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CORRIGENDUM

Equal-tailed confidence intervals for comparison of rates

The author would like to acknowledge that the skewness correction for Odds Ratio (OR) is not an entirely new development, having been derived previously (using the efficient score approach) by Gart[1]. The same paper also identified a non-zero asymptotic bias in the score, which explains the deficiency noted for the SCAS method in Section 3.3. The author is very grateful to the Biostatistics Branch of the Division of Cancer Epidemiology & Genetics, National Cancer Institute, USA for their assistance in unearthing that paper. As a result, an improved formulation of the SCAS method for OR is obtained.

Gart's formulation begins by deriving a *z*-statistic (uncorrected z_G) for stratified datasets based on the efficient score:

$$z_G(\theta) = \sum_j \left(X_{1j} - n_{1j}\tilde{p}_{1j}\right) / \tilde{V}_{G\bullet}^{1/2}$$

where $\tilde{V}_{G\bullet} = \sum_j \tilde{V}_{Gj} = \sum_j \left[1/(n_{1j}\tilde{p}_{1j}(1-\tilde{p}_{1j})) + 1/(n_{2j}\tilde{p}_{2j}(1-\tilde{p}_{2j}))\right]^{-1}$, and $\tilde{p}_{1j}$ and $\tilde{p}_{2j}$ are the restricted maximum likelihood estimates of p_1 and p_2 in stratum j for a given θ.

Gart identified the bias of $z_G(\theta)$ as:

$$B_G(\theta) = \tilde{V}_{G\bullet}^{-1/2} \sum_j \left[\tilde{V}_{Gj}^2\left(\tilde{p}_{1j} - \tilde{p}_{2j}\right) / \left(n_{1j}\tilde{p}_{1j}(1-\tilde{p}_{1j}) n_{2j}\tilde{p}_{2j}(1-\tilde{p}_{2j})\right)\right]$$

Since the z-statistic used for SCAS is equivalent to $z_G(\theta)$,[2, p.217] this bias translates to the following corrected formula for the score $S(\theta)$ in Appendix A.3, which applies both with and without stratification:

$$S(\theta) = \frac{(\hat{p}_1 - \tilde{p}_1)}{\tilde{p}_1(1-\tilde{p}_1)} - \frac{(\hat{p}_2 - \tilde{p}_2)}{\tilde{p}_2(1-\tilde{p}_2)} - b(\theta)$$

where the estimated bias $b(\theta) = (\tilde{p}_1 - \tilde{p}_2)/(n_1\tilde{p}_1(1-\tilde{p}_1) + n_2\tilde{p}_2(1-\tilde{p}_2))$

With this modification, the one-sided coverage probability surface is satisfactorily levelled, as demonstrated in the corrected versions of Figure 3, Figure S9, Figure S10, and Table 1.

Table 1 also contained errors in the results for the MOVER-J method, due to using boundary case modifications [3, p.2827] that are not needed for MOVER-J. These affect the results because the MOVER formulae for OR do not produce consistent intervals when the rows of the input data are interchanged, unless $n_1 = n_2$.

TABLE 1: Summary of one-sided moving average % proximate with various sample sizes

	OR				
	30,30	45,15	100,100	150,50	50,150
SCAS	**81.2**	**75.0**	**95.1**	**93.7**	**93.7**
MN	60.8	40.8	76.0	60.4	60.3
MOVER-J	**44.4**	**28.1**	**70.7**	**59.9**	**75.7**
AN	51.4	32.1	72.5	56.6	56.6

The revised SCAS intervals for OR, for the three examples shown in Table B1 on page 348, are (5.192, 949.8), (0.225, 2.101), and (0.759, inf).

The continuity-corrected SCAS method (see supplementary appendix S2 and S3.4) with $\gamma = 0.5$ is over-conservative for OR, but the 'compromise' continuity correction appears to achieve good conservative coverage with $\gamma = 0.125$.

There was also an omission in the evaluation of methods for the single rate (Section 3.5 on page 340). The 'mid-p' method is known to perform very well for achieving proximate and symmetrical coverage[4,5], although it requires iterative calculations. As shown in this updated version of Figure S15, the closed-form SCAS method for the single binomial or Poisson rate performs favourably compared to the corresponding mid-p method, with average RNCP slightly closer to $\alpha/2$.

REFERENCES

[1] Gart JJ. Analysis of the common odds ratio: Corrections for bias and skewness. Bulletin of the International Statistical Institute **1985**, 45th Session, Book 1, 175-176.

[2] Miettinen OS and Nurminen M. Comparative analysis of two rates. Statistics in Medicine **1985**, 4:213–226.

[3] Fagerland MW and Newcombe RG. Confidence intervals for odds ratio and relative risk based on the inverse hyperbolic sine transformation. Statistics in Medicine **2013**, 32 2823–2836.

[4] Newcombe RG. Two-sided confidence intervals for the single proportion: comparison of seven methods. Statistics in Medicine **1998**, 17, 857-872.

[5] Newcombe RG. Measures of Location for Confidence Intervals for Proportions. Commun. Stat. - Theory Methods **2011**, 40(10), 1743.