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ARTICLE TEMPLATE

Back analysis of the multilayer cylindrical HMA samples – height reduction method

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ABSTRACT
Although the complex modulus is one of the most basic properties used in pavement analysis and design, its accurate measurement for existing pavement layers has always been a challenging task. When samples cut out from the pavement asphalt layers are used for the HMA complex modulus tests, they are performed separately for each layer. This paper describes an original method for determining the complex moduli of individual asphalt layers. The new idea is tested by applying uniaxial loading-unloading cycle tests to the HMA specimens combined of multiple layers. It was observed that changing the thickness ratios in samples' layers, allows obtaining the sets of load and displacement values (F,u), which effectively enlarge the database needed for the back analysis. For now, the conducted analysis presented in the paper focused on numerical modelling of HMA specimens. The simulated numerical testing conditions were based on viscoelastic parameters of asphalt concrete samples whose values were determined in real laboratory tests. In the case of noisy results of laboratory test simulations with a stochastic Gaussian process, by applying multiple cuts and changing sample's height, the determined values of stiffness moduli of the individual layers do not vary from the reference values by more than 10%.

KEYWORDS
back analysis; HMA; multilayer samples; height reduction; complex modulus;

1. Introduction

The material stress-strain relationship is one of the most basic properties used for computer modelling of pavement layers. Various laboratory testing procedures have been proposed in the past for determining the stiffness properties, such as complex modulus of hot mix asphalt materials (HMA) as discussed in the ASTM D3497-79 (2003) and EN 12697-26 (2012) standards. Depending on the choice of the static scheme in both laboratory and numerical tests, the complex modulus is determined in relation to a forced stress state (compression, tension, bending, shearing) Di Benedetto *et al.* (2001), Kim *et al.* (2004), Zhanping and Qingli (2007), Kim (2009), Lee *et al.* (2012). For the design of new pavements, a material sample of required dimensions is prepared in the laboratory. For determining stiffness properties of the existing asphalt

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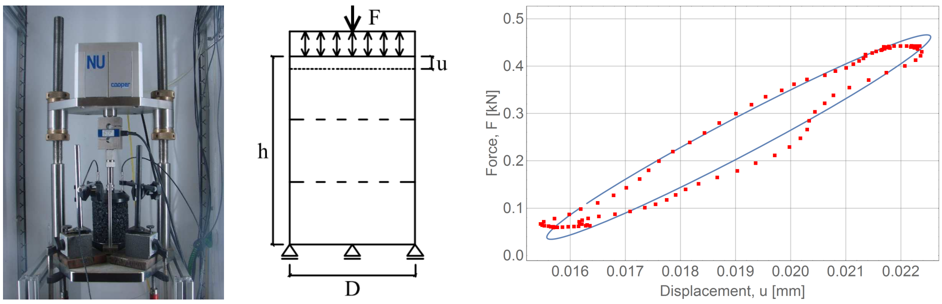
layers, testing is conducted on the cores cut out from pavement. However, modern asphalt pavements are often constructed with multiple layers, some of them are too thin to be tested individually (see Figure 1).

Figure 1. Examples of specimens with layers for which it is impossible to determine the values of their complex moduli in the laboratory

This paper proposes a method for determining complex moduli of individual asphalt layers, based on back analysis of sample‘s vertical displacements subjected to the load. The numerical testing conditions were simulated, based on viscoelastic parameters of asphalt concrete samples whose values were determined in real laboratory tests.

2. Leaps and bounds method of reducing the HMA sample’s height

The proposed method involves measuring the complex modulus of HMA lifts in the laboratory using a standard procedure for determining cylindrical HMA specimen stiffness moduli (EN 12697-26:2012). The testing procedure was assumed to be similiar to the uniaxial compression-tension method of HMA samples with, the samples subjected to repeated loading-unloading cycles (Figure 2).



(a) View of a sample prepared for tests.

(b) Calculation model of the sample.

(c) Example of test results for the specimen combined of 3 layers of HMA (testing temperature was T=25°C).

Figure 2. The scheme of loading-unloading mode.

Using the procedure illustrated in Figure 2, a set of maximum applied forces and maximum top surface displacements for various temperature can be obtained and the complex modulus of the specimen can be calculated using an analytical solution in the following form:

$$E^* = f(F, u, T, D, h) \tag{1}$$

where:
E^* - complex modulus [MPa]
$f(F, u, T, D, h)$ - functions of variables of respectively: force [kN], displacement [mm], sample temperature [deg. C], and sample dimensions (D - diameter [mm], h - height [mm]).

If an HMA core extracted from a pavement consists of at least two different HMA layers, the standard testing procedure allows calculating only the so-called equivalent value of the complex modulus, which describes stiffness of the whole sample, by analogy to the relation (1):

$$E^*_z = f(F, u, T_w, D_w, h_w) \tag{2}$$

where:
E^*_z - equivalent complex modulus [MPa]
$f(F, u, T_w, D_w, h_w)$ - function of variables of, respectively: force [kN], displacement [mm], temperature [°C] and dimensions of a multilayer sample [mm].

The equivalent complex modulus of a multilayer sample depends on the height (thickness) of its layers and their stiffness moduli. Therefore, it is possible to describe the equivalent complex modulus of a multilayer sample with relation (3).

$$E^*_z = g(E_1, E_2, ..., E_{j-1}, E_j, h_1, h_2, ..., h_{j-1}, h_j) \tag{3}$$

$$h_w = \sum_{i=1}^{j} h_i \tag{4}$$

where:
E_i - complex modulus of i^{th} layer [MPa]
h_i - thickness of i^{th} layer [mm]
j - number of pavement layers in a multilayer sample, $j \geq 2$.

To determine the complex moduli of individual layers, it is proposed to perform multiple testing on a cylindrical core extracted from a pavement. Each testing should be performed with the same loading protocol, but the height of core is reduced in each subsequent testing by cutting off a portion of the cores. From each testing, pairs of applied load and displacement histories are recorded and equivalent complex modulus is determined according to equation (2). The set constituting the basis for back analysis used for determining complex moduli for all layers in the sample can be easily created by the application of the leaps and bounds method for changing the height of the sample (by reducing the thickness of one of the outermost layers). The task can be expressed with an equation system (5).

$$\begin{cases} E_{z(0)}(E_1, E_2, ..., E_{j-1}, E_j, h_0) & = f_{(0)}(F, u, T, D, h_0) \\ E_{z(1)}(E_1, E_2, ..., E_{j-1}, E_j, h_1) & = f_{(1)}(F, u, T, D, h_1) \\ ... & ... \\ E_{z(i-1)}(E_1, E_2, ..., E_{j-1}, E_j, h_{i-1}) & = f_{(i-1)}(F, u, T, D, h_{i-1}) \\ E_{z(i)}(E_1, E_2, ..., E_{j-1}, E_j, h_i) & = f_{(i)}(F, u, T, D, h_i) \end{cases} \quad (5)$$

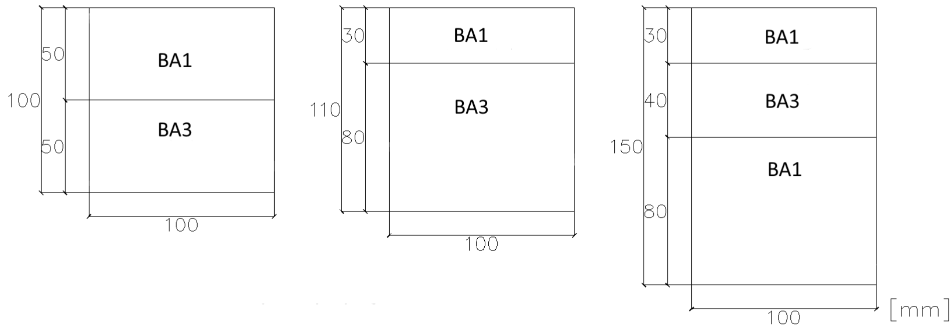
where:
$E_{z(n)}$ - equivalent elasticity moduli [MPa] for the samples' numerical models of h_n height [mm],
$f_{(n)}(F, u, T, D, h_n)$ - functions of: force [kN], displacement [mm], temperature [°C] and dimensions of the sample (D - diameter [mm], h_n - sample height [mm]),
$E_1, E_2, ..., E_{j-1}, E_j$ - stiffness moduli [MPa] of subsequent layers from 1 to j,
i - number of sample cuts,
j - number of sample layers when $i \geq j - 1$,
$n = 0$ - sample cut out from the pavement of the height of h_0 [mm],
$n = 1..i$ - sample with a reduced height of h_n [mm].

3. Verification of the algorithm

The method was verified based on the sets of (F, u) values, prepared using the numerical sample models. The developed models allowed for a simulation of the laboratory conditions of one-axis cyclic loading – unloading tests performed on cylindrical HMA samples.

3.1. *Numerical models of the samples*

Three examples of core configurations were considered. All three consisted of layers with materials properties either BA1 or BA3, and various thicknesses. Figure 3 shows three chosen specimens A, B and C, and the corresponding locations of cuts, for which numerical simulations of tests were conducted. Specimens A and B consist of two layers made of BA1 and BA3 materials. Specimen C consists of three layers.



(a) Specimen A. (b) Specimen B. (c) Specimen C.

Figure 3. Examples of the specimen models designated for tests of leaps and bounds method of reducing the sample height.

3.1.1. The loading model

The authors assumed a static load described with Gaussian white noise in all the calculations, which is a stationary stochastic process based on a normal distribution with the mean value of 0 and standard deviation of σ. The general relation derived by means of load values' calculations, is given in the form of (6).

$$\mathbf{P} = WN(\sigma) + F = WN(0.025) + A \cdot Sin(2\pi 10 \cdot t + \varphi) \quad [kN] \tag{6}$$

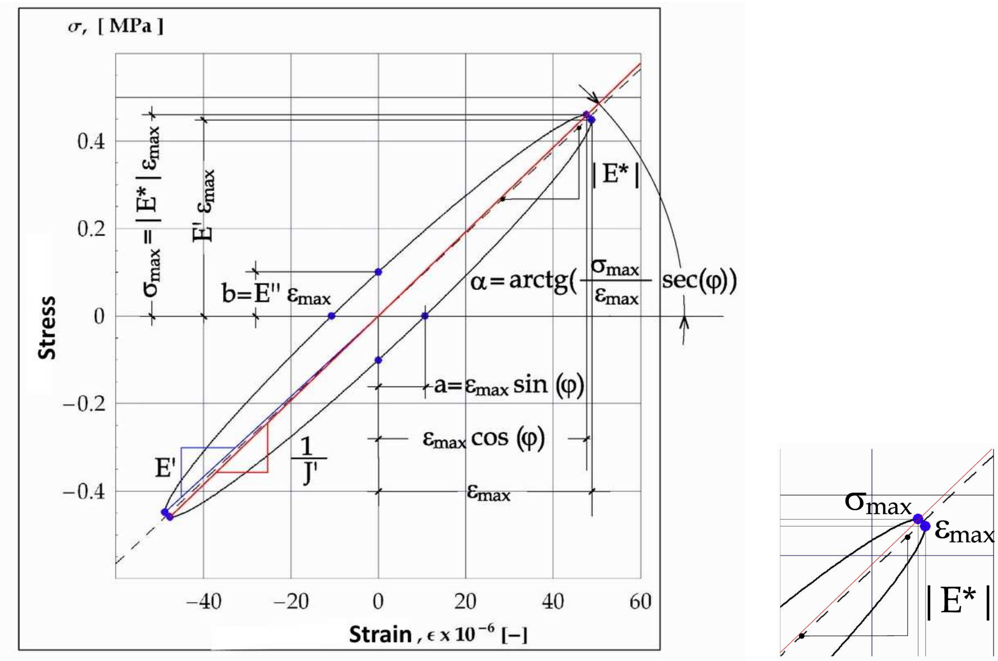
where:

$\mathbf{P}$ - one-row matrix with noisy F load values, that is (P_1, P_2, ..., P_{n-1}, P_n), A - an amplitude of a sinusoidal load function of a cylindrical sample with a diameter of r = 5 cm, corresponding to the maximum compressive stresses on the contact surface equal to 100 kPa (a priori assumption),
$WN(\sigma)$ - white noise process, understood as a series of uncorrelated random variables of the expected value of zero and constant variance of σ^2,
φ - phase shift angle between the initial function value, and the value at which point the function reaches the value of 0.

3.1.2. The reference values of complex moduli

The materials comprising individual cylindrical sample layers were described with a Linear Visco – Elastic model (LVE). For the loading scheme shown in Figure 2 (for a one-layer sample), the value of complex modulus is the relation of stress and deflection. Following our preliminary laboratory investigation, the stress-strain model, shown in Figure 4, was assumed. The quantities shown in the figure are as follows:
$|E^*|$ – complex modulus (absolute value of complex number) [MPa],
$E^{'}$ – the real part of complex modulus [MPa],
$E^{''}$ – the imaginary part of complex modulus [MPa],
σ – stresses in the loaded – unloaded cylindrical sample [MPa],
ϵ – strains in the loaded – unloaded cylindrical sample [–],

φ – phase angle [radians].



(a) The Lissajous material model

(b) Zoom of the max values

Figure 4. Model of stress-strain relationship assumed for the general case of loading-unloading test of HMA samples

Viscoelastic properties in back analysis were described by means of a relaxation function. The authors used normalized forms of the functions, which, for the generalized Maxwell model in the load frequency domain can be described with Prony series Di Benedetto *et al.* (2004). Prony series for the real G' part and the imaginary G'' part of the $G(\omega)$ shear modulus can be written as follows (7):

$$G(\omega) \quad \rightarrow \quad G' = G_0 \cdot (1 - \sum_{i=1}^{N} g_i + \sum_{i=1}^{N} \frac{g_i \cdot \tau_i^2 \cdot \omega^2}{1 + \tau_i^2 \cdot \omega^2}) \text{ and } G'' = G_0 \cdot (\sum_{i=1}^{N} \frac{g_i \cdot \tau_i \cdot \omega}{1 + \tau_i^2 \cdot \omega^2}) \quad (7)$$

where:

G_0, g_i, τ_i - Prony series parameters, presented for BA1 and BA3 materials in Table 1.

Table 1. LVE parameters for BA1 and BA3 materials at temperature T=15° C.

	Parameter				
Material name	G_0 [MPa]	g [-]	τ[-]	ν [-]	ρ [kg/m^3]
BA3	2 017.7	0.5422	0.0221	0.34	2 500
BA1	2 430.8	0.5789	0.0166	0.34	2 500

For each FEM numerical simulation of the laboratory test, the values of the force and vertical displacement (F(t), u(t)), are recorded and the effective complex modulus is determined by assuming that the sample is uniform using the equations (8).

$$|E^*| \approx \frac{F(t)_{max}}{u(t)_{max}} \cdot \gamma \ , \qquad \gamma = \frac{h}{\pi D^2} \tag{8}$$

where:
γ – the shape coefficient for the cylindrical sample $[\frac{1}{m}]$,
h – height of the single layered cylindrical sample [m],
D – diameter of the cylindrical sample [m].

The values of complex moduli of numerically modeled cylindrical samples made of BA1 and BA3 materials are given in Table 2 (for readability purposes their quantities are reported without the asterisk symbol).

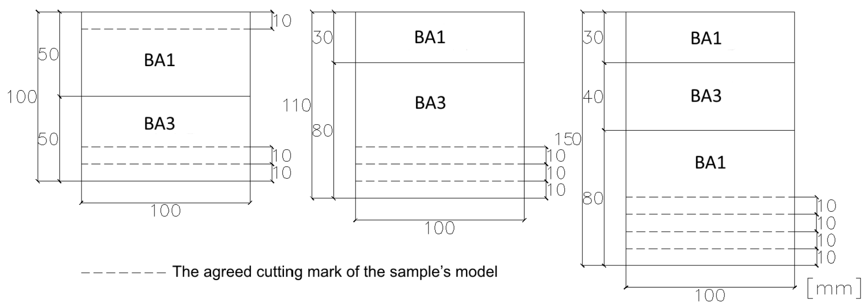
Table 2. Set of the true complex moduli values determined for samples made of materials BA1 and BA3.

	Complex modulus [MPa]
E_{BA1}	5 289
E_{BA3}	4 742

Further, in the paper, they are treated as true values approximating the moduli of samples made of mixtures used in HMA layers.

3.2. *Preparation of the dataset (F, u)*

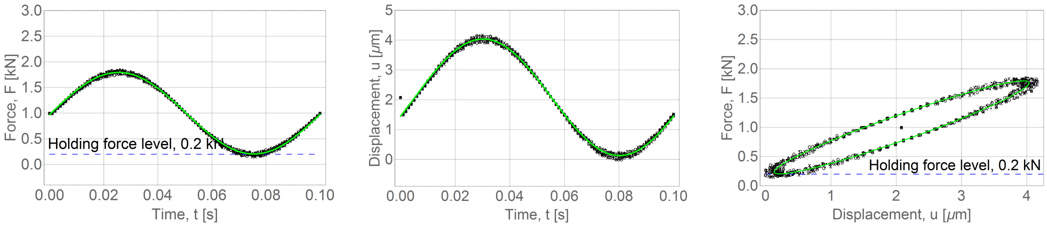
The choice of random A, B, and C samples shown in Figure 3 was based on the analysis of a system comprised of asphalt layers in flexible pavements, and the analysis of typical solutions applied in practice. In order to determine the values of complex moduli for individual specimen layers, the authors assumed a necessary number of cuts i, for each of the specimens A, B, and C ($i = j + 1$) and their locations. The assumption is to address the issue of noise in the input data set, according to the hypothesis that the accuracy of estimating the parameters of the model increases with the increasing sample population size.



(a) Specimen A. (b) Specimen B. (c) Specimen C.

Figure 5. Indicated location of the cutting mark forcing a change in the height of the HMA specimen.

For individual series of numerical test models, the authors considered the noisy values of the load function and calculated displacements. An example of the individual series of test results obtained for the specimen A before it was cut is illustrated in Figure 6.



(a) The relation of load in the function of time. (b) The relation of displacements in the function of time. (c) Load-displacement relation.

Figure 6. Graphic representation of the (F, u) data set (black markers on the charts symbolize the results of the numerical test model and the continuous green line - its approximation.

Thanks to comprehensive data, it is possible to complete the result set of measured values (F, u) for modeled multilayer A, B, and C samples, and to introduce them later on to the algorithm schematically presented in Figure 7. The mean values of maximum displacements for all three samples and cutting variants shown in Figure 5 are given in Table 3.

Table 3. Displacement values from the dataset for back analysis.

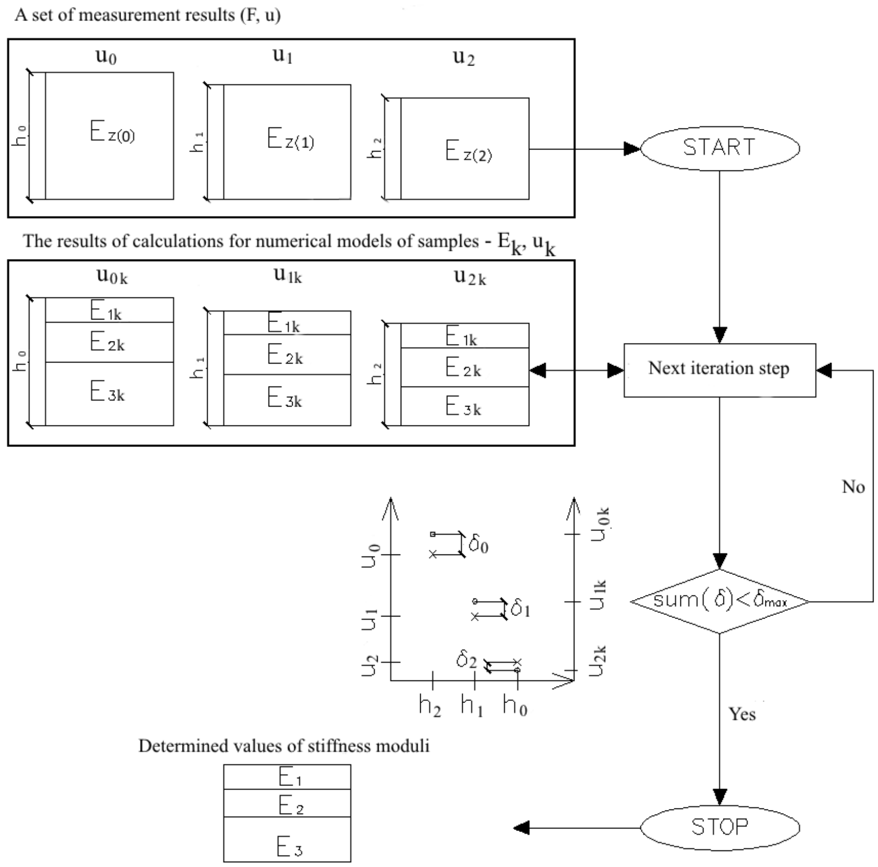
Specimen status	Mean of max. amplitude of displacements, [μm]		
(No. of cut)	Specimen A	Specimen B	Specimen C
No cut	2.027	2.222	3.110
First cut	1.817	2.007	2.807
Second cut	1.622	1.776	2.620
Third cut	1.413	1.586	2.411
Fourth cut	—	—	2.260

3.3. *Back analysis*

The back analysis algorithm used in analysis is shown in the Figure 7. Although it refers to a three-layer specimen, the method can be applied for determining the complex moduli of specimens with a greater number of layers. The calculations are iterative, and the number of iterations depends on the number of considered sets of values (F, u) for a given core ("(F, u) couples"). As the criterion for convergence of the back analysis, the authors assumed the best fit between the known displacements u and u_k which were the calculated displacements values for the samples using the numerical model after the $k-th$ iteration. The criterion was expressed in the form of an objective function described with condition (9). For calculating u_k displacement values, the authors used the finite element method described in Górnaś and Pożarycki (2014).

$$sum(\delta) = \frac{\mathbf{u} - \mathbf{u_k}}{\mathbf{u}} < \delta_{max} = 0.01 \tag{9}$$

where:
$sum(\delta)$ - sum of the values of the relative differences between referenced displacement values u (those from numerical simulation) and displacement values u_k, which are calculated in each iteration k,
δ_{max} - specified maximum value of the relative differences, which can be accepted in engineering calculations.



(a)

Figure 7. The procedure for determining complex moduli of individual lifts in a core by use of back analysis (used symbols correspond to the description in text, symbol k stands for iteration step in the back analysis procedure of searching for the moduli E_1, E_2 and E_3).

The assumed method uses a static load scheme. In the back analysis, the optimization procedure was performed using the Nelder Mead optimizing algorithm Fuchang and Lixing (2012). It is a derivative-free, direct search method, which in cases of numerically – oriented problems is its big advantage. Generally, the Nelder Mead algorithm performs well (considering the speed and accuracy) for solving low dimensional problems (the indicated limit is $n < 10$) and without many local minima. Dimensionality stands here for the number of input independant variables in the objective function. Considering the search variables declared for specimen C, equation (9) generates the two dimensional optimization problem expressed by formula (10).

$$\delta(E_{BA1}, E_{BA3}) \quad \rightarrow \quad min. \tag{10}$$

However standard form of Nelder Mead algorithm becomes inefficient in high dimensions and lacks of a satisfactory convergence theory. Therefore the Adaptive Nelder-Mead Simplex was chosen for our calculations such as described in already mentioned Fuchang and Lixing (2012), where the basic algorithm parameters (expansion, contraction and shrink) depend on the dimension (n) of the optimization problem. In this paper, the calculations were performed assuming:
- reflection $\alpha = 1$
- expansion $\beta = 1 + 2/\mathrm{n}$
- contraction $\gamma = 0.75 + 1/2n$
- shrink $\delta_{NM} = 1\text{-}1/n$
- the tolerance $tol = 1\text{e-}6$
- maximum number of iterations $maxiter = 200$.

The obtained values of searched variables are given in Table 4. Absolute value of the relative error of determining the complex modulus of individual specimen layers was assumed as the relation described with the general formula (11).

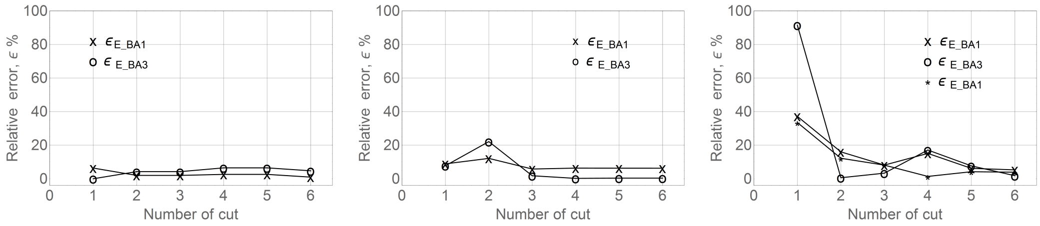
$$\epsilon = |\frac{E - \text{True value}}{\text{True value}}| \cdot 100\% \tag{11}$$

where:
E - calculated values of complex modulus referred to individual sample layers incorporated in studies of the samples A, B or C [MPa],
True value - the values of reference complex modulus [MPa] referred to individual sample layers considered in particular configuration of studied specimens A, B and C.

Table 4. The results of determining complex moduli of asphalt mineral mixtures for multilayer specimens A, B, and C

	Calculated E, [MPa]			Criterion, sum(δ), [%]	Error ϵ, [%]		
Specimen	BA1	BA3	BA1		BA1	BA3	BA1
A	5182	4540	...	0.65	2.02	4.26	—
B	4981	4656	...	0.40	5.82	1.81	—
C	6089	3929	5364	0.25	15.13	17.14	1.42
True value	5289	4742	5289	—	—	—	—

In the first approximation, the values in Table 4 come solely from calculations in which it was assumed that the number of cuts is greater by 1 than the number of layers in the sample ($j + 1$). This means that in the case of a sample with two layers (specimens A and B), the maximum value of errors for determining complex moduli of individual layers is lower than 6%. Assuming the admissible limit value of error equal to 10%, the results meet the requirements at the engineering assessment level. However, as far as specimens with three layers are concerned (specimen C), the value of error exceeds 10%. Figure 8 shows that the resulting relative error depends on the number of consecutive cuts.



(a) Specimen A. (b) Specimen B. (c) Specimen C.

Figure 8. Implemented values of relative differences ϵ [%] between the values of moduli calculated by the leaps and bounds method of reducing the HMA sample height, and the true values.

In fact, as the three-layer samples show variability, back analysis calculations were performed for further cuts. For the number of cuts larger than 6, the calculated coefficient of variation of relative error values ϵ, turned out to be less than 1%. It was determined that in the case of constituting sample C the value of error below 10% is achieved by applying a number of cuts that fulfills the condition of $i \geq 6$.

Following further investigations, an additional interesting insight of the presented method is to determine the properties of specimen C considering the consecutive cuts removing the top layer BA1 and then the top 10 mm of the layer BA3. Next, for the remaining sum of BA3 and BA1 thickness, consecutive cuts of the bottom layer BA1 were performed. This procedure likely gives "the exact" properties of BA3, even if the data are noisy. The cut configuration which meets such a cutting procedure is shown in the Table 5 as case 1. For the comparative purposes case 2 was used (discussed earlier according to Figure 8c).

Table 5. Cuts configuration for the further analysis of specimen C.

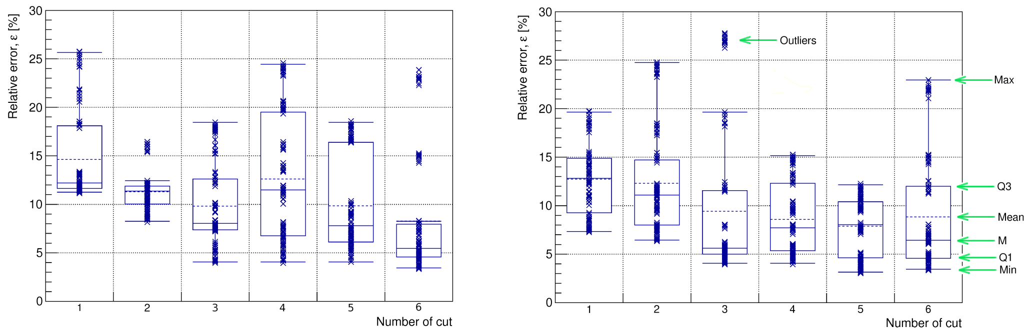
Sample status	Number of	Layer height [m] (case 1)			Layer height [m] (case 2)		
(No. of cuts)	(F,u) couples	BA1	BA3	BA1	BA1	BA3	BA1
No cut	1	0.03	0.04	0.08	0.03	0.04	0.08
1	2	0.02	0.04	0.08	0.03	0.04	0.07
2	3	0.01	0.04	0.08	0.03	0.04	0.06
3	4	0.00	0.03	0.08	0.03	0.04	0.05
4	5	0.00	0.03	0.07	0.03	0.04	0.04
5	6	0.00	0.03	0.06	0.03	0.04	0.03
6	7	0.00	0.03	0.05	0.02	0.04	0.03

In a standard deterministic modelling approach, after each consecutive cut on the specimen C, one would enlarge the input data set at most by one (F,u) couple. Therefore, the probabilistic methods were used to address once again the rule given by equation (6). In consequence, for each number of cuts, one hundred independent samples with the random noise were calculated to model the population of the results that would be obtained in a laboratory practice. Figure 9 shows a candle plots for the obtained data distributions (D), represented by the cross symbol and characterized with only five numbers. It is a convenient way to present the population properties, and these five numbers are:

- Min: the minimum value observed in the distribution D
- Q1: the lower quartile informing that 25% of the data points in D are less than Q1

- M: the median which indicates that 50% of the data points in D are less than M
- Q3: the upper quartile showing the limit of 75% of the data points in D which are less than Q3
- Max: the maximum value of the distribution D.

The outermost part that is not covered by the underlying distribution is typically represented by the outliers. As a criterion for the outlier detection the interquartile method was applied.



(a) Case 1. (b) Case 2.

Figure 9. A candle plots describing the relative error distribution D in function of the number of cuts for cases 1 and 2 of specimen C.

Back analysis results shown in Table 6 refer to the last made cut. As it was expected, the case 1 scheme results in the smallest error value for the BA3 layer. However, considering it is admissible engineering value below 10%, the results for cuts number of 5 and 6 are satisfactory in both cases.

Table 6. The results of determining complex moduli of C specimen layers (6^{th} cut)

Calculated E, [MPa]				Criterion, sum(δ), [%]	Error ϵ, [%]		
Specimen C	BA1	BA3	BA1		BA1	BA3	BA1
Case 1	4873	4627	5010	0.56	7.86	2.43	5.27
Case 2	4989	4329	4785	0.48	5.67	8.71	9.52
True value	5289	4742	5289	—	—	—	—

Numerically generated noisy data are still rather pseudo – random values than the true ones. In order to work with a high quality noisy data, the calculations were performed using the machine independent random number generator such as described in Matsumoto and Nishimura (1998). For calculation purposes a C++ code was implemented using the modular scientific software framework ROOT Antcheva *et al.* (2009).

3.4. *Discussion*

The maximum matching value of displacements max(sumδ) obtained from back analysis and laboratory tests simulations for particular examples A, B and C is lower than 0.65%. In the first approximation, during the method verification, the authors used a minimum number of necessary cuts of numerical models of cylindrical samples increased by 1. For conditions shaped in this manner, the relative values of the difference between determined and true values of complex moduli of asphalt mineral mixtures for samples A and B are satisfactory and amount to much less than 10%. In the case of sample C, that is the sample with 3 layers, the obtained values below this percent required at least 6 cuts. As far, in order to keep the acceptable engineering accuracy of laboratory test results, the following conditions are recommended:

- HMA specimen temperature should be $\leq 15°C$
- loading frequency meets the condition ≤ 10 Hz
- the amplitude of the strain value is $\leq 50 \cdot 10^{-6}$
- the layers in the specimen are fully bonded
- value of Poisson's ratio corresponds to the HMA specimen temperature. In the current study Poisson's ratio of 0.3, corresponding to the HMA temperature of $15°C$, was assumed.
- to address the issue of noise in the laboratory input data, the number of cuts should be larger than the number of layers in the specimen. However, a further increase in the number of cuts reduces the variability (or the error) of the data. For example, in the current study at least 6 cuts were required to reduce the value of error below 10%.

This methodology has also some limitations: (1) it will only be useful for relatively small displacement values, (2) cutting the samples in the laboratory must be precise, (3) the smallest height that a multilayered specimen can have after all cut configurations being applied in a real laboratory conditions should be at least equal to 3 times the diameter of the largest grain in the specimen layers. The first restriction is related to the commonly used practice where the elastic range of asphalt concrete samples displacement is assumed. The second limitation arises from the proposal of testing the samples in axial loading – unloading (compression – traction) mode. If the top and bottom surfaces of the tested samples are not horizontal and mutually parallel, the accuracy of the laboratory test results will be significantly affected. The third limitation is in a close relation to the minimum height of cylindrical specimens which must be preserved when testing its stiffness in the laboratory (the lower limit of specimen height required by the Indirect Tensile Stiffness Modulus procedure is ≥ 30 mm). It is worth to mention that this condition does not apply to the situation in which the required number of the sample's cuts can be obtained when cutting relatively high specimens (≥ 10 cm).

4. Conclusions

The paper discusses the calculation and testing procedure called the leaps and bounds method of reducing the height of cylindrical samples made of several different mixtures, such as used in HMA pavement layers. The method can be used to determine complex moduli of individual layers of a specimen and not only the values of the equivalent modulus. Based on the results of the laboratory tests simulated by the numerical models of multilayer specimens, the authors verified a procedure which uses a method of determining the values of complex moduli of cylindrical samples in the one-axis cyclical loading-unloading mode. It was proven that in the case of noisy results of laboratory test simulations with a stochastic Gaussian process, by applying multiple cuts, and changing sample's height, the determined values of stiffness moduli of the individual layers do not vary from the true values by more than 10%. To address the issue of noise in the laboratory input data, the number of cuts should be larger than the number of layers in the specimen. However, a further increase in the number of cuts reduces the variability (or the error) of the data. For example, in the current study at least 6 cuts were required to reduce the value of error below 10%.

The most important direction for further development of the described methodology is to carry out the laboratory tests. In the first try, the cylindrical samples prepared in the laboratory should be studied. There are two possibilities here. The first is the forming the multilayer plate, from which the cylindrical samples could be drilled out. An interesting alternative is to develop a procedure for making the multilayer cylindrical samples by extending the standard procedure of the Marshal method. Only after the laboratory phase is completed, one can try to use this method for testing the samples cut from the asphalt pavement layers. An important development for the proposed scheme is the investigation if the method can be adopted for materials with dissimilar phase angles.

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References

Antcheva, I., *et al.*, 2009. ROOT — A C++ framework for petabyte data storage, statistical analysis and visualization. *Computer Physics Communications*, 180, 2499–2512.

Di Benedetto, H., *et al.*, 2004. Linear viscoelastic behaviour of bituminous materials: From binders to mixes. *Road Materials and Pavement Design*, 5 (1), 163–202.

Di Benedetto, H., *et al.*, 2001. Stiffness testing for bituminous mixture. *Materials and Structures*, 34, 66–70.

Fuchang, G. and Lixing, H., 2012. Implementing the Nelder-Mead simplex algorithm with adaptive parameters. *Computational Optimization and Applications*, 51 (1), 259 – 277.

Górnaś, P. and Pożarycki, A., 2014. Selected properties of FEM numerical models for inverse analysis of road pavement structure. *Roads and Bridges*, 13 (3), 203–222.

Kim, Y., *et al.*, 2004. Dynamic modulus testing of asphalt concrete in indirect tension mode. *Transportation Research Record: Journal of the Transportation Research Board*, 1891, 163–173. Available from: https://doi.org/10.3141/1891-19.

Kim, Y.R., 2009. *Modeling of asphalt concrete*. ASCE Press ; McGraw-Hill, Reston, VA; New York.

Lee, H., *et al.*, 2012. Construction of dynamic modulus master curves with resilient modulus and creep test data. *Transportation Research Record: Journal of the Transportation Research Board*, 2296, 1–14. Available from: https://doi.org/10.3141/2296-01.

Matsumoto, M. and Nishimura, T., 1998. Mersenne Twister: A 623-Dimensionally Equidistributed Uniform Pseudo – Random Number Generator. *ACM Transactions on Modeling and Computer Simulation*, 8, 3–30.

Zhanping, Y. and Qingli, D., 2007. Dynamic complex modulus predictions of hot-mix asphalt using a micromechanical-based finite element model. *Canadian Journal of Civil Engineering*, 34 (12), 1519–1528. Available from: https://doi.org/10.1139/L07-064.