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# An improved $k-\omega$ turbulence model for the simulations of the wind turbine wakes in a neutral atmospheric boundary layer flow

Ioannis Bouras, Lin Ma, Derek Ingham, Mohamed Pourkashanian

Energy 2050, Faculty of Engineering, The University of Sheffield, S10 2TN, England

## Abstract

Correct prediction of the recovery of wind turbine wakes in terms of the wind velocity and turbulence downstream of the turbine is of paramount importance for the accurate simulations of turbine interactions, overall wind farm energy output and the impact to the facilities downstream of the wind farm. Conventional turbulence models often result in an unrealistic recovery of the wind velocity and turbulence downstream of the turbine. In this paper, a modified  $k-\omega$  turbulence model has been proposed together with conditions for achieving a zero streamwise gradient for all the fluid flow variables in neutral atmospheric flows. The new model has been implemented in the simulation of the wakes of two different wind turbines and the commonly used actuator disk model has been employed to represent the turbine rotors. The model has been tested for different wind speeds and turbulence levels. The comparison of the computational results shows good agreement with the available experimental data, in both near and far wake regions for all the modeled wind turbines. A zero streamwise gradient has been maintained in the far wake region in terms of both wind speed and turbulence quantities.

## 1. Introduction

Large Eddy Simulation (LES), with the advances in computational power, is being more and more popular and is employed mainly in academia. Many researchers such as Goodfriend et al. (2015), Porte – Agel et al. (2011), Churchfield et al. (2012) have employed LES to simulate successfully the neutral atmospheric boundary layer as well as the wind turbine wakes. However, despite the enormous advances in Computational Fluid Dynamics (CFD) techniques in recent years, RANS simulations still dominate the simulations in many engineering applications, especially in industry.

Accurate simulations of the atmospheric boundary layer (ABL) flows is still a challenge, in particular when the focus is on the flow over manmade structures such as wind turbines, where large differences in the length scales are considered. The difficulty in simulating a homogeneous ABL with RANS has been widely reported (Richards and Hoxey, 1993; Blocken et al., 2007; Franke et al., 2007; Hargreaves and Wright, 2007; Yang et al., 2009; O’Sullivan et al. 2011; Yan et al. 2016). Since the ABL can be as high as 1km and there is no boundary in the streamwise and spanwise directions, in the computational modeling of the flow over a structure, e.g. a wind turbine, reasonable distances from the region of interest have to be taken in order to reduce the computational time and efforts, and assumptions in the

35 conditions at the boundaries of the computational domain have to be made which can be inconsistent  
36 with the physics of the ABL flow. As a result, when the RANS approach is employed with conventional  
37 turbulence models, undesirable streamwise gradients of the primitive variables and turbulence  
38 quantities occur primarily due to the inconsistencies in the turbulence model with the boundary  
39 conditions employed.

40 In order to satisfy the flow conditions of a neutrally stratified ABL, the upstream and downstream  
41 boundaries of the computational domain should be assumed to have the same flow characteristics  
42 regarding the ground roughness and friction velocity, so that the ABL is fully developed at the  
43 downstream boundary and consistent with the prescribed inlet flow conditions. Any streamwise  
44 gradient of any variable is undesirable when compared to the flow conditions at the upstream and  
45 downstream boundaries. For the upper boundary of the computational domain, since the wind flow is  
46 driven by geostrophic winds, the imposition of a zero stress boundary condition at the upper boundary  
47 of the solution computational domain is not, theoretically, an appropriate choice.

48 Richards and Hoxey (1993) proposed a shear stress boundary condition together with a set of inlet flow  
49 profiles and they successfully simulated the neutral ABL without any undesirable streamwise gradients  
50 in their solutions. Their model is mathematically consistent, and the implementation of this model in the  
51 commercial CFD software ANSYS CFX and FLUENT by Hargreaves and Wright (2007) was successful in  
52 achieving a zero streamwise gradient by slightly modifying the standard grain sand rough wall function  
53 and the inclusion of a momentum source on the upper layer of cells of the computational domain.  
54 Furthermore, Blocken et al. (2007) have suggested 4 basic requirements for the homogeneity of the ABL  
55 and proposed some remedial measures to mitigate the problem with the inconsistency of the inlet  
56 profiles with the wall functions employed in the commercial CFD software FLUENT and CFX. Also, they  
57 used essentially a Dirichlet boundary condition at the upper boundary of the solution domain by directly  
58 specifying the values of the velocity and turbulence. This method recovers, to some extent, the desirable  
59 profiles of the velocity and turbulence quantities but it has the drawback that it does not allow mass to  
60 enter or exit the upper boundary (O'Sullivan et al., 2011) which is not ideal. Yang et al. (2009) used a  
61 dissipating profile for the turbulent kinetic energy with the height based on laboratory experimental  
62 data and they implemented them in the commercial CFD software FLUENT and their computational  
63 results have shown good agreement with their experimental data. Parente et al. (2011) modified the  
64 standard  $k - \epsilon$  turbulence model by adding source terms for the turbulent kinetic energy and turbulent  
65 dissipation rate to allow flexibility on the imposed profiles as in the Richards and Hoxey approach of a  
66 steady value for the turbulent kinetic energy was a rough approximation of the neutral ABL (Richards  
67 and Norris, 2015). O'Sullivan et al. (2011) performed an error analysis on the profiles of the velocity  
68 magnitude, turbulent kinetic energy and eddy dissipation rate which are produced by the inconsistent  
69 boundary conditions employed and they proposed an extension to the shear stress boundary condition  
70 on the upper boundary of the domain based on the profiles for turbulent kinetic energy and eddy  
71 dissipation rate generated by Yang et al. (2009). Their results showed improvement by minimizing any  
72 streamwise gradients for both Yang et al. (2009) and Richards and Hoxey (1993) profiles and proven that  
73 regardless of the type of the boundary condition at the upper boundary the increased height of the  
74 computational domain can decrease the errors.

75 The importance of accurate predictions of the homogeneous ABL is related to with various applications,  
76 such as pollutant dispersion and meteorological models (Mokhtarzadeh et al., 2012, Juretic and Kozmar,  
77 2013). By summarizing various papers, Tominaga et al. (2008) made some recommendations for the  
78 simulations of flows around buildings regarding the inlet conditions, the turbulence models, the  
79 boundary conditions, as well as the appropriate domain size, while the type of the zero streamwise  
80 gradient condition does not appear to play any role due to the strong velocity gradients and  
81 consequently, high turbulence generation.

82 Research on achieving the streamwise gradient condition for the simulations of the wind turbine wakes  
83 has not been fully investigated. The importance of the zero streamwise gradient condition, along with  
84 the correct recovery in the very far wake region, for the simulation of the wind farms, is of paramount  
85 importance. This is because the velocity and turbulence of the first turbine become the inlet for the  
86 turbines at the rear of the first turbine. Consequently, failure in achieving the streamwise gradient  
87 condition, depending on the consistency of the employed model with the inlet values and boundary  
88 conditions, may have disastrous consequences in the predicted power output of the wind farm as well  
89 as in the structural damage of the wind turbines.

90 There are many researchers who have noticed the problems of modeling flow and turbulence behind  
91 the wind turbines. Prospathopoulos et al. (2010) modeled 2 wind farms, one on a flat terrain and  
92 another on a complex terrain for various wind directions, in neutral atmospheric conditions, using the  
93 actuator disk approach. They applied the  $k - \omega$  turbulence model with the Boussinesq eddy viscosity  
94 assumption, as well as another definition of the eddy viscosity, which is based on the Durbin correction  
95 (1996), to show the differences in the power production with the conventional and the modified  
96 definition of the eddy viscosity for both types of terrain. Cabezon et al. (2010) simulated a 43 wind  
97 turbine wind farm on a complex terrain with the wake model CFDWake 1.0 in order to validate and  
98 compare their results with the available experimental data. Makridis and Chick (2013) used the  
99 guidelines of Blocken et al. (2007) to simulate a wind turbine with the actuator disk model over a  
100 complex terrain as well as a small coastal wind farm and compared their results with experimental data.  
101 They used the commercial CFD software FLUENT and in order to take into account the anisotropy of the  
102 atmospheric turbulence, they used the RSM model. Castellani and Vignaroli (2013) also applied the  
103 actuator disk technique for a small wind turbine using the CFD code Phoenics and the comparison of  
104 their results with the available experimental data was generally good, however, no discussion was  
105 presented on the zero streamwise gradient condition. Simisiroglou et al. (2016) modeled various large  
106 horizontal axis wind turbines using the commercial CFD software PHOENICS. They made a few  
107 parametric studies based on the convergence criteria, the turbulence model, the grid resolution and the  
108 actuator disk thickness. They validated their results with the thrust and power curve for one of the  
109 turbines they used. However, in the absence of experimental data for the wake region, they used results  
110 from large eddy simulations for validation. Similarly to Makridis and Chick (2013), Nedjari et al. (2017)  
111 examined the actuator disk model with the standard  $k - \epsilon$  model on a flat and a complex terrain and  
112 validated their results with experimental data. The validation of the model with experimental data was  
113 very good in the near or far wake region, however in the very far wake region the normalized velocity  
114 appears to recover to approximately 85% of the inlet velocity and remains at this value until the outlet.

115 Also, no results for the turbulent kinetic energy were shown. It is characteristic that none of the above  
116 researchers performed any simulations of an empty domain in order to show the changes of their inlet  
117 conditions on the velocity, turbulent kinetic energy and eddy dissipation rate within the domain.

118 Many researchers, such as Kasmi and Masson (2008) and Simisiroglou et al. (2016) have shown that 2  
119 equation turbulence models fail to predict the velocity and turbulence quantities in the near or the far  
120 wake regions of the wind turbine. Kasmi and Masson (2008) proposed a remedy to this problem by  
121 adding a source term in the region of the turbine in the equation for the eddy dissipation of the  
122 standard  $k - \epsilon$  model, based on the work done by Chen and Kim (1987). Their proposed model showed  
123 significant improvement in predicting the velocity downstream of the turbine over the standard  $k - \epsilon$   
124 model when comparing their results with experimental data for 3 wind turbines, however, no  
125 quantification of their results has been reported. Recently, El - Askary et al. (2017) have implemented  
126 Kasmi and Masson (2008) model and achieved some improvement of the results when compared to  
127 experimental results. This can partially be explained by the fact that Kasmi and Mason (2008) have also  
128 included the nacelle in their simulations while El - Askary et al. (2017) have not included it. Also, Kasmi  
129 and Masson (2008) added 2 extra terms in the transport equations of the  $k - \epsilon$  equation while El -  
130 Askary et al. (2017) have not used them. However, these 2 extra terms in the transport equations of the  
131  $k - \epsilon$  model violate the zero streamwise gradient condition. Finally, Kasmi and Masson (2008) simulated  
132 3 different wind turbines but with the same relative inlet turbulence levels, and therefore it is unknown  
133 how their model will perform for different relative inlet turbulence levels.

134 The standard  $k - \epsilon$  model has the theoretical advantage of being suitable for free shear fully turbulent  
135 flows, which is the case for this application, so it is the most obvious model to use. However, one of its  
136 most important weaknesses is its lack of sensitivity to adverse pressure gradients (Menter, 1994). On  
137 the other hand, the standard  $k - \omega$  model is suitable for wall bounded flows and for flows where adverse  
138 pressure gradients occur. Although there are no strong adverse pressure gradients involved for the wind  
139 turbine wakes, there is a small increase in the pressure upstream and downstream of the turbine at the  
140 hub - height, a fact which makes the standard  $k - \omega$  model, theoretically, the optimal solution for this  
141 application. Finally, the modification of Chen and Kim (1987), which is employed around the wind  
142 turbine in the Kasmi and Masson (2008) model, is highly dependent on the relative turbulent kinetic  
143 energy of the field in the standard  $k - \epsilon$  model, while in the standard  $k - \omega$  model is independent. Details  
144 are presented later in theory section.

145 In this paper, the 3D Reynolds Averaged Navier - Stokes equations are solved with the standard  $k - \omega$   
146 turbulence model to examine an empty domain for a neutrally stratified atmospheric boundary layer. An  
147 equation for the zero streamwise gradient condition is proposed by solving the transport equations for  
148 the standard  $k - \omega$  model, and simulations have been performed for various turbulence levels.  
149 Validation of the results is based on theoretical values for a neutral atmosphere proposed by Richards  
150 and Hoxey (1993). Then, the model is applied to the simulations of wind turbine wakes with a small  
151 modification in the transport equation of the specific dissipation rate based on the work performed by  
152 Chen and Kim (1987) in the region around the wind turbine. The rotor of the wind turbine is modeled  
153 using the actuator disk approach based on the blade element theory and 2 small wind turbines are  
154 simulated for various inlet velocity and turbulence levels. The model performs well in both near and far

155 wake regions and the properties of the neutral atmosphere are recovered to the undisturbed inlet  
156 conditions far away downstream of the wind turbine. The simulations were performed with the  
157 commercial CFD software FLUENT and the grid generation in the software ICEM.

## 158 **2. Modifications to the standard $k - \omega$ model**

159 For a neutral atmospheric boundary layer flow, the following assumptions can be made for a flat empty  
160 computational domain, see Richards and Hoxey (1993):

- 161 (a) The vertical velocity is zero throughout the domain
- 162 (b) The pressure is constant throughout the domain
- 163 (c) The shear stress is constant throughout the domain, being independent of the height and it is  
164 given by:

$$\tau_0 = \rho u_*^2 \quad (1)$$

165

166 where  $\rho$  is the density of the air, which is considered as a constant throughout the atmospheric  
167 boundary layer and  $u_*$  is the friction velocity.

168 The profiles for the velocity, turbulent kinetic energy and eddy dissipation rate, respectively are as  
169 follows:

$$U_{(y)} = \frac{u_*}{\kappa} \ln\left(\frac{y + y_0}{y_0}\right) \quad (2)$$

170

$$k = \frac{u_*^2}{\sqrt{C_\mu}} \quad (3)$$

171

$$\varepsilon_{(y)} = \frac{u_*^3}{\kappa(y + y_0)} \quad (4)$$

172

173 where  $U_{(y)}$  and  $\varepsilon_{(y)}$  is the velocity magnitude and the eddy dissipation rate, respectively, as a function  
174 of the height,  $y_0$  is the roughness length of the ground and  $\kappa$  is the von Karman constant.  $k$  is the  
175 turbulent kinetic energy.

176 The assumption of a constant value of the turbulent kinetic energy throughout the domain has been  
177 criticized by some researchers, such as Yang et al. (2009), Parente et al. (2011) and Richards and Norris  
178 (2015). However, the turbulent kinetic energy appears to have an almost steady value for the first 100  
179 meters within the ABL (Juretic and Kozmar, 2013), and it dissipates further away with the height and  
180 reaches a value of approximately 5% of the value that it has close to the ground at the height of the ABL  
181 (Allaerts and Mayers, 2015). Also, most researchers, such as Kasmi and Mason (2008), Prospathopoulos

182 et al. (2010), Cabezon et al. (2010), Makridis and Chick (2013) and Simisiroglou et al. (2016) used a  
 183 steady value for the turbulent kinetic energy at the inlet of the domains in order to simulate the wake  
 184 region around a wind turbine with the actuator disk model. The assumption of a constant value of the  
 185 turbulent kinetic energy is a good approximation for the simulations of small wind turbine wakes since,  
 186 in many cases, for economic issues, experimental data are measured only at the hub – height at various  
 187 locations upstream or downstream of the turbine, although, as explained earlier, it is not consistent with  
 188 the neutral ABL.

189 Richards and Hoxey (1993) discovered a condition for the standard  $k - \epsilon$  model that satisfies the  
 190 equations (2) – (4). In a similar way, a condition for the elimination of the streamwise gradients for any  
 191 variable in the standard  $k - \omega$  (Wilcox, 1988) model can be found.

192 The formulation of the standard  $k - \omega$  model (Wilcox, 1988) is given as follows (see FLUENT Theory  
 193 Guide (2011)):

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_i} \right] + G_k - Y_k + S_k \quad (5)$$

194

$$\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_i}(\rho \omega u_i) = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_i} \right] + G_\omega - Y_\omega + S_\omega \quad (6)$$

195

196 The eddy viscosity is defined as:

$$\mu_t = a^* \frac{\rho k}{\omega} \quad (7)$$

197

198 where

$$a^* = a_\infty^* \left( \frac{\frac{\beta_i}{3} + \frac{\mu \omega}{6}}{1 + \frac{\mu \omega}{6}} \right) \quad (8)$$

199

200 The equations (5) and (6), on taking into account the fact that the flow in an empty domain is essentially  
 201 one dimensional and time independent, there are no buoyancy or compressibility effects, and the  
 202 turbulent kinetic energy is constant for any direction within the domain, may be simplified as follows:

$$0 = G_k - Y_k \quad (9)$$

203

$$0 = \frac{\partial}{\partial y} \left[ \left( \mu_t + \frac{\mu_t}{\sigma_k} \right) \frac{\partial \omega}{\partial y} \right] + G_\omega - Y_\omega \quad (10)$$

204

205 Finally, the connection between the eddy dissipation rate and the specific dissipation rate (or eddy  
206 frequency) is given by (Wilcox, 1988):

$$\omega = \frac{\varepsilon}{k\beta_{\infty}^*} \quad (11)$$

207

208 By making some mathematical calculations, it can be easily concluded that the equations (2) – (4) satisfy  
209 automatically the equation (5) but satisfy the equation (6) only if the following expression is satisfied:

$$\frac{1}{\sigma_{\omega}\sqrt{\beta_{\infty}^*}} + \frac{1}{\kappa^2} = \frac{\beta_i}{\beta_{\infty}^*\kappa^2} \quad (12)$$

210

211 Therefore, to achieve a zero streamwise gradient, equation (12) must be satisfied and it is independent  
212 of the friction velocity, the height of the domain or the roughness of the ground, in a similar way as that  
213 is employed in the standard  $k - \varepsilon$  model (Richards and Hoxey, 1993).

214 The constant  $\beta_{\infty}^*$  is defined by the existent turbulence levels in the field (equation (3)). Instead of the  
215 coefficient  $C_{\mu}$  that the standard  $k - \varepsilon$  model uses, the turbulence levels for the standard  $k - \omega$  model are  
216 defined by:

$$k = \frac{u_*^2}{\sqrt{\beta_{\infty}^*}} \quad (13)$$

217

218 Consequently, for specific turbulence levels, which are defined by the coefficient  $\beta_{\infty}^*$ , the constants  $\sigma_{\omega}$   
219 and  $\beta_i$  have to be chosen accordingly in order to satisfy the expression (12) in order to avoid streamwise  
220 gradients for any variable within the solution domain.

221 Finally, the following consideration was taken in order to conclude to the expression (12):

$$\mu_l \ll \mu_t \quad (14)$$

222

223 i.e. the laminar viscosity was omitted from the transport equations for turbulent kinetic energy and  
224 specific dissipation rate. The error in using this simplification is expected to be negligible since the flow  
225 is highly turbulent.

226 As discussed previously, since 2 equation turbulence models fail to predict the velocity and turbulence  
227 quantities in the near or the far wake regions of the turbine, Kasmir and Masson (2008) proposed a  
228 remedy to this problem by adding a source term in the vicinity of the turbine in the equation for the  
229 eddy dissipation in the standard  $k - \varepsilon$  model, and this is based on the work performed by Chen and Kim  
230 (1987). This source term is described by the following formula:

$$S_\varepsilon = C_{\varepsilon 4} \frac{G_k}{\rho k} \quad (15)$$

231

232 The coefficient  $C_{\varepsilon 4}$  was set at the default value of 0.25. The main idea behind this source term is the fact  
 233 that the equation for the eddy dissipation rate for the family of the  $k - \varepsilon$  models is empirical, and  
 234 therefore there are many applications that the standard  $k - \varepsilon$  model fails to accurately predict the flow  
 235 (e.g. the backward facing step, swirling flow problems etc.) and gives highly diffusive results. Therefore a  
 236 second time scale (equation (15)) is added to the eddy dissipation equation to represent the energy  
 237 transfer from the large to the small scales more effectively. In particular, the energy transfer from the  
 238 large scales to the small ones is controlled by the production range scale and the dissipation rate time  
 239 scale (Chen and Kim, 1987). Consequently, Chen and Kim (1987) added a second time scale in the eddy  
 240 dissipation equation of the standard  $k - \varepsilon$  model and they found a significant improvement for a wide  
 241 range of engineering applications.

242 Although this term was designated to be used in the family of  $k - \varepsilon$  models, it appears that it improves  
 243 the results in the standard  $k - \omega$  model as will be shown later. As Kasmi and Masson (2008) showed that  
 244 the standard  $k - \varepsilon$  model overestimates the turbulent kinetic energy for the wind turbine wakes, the  
 245 same applies for the standard  $k - \omega$  model. This may be explained by the fact that Wilcox (2006) used a  
 246 slightly different version of his initial  $k - \omega$  model by adding a cross diffusion term in the specific  
 247 dissipation rate equation along with a stress limiter modification to the definition of the eddy viscosity,  
 248 as many researchers have shown improved results of this version.

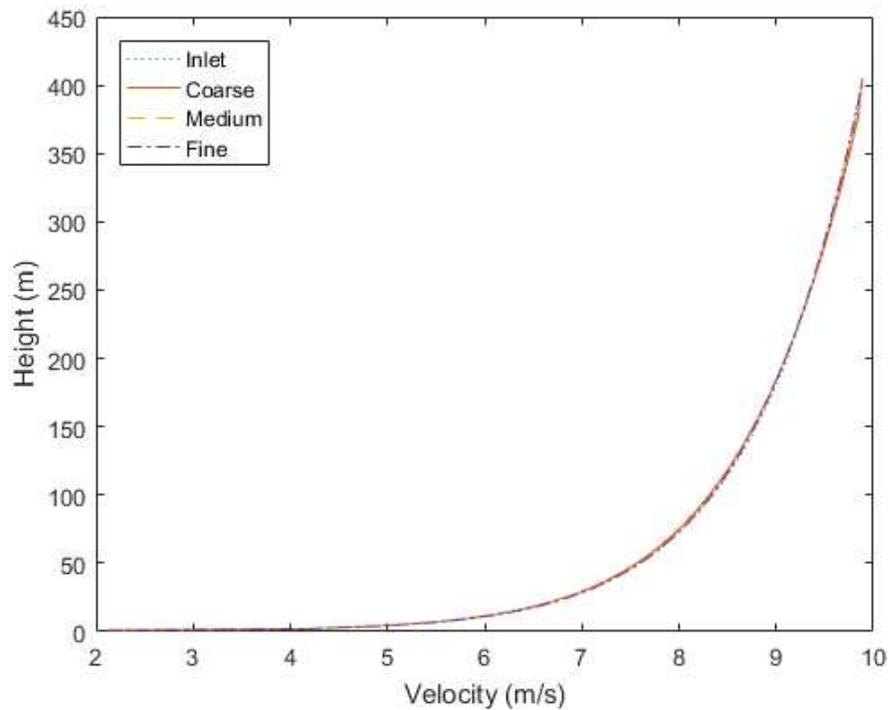
249 The most important theoretical advantage of the  $k - \omega$  model, in relation to the  $k - \varepsilon$  model, is that it  
 250 does not include any constant in the definition of the eddy viscosity. In fact, in the standard  $k - \varepsilon$  model  
 251 the production term ( $G_k$ ) that is included in equation (15) includes the eddy viscosity which depends  
 252 highly on the constant  $C_\mu$  which defines the turbulence levels of the field. However, in the  $k - \omega$  model  
 253 there is no  $C_\mu$  constant (or  $\beta_\infty^*$  as the turbulent kinetic energy in the family of  $k - \omega$  models is defined by  
 254 the coefficient  $\beta_\infty^*$  in the neutral atmosphere as described earlier) so the model is independent of the  
 255 relative to the velocity turbulent kinetic energy.

### 256 **3. Examination of the empty domain**

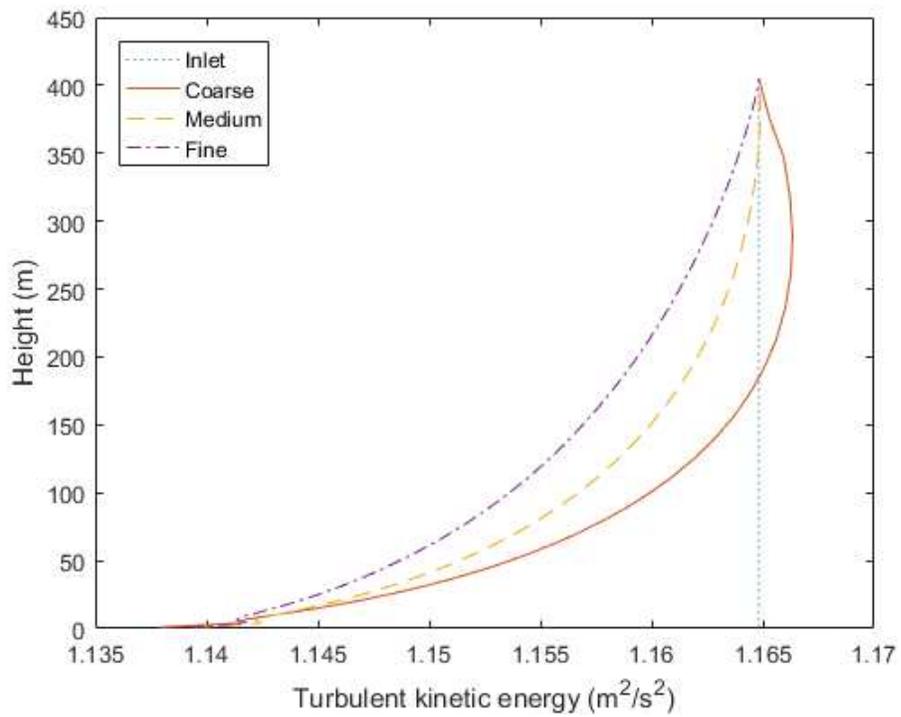
257 In order to validate the modified  $k - \omega$  model and check if the zero streamwise gradient of the fluid flow  
 258 properties can be maintained, simulations have been performed for an ABL flow throughout an empty  
 259 domain. The dimensions of the computational domain employed are 10,000m, 405m and 50m in the x,y  
 260 and z directions, respectively. The y direction refers to the height of the domain from the ground. The  
 261 10km length of the domain has been selected in order to make sure that the flow will be fully developed  
 262 within this long domain while the 405m height has been selected because it is considered as an  
 263 adequate height for the simulation of any small or medium size wind turbine. Finally, a very short  
 264 distance in the spanwise direction was selected because there are no gradients for any variable in this  
 265 direction. A velocity inlet boundary condition was imposed at the inlet of the domain based on the  
 266 equations (2) – (4). The friction velocity of the wind flow is  $u_* = 0.46m/s$  and the roughness length is

267 0.05m, which is valid for a relatively low roughness terrain. A value of  $\beta_{\infty}^* = 0.033$  is used to define the  
268 turbulence levels at the inlet of the domain based on Panofsky and Dutton (1984) as well as other  
269 researchers, such as Makridis and Chick (2013) and Kasmi and Masson (2008). Regarding the rest  
270 boundary conditions, a pressure outlet boundary was imposed at the outlet, a symmetry (or zero  
271 gradients) at the lateral sides of the domain and a Dirichlet boundary condition at the upper boundary  
272 based on the equations (2) – (4).

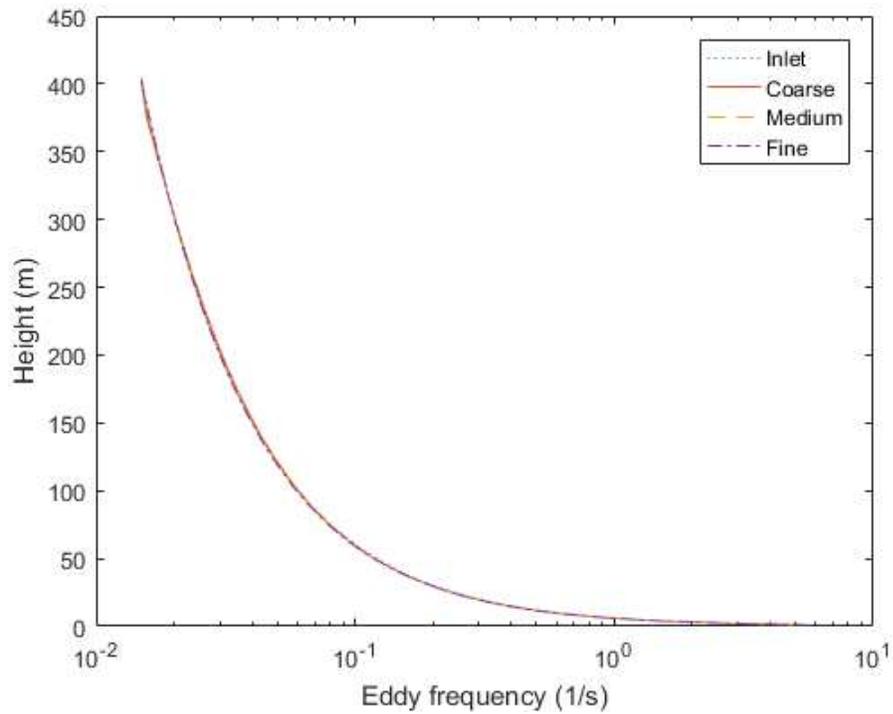
273 The third order MUSCL scheme was used for the discretization of the momentum equations and the  
274 second – order upwind scheme for the transport equations for the turbulent kinetic energy and specific  
275 dissipation rate and the SIMPLE algorithm was implemented for the pressure velocity coupling, while  
276 the convergence criteria were set to  $10^{-7}$  for all the equations and this was found to be small enough to  
277 obtain graphically indistinguishable results. Mass imbalance has also been checked to make sure that all  
278 simulations have converged. Finally, regarding the grid resolution, 3 different grid sizes have been  
279 employed consisting of approximately 200,000, 600,000 and 1,800,000 elements. The numerical grids  
280 were fully structured and the refinement of the grid has been equally done in all directions.



281 (a)



282 (b)



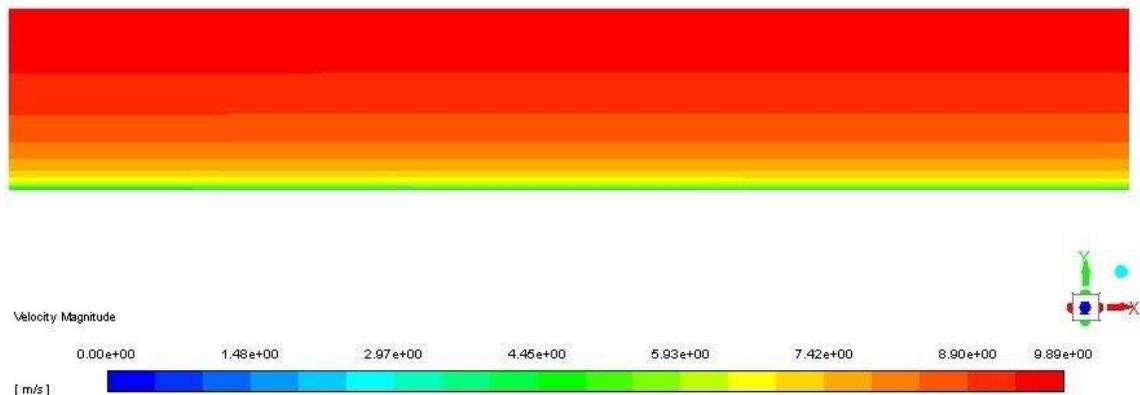
283 (c)

284 Figure 1: Comparison of (a) velocity, (b) turbulent kinetic energy, and (c) specific dissipation rate between the inlet  
 285 and outlet in a 10km domain for 3 different grid sizes.

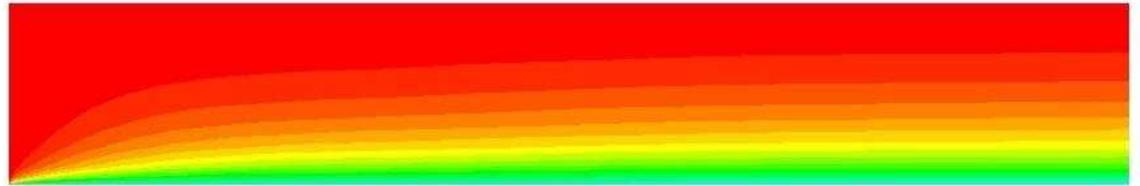
286 Figure 1 compares the solutions for the velocity, turbulent kinetic energy and specific dissipation rate,  
 287 respectively, for an empty domain at the inlet and outlet of the domain with various different grid sizes.

288 Due to the rapid change of the eddy frequency with the height, the logarithmic scale is used in Figure 1  
 289 (c), as well as in the contour map in Figure 2 (c). An error analysis showed that the difference between  
 290 the inlet and outlet for the turbulent kinetic energy is approximately 2% on the ground, for any grid size  
 291 and it decreases with the height. Figure 2 illustrates contour maps of the velocity, turbulent kinetic  
 292 energy and eddy frequency to show the development of these variables within the domain. The height  
 293 of the domain was scaled up 4 times due to its initial small perpendicular to the ground direction, in  
 294 relation to the length of the domain. A similar situation exists for the eddy frequency where the error  
 295 appears to reach an error of approximately 4% close to the ground and it becomes gradually smaller  
 296 with the height. Finally, regarding the velocity, it appears to have an error of approximately 2% close to  
 297 the ground but it becomes less than 1% within the first 10m from the ground. There are 2 reasons for  
 298 the errors close to the ground for any variable. The first reason is due to the wall formulation which is  
 299 not consistent with the profiles of the equations (2) – (4) and it appears that the calculation of the  
 300 turbulence quantities is a function of the friction velocity (Ansys FLUENT, 2011). Another reason is  
 301 attributed to the assumption of the negligence of the laminar viscosity (equation (14)) which is not valid  
 302 on the ground. However, the differences are in general small, and it can be concluded that the velocity,  
 303 turbulent kinetic energy and specific dissipation rate are maintained from the inlet to the outlet of the  
 304 domain with a good accuracy. Moreover, parametric studies based on the friction velocity from 0.4 –  
 305 0.62 m/s and turbulence levels for values of  $\beta_{\infty}^*$  from 0.033 to 0.1 showed small dependence and the  
 306 comparison of the results with the theoretical values based on the equations (2) – (4) was similar to the  
 307 ones present in Figure 1. The small errors far away from the ground are attributed to the simplifications  
 308 that have been made in theory, numerical and convergence issues. Finally, the results show negligible  
 309 sensitivity to the grid size and this is due to the simplicity of the geometry. In particular, the maximum  
 310 differences between the coarse and medium sized grid for the velocity, turbulent kinetic energy and  
 311 eddy frequency were 0.12%, 0.16% and 0.72%, while the maximum differences in the same variables  
 312 between the medium sized grid and fine grid were 0.06%, 0.11% and 0.57%, respectively, and  
 313 consequently the numerical grid consisting of 600,000 elements has been used.

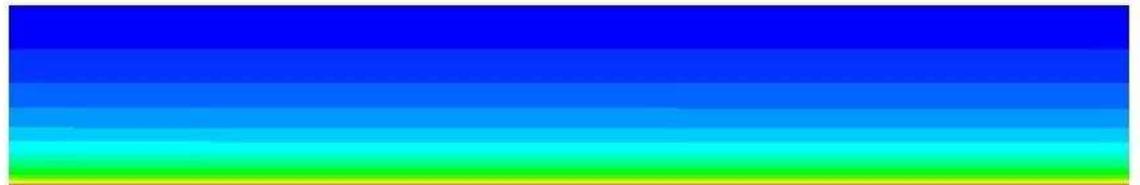
314



315 (a)



316 (b)



317 (c)

318 Figure 2: Results of (a) velocity, (b) turbulent kinetic energy and (c) eddy frequency along the domain.

319 Most researchers who studied the characteristics of wind turbine wakes did not examine the zero  
 320 streamwise gradient condition. It appears, although it has not been proven, that it is not important  
 321 when a single turbine is examined due to the fact that the undisturbed wind conditions do not change  
 322 significantly within a few characteristic lengths of the domain when the zero streamwise gradient  
 323 condition is not satisfied. However, when a large domain is examined with multiple wind turbines in any  
 324 arrangement, it is of paramount importance that the velocity and turbulence levels have a correct  
 325 recovery and, in the long run, recover to the undisturbed inlet conditions and be maintained as happens  
 326 in nature.

327 In the next section, 2 different small wind turbines are examined and the importance of the zero  
 328 streamwise gradient for all variables is illustrated.

#### 329 **4. Modeling of a single wind turbine using the actuator disk theory**

330 A full – scale detailed aerodynamic simulation of a wind turbine is very time consuming since it requires  
 331 a transient simulation as well as a very refined numerical grid around the blades, the nacelle, the tower  
 332 of the wind turbine etc. Consequently, many other computationally cheaper ways of simulating the wind  
 333 turbine wakes have been developed. The simplest model is the actuator disk model without rotation  
 334 and is based on the blade element method.

335 Mikkelsen (2003) has analyzed many models for the modeling of the rotor of the wind turbines. The  
336 simplest of all models, when the aerodynamics of the wind turbine is unknown, is the actuator disk  
337 model without rotation and based on the thrust coefficient ( $C_T$ ) of the turbine. The pressure drop  
338 through the wind turbine can be calculated by the following equation:

$$\Delta P = 0.5\rho AC_T U_\infty^2 \quad (16)$$

339

340 where  $A$  is the rotor disk area and  $U_\infty$  is the undisturbed wind velocity upstream of the turbine. The only  
341 information that is needed is the thrust coefficient and the diameter of the wind turbine.

#### 342 **4.1 Nibe – B 630kw turbine**

343 The first wind turbine that is examined is a Nibe – B 630kw turbine and this is a horizontal 3 bladed wind  
344 turbine operating at 33rpm with a 40m diameter at 45m hub – height. In the simulations performed in  
345 this paper, the actuator disk model without rotational effects was employed.

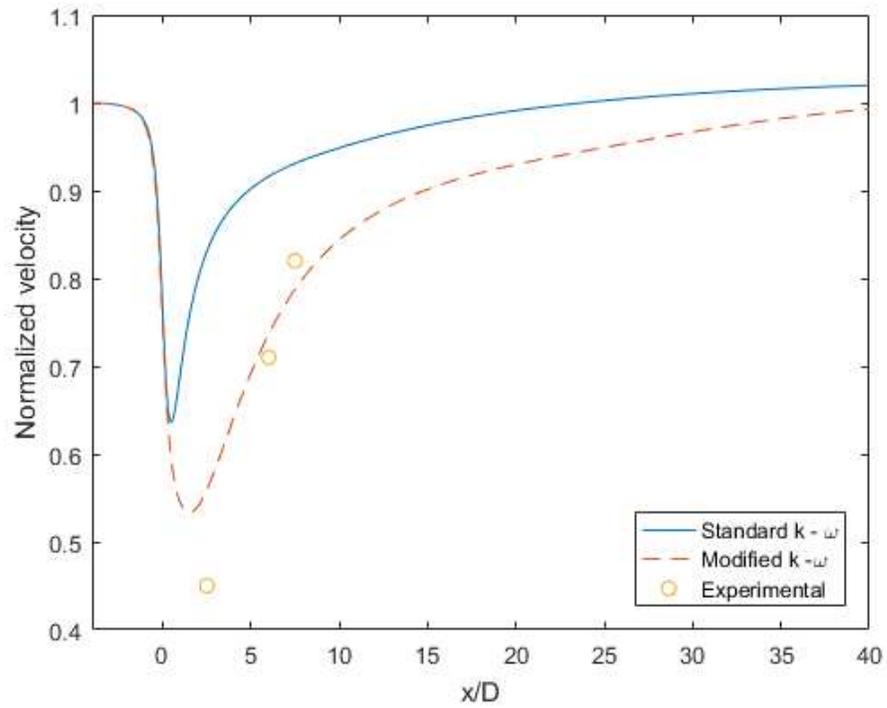
346 Regarding the size of the computational domain, the distance from the inlet to the turbine is 4D, the  
347 distance from the turbine to the outlet is 40D, the distance from the turbine to the upper boundary is  
348 5D and the distance between the turbine and the lateral sides of the domain is 4D, where D is the  
349 diameter of the wind turbine. The boundary conditions were the same as in the empty domain  
350 examined earlier along with the other settings of the solver. The pressure drop along the wind turbine  
351 was calculated from the equation (16).

352 As stated in theory, the condition (12) must be satisfied in order to ensure the recovery of the velocity  
353 and turbulence quantities in the far wake region. The zero streamwise gradient condition is important in  
354 the far wake region, however, the recovery of the velocity and turbulence are highly sensitive on the  $\sigma_\omega$   
355 coefficient. By performing some parametric studies, a value of  $\sigma_\omega = 1.3$  was chosen as the optimum  
356 coefficient for all wind turbines. Given a coefficient of  $\beta_\infty^* = 0.033$  for the definition of the turbulence  
357 levels and a value of  $\sigma_\omega = 1.3$ , the value of  $\beta_i = 0.0575$  satisfies the equation (12). The Von Karman  
358 constant that is used is  $\kappa = 0.4187$ . The standard k –  $\omega$  model also has been employed to illustrate the  
359 differences between the 2 models against the experimental data. The only modification that has been  
360 done to the standard k –  $\omega$  model is the coefficient  $\beta_\infty^*$  and it has been given the same value as in the  
361 modified k –  $\omega$  model in order to match the inlet turbulent kinetic energy at the inlet of the domain  
362 (equation (12)).

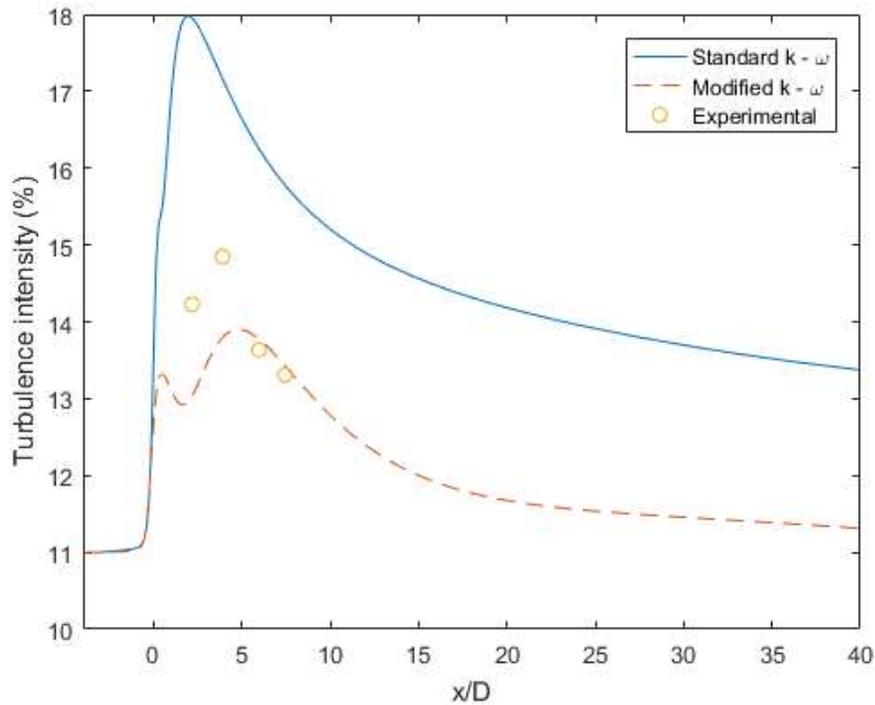
363 A grid independence study has been carried out. Given the simplicity of the geometry, the requirement  
364 of the simulation regarding its number of cells was not very demanding. 3 different grid sizes have been  
365 simulated consisting of approximately 140,000, 600,000 and 1,560,000 cells. All of them were fully  
366 structured numerical grids and the refinement from the coarse to the fine grid has been done  
367 everywhere in the domain but mainly in the region around the wind turbine and at a few characteristic  
368 lengths upstream and downstream of it. The maximum difference between the 2 coarser numerical  
369 grids was found to be approximately 2.5% for the velocity and 3.5% for the turbulent kinetic energy,  
370 while the maximum difference in the results obtained using the 2 finer grids were less than 0.2% for the  
371 velocity and less than 0.5% for the turbulent kinetic energy. Consequently, the numerical grid consisting

372 of 600,000 elements was employed and a similar grid has been created with a similar number of cells  
373 and spacing between the nodes for the second wind turbine that is examined later.

374 It should be noted that the near wake region is considered as the region within 3D at the rear of the  
375 turbine, the far wake region as the region within 5.5D and 8D at the rear of the turbine and the very far  
376 wake region as the region from 8D up to the outlet.



377 (a)



378 (b)

379 Figure 3: (a) Normalized velocity and (b) turbulence intensity along the streamwise direction at the hub – height of  
 380 the domain.

381 Figure 3 illustrates the predicted normalized velocity and turbulence intensity in comparison to  
 382 experimental data and the standard  $k - \omega$  model for  $U_{hub} = 8.5m/s$ ,  $C_T = 0.82$  and  $TI_{hub} = 11\%$   
 383 along the centerline at the hub height of the turbine. This is the condition when the turbine is operating  
 384 at 630kw. The velocity is normalized with the inlet velocity value and the experimental data are  
 385 provided by Taylor et al. (1985).

386 It is observed in the far wake region that the modified  $k - \omega$  model is able capture the correct  
 387 turbulence levels, according to the experimental data. Also, in the very far wake region, close to the  
 388 outlet boundary, the turbulence levels drop to the undisturbed values that are applied at the inlet  
 389 boundary. It is interesting that the highest value of the turbulent kinetic energy does not appear in the  
 390 near wake region of the turbine but, rather, a few characteristic lengths downstream of the turbine  
 391 ( $\approx 4D$ ). This observation is also visible in other experimental data for the second wind turbine that is  
 392 presented later. This trend of the turbulence intensity is captured by the modified  $k - \omega$  model, while  
 393 the standard  $k - \omega$  model failed to capture the turbulent kinetic energy anywhere within the domain.

394 The velocity also shows a similar trend to the turbulent kinetic energy. At the near wake region ( $2.5D$ )  
 395 the modified  $k - \omega$  model closely predicted the wind velocity, and in the far wake region the velocity is  
 396 predicted very well, while the standard  $k - \omega$  model failed to predict the velocity anywhere within the  
 397 domain. It is also noticeable that the velocity and turbulent kinetic energy did not converge to the  
 398 undisturbed inlet values according to the standard  $k - \omega$  model, which was expected since it does not  
 399 satisfy the equation (12). These results are indicative of the very simplistic model that is used to

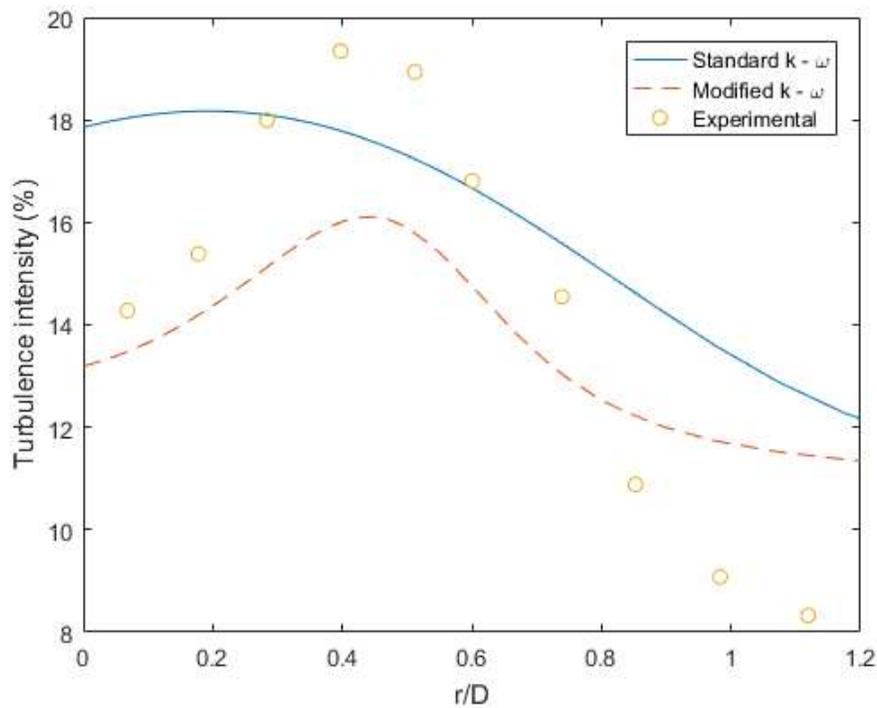
400 simulate the wind turbine. A more accurate or elaborative model, instead of the actuator disk model  
401 without rotational effects based on the thrust coefficient, would have given more accurate predictions  
402 for the velocity in the near wake region.

403 According to the original source of the experimental data, the mast located  $7.5D$  downstream of the  
404 turbine was not aligned exactly with the wind direction (Taylor et al., 1985). This statement can be seen  
405 from the almost linear behavior, if these 3 points are connected, of the velocity according to the  
406 measurements. Also, as will be shown later, the velocity of the wind does not have such a steep  
407 recovery for other wind turbines, or even for the same turbine under different operational conditions.

408 As far as the errors are concerned, the difference of the velocity in the near wake region with  
409 experimental data was more than 20% while in the far wake region this reduced to less than 5%, and the  
410 difference in the turbulence intensity was less than 10% in the near or far wake region.

411 Figure 4 shows the turbulence intensity perpendicular to the ground from the hub – height up to  $1.2D$   
412 above the centerline of the hub – height of the turbine located at  $2.5D$  at the rear of the turbine. There  
413 appears to be a peak in the turbulence intensity at  $0.5D$  and probably this arises from the tip of the  
414 turbine blades. The modified  $k - \omega$  model is able to capture this increase in the turbulence in this region  
415 but it fails to predict the magnitude of it, which is indicative of the very simplistic model that is used to  
416 simulate the wind turbine. Another explanation may lie to the fact that the pressure drop that has been  
417 applied on the disk is based on the undisturbed velocity value at the hub – height of the turbine.  
418 However, the undisturbed velocity changes with the height based on the logarithmic velocity profile as  
419 given equation (6). Consequently, a higher pressure drop from the hub – height up to the tip of the  
420 turbine would, theoretically, give higher turbulence levels. Another interesting fact is that the measured  
421 turbulence intensity drops less than 10% while the inlet turbulence intensity is 11%. The only possible  
422 explanation could be that the turbulent kinetic energy slightly decreases with the height of the domain,  
423 although nothing is stated about this in the report. In any case, as stated in the introduction, employing  
424 a constant turbulent kinetic energy at the inlet may be a special or a simplified case, however, it  
425 approximates the neutral atmospheric conditions and it has been the view of many researchers for the  
426 simulation of small wind turbines (Kasmi and Masson, 2008; Makridis and Chick; 2012).

427

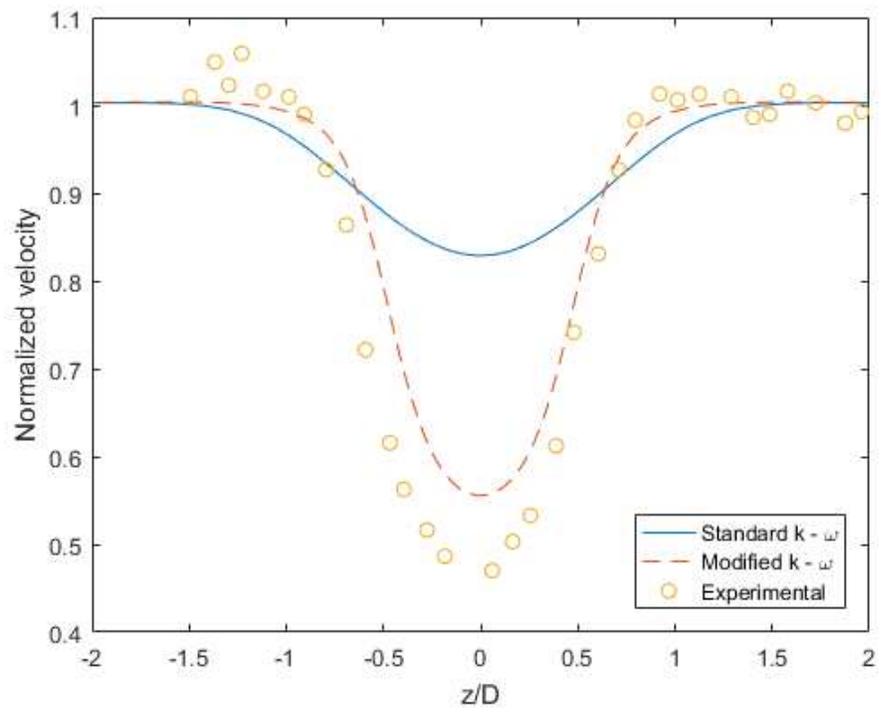


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429 Figure 4: Turbulence intensity distribution along a line perpendicular to the ground from the hub – height up to  
 430 1.2D placed at 2.5D at the rear of the turbine.

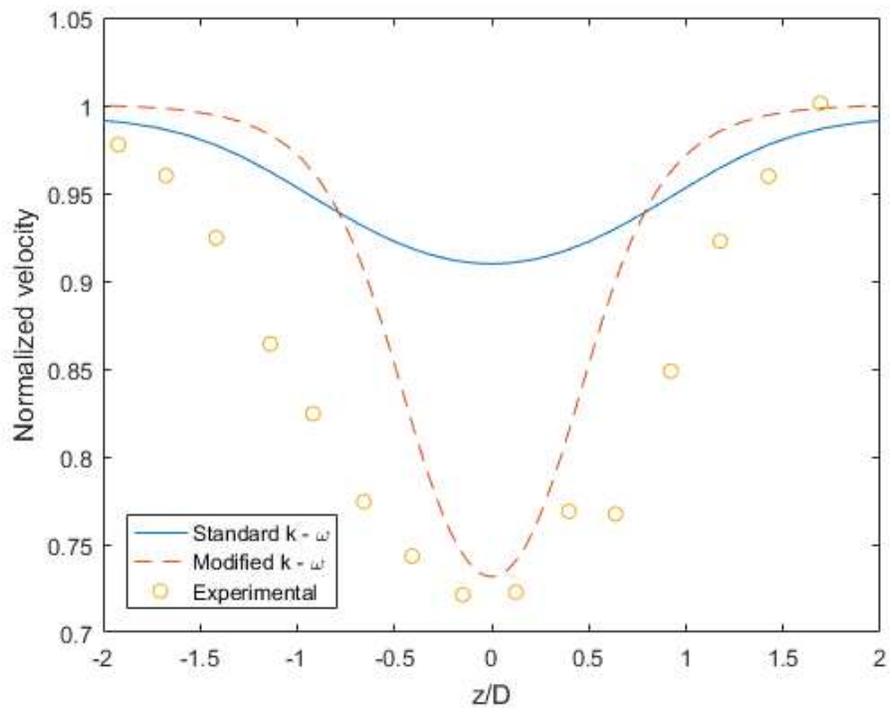
431 Taking into account the fact that the turbulence generation depends on the velocity gradients in the 2  
 432 equation turbulence models based on the Boussinesq assumption for isotropy, it can be concluded that  
 433 a more elaborative model for the wind turbine, e.g. the inclusion of the nacelle and the tower, would  
 434 have given even better results for both the velocity and turbulence because the minimum velocity  
 435 would have been lower, and consequently, the turbulence levels in the near wake region would have  
 436 been larger, due to the higher pressure drop imposed at the disk.

437 Figure 5 illustrates the normalized velocity distribution from one lateral side to the other of the domain  
 438 at the hub – height located at (a) 2.5D, (b) 6D and (c) 7.5D at the rear of the turbine.



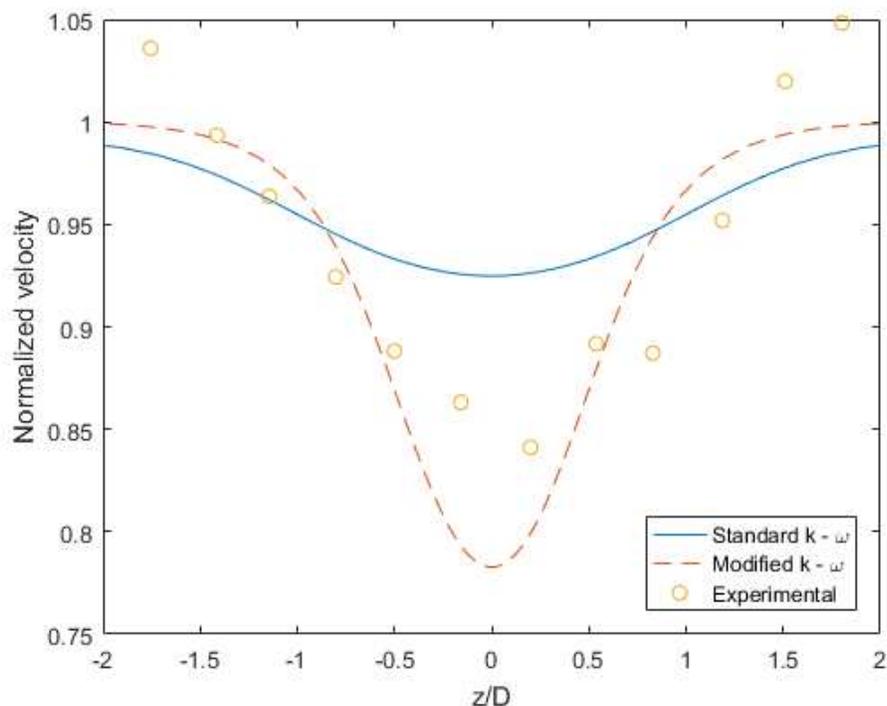
439 (a)

440



441 (b)

442

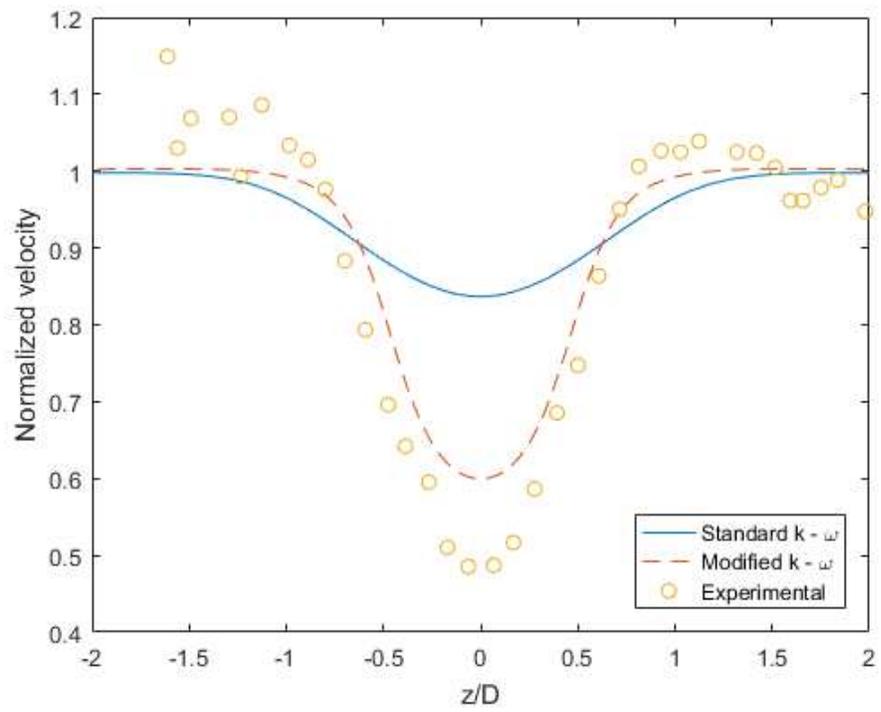


443 (c)

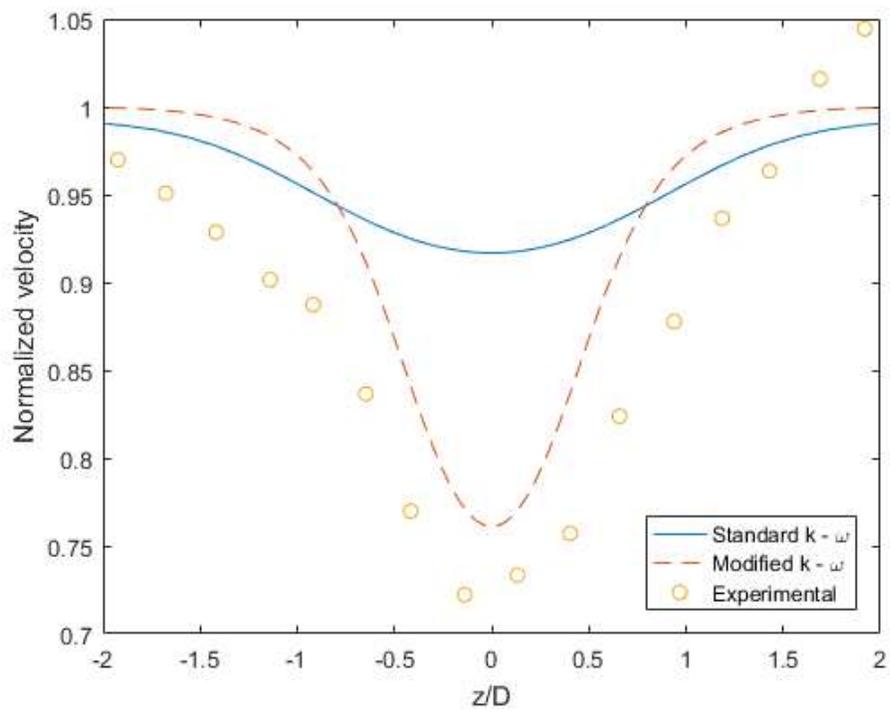
444 Figure 5: Distribution of the normalized velocity along the lateral sides of the domain at the hub – height at (a)  
 445 2.5D, (b) 6D and (c) 7.5D.

446 The modified  $k - \omega$  model in the far wake region at  $6D$  and  $7.5D$  at the rear of the turbine predicts the  
 447 velocity very well although the width of the velocity deficit is larger according to the experimental data  
 448 at a distance of  $6D$  at the rear of the turbine. In the region  $7.5D$  downstream of the turbine the velocity  
 449 appears to be slightly underestimated, however, as stated earlier, the actual velocity is lower than the  
 450 values that appear in the Figure 5 because the mast was not 100% aligned with the wind turbine. This  
 451 statement is also enforced by the fact that the normalized velocity, according to the experimental data  
 452 appears to be higher than 1 close to the lateral sides of the domain. If the computationally predicted  
 453 results had been normalized with a lower value, the validation would have been even better.

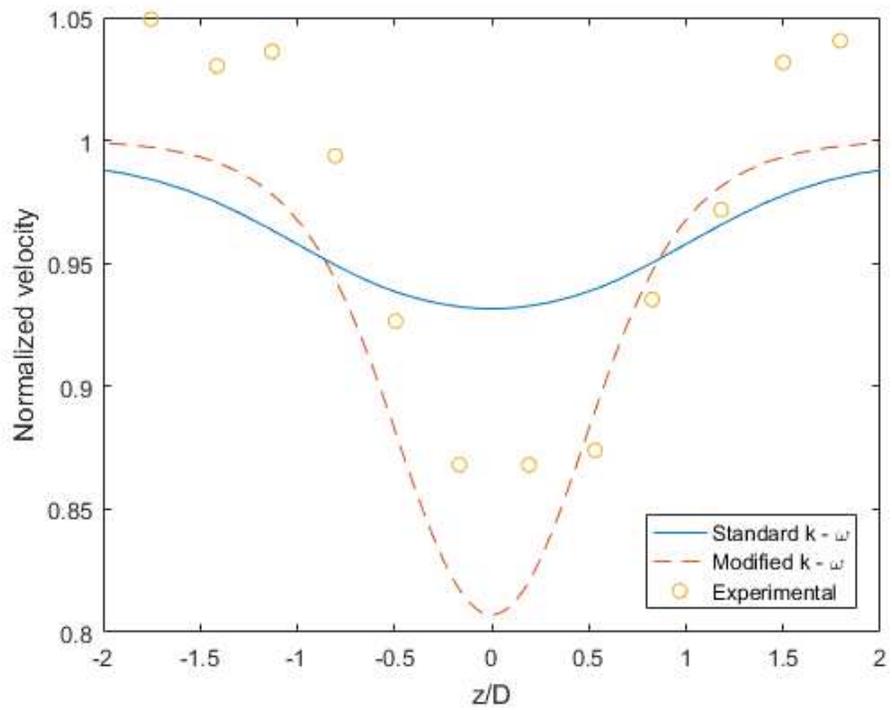
454 Figures 6 and 7 illustrate the normalized velocity from one lateral side of the domain to the other lateral  
 455 side at the hub – height at (a)  $2.5D$ , (b)  $6D$  and (c)  $7.5D$  for the same turbine but for different wind  
 456 velocity and turbulence levels. Figure 6 shows the normalized velocity for  $U_{\infty, hub} = 9.56m/s$ ,  $TI =$   
 457  $11\%$  and  $C_T = 0.77$  and Figure 7 shows the normalized velocity for  $U_{\infty, hub} = 11.52m/s$ ,  $TI = 10.5\%$   
 458 and  $C_T = 0.67$ .



459 (a)

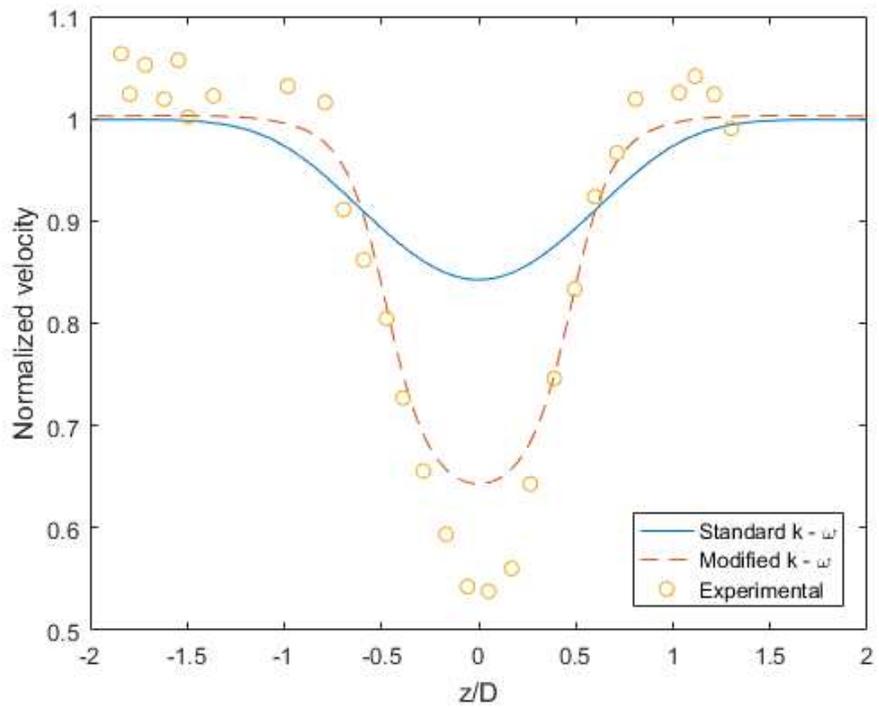


460 (b)

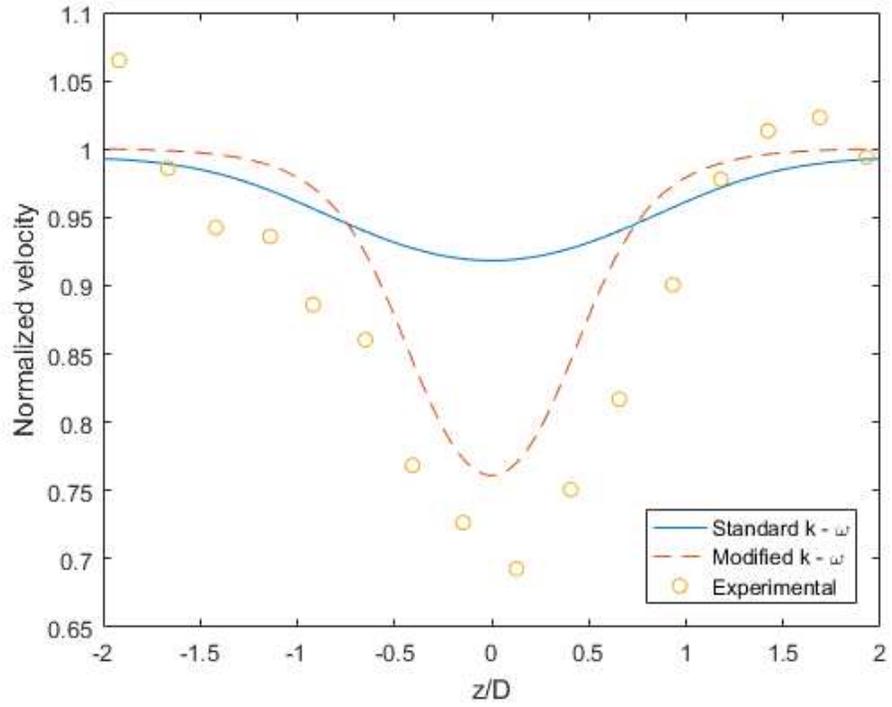


461 (c)

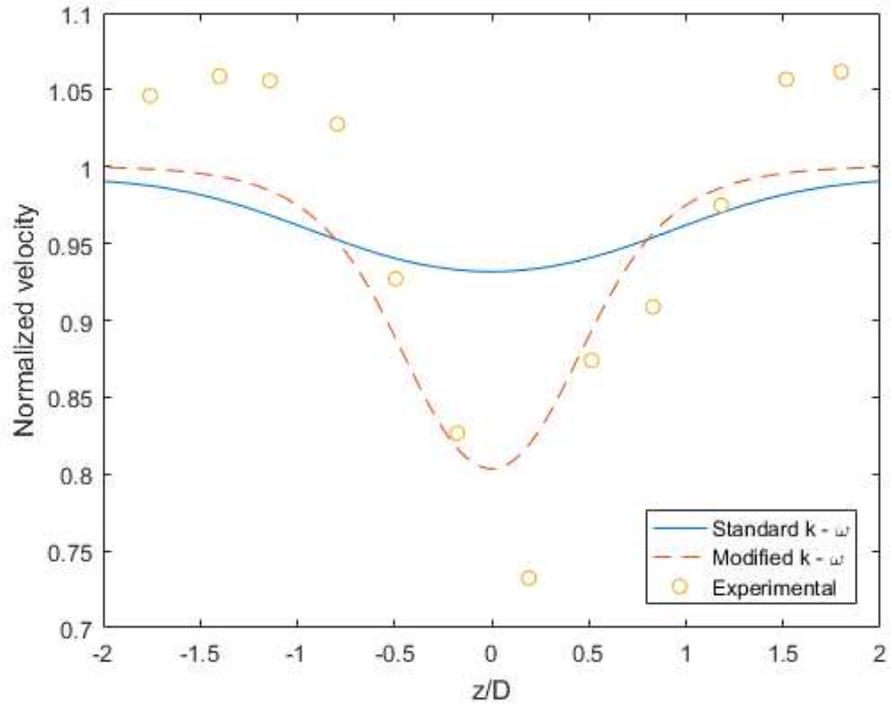
462 Figure 6: Distribution of the normalized velocity along the lateral sides of the domain at the hub – height at (a)  
 463 2.5D, (b) 6D and (c) 7.5D for  $U_{\infty,hub} = 9.56m/s$ .



464 (a)



465 (b)



466 (c)

467 Figure 7: Distribution of the normalized velocity along the lateral sides of the domain at the hub – height at  
 468 (a) 2.5D, (b) 6D and (c) 7.5D for  $U_{\infty,hub} = 11.52m/s$ .

469 The results for  $U_{\infty,hub} = 9.56m/s$  and  $U_{\infty,hub} = 11.526m/s$  have a similar behavior to the results  
470 presented in Figure 4 for  $U_{\infty,hub} = 8.5m/s$ .

471 In general, the modified  $k - \omega$  model predicts well the velocity at  $6D$  and  $7.5D$  at the rear of the turbine  
472 while it rather overestimates the velocity  $2.5D$  at the rear of the turbine, while the standard  $k - \omega$   
473 model failed to predict the velocity correctly anywhere within the domain. The problem with the  
474 normalized velocity being over 1 according to the measurements is still present for all inlet velocity  
475 values as seen in Figures 5(c), 6(c) and 7(c).

476 Also, it is observed from Figures 5 – 7 that, as the undisturbed inlet velocity decreases, the results close  
477 to the turbine become better when compared to experimental data, although the difference is generally  
478 small. The explanation for this behavior lies to the thrust coefficient. As stated earlier, the pressure drop  
479 that is applied on the disk is based on the equation (16) and the model does not include any fixed parts  
480 of the wind turbine such as the nacelle or the tower. The pressure drop of any fixed part of the turbine  
481 would have been calculated by the same formula, equation (16), but it would have included the drag  
482 coefficient instead of the thrust coefficient. However, as the velocity increases, the thrust coefficient,  
483 based on the power curve of the turbine, decreases, while the drag coefficient of the bluff bodies is not  
484 that sensitive to the inlet velocity, at least for fully turbulent flows, which is the case in the present  
485 investigation. Taking into account the fact that the drag coefficient of the fixed parts of the turbine is  
486 higher than the thrust coefficient, and almost steady regardless of the velocity, it can be concluded that  
487 for low velocities, where the thrust coefficient is higher, the omission of the fixed parts of the turbine  
488 affects the results to a smaller extent than for the cases of the higher velocities. This statement will be  
489 validated later when the results of the Holec turbine are presented, although the difference is smaller  
490 due to the small differences in the thrust coefficient.

## 491 **4.2 The Holec wind turbine**

492 The second wind turbine that is examined is a small Holec horizontal axis three – bladed turbine with a  
493 rated power of approximately 300kW. A wind farm of these turbines is located at Sexbierum, a village in  
494 northern Holland. The examination of another wind turbine is important in order to show the  
495 universality of the modified  $k - \omega$  model and in order to show that the model is not sensitive to the inlet  
496 turbulence levels, and this is because the turbulence levels in this region are relatively low. The  
497 measurement data are taken from Cleinje (1992).

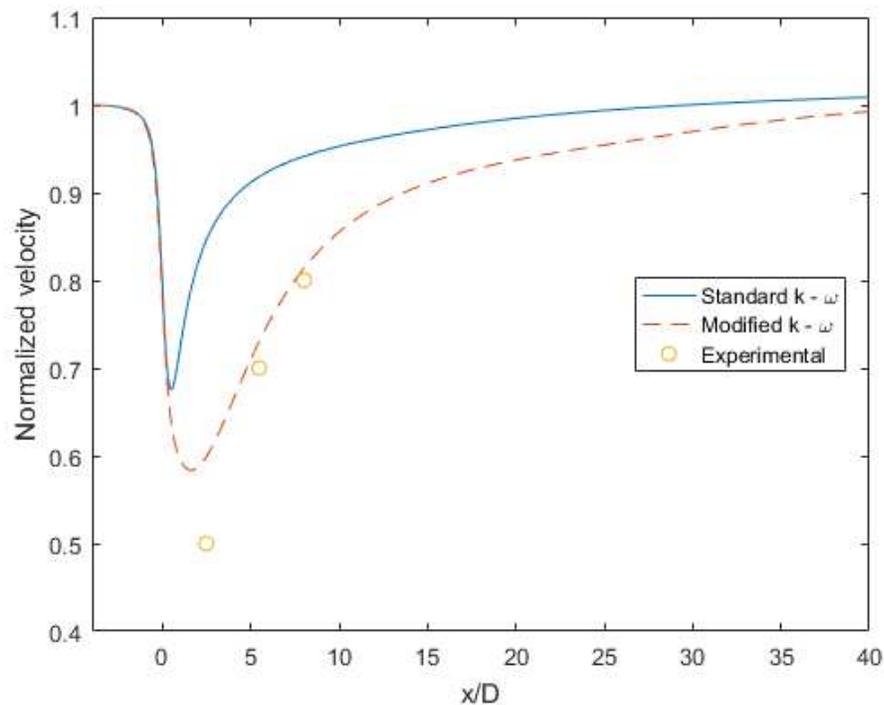
498 The computational setup is similar to the Nibe – B 630kw wind turbine explained previously. The flow  
499 conditions are based on Cleinje (1992). The logarithmic velocity profile is still valid, as in all atmospheric  
500 flows under neutral stratification within the surface layer where small wind turbines are located, but the  
501 turbulence levels are quite lower in relation to the previous wind turbine. However Cleinje (1992), is  
502 rather vague when it comes to the inlet turbulence levels. They took measurements at 3 different  
503 heights and analytically expressed the turbulence intensity and the corresponded roughness length, but  
504 in the results section for high yaw angles ( $25^{\circ} - 30^{\circ}$ ) the turbulence levels appear to be far lower than  
505 the initially estimated ones for every mast. For this reason, the results for the turbulence intensity based  
506 on the results of the wind turbine for high angles of attack of the wind will be considered. In any case,

507 the constant value for the turbulent kinetic energy, regardless of the height, appears to be almost valid  
508 based on all of their measurements.

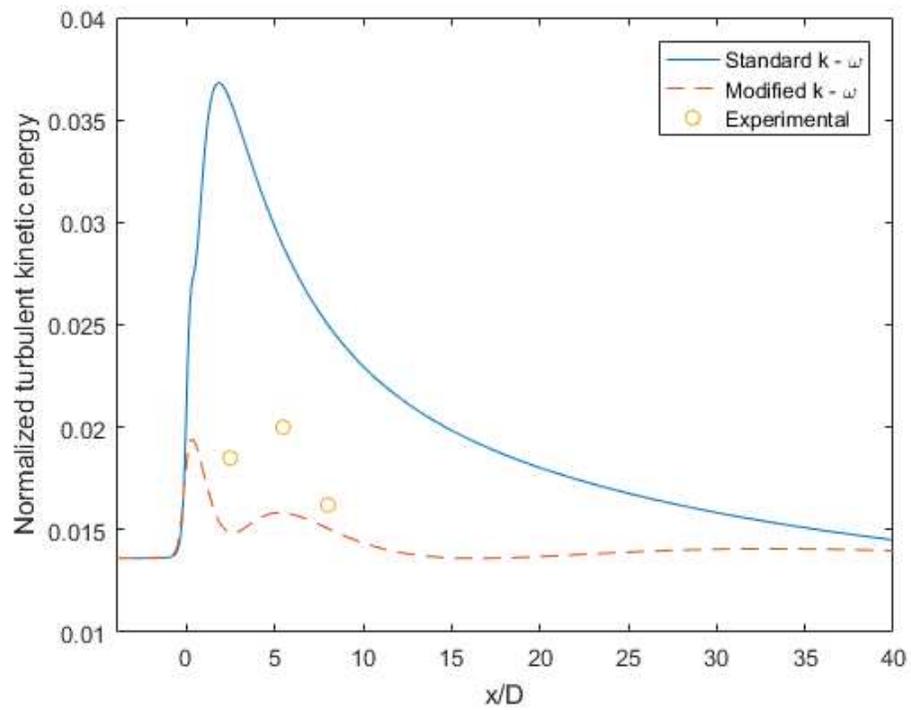
509 The measured undisturbed normalized turbulent kinetic energy appears to be in the range 0.011 –  
510 0.014. The normalization of the turbulence intensity was achieved with the squared undisturbed velocity  
511 inlet. These inlet turbulence levels correspond to a value of approximately  $\beta_{\infty}^* = 0.09$  for the standard k  
512 –  $\omega$  model, and consequently the standard k –  $\omega$  model has been used without any modifications for the  
513 simulations. This value of  $\beta_{\infty}^*$  gives a normalized turbulent kinetic energy of 0.0136 and this agrees well  
514 with the measurement data. For the zero streamwise gradient condition, the value of  $\beta_i$  has to be  
515 changed according to the equation (12) and the corresponding value is  $\beta_i = 0.1263$  for the modified k –  
516  $\omega$  model. Regarding the eddy frequency, the profile based on the equation (4) is chosen and modified  
517 according to the equation (11) while the logarithmic velocity profile is employed at the inlet by the  
518 equation (2), as in the previous wind turbine.

519 The average undisturbed velocity magnitude during the measurements at the hub – height of the  
520 turbine was  $7.6 \text{ m/s}$ . Consequently, in this paper, to show the universality of the model, a value of  
521  $8.6 \text{ m/s}$ , as well as a lower velocity of  $6.2 \text{ m/s}$  is employed. The thrust coefficient is 0.75 for a range of  
522 hub – height velocities from  $7 \text{ m/s}$  to  $10 \text{ m/s}$  and it increases to 0.78 for the  $6.2 \text{ m/s}$  inlet velocity at  
523 the hub – height.

524 Figures 8 and 9 show the computed normalized velocity and turbulence along the hub – height for both  
525 the velocity and turbulence inlets and the experimental data.

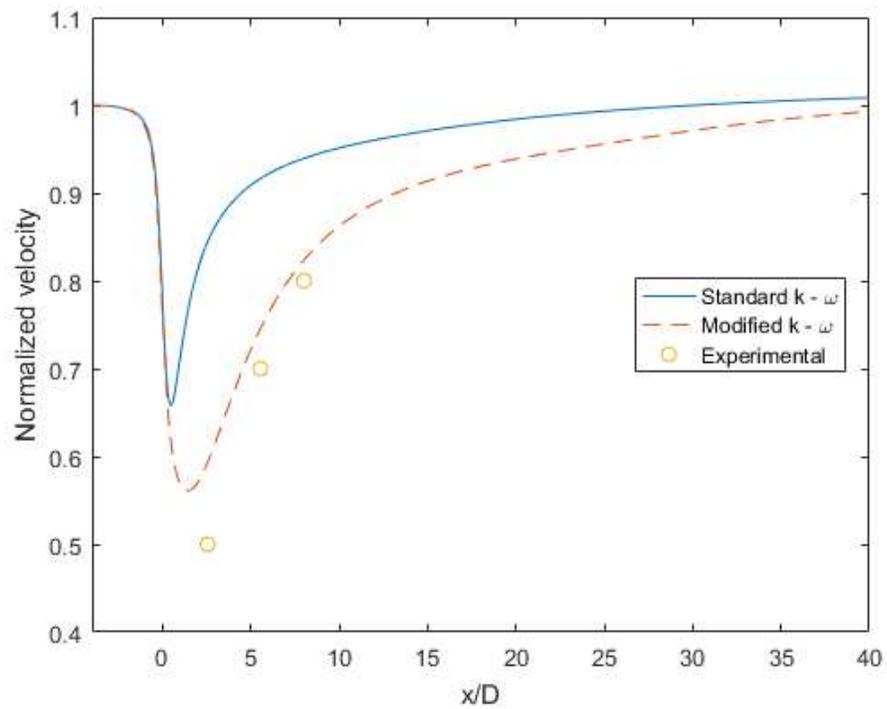


526 (a)

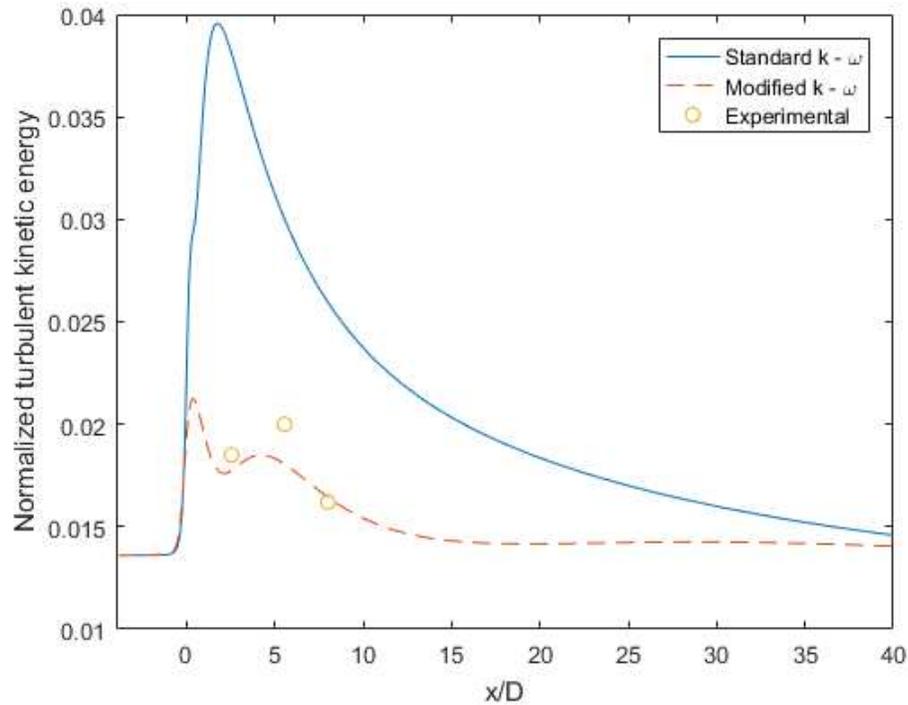


527 (b)

528 Figure 8: (a) Normalized velocity and (b) turbulent kinetic energy along the streamwise direction at the hub –  
 529 height for  $U_{hub} = 8.6m/s$ .



530 (a)



531 (b)

532 Figure 9: (a) Normalized velocity, and (b) turbulent kinetic energy along the streamwise direction at the hub –  
 533 height for  $U_{hub} = 6.2m/s$ .

534 The velocity has a similar trend as in the previous investigated wind turbine. The velocity drops to half of  
 535 the undisturbed value at a distance  $2.5D$  downstream of the wind turbine and then it gradually  
 536 increases, reaching 80% of the value of the undisturbed velocity at  $8D$  downstream of the turbine. It is  
 537 noticeable that the behavior of the computationally predicted velocity appears to be the same for both  
 538 velocity inlet values. The model, like in the previous wind turbine, predicts the recovery of the velocity  
 539 and turbulence kinetic energy with a very good accuracy as seen in the Figures 8 and 9, while the results  
 540 are as good in the near wake region.

541 As far as the turbulent kinetic energy is concerned, a similar behavior with the Nibe turbine is illustrated.  
 542 The maximum value does not appear in the near wake region but rather a few characteristic lengths  
 543 away from the turbine, and this is not predicted by the model. However, in the far wake region the  
 544 correct values of the turbulent kinetic energy are recovered and maintained along the domain until the  
 545 outlet.

546 The small differences in the velocity and turbulent kinetic energy at the rear of the turbine between the  
 547 2 different inlet velocities are related to the very small difference in the thrust coefficient of the turbine.  
 548 As shown in the Nibe wind turbine, the results are more reliable for high thrust coefficients. The same  
 549 applies here for the Holec turbine but the differences are very small, especially for the velocity and this  
 550 is due to the very small difference in the thrust coefficient. It is also noticeable again that the standard k  
 551 –  $\omega$  model failed to predict the velocity and the turbulent kinetic energy everywhere throughout the  
 552 domain as expected.

553 Regarding the errors in the turbulent kinetic energy with the experimental data, although a significant  
554 improvement has been attained when compared with experimental data, the differences were generally  
555 high. These errors were of the order of magnitude of 20% for the case of 8.6  $m/s$  velocity at the hub –  
556 height at distances  $2.5D$  and  $5.5D$  downstream of the turbine while for the case of 6.2  $m/s$  velocity at  
557 the hub – height at the same distances, the error was less than 10%. In both velocity inlets, however, the  
558 turbulent kinetic energy at a distance  $8D$  downstream of the turbine, the errors were approximately 2%  
559 and 6% for the 6.2  $m/s$  and 8.6  $m/s$  velocity inlet, respectively.

560 Finally, regarding the errors in the velocity, as is the case of the Nibe turbine, the error in the velocity  
561 was approximately 20% in the near wake region, while in the far wake region it was about 6% or smaller  
562 regardless of the velocity inlet. In any case, for both turbines, for higher thrust coefficients, the results  
563 for both the velocity and turbulent kinetic energy were closer to the experimental data and far closer  
564 than the standard  $k - \omega$  model.

565 In general, the modified  $k - \omega$  model showed significant improvement when compared with the  
566 standard  $k - \omega$  model which is of paramount importance, especially for wind farm simulations where the  
567 power output and possible future structural damage will be better predicted.

## 568 **Conclusions**

569 For wind farm simulations, using a steady RANS model, the recovery of the wind properties in the  
570 turbine wakes can affect the accurate prediction of the performance of the downstream turbine. In this  
571 paper, a modified  $k - \omega$  model for simulating small wind turbine wakes for a uniform roughness flat  
572 terrain in a neutrally stratified atmospheric boundary layer is proposed. A condition for achieving the  
573 zero streamwise gradients for all flow variables has been mathematically produced. The model has been  
574 successfully implemented and tested in an empty domain for various turbulence levels and friction  
575 velocity values. The modified  $k - \omega$  model has been employed for the simulation of 2 small wind  
576 turbines for different inlet conditions with the actuator disk model based on the thrust coefficient of the  
577 turbines. The comparison of the results in the near wake region for both wind turbines with available  
578 experimental data was mediocre which may have been expected due to the very simplistic model that  
579 has been employed to represent the wind turbines. For higher thrust coefficients, the results were more  
580 accurate than for lower thrust coefficients for both the velocity and turbulent kinetic energy although  
581 the difference was small. In the far wake region, however, the comparison of the velocity and  
582 turbulence levels for both wind turbines with the experimental data was relatively good due to the  
583 imposition of the zero streamwise gradient condition for all variables. In all cases, the modified  $k - \omega$   
584 model produced results far closer to the experimental data rather than the standard  $k - \omega$ .

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