Azide | a | b | c | α | β | γ | V | % Vol deviation |
|---------------------------------------|-------|-------|-------|--------|--------|-------|---------|-----------------|
| AgN ₃ (Exp) | 5.600 | 5.980 | 5.998 | 90.0 | 90.0 | 90.0 | 200.861 | +1.5 |
| AgN ₃ (Calc) | 5.717 | 5.962 | 5.979 | 90.0 | 90.0 | 90.0 | 203.815 | |
| BaN ₃ (Exp) | 9.59 | 4.39 | 5.42 | 90.0 | 99.75 | 90.0 | 224.89 | +6 |
| BaN ₃ (Calc) | 9.826 | 4.444 | 5.532 | 90.0 | 99.14 | 90.0 | 238.50 | |
| HN ₃ (Exp) | 8.208 | 8.208 | 6.782 | 110.42 | 110.42 | 90.01 | 397.42 | +5.9 |
| HN ₃ (Calc) | 8.381 | 8.381 | 6.903 | 110.51 | 110.51 | 90.00 | 421.18 | |
| NH ₄ N ₃ (Exp) | 8.933 | 3.808 | 8.661 | 90.0 | 90.0 | 90.0 | 294.62 | +0.1 |
| NH ₄ N ₃ (Calc) | 9.016 | 3.814 | 8.573 | 90.0 | 90.0 | 90.0 | 294.80 | |

| | | | | | | | | |
|-------------------------|-------|--------|-------|------|-------|------|--------|--------|
| LiN ₃ (Exp) | 5.627 | 3.319 | 4.979 | 90.0 | 107.4 | 90.0 | 88.73 | -0.005 |
| LiN ₃ (Calc) | 5.594 | 3.323 | 4.911 | 90.0 | 104.8 | 90.0 | 88.25 | |
| NaN ₃ (Exp) | 6.223 | 3.651 | 5.336 | 90.0 | 108.4 | 90.0 | 115.04 | -0.4 |
| NaN ₃ (Calc) | 6.161 | 3.685 | 5.198 | 90.0 | 103.8 | 90.0 | 114.61 | |
| SnN ₃ (Exp) | 6.776 | 11.059 | 6.232 | 90.0 | 94.67 | 90.0 | 465.51 | +0.7 |
| SnN ₃ (Calc) | 6.686 | 11.795 | 5.948 | 90.0 | 91.41 | 90.0 | 468.93 | |
| ZnN ₃ (Exp) | 3.459 | 16.261 | 6.931 | 90.0 | 95.9 | 90.0 | 387.8 | +1.5 |
| ZnN ₃ (Calc) | 3.439 | 16.469 | 6.997 | 90.0 | 96.42 | 90.0 | 393.84 | |

S2. CRYSTAL17 Computational Methods

The electronic structures of the crystalline azides were calculated using CRYSTAL17.⁶ For all azides, the nitrogen atoms were treated using a triple-zeta with polarised valence basis set.⁷ SCF convergence tolerance of 1×10^{-7} Ha was used, with integral tolerances of 7, 7, 7, 19, and 30 and TOLDEE 8. The basis sets, shrinking factors and resulting number of K-points are given in Table S2.1.

Table S2.1: Calculation parameters for CRYSTAL17 electronic structure calculations.

| Azide | Counter Ion BS | SHRINK | Num K-point in IBZ |
|----------------------------------|---|--------|--------------------|
| NaN ₃ | TZVP ⁷ | 20 | 4004 |
| NH ₄ N ₃ | 5-11G* ⁸ | 20 | 1331 |
| TAGZ | C: 6-31d1G ⁹ H: TZVP ⁷ | 15 | 1688 |
| LiN ₃ | TZVP ⁷ | 20 | 4004 |
| HN ₃ | 5-11G* ⁸ | 15 | 1688 |
| Zn(N ₃) ₂ | 4-11d31G ¹⁰ | 20 | 2222 |
| Ba(N ₃) ₂ | HAYWSC-3111(2d)G ¹¹ | 20 | 2222 |
| AgN ₃ | From Ref ¹² | 10 | 504 |
| Sn(N ₃) ₂ | DURAND-21G* ¹³ | 20 | 2222 |

S3. Phonon Calculations

S3.1 Phonon Calculation Parameters

Phonon calculations were performed using the same computational packages as outlined in Section S1. For structures calculated using the CASTEP v16.1 code, dynamical matrices were initially calculated on a coarse grid (q -point spacing $< 0.05 \text{ \AA}^{-1}$), and subsequently Fourier interpolated onto a finer grid. The acoustic sum rule was implemented in reciprocal space. For those calculated within the QuantumESPRESSO v 6.1 package, phonon calculations were performed using the ph.x routine, within the framework of DFPT. Self-consistency threshold of 1×10^{-15} Ry was used in each case. The ‘crystal’ acoustic sum rule was implemented. A coarse grid was selected with q -point spacing $< 0.05 \text{ \AA}^{-1}$ in each case, followed by interpolation along a dense set of q -points along all high symmetry paths in the Brillouin zone, as suggested by Ref ¹⁴, to no less than a total of 50 q -pts. For a few cases, considerably larger q -point integrations were attempted, and found to make negligible difference to the integration of $\Omega^{(2)}(\omega_{\tau})$, Section S8.1.

S3.2 Phonon Dispersion Curves

The phonon dispersion curves were generated for each of the azide materials, Figures S3.2.1-S3.2.9. In each case, the target frequencies (main text, Table 3) are found to be relatively flat, while the phonon bands show large dispersion.

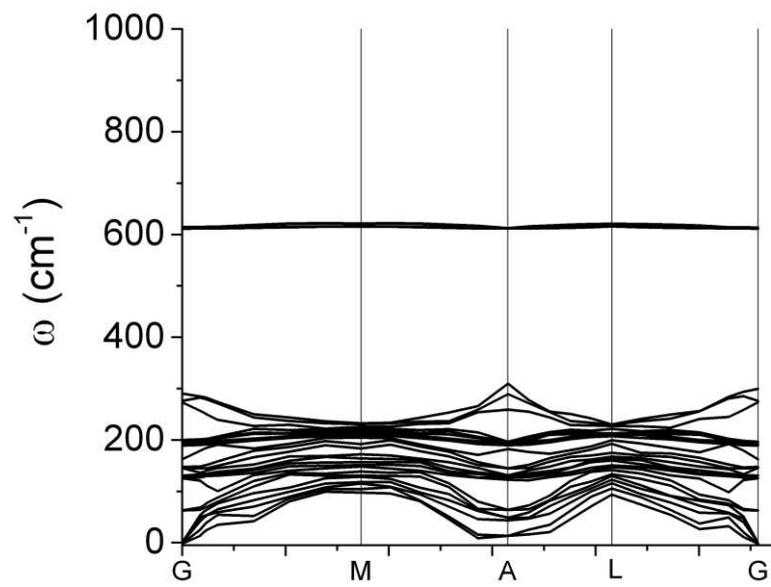


Fig S3.2.1: Phonon dispersion curve along the high symmetry paths for NaN₃.

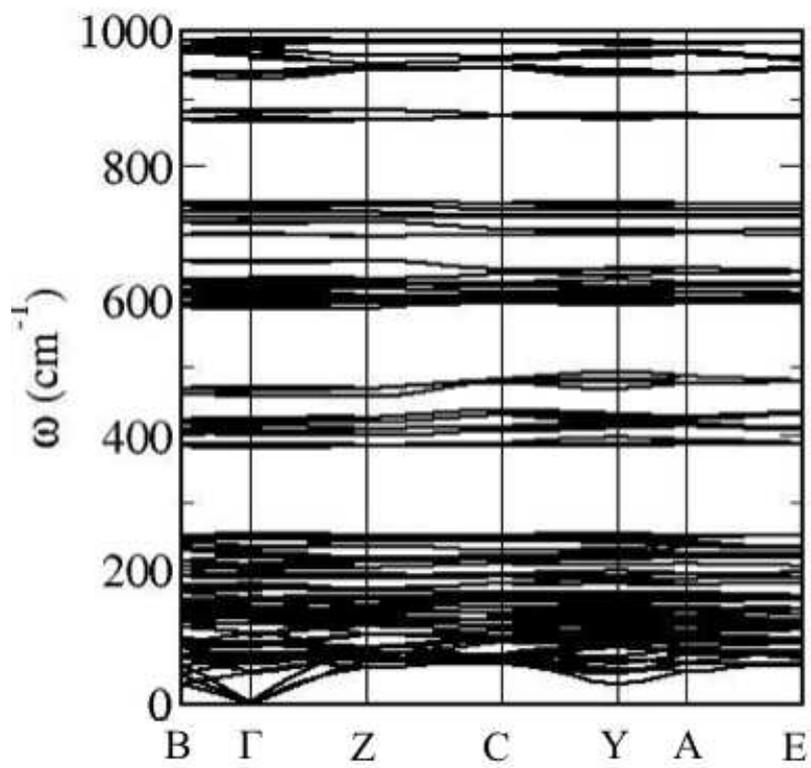


Fig S3.2.2: Phonon dispersion curve along the high symmetry paths for TAGZ.

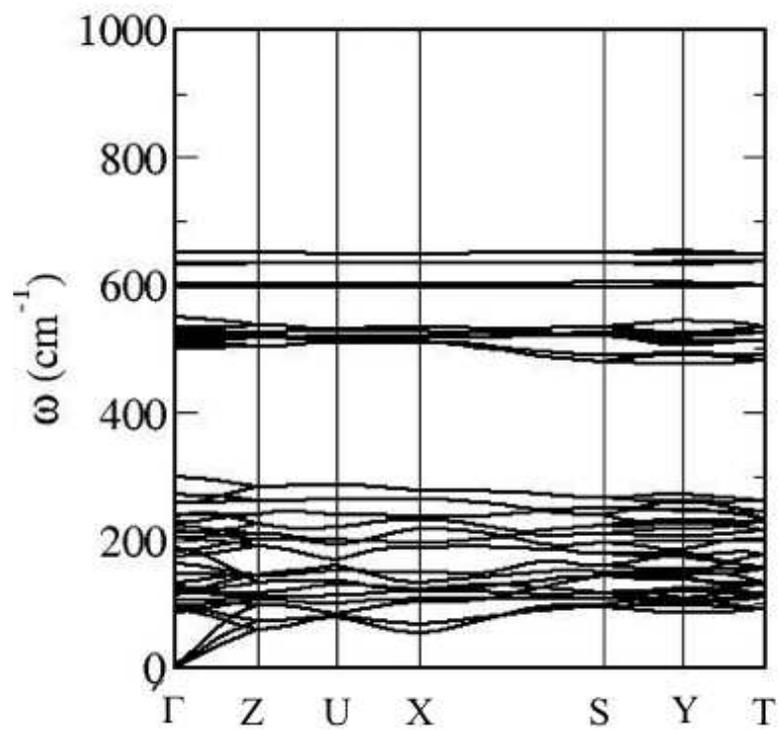


Fig S3.2.3: Phonon dispersion curve along the high symmetry paths for NH_4N_3 .

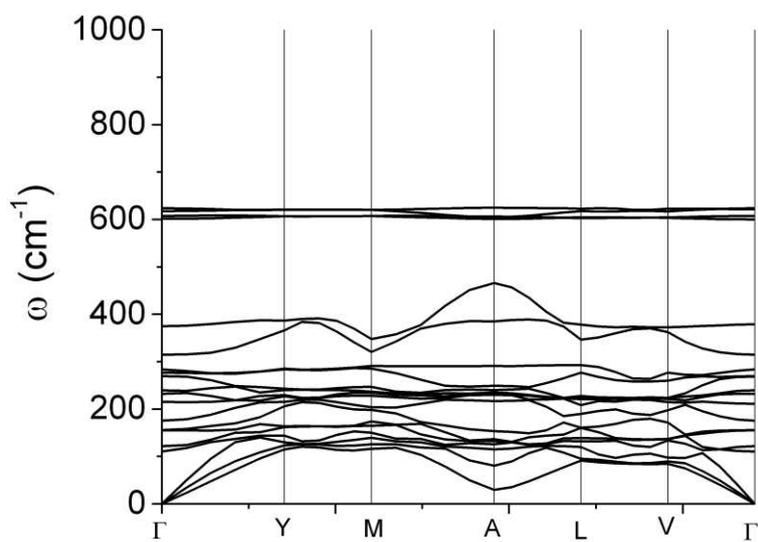


Fig S3.2.4: Phonon dispersion curve along the high symmetry paths for LiN_3 .

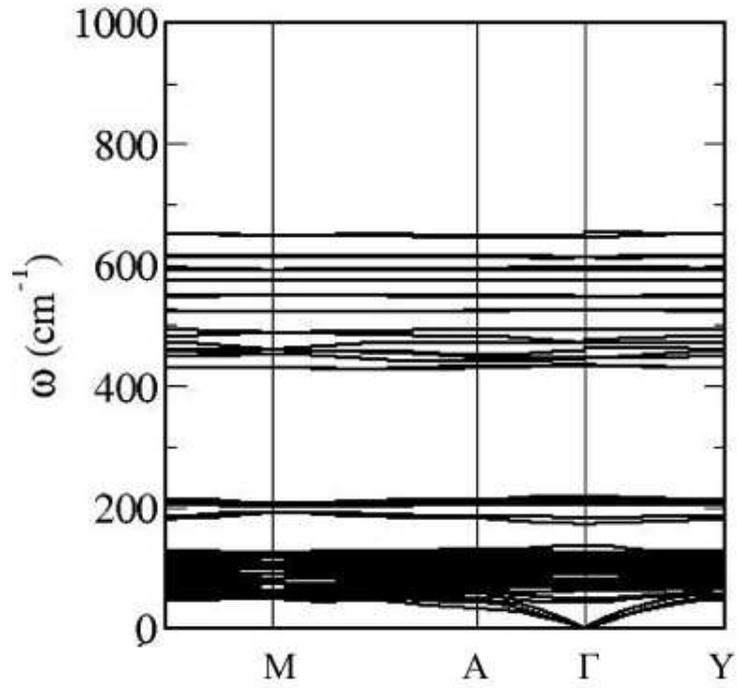


Fig S3.2.5: Phonon dispersion curve along the high symmetry paths for HN₃

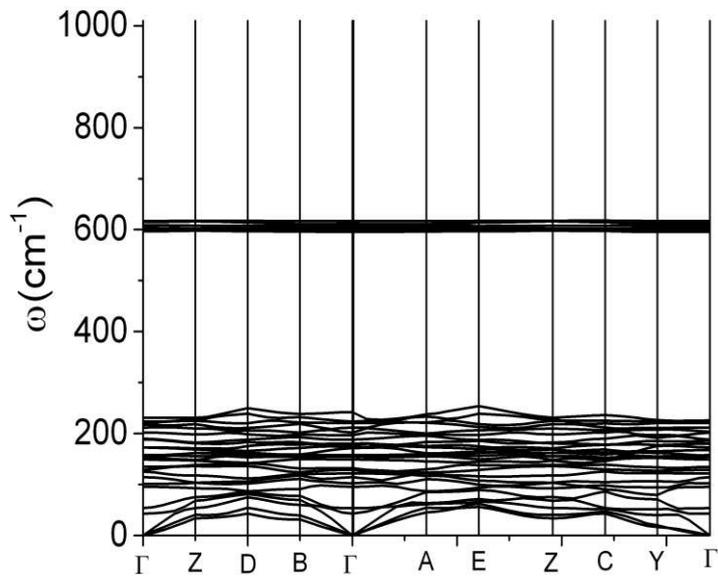


Fig S3.2.6: Phonon dispersion curve along the high symmetry paths for Ba(N₃)₂.

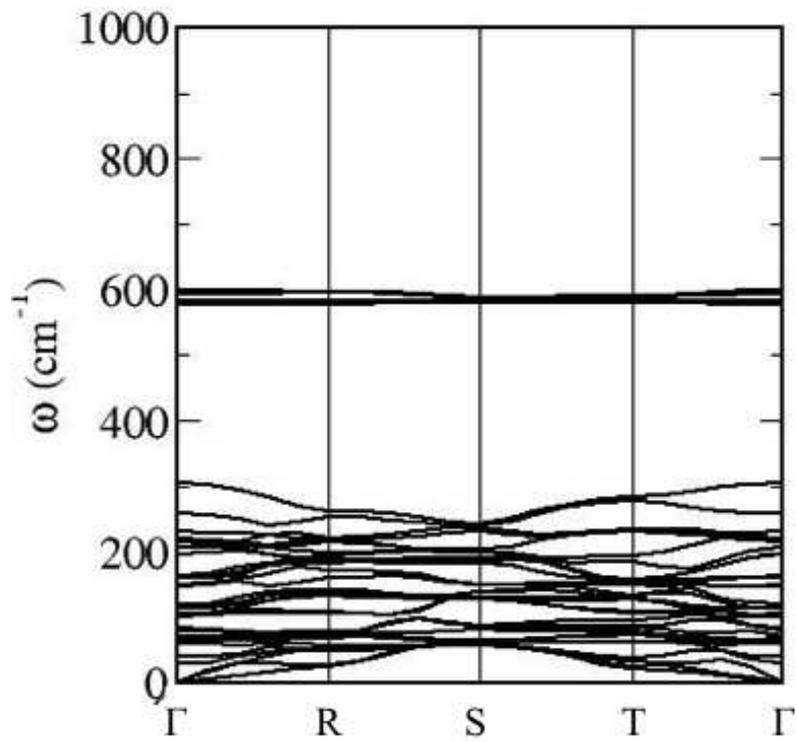


Fig S3.2.7: Phonon dispersion curve along the high symmetry paths for AgN₃.

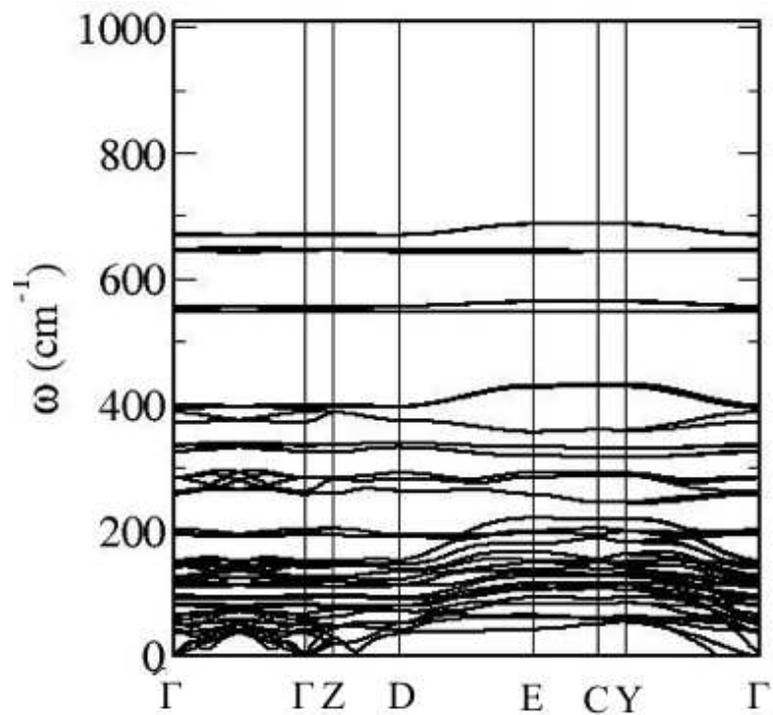


Fig S3.2.8: Phonon dispersion curve along the high symmetry paths for Zn(N₃)₂.

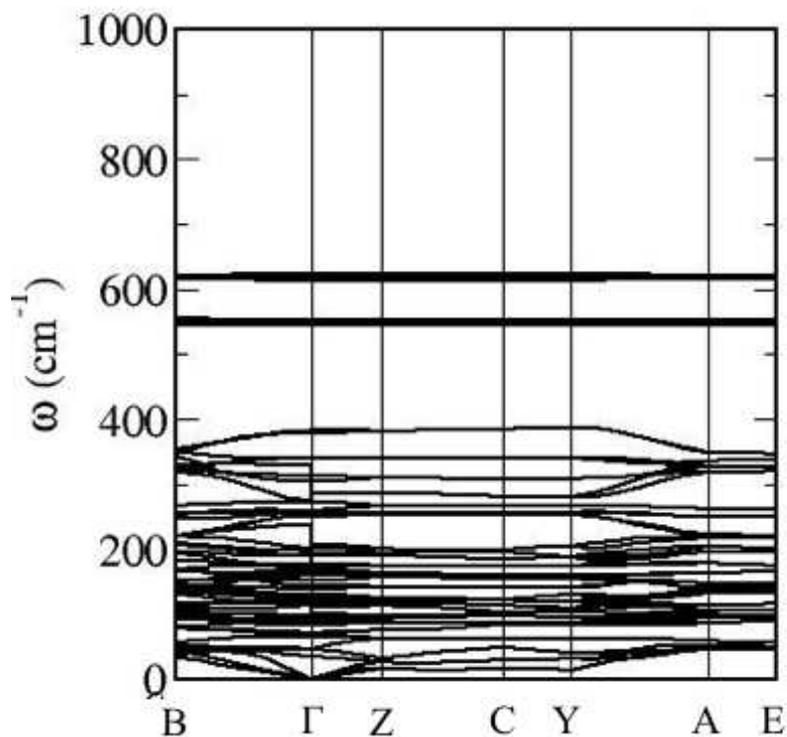


Fig S3.2.9: Phonon dispersion curve along the high symmetry paths for $\text{Sn}(\text{N}_3)_2$.

S4. Analysis of N-N interactions in Crystalline Azides (COHP)

Analysis of the projected Crystal overlap Hamilton population (pCOHP) was used to investigate the electronic structure of the N_3^- anion within the crystalline azides. In the main text, analysis is performed for the NaN_3 and AgN_3 systems: a non-covalent and a covalent system, respectively. Similar analysis has been performed for the remaining azides, which are presented in Figs S4.1-S4.6. The crystal canonical orbitals are also analysed in order to assist in the interpretation of the pCOHP. All pCOHP were generated using the HSE06 functional with minimum STO-3G basis set. Note that no suitable minimum basis set was available for $\text{Ba}(\text{N}_3)_2$, and its analysis is therefore not presented here.

For the ionic azides (NaN_3 , TAGZ, NH_4N_3 , LiN_3) pCOHP analysis of the N_3^- molecule in the crystalline state shows the same electronic structure as presented for the isolated anion in the main text, Figure 2. As the interaction with the counter cation becomes increasingly covalent in nature, notable hybridisation of the N_3^- canonical orbitals is observed. This is generally accompanied by an increase in the anti-bonding character of the individual N...N bonds within the anion. However, we note that in each case, the underlying N_3^- orbitals can be generally recovered.

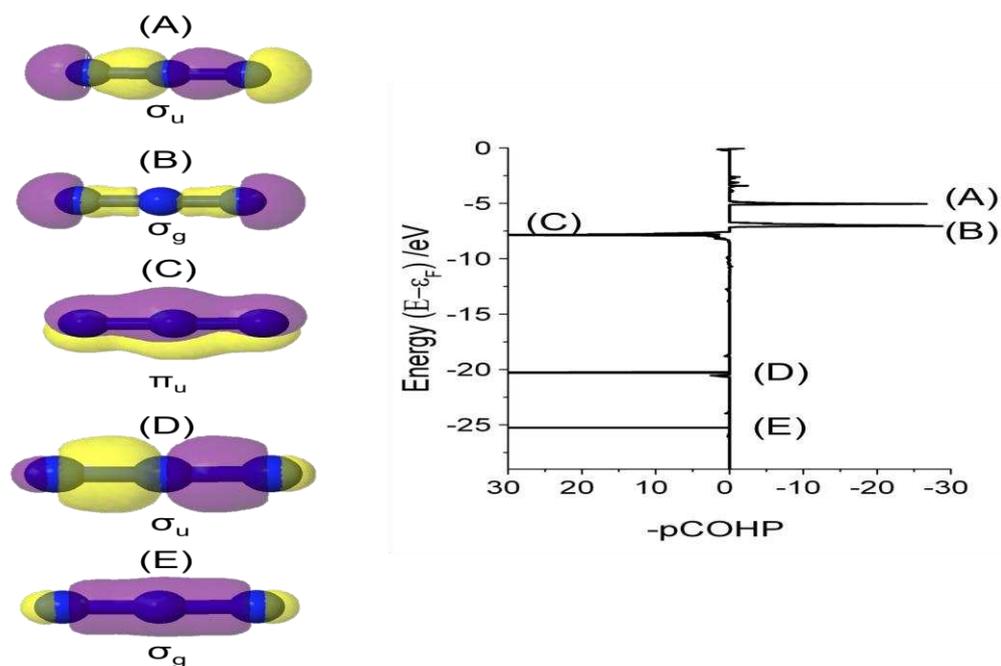


Fig S4.1: pCOHP analysis of N...N bonds in TAGZ. The crystalline canonical orbitals are given for selected features in the pCOHP curves. Symmetry classifications are based on the isolated N_3^- anion, and are used to reflect similarities with the pure anion.

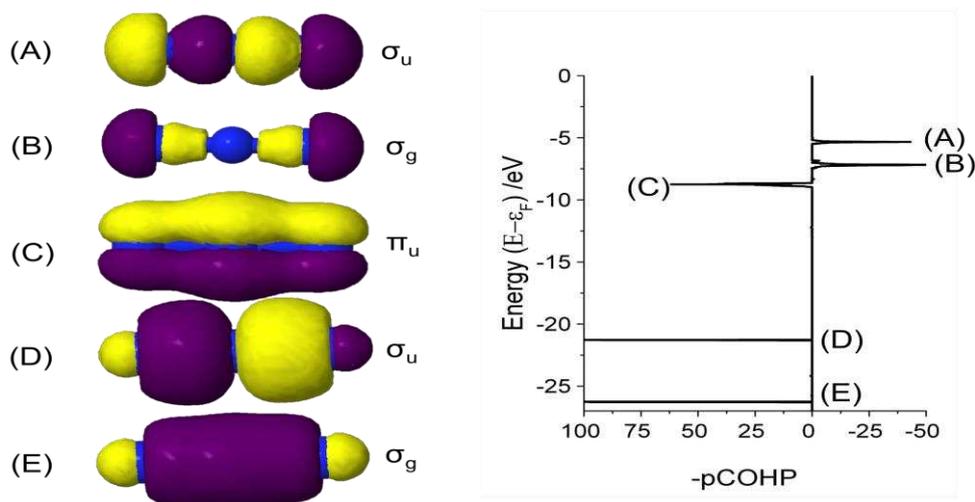


Fig S4.2: pCOHP analysis of N...N bonds in NH_4N_3 . The crystalline canonical orbitals are given for selected features in the pCOHP curves. Symmetry classifications are based on the isolated N_3^- anion, and are used to reflect similarities with the pure anion.

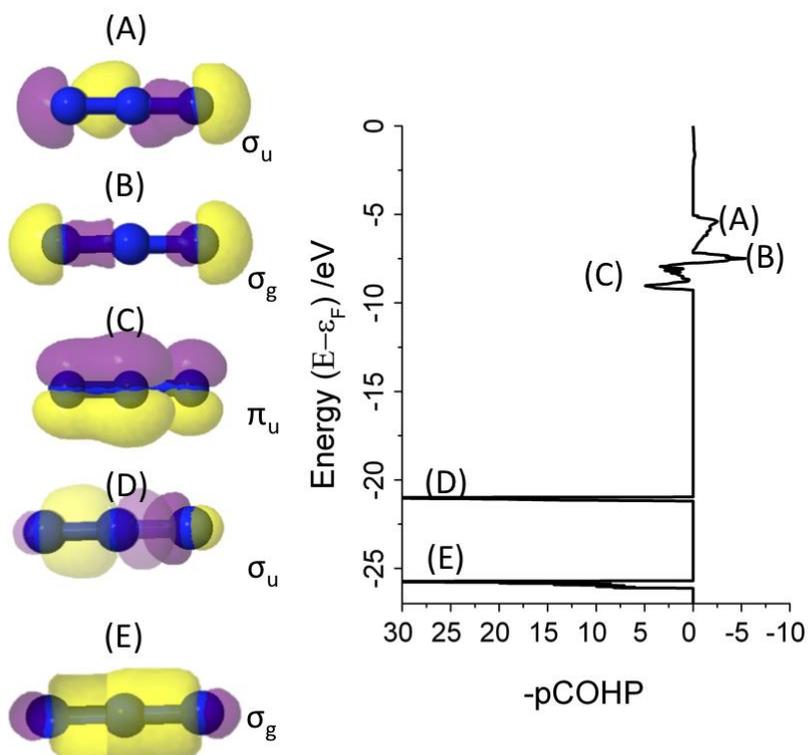


Fig S4.3: pCOHP analysis of N...N bonds in LiN₃. The crystalline canonical orbitals are given for selected features in the pCOHP curves. Symmetry classifications are based on the isolated N₃⁻ anion, and are used to reflect similarities with the pure anion. Note orbitals were manually reconstructed across the periodic boundaries.

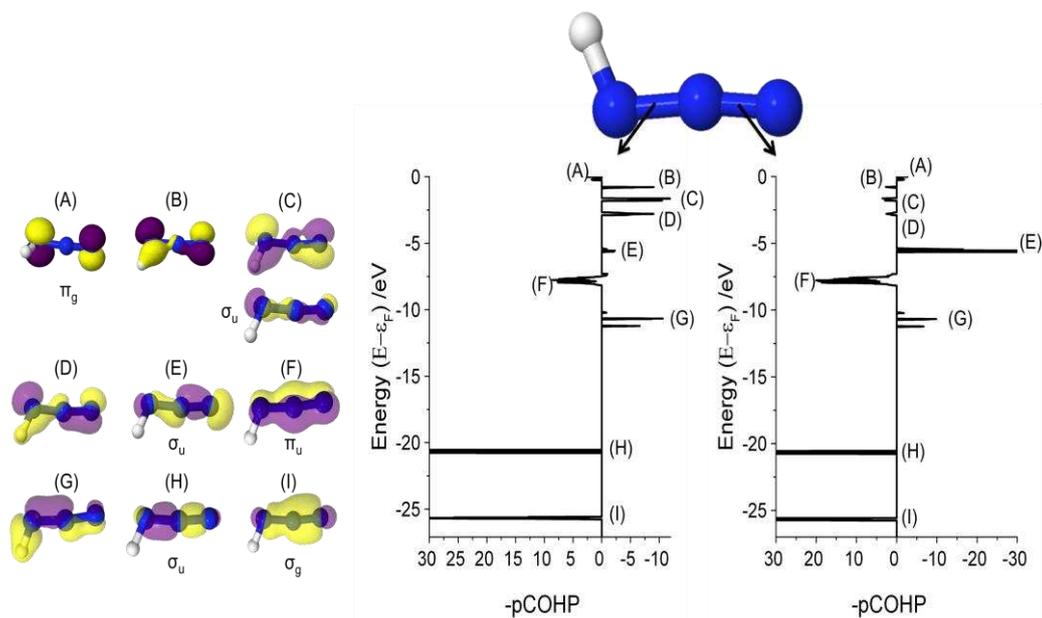


Fig S4.4: pCOHP analysis of HN₃ across the indicated N...N bonds. The crystalline canonical orbitals are given for selected features in the pCOHP curves. Symmetry classifications are based on the isolated N₃⁻ anion, and are used to reflect similarities with the pure anion.

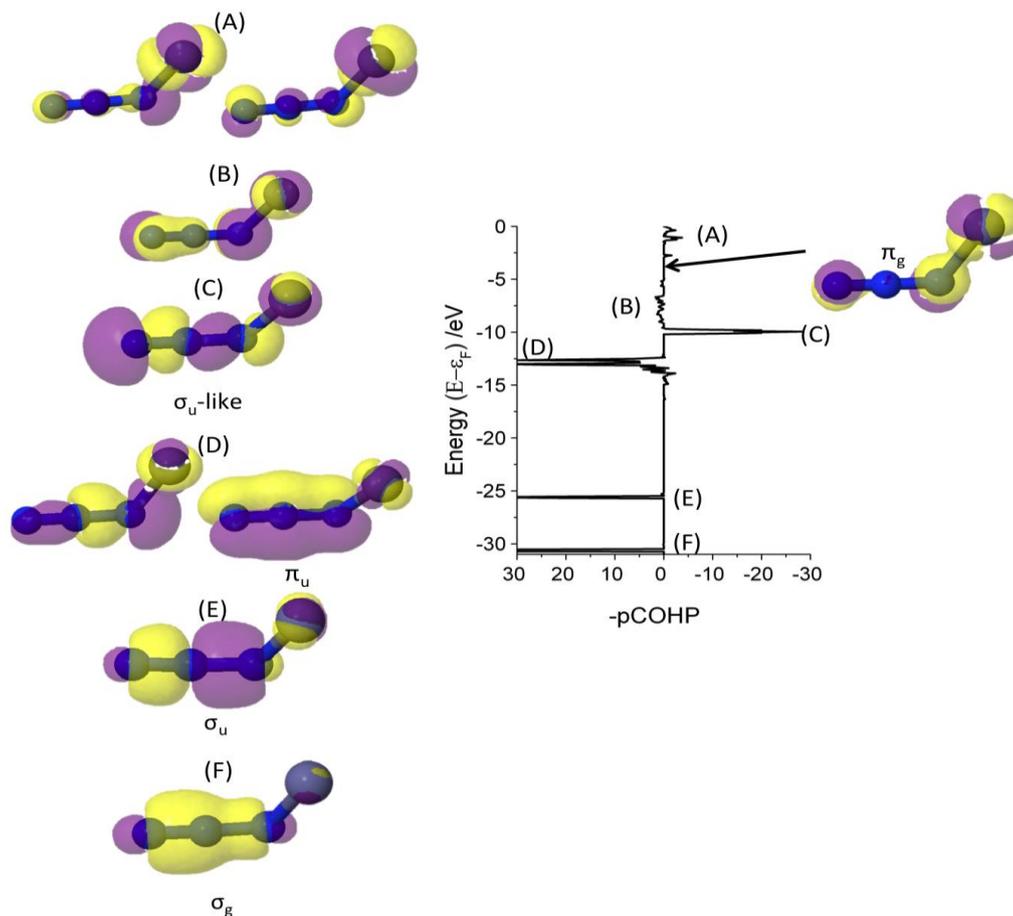


Fig S4.5: pCOHP analysis of N...N bonds in $\text{Zn}(\text{N}_3)_2$. Both N...N bonds exhibit the same major features. The crystalline canonical orbitals are given for selected features in the pCOHP curves. Symmetry classifications are based on the isolated N_3^- anion, and are used to reflect similarities with the pure anion.

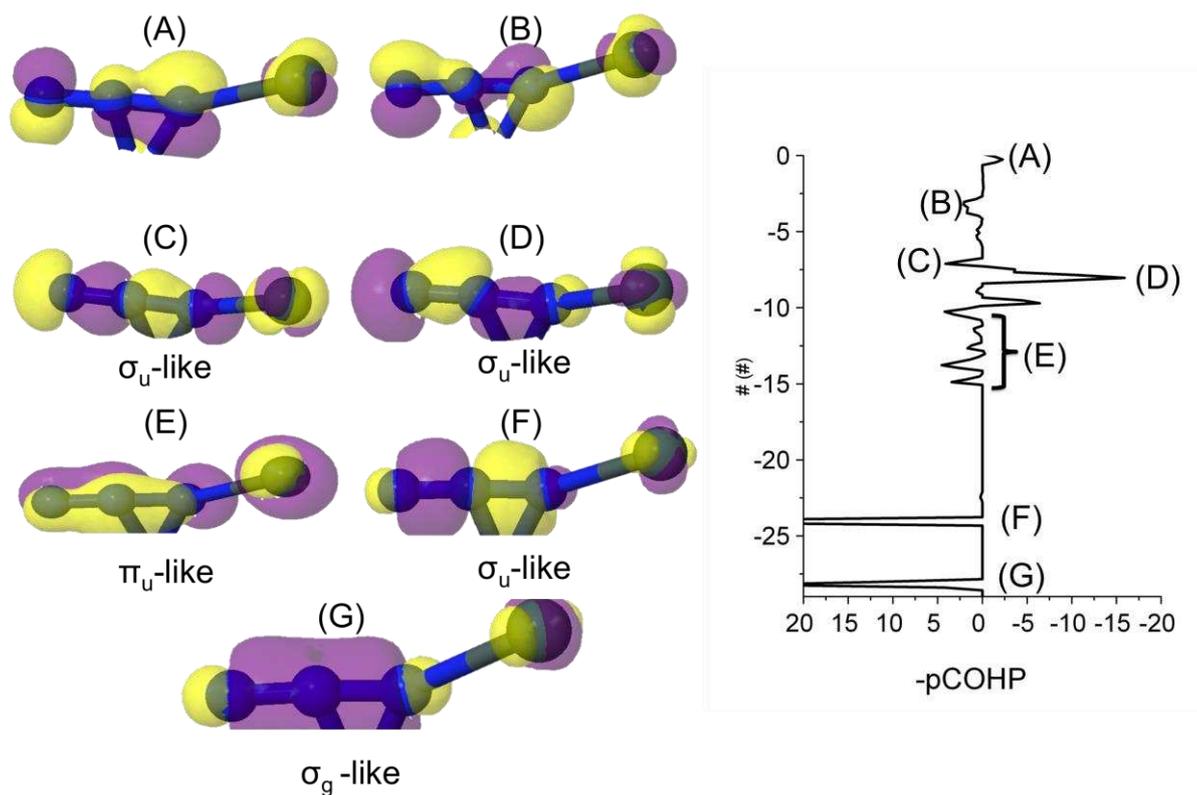


Fig S4.6: pCOHP analysis of N...N bonds in $\text{Sn}(\text{N}_3)_2$. The crystalline canonical orbitals are given for selected features in the pCOHP curves. Symmetry classifications are based on the isolated N_3^- anion, and are used to reflect similarities with the pure anion.

S5. Identification of Target Frequencies

In order to identify which vibrational modes correspond to θ_{NNN} , the Γ -point eigenvectors were analysed for each normal mode. Eigenvectors were artificially extended to accentuate internal molecular behaviour. The azide bend was therefore easily identified, Figure S5.1. Noting that the bands corresponding to each internal vibrational mode are relatively flat

means that the Γ -point frequencies are reliable indicators of the target frequency for each material.

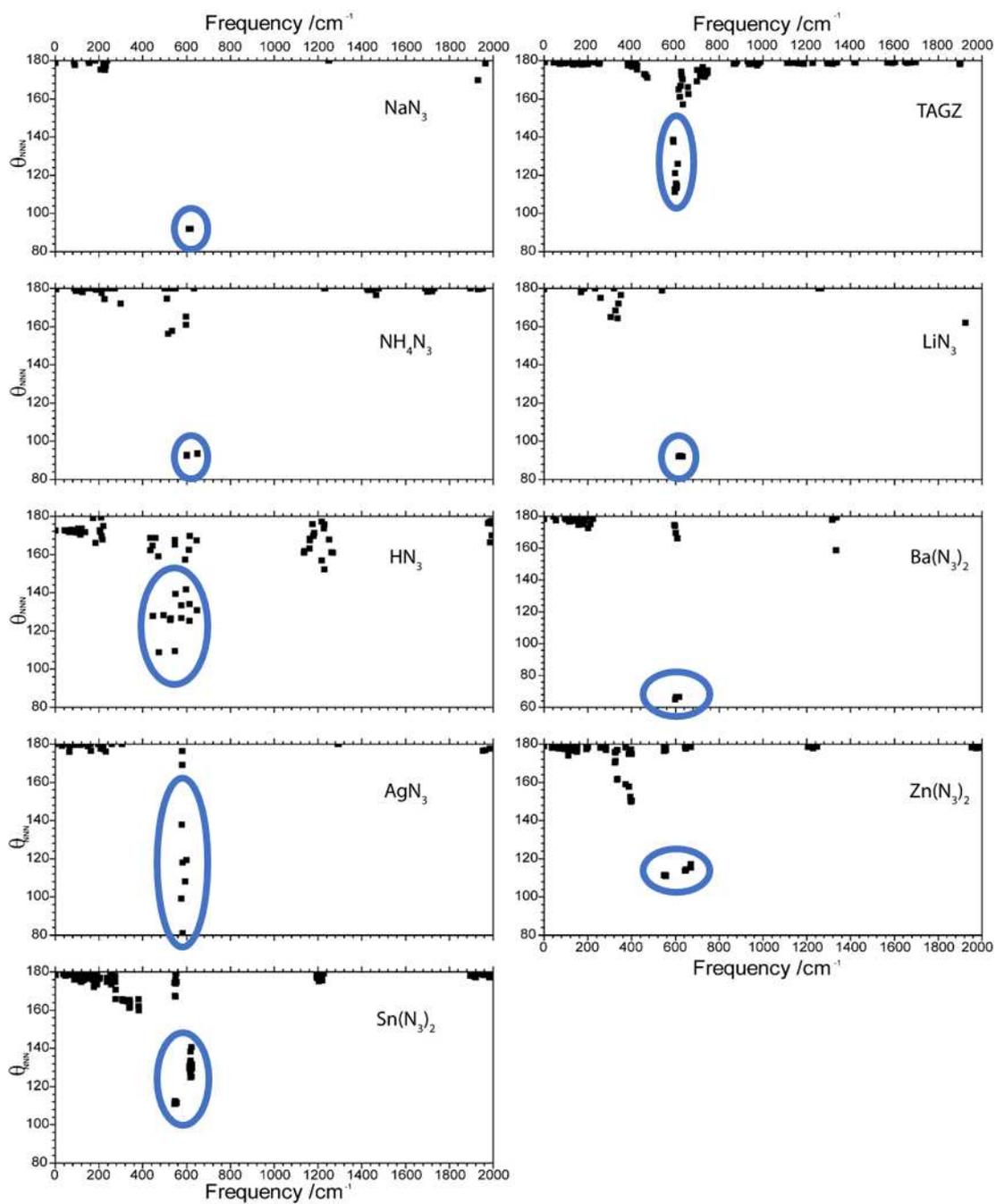


Fig S5.1: Identification of target modes. The azide bend at each Γ -point normal mode is highlighted.

S6. Anharmonic mode coupling in LiN_3 .

The solution of Equation 2 in the main text requires solution of the third-order scattering coefficients, $V^{(3)}$. This is, in principle, possible, and has been implemented in a variety of codes. Here, the D3Q code¹⁵ has been used, as implemented for QuantumESPRESSO v6.1. Given the constraints on unit cell size, we were not able to compute the third order coupling matrix elements for either of the molecular salt azides (TAGZ or NH_4N_3). However, these calculations have been performed for LiN_3 . We note that no single frequencies contained ‘cation-only’ modes, although Li-dominant modes were obvious (e.g. Γ -point *ca* 210, 220 and 230 cm^{-1}). Using the Γ -point vibrational mode at 602 cm^{-1} as the ‘target’ mode in the coupling calculations, the value of $V^{(3)}$ was calculated for zone-centre couplings, Table S6.1. We note that there remains some strong coupling (although rare) with Li-dominant modes, e.g. 160/224 cm^{-1} coupling. However, we again stress that these modes retain azide character. It is clear that less azide character leads to considerably weaker (orders of magnitude) than for modes with strong azide character. It follows that for materials with strongly decoupled external modes (molecular salts), this difference in coupling strengths should be expected to be even larger.

Table S6.1: Cubic anharmonic coupling constants for zone-centre couplings to 602 cm^{-1} target mode in LiN_3 . The coupling modes are labelled ‘1’ and ‘2’, and are indicated ‘Li’ if dominated by the Li cation.

| Mode 1 | Mode 2 | Order of Magnitude Coupling Constant |
|----------|----------|--------------------------------------|
| 160 | 134 | E-16 |
| 160 | 100 | E-38 |
| 160 | 150 | E-38 |
| 160 | 210 (Li) | E-41 |
| 160 | 224 (Li) | E-12 |
| 160 | 232 (Li) | E-39 |
| 160 | 243 | E-22 |
| 160 | 274 | E-24 |
| 160 | 280 | E-14 |
| 160 | 283 | E-11 |
| 210 (Li) | 134 | E-34 |
| 210 (Li) | 150 | E-15 |
| 210(Li) | 160 | E-34 |
| 210(Li) | 224 (Li) | E-34 |
| 210 (Li) | 232(Li) | E-21 |
| 210 (Li) | 243 | E-34 |
| 210 (Li) | 274 | E-39 |
| 210((Li) | 280 | E-38 |
| 210(Li) | 283 | E-38 |
| 210(Li) | 303 | E-35 |

S7. Integration Window Effect

Due to slight broadening, minor errors associated with vibrational frequency calculations, Gaussian broadening in production of the PDOS, and to account for resonance between neighbouring state, an integration window was used to include density $\pm\omega_T$. In the main text, an integration window of $\omega_T \pm 10 \text{ cm}^{-1}$ was employed, as this reflects the Gaussian broadening applied in generation of the PDOS. To ensure no artificial ordering was introduced, the integration window was varied, Figure S7.1. The relative ordering remains the same in all cases, barring HN_3 , whose integration window ($435\text{-}500 \text{ cm}^{-1}$) is not affected by the choice of window. However, we note that the total integral of HN_3 is *ca* 13.5. The sensitivity ordering $\text{HN}_3 > \text{AgN}_3$ remains for all integration windows $< \pm 20$.

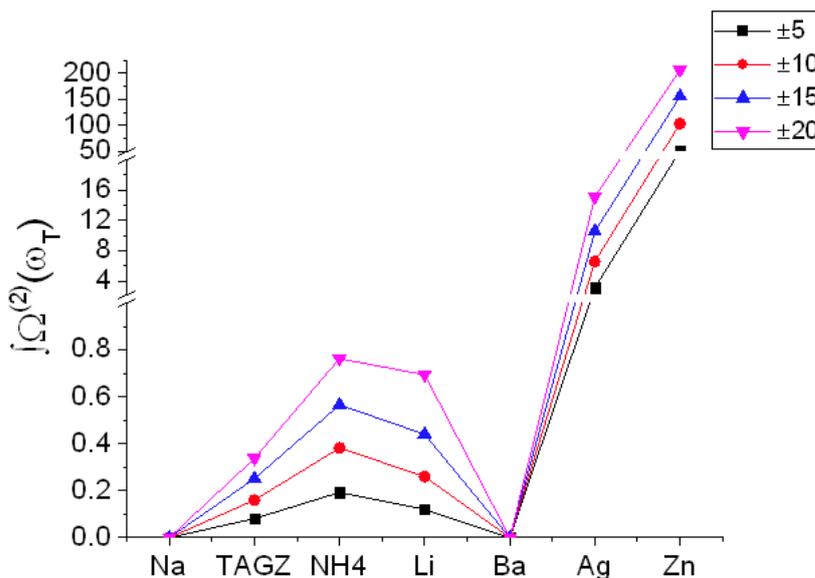


Fig S7.1: Effect of integration window on ordering of sensitivity

S8 Integration of multi-phonon Density of States

On examples tested, an increase in the fine grid used to produce the phonon density of states had negligible effect on the resulting integration of the two-phonon density of states. For example $\text{Zn}(\text{N}_3)_2$ PDOS generated using 8 q -points resulted in a $\Omega^{(2)}(\omega_T) = 102.57$. Increasing the number of fine q -points to 420 resulted in $\Omega^{(2)}(\omega_T) = 102.8$.

The integration values associated with Figure 8 in the main text are given in Table S8.1.

Table S8.1: Values for the integration of $\Omega^{(2)}(\omega_T \pm 10)$ for each azide. Values correspond to the data presented in the main text, Figure 8.

| Azide | $\Omega^{(2)}$ | |
|---------------------------|----------------|---------|
| | Fig 8 A | Fig 8 B |
| NaN_3 | 0.01 | 0.34 |
| TAGZ | 0.16 | 0.27 |
| NH_4N_3 | 0.39 | 0.71 |
| LiN_3 | 0.28 | 0.81 |
| HN_3 | 13.58 | 13.77 |
| $\text{Ba}(\text{N}_3)_2$ | 0.01 | 1.41 |
| AgN_3 | 6.11 | 7.63 |
| $\text{Zn}(\text{N}_3)_2$ | 99.25 | 102.57 |
| $\text{Sn}(\text{N}_3)_2$ | 23.57 | 26.48 |

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