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Finite lifetime estimation of mechanical assemblies subjected to fretting fatigue loading

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Abstract. This paper proposes a new design method for predicting the finite lifetime of mechanical assemblies subjected to constant amplitude (CA) fretting fatigue loading. The proposed methodology is based on the use of the Modified Wöhler Curve Method (MWCM) applied in conjunction with the Theory of Critical Distance (TCD) and the Shear Stress-Maximum Variance Method (τ-MVM). In more detail, this engineering approach uses the τ-MVM to calculate the stress quantities relative to the critical plane, whose orientation is determined numerically by locating the plane containing the direction experiencing the maximum variance of the resolved shear stress. To estimate the fretting fatigue lifetime, the time-variable linear elastic stress quantities are post processed according to the MWCM applied in conjunction with the TCD. The proposed approach was checked against experimental data taken from the literature and generated by testing specimens made of aluminium alloy Al 7075-T6. The extensive validation supports the idea that the MWCM applied in conjunction with both the TCD and τ-MVM can be suitable to predict the finite lifetime of mechanical assemblies subjected to fretting fatigue loading.

Keywords: Fatigue lifetime estimation, Fretting fatigue, Multiaxial fatigue.

1 Introduction

Fretting is a type of failure commonly observed in industrial components that are in contact and subjected to small amplitude oscillatory movements in the contact region. In situations where one or both contacting components undergo cyclic loading, the damage resulting from fretting is called “fretting fatigue”. Fretting fatigue is a very complex phenomenon and leads to the premature failure of several mechanical assemblies. This premature failure results from the reduction of the fatigue limit of materials by up to 50% [1]. Therefore, the prediction of in-service lifetime of materials under fretting fatigue loading is of great interest in engineering. In this regard, extensive analytical and experimental work have been undertaken over the past years. For example, Nowell et al. [2] used the TCD [3] in conjunction with the short crack arrest criteria [4] to predict fretting fatigue lifetime of materials.

To investigate the fretting fatigue behavior of medium-carbon steel JIS S45C, Noraphaiphipaksa et al. [5] proposed the use of maximum shear stress range criterion (MSSRC) [6] and the maximum tangential stress range criterion (MTSRC) [7]. In more detail, the MSSRC predicted the location of the crack nucleation sites whereas, the MTSRC estimated the orientation of fretting fatigue crack paths. Finally, they estimated fretting fatigue lifetime by integrating the fatigue crack growth curve from an initial to a critical crack length.

Recently, to estimate the fretting fatigue lifetime of AL-4%Cu specimens subjected fretting fatigue loading, Araújo et al. [8-10] used the Modified Wöhler Curve Method (MWCM) in conjunction with the Theory of Critical Distance (TCD) [11- 12] and the Maximum Rectangular Hull concept [13-16]. In this approach, the stress quantities relative to the critical plane required to predict fretting fatigue lifetime according to the MWCM were calculated using the Maximum Rectangular Hull concept. To validate their predictions, they used the Hertzian fretting test results generated by Nowell [17].

This paper proposes a design methodology used to estimate the in-service lifetime of mechanical assemblies under constant amplitude (CA) fretting fatigue loading. Here, the fretting fatigue damage will be assessed according to the ‘‘notch analogue’’ concept [18], by directly post-processing the linear-elastic multiaxial stress field around the contact zone of the assessed mechanical assembly. The proposed design methodology makes use of the Modified Wöhler Curve Method (MWCM) applied in conjunction with the Theory of Critical Distance (TCD) in the form of Point Method (PM) and the Shear Stress-Maximum Variance Method (τ-MVM). The novelty characterizing this investigation is that, the τ-MVM [19] is used to calculate the critical plane and associated time-variable linear elastic stress quantities needed to estimate fretting fatigue lifetime.

2 The MWCM to estimate fatigue life under CA multiaxial fatigue loading

The MWCM [20-21] is a multiaxial stress-based approach which postulates that, under CA multiaxial loading, crack initiates on the material plane experiencing the maximum shear stress amplitude (i.e. the so-called critical plane). To perform fatigue assessment of materials, the MWCM is used to post-process the maximum shear amplitude, τ_a, the mean normal stress, $\sigma_{n,m}$ and the normal stress amplitude, $\sigma_{n,a}$, relative to the critical plane. The combined effect of the above stresses are used to formalize the MWCM through the effective critical plane stress ratio, ρ_{eff}, defined as [22]:

$$\rho_{eff} = \frac{m\cdot\sigma_{n,m}}{\tau_a} + \frac{\sigma_{n,a}}{\tau_a} \tag{1}$$

where $\sigma_{n,m}$, $\sigma_{n,a}$, and τ_a are the mean normal stress, normal stress amplitude and maximum shear stress amplitude relative to the critical plane, respectively. The mean stress sensitivity index, m, is a material property ranging between 0 and 1, to be determined experimentally [23]. The key feature of the stress ratio, ρ_{eff} is the fact that it can effectively account for the presence of superimposed static stresses and also the degree of non-proportionality of the loading applied to the assessed material [11].

The MWCM is defined by its negative inverse slope, $K_\tau(\rho_{eff})$, and the reference shear stress amplitude, $\tau_{A,Ref}(\rho_{eff})$, calculated at a number of cycles to failure N_A. Schematically, it is illustrated by the log-log diagram that plots the amplitude of shear stress, τ_A , relative to the critical plane against the number of cycles to failure, N_f (Fig. 1). Furthermore, under the fully-reversed uniaxial loading, the stress ratio, ρ_{eff} equals one, whereas under fully-reversed torsional loading, ρ_{eff} returns a value of zero. Fig. 1 also shows that, for a given the shear stress amplitude relative to the critical plane, τ_A, fatigue damage is seen to increase as the stress ratio, ρ_{eff} increases [24-25].

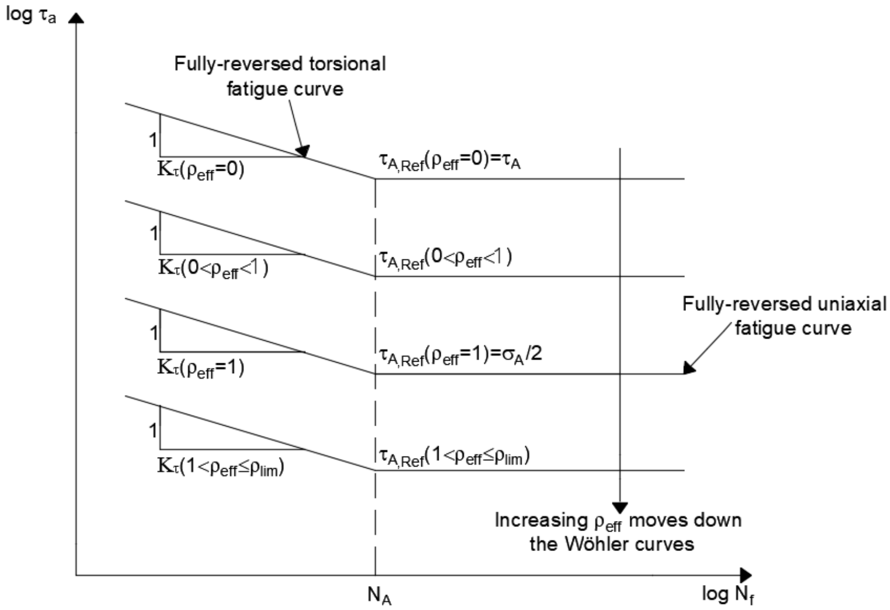

Fig. 1. Modified Wöhler diagram

Past researchers such as Susmel and Lazzarin carried out extensive investigations on the MWCM. They concluded that the MWCM parameters such as $K_\tau(\rho_{eff})$ and $\tau_{A,Ref}(\rho_{eff})$ can be expressed effectively by simply using the following linear equations [26-27]:

$$k_\tau(\rho_{eff}) = (k - k_0).\rho_{eff} + k_0 \qquad for\ \rho_{eff} \le \rho_{lim} \tag{2}$$

$$\tau_{A,Ref}(\rho_{eff}) = \left(\frac{\sigma_A}{2} - \tau_A\right).\rho_{eff} + \tau_A \qquad for\ \rho_{eff} \le \rho_{lim} \tag{3}$$

$$k_\tau(\rho_{eff}) = (k - k_0).\rho_{lim} + k_0 \qquad for\ \rho_{eff} > \rho_{lim} \tag{4}$$

$$\tau_{A,Ref}(\rho_{eff}) = \left(\frac{\sigma_A}{2} - \tau_A\right)\cdot\rho_{lim} + \tau_A \qquad for\ \rho_{eff} > \rho_{lim} \tag{5}$$

where:

$$\rho_{lim} = \frac{\tau_A}{2\tau_A - \sigma_A} \tag{6}$$

k_0 and τ_A are defined as the negative inverse slope and endurance limit (extrapolated at N_A cycles to failure) obtained from a fully-reversed torsional fatigue experiment whereas k and σ_A are the corresponding quantities associated with the fully-reversed uniaxial fatigue experiment. ρ_{lim} represents the material threshold limit. The use of the threshold limit allows the MWCM to be applied in situations where the critical plane experiences high values of ρ_{eff}. After estimating the required modified Wöhler curves from Eqs. (2) to (6), the number of cycles to failure can be estimated as follows [11]:

$$N_{f,e} = N_A \cdot \left[\frac{\tau_{A,ref}(\rho_{eff})}{\tau_a}\right]^{k_\tau(\rho_{eff})} \tag{7}$$

Where τ_a is the maximum shear stress amplitude relative to the critical plane.

3 The TCD approach

In general, fatigue failure of notch materials occurs when the reference stress, $\sigma_{eff,a}$, is greater than the material strength, σ_{Ref}. The reference stress, $\sigma_{eff,a}$, can be calculated using the TCD formalized in terms of line method (LM), area method (AM), volume method (VM) and point method (PM). As far as the PM is concerned, $\sigma_{eff,a}$ is obtained at half the critical distance, L, from the vicinity of the notch tip [3]. Because of its simplicity, the PM summarised in Fig. 2 is used in this investigation.

Experimental evidence has shown that, accurate estimation of the finite lifetime of notch components can be obtained by assuming that in a medium-cycle fatigue regime, the critical distance, L_M decreases as the number of cycles to failure, N_f increases. This idea is expressed mathematically by the following L_M versus N_f relationship [28-29]:

$$L_M(N_f) = A.N_f^B \tag{8}$$

Where A and B are material constants estimated from two fatigue curves. The first curve is generated by testing notch specimens under fully-reversed uniaxial fatigue loading whereas the second is obtained by testing plain specimens under a fully-reversed uniaxial loading [26-28]. Fig. 2 summarizes the procedure to be followed to determine the fatigue constants A and B. In more detail, given a generated plain and notch fatigue curve, by using the PM argument, the linear elastic stress field in the vicinity of the notch (calculated using the FE or analytical method) is used to calculate the distance, $L_M/2$, from the notch tip. At such a distance, the amplitude of the linear elastic maximum principal stress is equal to the stress amplitude, $\sigma_{1,a}$, which breaks the plain material at $N_f = N_{f,i}$ cycles to failure (Fig. 2b). The above method can be extended to any number of cycles between the low to high-cycle fatigue regime associated to the plain fatigue curve (Fig. 2a). This then allows the estimation of the material constants A and B.

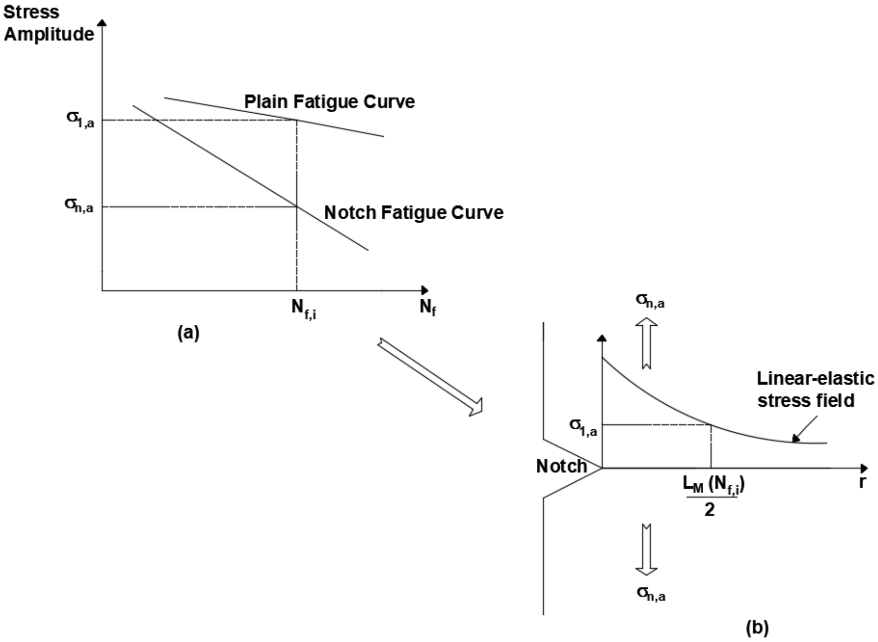

Fig. 2. In-field procedure to calculate the critical distance, L_M , in the medium-cycle fatigue regime.

4 Estimation of stress quantities relative to the critical plane under CA multiaxial loading

In order to use the MWCM in situation involving CA multiaxial fatigue loading, one of the tasks is to determine the orientation of the critical plane and its associated stress quantities, $\sigma_{n,m}$, $\sigma_{n,a}$ and τ_a. This has led Susmel [30] to reformulate the MVM to efficiently calculate the orientation of the critical plane in multiaxial fatigue problems. In detail, the Shear Stress–Maximum Variance Method (τ-MVM) postulates that the critical plane is the material plane containing the direction which experiences the maximum variance of the resolved shear stress, $\tau_{MV}(t)$ [19-20]. The full algorithm formulated and implemented by Susmel to calculate the orientation of the critical plane according to the τ-MVM can be found in [30] while its validation is found in [31]. The main advantage of the τ-MVM over existing methods is the fact that the computational time required to determine the critical plane does not depend on the length of the assessed input load history. In fact, as soon as the variance and co-variance, characterising the time-variable stress tensor at the critical location are known, the computational time required to calculate the critical plane becomes just a conventional multi-variable optimization problem [30]. Having calculated the critical plane according to τ-MVM, the mean value, $\sigma_{n,m}$, and the amplitude, $\sigma_{n,a}$, of the stress, $\sigma_n(t)$, normal to the critical plane can be estimated by using the following definitions [19]:

$$\sigma_{n,m} = \frac{1}{2}\left(\sigma_{n,max} + \sigma_{n,min}\right) \tag{9}$$

$$\sigma_{n,a} = \frac{1}{2}\left(\sigma_{n,max} - \sigma_{n,min}\right) \tag{10}$$

where $\sigma_{n,max}$ and $\sigma_{n,min}$ are the maximum and minimum values of $\sigma_n(t)$, respectively. Similarly to the above definition, the mean value, τ_m, and the amplitude, τ_a, of the shear stress relative to the critical plane, $\tau_{MV}(t)$, are calculated as follows [19]:

$$\tau_m = \frac{1}{2}\left(\tau_{MV,max} + \tau_{MV,min}\right) \tag{11}$$

$$\tau_a = \frac{1}{2}\left(\tau_{MV,max} - \tau_{MV,min}\right) \tag{12}$$

Where $\tau_{MV,max}$ and $\tau_{MV,min}$ are the maximum and minimum values of $\tau_{MV}(t)$, respectively. In this investigation, the fretting fatigue lifetime is estimated by calculating the stresses relative to the critical plane using the τ-MVM concept [19].

5 Formalization of the design methodology to estimate finite lifetime of mechanical assemblies under CA fretting fatigue loading

The proposed methodology makes use of the ‘notch analogue’ concept [18] by assessing the linear elastic multiaxial stress field, damaging the fretting specimen in the vicinity of the trailing edge of the contact region. In more detail, the approach takes as its starting point the assumption that the cracking behavior in metallic materials under fretting fatigue loading is similar to those observed in notched components, made of the same material and subjected to cyclic load history [29]. To estimate the finite lifetime under CA fretting fatigue loading, the proposed approach makes use of the MWCM applied in conjunction with the TCD and τ-MVM. In this setting, the τ-MVM calculates the orientation of the critical plane and its associated time-variable linear elastic stress quantities. The calculated stresses relative to the critical plane are post processed according to the MWCM applied in conjunction with the PM.

The proposed design methodology to be used to estimate the finite lifetime under CA fretting fatigue loading is summarized in Fig. 3. To apply this procedure in situations of practical interest, primarily, the linear elastic multiaxial stress distributions along the focus path has to be estimated. In the fretting fatigue context, the focus path is a straight line emanating from the crack initiation location, A, and normal to the contact surface. Next, the MWCM's functions $\tau_{A,Ref}(\rho_{eff})$, $K_\tau(\rho_{eff})$ Eq. (2) to (6) and the critical distance $L_M(N_f)$ Eq. (8) have to be calibrated as explained in sections 2 and 3. After the calibration process, the recursive procedure summarized in Fig. 3 can be used to estimate the finite lifetime under CA fretting fatigue loading. The starting point of this recursive procedure is the fact that at any given distance r from the assumed crack initiation location A (along the focus path), the stress quantities, $\tau_a(r)$, $\sigma_{n,a}(r)$ and $\sigma_{n,m}(r)$ relative to the critical plane and its associated critical stress ratio, $\rho_{eff}(r)$ (Eq. 1) can be calculated according to the τ-MVM. This then leads to the estimation of the corresponding MWCM's calibrating functions , $\tau_{A,Ref}(\rho_{eff})$ and $K_\tau(\rho_{eff})$, and the associated number of cycles to failure N_f (Eq. 7). Having calculated N_f corresponding to the r value under investigation, the critical distance, $L_M(N_f)$ is estimated using (Eq. 8). The fact that N_f can be estimated at any location r along the focus path, the material being assessed is expected to fail at the number of cycles to failure $N_{f,e}$, if the following convergence is reached [26]:

$$\frac{L_M(N_{f,e})}{2} = r \tag{13}$$

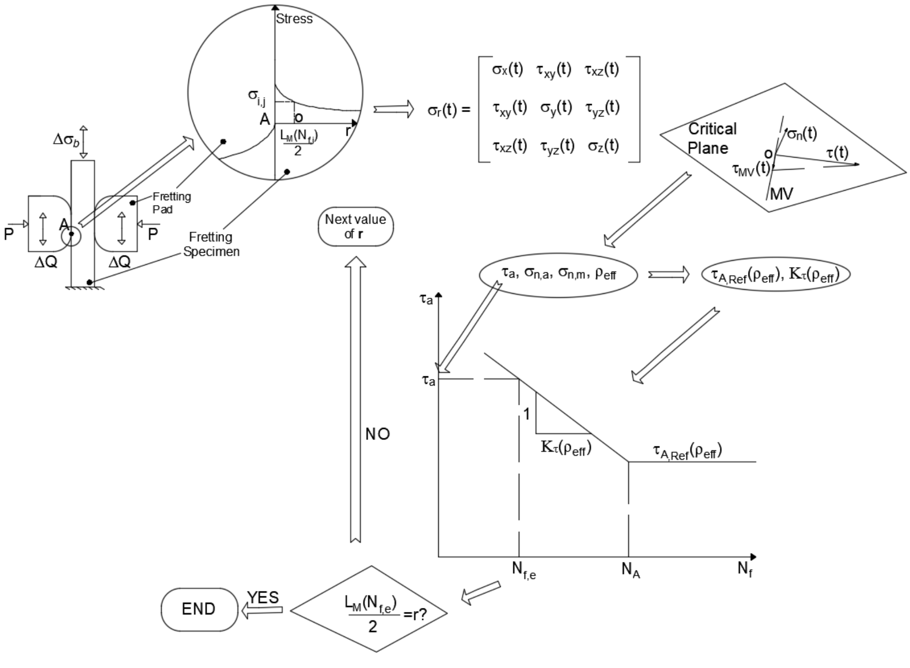

Fig. 3. The MWCM applied in conjunction with the PM and τ-MVM to estimate finite life-time under CA fretting fatigue loading.

6 Validation with experimental data

The fretting fatigue experiments under CA loading generated by Wittkowsky et al [32] were used to validate the proposed methodology. The schematization of the experimental rig used by Wittkowsky et al. is similar to the one below.

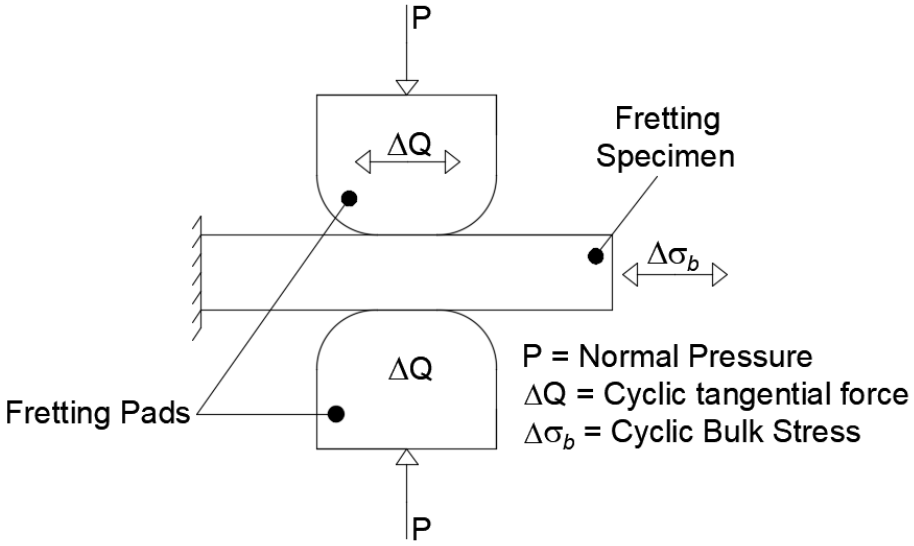

Fig. 4. Schematization of fretting fatigue

The tests were carried out using a pair of spherical fretting pads and a flat-dog-bone specimen, both made of Aluminium Alloy Al 7075-T6 with the following mechanical properties: $\sigma_{UTS} = 572\ MPa,\ \nu = 0.33$ and $E = 72\ GPa$ [32- 33]. The pads were manufactured with a contact radius of 25.4 mm whereas the specimens had a square section of 5 mm x 5 mm. During the tests, the two fretting pads were pushed against the specimens by a constant force P, as well as a fully-reversed CA cyclic tangential force, Q, applied in phase with a fully-reversed CA cyclic bulk stress, σ_b. A number of fretting fatigue tests were run in partial slip, i.e. $Q < f.P$, where the friction coefficient, f, was 1.2. Table 1 reports the experimental results generated by Wittkowsky et al [32] according to the experimental protocol summarized above.

Table 1. Summary of the experimental results generated by Wittkowsky et al., testing specimens of Al 7075-T6 [32].

R_p (mm)	P (N)	Q_{max} (N)	Q_{max}/P	$\sigma_{b,a}$ (MPa)	P_o (MPa)	N_f (Cycles)	$Spec.$ $status$
25.4	13.0	7.0	0.538	83	183	10000000	Run out
25.4	7.3	6.6	0.904	83	151	3450000	Run out
25.4	20.0	16.0	0.800	62.5	211	2190000	Run out
25.4	20.0	15.0	0.750	56	211	1540000	Run out
25.4	20.0	15.0	0.750	63	211	2940000	Run out
25.4	20.0	15.0	0.750	59	211	1780000	Run out
25.4	20.0	15.0	0.750	84.2	211	549000	Failure
25.4	10.3	7.5	0.728	83.6	171	2940000	Failure
25.4	30.0	15.0	0.500	85	241	480000	Failure
25.4	20.8	15.0	0.721	83	214	449000	Failure
25.4	15.6	15.0	0.962	85	194	395000	Failure
25.4	18.5	13.6	0.735	77	206	551000	Failure
25.4	16.0	11.7	0.731	83	196	530000	Failure
25.4	13.9	10.0	0.719	83	187	803000	Failure
25.4	20.0	15.0	0.750	70	211	516000	Failure

To predict the fretting fatigue finite lifetime of the above experiment, the first step was to estimate the linear-elastic stress field damaging the fretting specimen in the vicinity of the contact region. Due to the fact that the above fretting fatigue experiment was generated using standard testing configurations (Fig. 4), the required stress fields were determined according to the analytical framework formulated by Nowell et al. [34]. In more detail, this analytical framework consists of calculating around the contact zone, the sub-surface stresses resulting from the normal load, cyclic tangential load and remote bulk stress. Subsequently, by using the superposition principle, the sub-surface stress tensors were obtained at any given point along the focus path. For example the stress tensor component σ_{xx} at a location (x, y) along the focus path is given by:

$$\sigma_{xx}(x,y) = \sigma_{xx}^{n}\ (x,y) + \sigma_{xx}^{t}(x,y) - \frac{c}{a}\,\sigma_{xx}^{t}(x,y) + \sigma_{b,a} \tag{14}$$

Where:
σ_{xx}^{n} , σ_{xx}^{t} and $\sigma_{b,a}$ are the stresses due to normal load, tangential load and the remote bulk stress amplitude, respectively. The superscripts n and t denote the stresses due to normal and tangential tractions, respectively.

The material properties used to calibrate the MWCM's governing equations are summarized in Table 2. The ultimate tensile strength of 572 MPa, endurance limits, $\sigma_A = 166\,MPa$ and $\tau_A = 95.8\,MPa$ extrapolated at $N_A = 10^7$, were taken from the mechanical fatigue properties of Al 7075-T6 published by Talemi et al. [35]. Because the experimental information required to calculate the negative inverse slopes, k and k_0 was not available, these two constants were estimated according to the following equations [36-38]:

$$k = \frac{log(N_A/N_S)}{log(\sigma_S/\sigma_A)} \quad \text{with } \sigma_S = 0.75 \cdot \sigma_{UTS} \text{ at } N_S = 10^3\ cycles \tag{15}$$

$$k_0 = \frac{log(N_A/N_S)}{log(\tau_S/\tau_A)} \quad \text{with } \tau_S = 0.63 \cdot \sigma_{UTS} \text{ at } N_S = 10^3\ cycles \tag{16}$$

Table 2. Constant values used to calibrate the MWCM's equations and $L_M(N_f)$ vs. N_f.

Material	**σ_{UTS}** *[MPa]*	**σ_A** *[MPa]*	**k**	**τ_A** *[MPa]*	**k_0**	**N_A** *[Cycles]*	**A**	**B**
Al 7075-T6	572	166.0	9.7	95.8	7.0	10^7	169.89	-0.516

To calibrate of the L_M vs. N_f relationship in Eq. (8), ideally the procedure summarized in Fig. 2 should be used. Unfortunately, due to the unavailability of suitable fatigue curves obtained by testing conventional notched specimens, the calibration methodology described in Fig. 2 could not be used in this investigation. Hence, a different approach (summarized in Fig. 5) was used to estimate the constants A and B in the relationship L_M vs. N_f (Eq. 8). This consisted of post-processing the fretting fatigue experiment summarized in Table 1. In more detail, for a given value of the number of cycles to failure from the considered fretting fatigue test, (i.e. $N_f = N_{f,i}$ in Fig. 5c) and the linear-elastic stress field along the focus path, the PM argument [11-12] was used to calculate the distance from the assumed crack initiation point (i.e. point A in Fig. 5a and 5b), $L_M(N_{f,i})/2$. Furthermore, at this distance, the amplitude of the linear-elastic maximum principal stress, $\Delta\sigma_{1,i}$, was equal to the stress applied to generate the fretting fatigue failure at $N_f = N_{f,i}$ cycles. By applying the above strategy, the critical distance value in medium and high-cycle regimes for the considered material was estimated (Fig. 5c). Finally, the constants A and B were calculated by interpolating the values of the distance L_M using the least-squares method. The obtained values of A and B can be seen in Table 2.

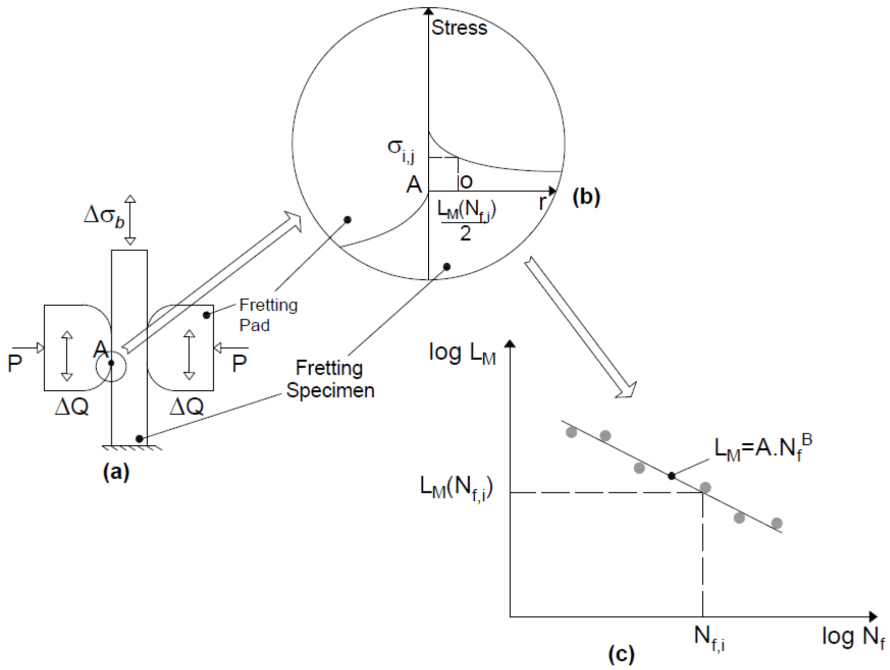

Fig. 5. Calibration of L_M vs N_f by post-processing fretting fatigue experimental results.

To conclude, because of the unavailability of the experimental data in the literature needed to determine the index m in Eq. (1), it was assumed that the Aluminium Alloy Al 7075-T6, was fully sensitive to superimposed static tensile stresses perpendicular to the critical plane (i.e. m equal to unity).

7 Results and discussion

The Figure below shows the observed, N_f versus estimated number of cycles to failure, $N_{f,e}$ for the fretting fatigue experiments (table 1) used to check the overall accuracy of the proposed design methodology. The solid diagonal line in the figure represents a perfect correlation between the observed and estimated lifetimes whereas the dashed lines represent a scattered band defined by a factor-of-two. The errors shown in Fig. 6 demonstrate that, the use of MWCM in conjunction with the PM and τ-MVM (Fig. 3) results in a reasonably good prediction despite the numerous assumptions made to calibrate the L_M vs. N_f relationship and the MWCM equations. It can be clearly anticipated that, the overall accuracy of the proposed design methodology is expected to increase remarkably provided the MWCM equations, and L_M vs. N_f are calibrated experimentally according to the methodology discussed earlier.

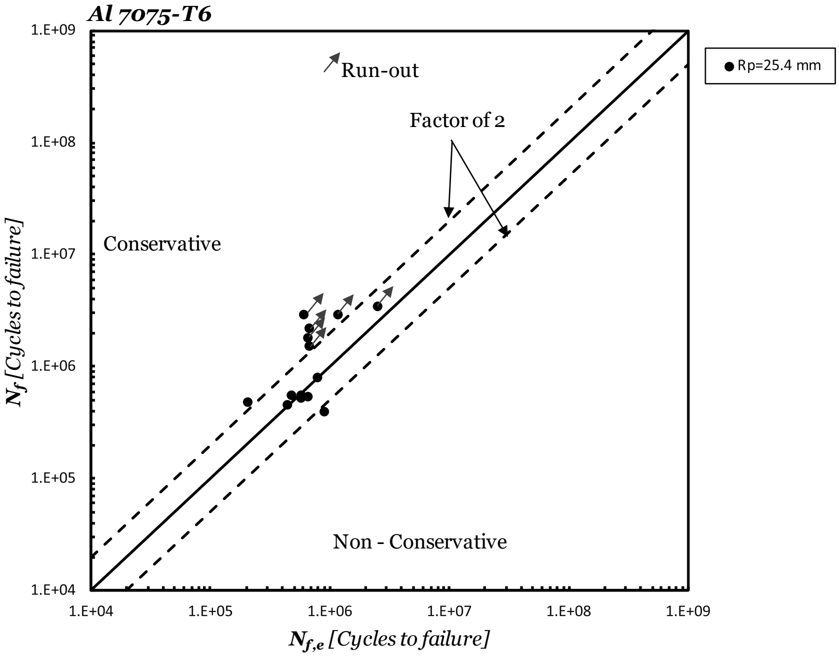

Fig. 6. Overall accuracy of the proposed methodology in estimating fretting fatigue tests reported in Ref. [32].

8 Conclusion

- The proposed methodology is seen to give good estimation of the finite lifetime of mechanical assemblies subjected to CA fretting fatigue loading.
- This approach has the advantage that the MWCM accounts effectively for the presence of superimposed static stresses and also the degree of non-proportionality of the load applied to the assessed material
- The use of τ-MVM to calculate the orientation of the critical plane, is very efficient from a computational point of view. It can therefore minimize the cost associated with the design process.
- The proposed approach is suitable for designers because of its ability to post-process a simple-linear elastic FE model.

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