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The Diffusion of Mobile Social Networking: Further Study

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Abstract

In a recent study, Scaglione, Giovannetti, and Hamoudia (2015) analyze the diffusion of mobile social networking in four G7 countries. Using Bass's model and Bemmaor's Gamma/Shifted Gompertz (G/SG) model, they find evidence for a left skew in the right-censored distributions of the times to adoption in three countries out of four. However, they rely on the skewness parameter of Bemmaor's model to draw their conclusion. With the use of three special cases of the G/SG as well as the full version, we reanalyze the data. Extending the data basis to six countries, we show that (i) fitting the four models to the data does not allow us to discriminate between models, but (ii) forecasting subsequent adoptions provides strong support of right skew in the data set: in each country (except France), after an initial embrace of the access, there appears a substantial mass of later adopters of mobile social networking.

Keywords: Gamma/Shifted Gompertz (G/SG); Skew; Chasm; Chilling effects; Social media

. . It is thus of interest to understand how attention to novel items propagates and eventually fades among large populations" (Wu and Huberman, 2007).

1. Introduction.

A recent study by Scaglione, Giovannetti, and Hamoudia – hereafter SGH (2015) focuses on the diffusion of mobile social networking (MSN) in four G7 countries. Using Bass's (1969) model and Bemmaor's (1994) Gamma/Shifted Gompertz model (G/SG) on 67 monthly data points, they find that the adoption curves were left skewed in three countries: France is the exception (no skew). This pattern depicts an apparently increasing fervour for social media at an increasing rate as the pool of active and unique MSN users increases (for France, the rate of change is constant). However, their finding is based on the fit of the more flexible model, the Bemmaor model, to the data when the special case, the Bass model, provides an equal fit. We reanalyse their data by (i) including a broader range of models, and (ii) relying on the forecasting accuracy to assess the skew of the right-censored diffusion curves. Similar to the original interpretation of the Bass model, the study relies on the assumption of complete homogeneity of the densities of the times to adoption across the population. The interpretation of the G/SG differs from that given in Bemmaor (1994).

The data used are the monthly numbers of active and unique MSN users over an observation period of 67 months starting in April 2007[1]. We added two countries (Spain and Italy) from the same data source (comScore) to extend the scope of the analysis. Using three two-parameter models that allow for left-skew, symmetry and right-skew respectively as well

[1] SGH made a data handling error in three countries out of four by summing the numbers of active and unique MSN users in months t and t – 1 in order to obtain the number of active and unique MSN users in month t. (The US is the exception). The error resulted in the doubling of MSN users at the end of the observation period (October 2012).

as estimating the full three-parameter version of the G/SG, excluding the market size parameter, we show that all four models provide a comparable fit to the data set despite their apparently divergent implications. However, when used for forecasting purposes, the shifted Gompertz leads to superior forecasts to the other models, in five countries out of six (France is the exception). Therefore, from a predictive standpoint, the data are mostly consistent with right skew which corresponds to a relatively thick right-hand tail. The following section provides a brief introduction to the three nested models and to the generalized G/SG, and to their characteristics in terms of implied effect of network externalities. The third section reanalyzes the data set. The fourth section is the conclusion.

2. The models and their implied effect of network externalities

SGH tested the Bass model versus the G/SG. Here, we estimate the G/SG but also three constrained two-parameter versions. The reason for this is that parameter identification issues can arise when the data are right censored which is typically the case with diffusion data.

The G/SG is a three-parameter model whose cumulative distribution function takes the following form:

$$F(t) = \frac{1 - e^{-bt}}{\left(1+\beta e^{-bt}\right)^{\alpha}}\ , b, \alpha, \beta > 0, t > 0. \tag{1}$$

The advantage of the formulation is that it is relatively flexible: the probability density function (p.d.f.) can be skewed to the right, to the left or it can be symmetric depending on the value of α. The model reduces to the Bass model when $\alpha = 1$. Letting f(t) be the p.d.f, we can parametrize it as a function of (i) a coefficient of external influence, $f(0) = p$, which captures the likelihood to adopt at time $t = 0$, and (ii) a coefficient of of internal influence q, with b = p + q and β = q/p for the Bass model. Evaluated at t = 0, the p.d.f. of the G/SG is such as:

$$f(0) = p = \frac{b}{(1+\beta)^{\alpha}}. \tag{2}$$

Letting z(t) be the conditional likelihood to adopt at time t given that one has not adopted yet with z(t) = f(t)/(1 – F(t)), it can be shown that z(t) approaches b as t gets close to ∞. It follows that:

$$b = p + q \tag{3}$$

regardless of the value of α, and it follows that

$$\beta = (1 + q/p)^{1/\alpha} - 1. \tag{4}$$

The G/SG can be parametrized as a function of p and q in the following way[2]:

$$F(t) = \frac{1 - e^{-(p+q)t}}{\{1 + [(1 + q/p)^{1/\alpha} - 1]e^{-(p+q)t}\}^{\alpha}}\ , t > 0, p, q, \alpha > 0. \tag{5}$$

Such parametrization offers a common interpretation to the parameters of the nested versions and of the general version. Depending on the value of α, the shape of the conditional likelihood to adopt given one has not adopted yet can vary substantially as a function of the cumulative proportion of adopters.

We study three special cases that include two parameters for estimation only and the generalized case (Eq. 5). The cases are as follows:

- $0 < \alpha < 1$: Skew to the left with 0.5 < F(t*) < 1 (t*: mode of f(t)).

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[2] SGH (2015, p. 1162) parametrize the G/SG differently from us: in Eq. (1), they replace b with p + q and β with q/p for all the values of α. However, in this case, the parameters p and q cannot be interpreted as the coefficient of external influence and the coefficient of internal influence respectively since f(0) is a function of α (Eq. 2). Hence, our estimates of p and q are not comparable with theirs (unless α = 1). In our case, the interpretations of p and q are consistent regardless of the value of α.

The selected case is G/SG(α = 1/2) which exhibits a slight skew to the left ($0.5 < F(t^*) < 0.58$). The implied hazard rate is a convex function of the cumulative proportion of active MSN users: according to the model, the rate of change of the conditional likelihood to adopt (given one has not adopted yet) increases with the cumulative proportion of adopter. The model captures an increasingly warming effect of network externalities: Later adopters carry more weight than early adopters in the diffusion curve. On average, the rate of change is equal to q over time. Such pattern in the effect of network externalities can apply when adoption induces switching costs, for example from one generation of the product to the next one, that the attraction of the new version gradually overcomes;

- $\alpha = 1$: Right-skewed distribution that approaches symmetry as p/q gets close to 0 ($0 < F(t^*) < 0.5$)

This is the Bass model. Its shape has been studied by Mahajan, Muller, and Srivastava (1990). Here, the rate of change in the conditional likelihood to adopt (given one has not adopted yet) as the cumulative proportion of adopters increases is constant; it is equal to q. This is the case where the hazard rate is a linear function of the cumulative proportion of adopters. Network externalities operate as a warming effect at a constant temperature. This can appear as a relatively strong assumption.

- $\alpha = \infty$: Right skew: $0 < F(t^*) < e^{-1}$.

In this case, the G/SG reduces to the shifted Gompertz (SG) distribution[3]. The conditional likelihood to adopt a social service given one has not adopted it yet is a concave function of the cumulative proportion of adopters: the marginal effect of the cumulative proportion of adopters on the conditional likelihood to adopt (given one has not adopted yet) decreases as

[3] When α gets close to ∞, the G/SG approaches a SG. There is an error in SGH (p. 1162) on the limit distribution. The "Bass model" is a shifted logistic curve.

the cumulative proportion of adopters increases. The warming effect declines over time. On average, the rate of change is equal to q over the diffusion process. The effect of network externalities tapers off as the number of active MSN users builds up: Early adopters carry more impact on potential adopters than later adopters. This is consistent with the "decay factor" in collective attention that Wu and Huberman (2007) refer to. Recently, the SG distribution has been shown to be superior to the Bass model to describe the search frequencies from 45 countries related to 175 social media services and Web businesses (Bauckhage and Kersting, 2016).

- Free α: Skewed to the left when $0.5 < F(t^*) < 1$, to the right when $0 < F(t^*) < 0.5$ or symmetric when $F(t^*) = 0.5$

This is the most flexible distribution. SGH refer to it as the Bemmaor model (with a different parametrization)[4]. Interestingly, it captures a scenario which differs from the preceding ones: $0 < \alpha < 0.5$. In such case, the p.d.f. exhibits two modes (one at 0 and another away from 0 with a local minimum in-between). Such apparently odd pattern is consistent with the existence of a chasm in the data set which means a time gap (a crack) between the early adopters and the later adopters (see, e.g., Chandrasekaran and Tellis, 2011; Goldenberg, Libai, and Muller, 2002; Libai, Mahajan, and Muller, 2008; Moore, 1991, Figure in p. 17; Peres, Muller, and Mahajan, 2010). The consequence is that the rate of change of the conditional likelihood to adopt (given one has not adopted yet) is negative and it decreases (in

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[4] SGH use the parameter α of the fitted Bemmaor model to the data set in order to infer the skewness of the times-to-adoption distribution. Since they find that in three cases out of four, α is less than one, they conclude that the curves exhibit left skew – when the diffusion curves are right censored. Our argument here is that one cannot infer skewness from the fit of the models, including Bemmaor's model, since all the four models lead to relatively close fits (R^2). As forecasting accuracy appears as a better discriminatory device than a model's fit, we assess skewness from the relative forecasting accuracy.

absolute value) to a minimum as the cumulative proportion of adopters increases, prior to increasing with the cumulative proportion of adopters. This shape is compatible with the existence of a chilling effect of network externalities prior to a warming effect beyond a cut-off value of the cumulative proportion of adopters (Goldenberg, Libai, and Muller, 2010). The analysis allows us to assess the required cumulative proportion of adopters for the warming effect to take place. It also demonstrates the potential co-existence of both effects in time over the whole diffusion process. (Note that some products may fail before the warming effect takes place). Hence the G/SG(α) shows that the existence of a chasm and the chilling effect of network externalities can be considered as two alternative facets of the same process.

Overall, all three models and the full G/SG(α) capture a broad pattern of effects. All four models and their characteristics are depicted in Table 1.

[Insert Table 1 about here]

3. The data analysis

We use the same data set as SGH and add two countries (Italy and Spain) over the same observation window: April 2007 to October 2012, i.e., a total of 67 monthly observations. Active users in month t logged in to the social network at least once in the month via their mobile phone. Figure 1 shows the corrected numbers of active and unique MSN users for France, Germany and the UK (see also the six curves separately at the Github address in the Acknowledgements section).

[Insert Figure 1 about here]

We used Srinivasan and Mason's (1986) nonlinear least square method to estimate the parameters of all four models. Letting N_t be the number of active and unique MSN users in month t and m be the eventual number of adopters (i.e., registered individuals), the method consists of minimizing the following sum with respect to the parameters:

$$\text{Min.} \sum_{t=1}^{T}[m\big(F(t)-F(t-1)\big)-(N_t-N_{t-1})]^2 \qquad (6)$$

with $F(0) = 0$, $N_0 = 0$ and $T = 67$ for all four models[5]. The parameters m, p and q or m, p, q and α, can be obtained with a search algorithm.

Table 2 shows the parameter estimates of the four models, i.e., the three special cases and the full G/SG, as well as the corresponding measures of skewness, the relative impact of network externalities, and the sizes of the right-hand tails of the implied adoption curves. For example, for Germany, the full G/SG with $\alpha = .0495$ may support a heavily left-skewed ($F(t^*) = 0.86$) distribution but the improvement in fit relative to the SG is quite small[6]: the SG is right-skewed ($F(t^*) = 0.36$). The same applies to the US where α is estimated at 0.2066 with $F(t^*) = 0.69$ but the root mean squared error corresponding to the SG ($F(t^*) = 0.32$) is marginally larger. Based on those results, we can make the following additional observations:

[Insert Table 2 about here]

(i) As the skewness parameter α increases, the predicted market potential m increases whereas the parameters p and q decrease; the increase in market size can be modest as in France or it can be substantial as in Spain. The marginal effect of the cumulative number of

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[5] By comparison, SGH (2015) used the Srinivasan and Mason (1986) estimation method for the Bass model only. Our estimates for the US data differ from theirs because they either started the summation with $t = 2$ or they inadvertently set $N_1 – N_0$ to 0 when t equals one. Both procedures lead to about the same parameter estimates when the total number of observations equals 66 or 67. For the Bemmaor model which corresponds to the G/SG with free α here, they fitted the theoretical cumulative number of MSN users in month t, $mF(t)$, to the actual number of active and unique MSN users N_t to obtain the parameter estimates. Again, our estimates are not comparable with theirs due to the difference in the estimation procedures. For the US, the first-order correlation among the residuals when one applies the SGH estimation method equals .833 versus -0.037 when one uses the differences between the numbers of active and unique MSN users in months t and $t – 1$ as shown in Eq. (6) (see Schmittlein and Mahajan, 1982, Table 7).

[6] As shown in Table 2, R^2 measures the fit to the increments in the number of active and unique MSN users. In contrast, SGH (2015) assess the fit of the models to the number of active and unique MSN users.

adopters as measured with the q parameter varies sensibly across models. This shows that the formulation of the diffusion model matters to capture the size of the eventual market size despite the assumption of exogeneity, but also the key characteristics of the process;

(ii) The predicted times to peak adoption are rather close between formulations and they belong to the observation window, except for Spain (SG); the peak magnitude decreases as α increases (except for Spain);

(iii) As shown by the 95-th percentile, the level of right censoring increases with α: the right-hand tail becomes all the fatter as α is large. The parameter α becomes a signal for the implied speed of diffusion as it tends to vary with the coefficient of internal influence q: the larger it is, the smaller q is, and the slower the speed. For example, for the US, the expected difference between two randomly picked adoption times equals 5.2 months for G/SG(α = 0.2066, q = 0.1595), 10.1 months for the G/SG(α = 0.5, q = 0.0783), 15.2 months for the Bass model (q = 0.0477), and 27.3 months for the SG (q = 0.0202) - see Trajtenberg and Yitzhaki, 1989, Eq. 8[7]. Hence, the G/SG(α = 0.2066) predicts a diffusion which is more than five times faster than that implied by the SG. Still, except for Germany and the US, the standard errors of the α parameters are quite large. This lack of reliability may be due to a very short data duration;

(iv) There appears a monotonic relationship between α and the relative impact of network externalities as measured with q/p: it can be increasing with α as in the case of Germany or it can be decreasing as α increases as shown with Italy (the deviation in the case of Spain can be due to sampling error);

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[7] The speed corresponds to the Gini index which is such as:

$$\Gamma = \int_0^\infty F(t)(1 - F(t))dt$$

(v) As depicted by the R^2 and the root mean squared error, the fits of all four models are quite close. Figure 2A shows an example of the fit of the four models to the US data.

[Insert Figure 2 about here]

As shown in Figure 2B, the analysis supports the existence of a chasm for the US data. (The same applies to Germany). The conditional likelihood to adopt (given one has not adopted yet) decreases to a minimum as the cumulative proportion of adopters increases, prior to increasing. It reaches the minimum when the cumulative proportion of MSN users equals 18.5% in the US and 6.8% in Germany. Chilling effects precede the warming effects of network externalities. In the early stages of the diffusion process, some individuals may disassociate themselves from the group of the early active MSN users, perhaps in part to protect their privacy, until they weigh the positive sides more heavily. Note that in two countries out of six, a chasm seems to exist between the early active MSN users and the later users. (The US data show that the number of active and unique MSN users decreased by 3.1% in July 2007. The data for Germany exhibit a decline by 14% between June and August 2007).

Overall, despite the analysis of relatively extreme cases, the skewness of the diffusion curves cannot be identified from the mere fit to the data: Models with varying implied skewness and speed of diffusion tend to fit the data about the same.

To discriminate between models more forcefully and in particular, to identify skewness, we carry out forecasts using the same setting as SGH: We make forecasts starting with observation 33 (December 2009), and use a rolling estimation period to make from one-month-ahead up to 18-month-ahead forecasts. We also use a linear (“naïve”) trend model and a seasonal model for comparison (SGH, p. 1165). The results are shown in Table 3.

[Insert Table 3 about here]

Starting with about the same accuracy as the other models, with the exception of France, the SG appears as the model whose predictions deteriorate least as the length of the forecasting horizon increases[8]. This pattern applies to both error measures (median and geometric mean of the absolute percentage errors). Note the good overall forecasting performance of the linear trend model in the 35-month time window, except for Germany and Spain, as compared with the four models[9]. When one looks at the right-hand half of Figure 1 on its own, a linear trend seems appropriate over the 5+-year observation period. If the use of MSN is really a diffusion process, then we are looking at this process before any sort of shift becomes apparent. Over this range, the relative value of diffusion models appears quite limited.

Lastly, the G/SG with free α performs rather poorly which raises the issue of parameter identification when (i) diffusion curves are right-censored, and (ii) both m and α are included as free parameters.

In sum, from a forecasting standpoint, the data seem mostly consistent with right-skewed distributions. The pooled measures of errors across all six countries support this finding. The initial spark created with the building of the installed base of MSN users seems to fade away with later adopters. Resistance to change may also become stronger and stronger as the novelty diffuses through the intended audience. Such result holds across five out of the six countries under study.

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[8] For the US data, the difference with the results shown in Table 2 of SGH for the Bass model and the Bemmaor model can be explained by the difference in the estimation procedures (see footnote 5). We cannot explain the differences with the "naïve" model. We checked our own code and made it available.

[9] We also computed the forecasts of the seasonal model but they were totally uncompetitive.

4. Conclusion

We reanalyze the data on the active and unique MSN users in four countries as reported in SGH and add Italy and Spain. In addition to providing an analytical framework for testing for the existence of a chasm, the study shows that the distributions of the times to adoption are heavily skewed to the right: Increments in the pool of potential MSN users tend to be smaller and smaller as the installed base builds up. The fading of novelty as it applies to social media appears as a prevalent phenomenon, perhaps in combination with an increasing inertia through the layers of the targeted population. The next step is to provide alternative explanations.

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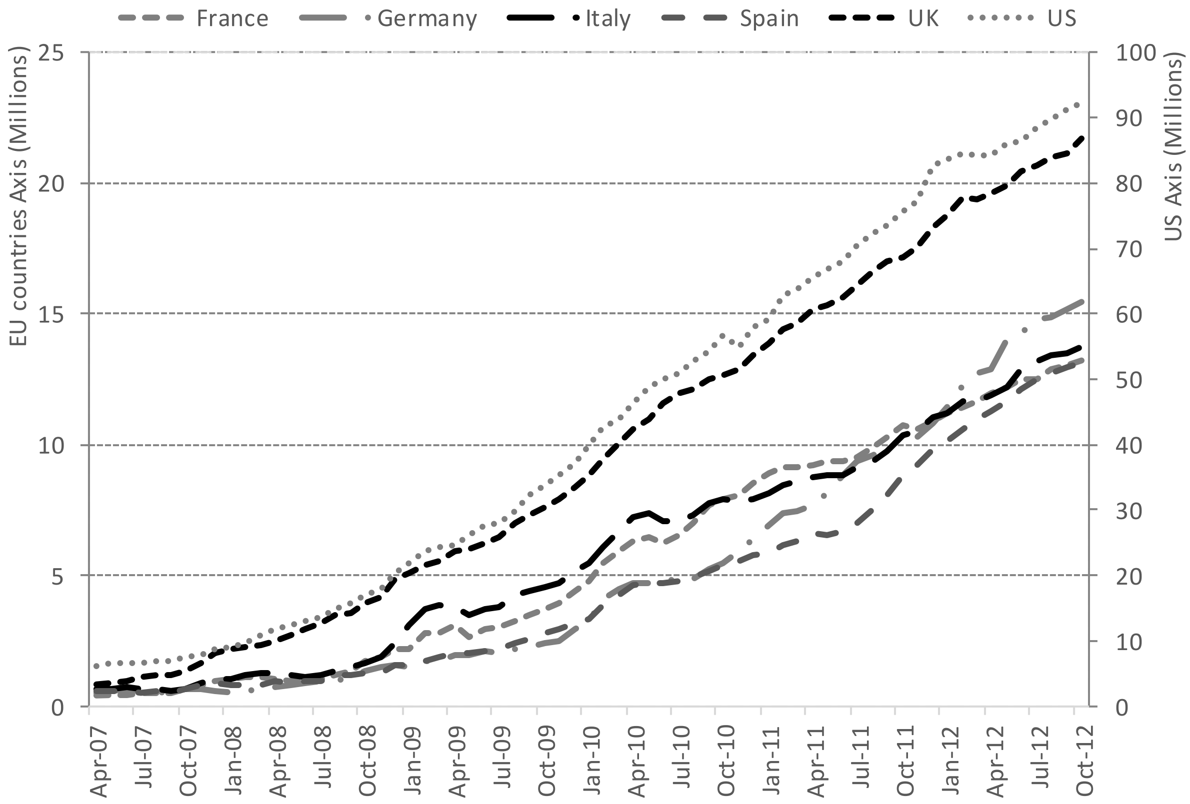
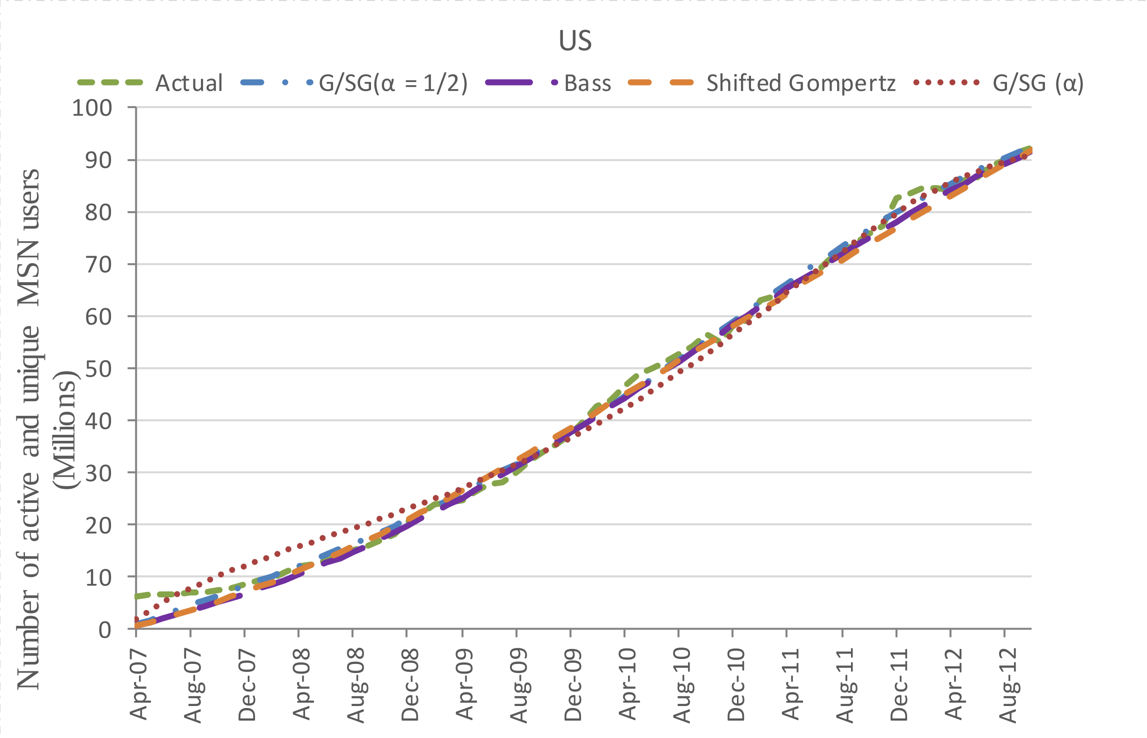
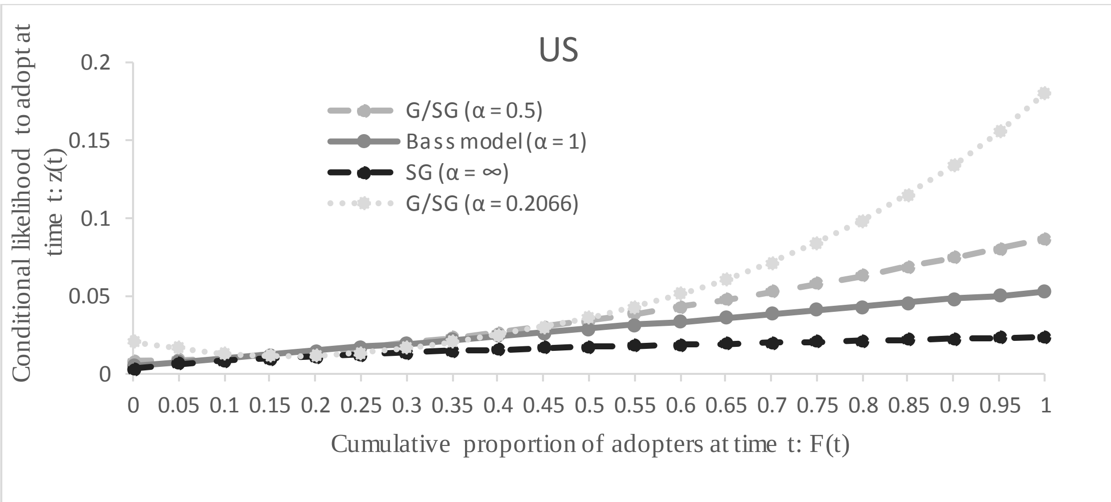


Fig. 1. Mobile social networking (MSN) in the US (right axis), UK, Germany, Italy, France, and Spain, (left axis) - from top to bottom in October 2012.

A. Fit of the four models



B. Hazard rates



Note: For the G/SG(α = 0.2066), the maxima of the hazard rate are equal to 0.0205 and 0.180 when F(t) equals 0 and 1 respectively, with a minimum at 0.01142. The minimum takes place when F(t) = 0.185. It can be obtained with a numerical search**.**

Fig. 2. Data fitting and hazard rates for Bass, Shifted Gompertz, G/SG(α = 1/2) and G/SG(α): US.

Table 1
Description of G/SG and three special cases.

Models of diffusion	Type of skew	Cumulative distribution function	Probability density function	Conditional likelihood to adopt at t: z(t) = f(t)/(1 – F(t))
G/SG (α)	0 < F(t*) < $(1 + 1/\alpha)^{-\alpha}$, where t* is the mode of f(t)	$F(t) = \frac{1 - e^{-(p+q)t}}{\{1 + [(1+q/p)^{1/\alpha} - 1]e^{-(p+q)t}\}^{\alpha}}$ $p > 0, q, \alpha \geq 0, t > 0$ When q = 0, F(t) reduces to an exponential distribution.	$f(t) = \frac{(p+q)e^{-(p+q)t}}{\{1 + [(1+q/p)^{1/\alpha} - 1]e^{-(p+q)t}\}^{\alpha+1}} \times$ $\{1 + [(1+q/p)^{1/\alpha} - 1][\alpha + e^{-(p+q)t}(1-\alpha)]\}$ Bi-modal curve with one mode at zero when 0 < α < 0.5	0.5 ≤ α < 1: Convex function of F(t) from z(0) = p to z(∞) = p + q 0 < α < 0.5: Nonmonotone function of F(t), first decreasing from z(0) = p towards a minimum and then increasing towards z(∞) = p + q α = 1: Linear function of F(t) α > 1: Concave function of F(t) from z(0) = p to z(∞) = p + q
G/SG (α = 1/2)	Left 0.5 < F(t*) < 0.58	$F(t) = \frac{1 - e^{-(p+q)t}}{[1 + (q/p)(2+q/p)e^{-(p+q)t}]^{1/2}}$ $p > 0, q \geq 0, t > 0$	$f(t) = \frac{(p+q)e^{-(p+q)t}}{[1 + (q/p)(2+q/p)e^{-(p+q)t}]^{3/2}} \times$ $[1 + (q/p)(1+q/2p)(1+e^{-(p+q)t})]$	Convex function of F(t)
G/SG (α = 1): Bass model	Right skewed. Approaches symmetry as p/q gets close to 0: 0 < F(t*) < 0.5	$F(t) = \frac{1 - e^{-(p+q)t}}{1 + (q/p)e^{-(p+q)t}}$ $p > 0, q \geq 0, t > 0$	$f(t) = \frac{(p+q)^2}{p} \frac{e^{-(p+q)t}}{[1 + (q/p)e^{-(p+q)t}]^2}$	$z(t) = p + qF(t)$.
G/SG (α = ∞): Shifted Gompertz	Right 0 < F(t*) < e^{-1}	$F(t) = (1 - e^{-(p+q)t})(1+q/p)^{-\exp(-(p+q)t)}$ $p > 0, q \geq 0, t > 0$	$f(t) = (p+q)e^{-(p+q)t}(1+q/p)^{-\exp(-(p+q)t)} \times$ $[1 + \ln(1+q/p)(1 - e^{-(p+q)t})]$	Concave function of F(t)

Table 2

Gamma/Shifted Gompertz (G/SG) models: α = 0.5, Bass (α = 1), Shifted Gompertz (α = ∞) and full version with free α (N = 67).

Country	Model	Market potential: m	Coefficient of external influence: p	Coefficient of internal influence: q	Relative impact of network externalities: q/p	Skew parameter: α	R^{2a}	Root mean squared error(x10^3)	Time to peak adoption t* (Months)	Skewness F(t*)	Peak magnitude mf(t*)	95-th percentile (Months)
France	α = 0.5	14,816,812	0.00707	0.0935	13.225		0.118	186.0	46	0.57	292,467	75
		(2,158,914)[b]	(0.0019)	(0.022)								
	α = 1	15,729,027	0.00329	0.067	20.365		0.134	184.3	43	0.48	289,804	86
		(2,700,906)	(0.0015)	(0.019)								
	α = ∞	17,816,480	0.000446	0.0418	93.722		0.133	184.5	40	0.36	282,578	111
		(3,977,197)	(0.00081)	(0.013)								
	Free α	16,424,121	0.0019	0.0553	29.105	1.7879	0.136	184.1	42	0.43	287,594	94
		(4,582,523)	(0.0035)	(0.0436)		(4.2210)						
Germany	α = 0.5	19,381,617	0.00269	0.1192	44.312		0.35	201.2	57	0.58	455,456	81
		(3,296,772)	(0.0011)	(0.025)								
	α = 1	22,558,689	0.000946	0.0754	79.704		0.342	202.4	57	0.49	435,719	96
		(5,637,956)	(0.00058)	(0.021)								
	α = ∞	36,569,559	0.0000585	0.0313	535.043		0.331	204.2	63	0.36	426,223	158
		(19,322,209)	(0.00016)	(0.014)								
	Free α	15,339,020	0.0477	0.8905	18.669	0.0495	0.407	192.2	61	0.86	583,526	64
		(1,504,014)	(0.0235)	(0.4343)		(0.0231)						
Italy	α = 0.5	21,338,374	0.00637	0.0583	9.152		0.058	214.8	60	0.55	276,191	106
		(10,872,639)	(0.0025)	(0.029)								
	α = 1	28,334,911	0.00365	0.0304	8.329		0.064	215.5	62	0.44	270,461	152
		(25,810,042)	(0.0023)	(0.026)								
	α = ∞	41,600,959	0.00204	0.0142	6.961		0.065	214.6	67	0.33	269,302	252
		(51,332,896)	(0.0017)	(0.0158)								
	Free α	39,280,378	0.0022	0.0158	7.182	6.8230	0.065	214.6	66	0.34	269,437	233
		(193,244,947)	(0.0127)	(0.1207)		(398.1733)						
Spain	α = 0.5	17,039,980	0.00297	0.1101	37.071		0.384	143.1	58	0.58	371,835	84
		(2,808,661)	(0.00092)	(0.0208)								
	α = 1	22,555,944	0.00129	0.0609	47.179		0.363	145.6	62	0.49	358,224	110

		(7,324,805)	(0.00052)	(0.0179)								
	$\alpha = \infty$	71,168,140	0.000317	0.0162	51.104		0.348	147.3	96	0.36	444,741	276
		(94,008,402)	(0.00023)	(0.012)								
	Free α	16,371,747	0.0028	0.1153	41.179	0.4919	0.383	143.1	59	0.58	370,055	83
		(4,488,860)	(0.0038)	(0.0800)		(0.4356)						
UK	$\alpha = 0.5$	26,318,806	0.0081	0.0719	8.877		0.143	181.8	48	0.55	422,264	85
		(2,985,809)	(0.0011)	(0.014)								
	$\alpha = 1$	30,933,350	0.00493	0.042	8.519		0.141	181.9	46	0.44	405,392	111
		(5,672,411)	(0.0010)	(0.0124)								
	$\alpha = \infty$	44,192,144	0.00307	0.0189	6.156		0.129	183.3	47	0.32	388,331	185
		(15,142,872)	(0.0010)	(0.0084)								
	Free α	27,821,425	0.0064	0.0582	9.094	0.6514	0.145	181.6	47	0.51	414,556	94
		6,152,766	(0.0036)	(0.0369)		(0.4591)						
US	$\alpha = 0.5$	106,116,751	0.00844	0.0783	9.277		0.082	1,108.9	45	0.55	1,839,306	80
		(14,692,512)	(0.0017)	(0.020)								
	$\alpha = 1$	119,975,856	0.0051	0.0477	9.353		0.067	1,117.7	42	0.45	1,753,711	100
		(24,418,252)	(0.0016)	(0.017)								
	$\alpha = \infty$	170,930,724	0.00348	0.0202	5.805		0.05	1,128.2	43	0.32	1,626,618	171
		(71,412,855)	(0.0016)	(0.012)								
	Free α	94,526,976	0.0205	0.1595	7.780	0.2066	0.09	1,104.2	50[c]	0.69	2,024,824	65
		(11,011,079)	(0.0084)	(0.0573)		(0.0768)						

[a] R^2 assesses the fit to the incremental number of active and unique MSN users.

[b] The asymptotic standard errors are reported in parentheses.

[c] Since α is less than 0.5, the probability density function is bi-modal with the other mode at 0. The time to the minimum between the two modes t** is equal to 15 months

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Table 3

Forecasting accuracy levels in the six countries: Absolute percentage errors[a].

L-step-ahead (sample size)	France										Germany									
	G/SG (α = 1/2)		Bass		SG		G/SG (α)		Naïve trend		G/SG (α = 1/2)		Bass		SG		G/SG (α)		Naïve trend	
	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean	Median	Geo. Mean
1 (35)[b]	**2.32**	**1.62**	3.26	2.73	4.97	3.66	3.20	3.24	1.31	1.24	3.32	3.97	3.35	3.78	**3.03**	**2.67**	3.05	2.92	3.30	2.60
2 (34)	**3.50**	**2.38**	4.07	3.37	5.12	4.55	6.54	5.09	2.30	2.26	5.04	6.08	4.42	5.23	4.64	3.58	5.30	4.46	5.41	3.43
3 (33)	**4.98**	**3.91**	5.61	4.77	6.36	4.75	6.43	6.97	2.50	2.38	8.07	8.92	8.18	8.12	**6.45**	**5.77**	8.47	6.36	6.66	5.83
4 (32)	**6.34**	**5.37**	7.23	5.97	7.04	6.67	8.31	8.35	2.96	2.70	8.59	11.17	8.83	10.01	**6.51**	**6.31**	11.06	9.39	9.49	7.50
12 (24)	16.31	12.75	13.72	13.31	17.52	15.63	27.95	27.38	6.54	5.50	23.31	31.17	17.19	24.24	**12.09**	**8.33**	32.37	24.45	26.84	27.59
18 (18)	24.05	22.10	**19.80**	**17.49**	36.21	26.12	40.91	54.54	11.87	9.68	60.99	61.50	43.17	51.35	**17.52**	**18.98**	48.49	41.80	38.31	39.27
Pooled[c]	12.95	9.14	**12.75**	**10.00**	14.82	11.02	22.47	18.02	4.93	4.60	18.55	20.92	13.08	17.09	**8.78**	**7.63**	26.53	16.64	20.90	16.53
Italy											Spain									
1 (35)	**2.30**	**1.70**	2.67	2.06	4.17	2.97	4.18	2.98	1.52	1.19	1.99	1.80	3.65	2.60	1.98	2.10	2.94	3.18	2.12	1.46
2 (34)	**3.19**	**3.01**	3.19	3.20	6.76	4.33	4.94	4.72	2.62	2.40	3.35	3.09	4.95	3.60	**2.95**	**2.25**	3.17	2.61	3.71	3.21
3 (33)	**4.26**	**4.00**	4.44	4.58	7.55	5.94	9.13	6.11	3.56	2.37	5.87	3.85	6.46	4.48	**3.75**	**2.95**	3.56	3.50	5.02	4.62
4 (32)	**4.57**	**4.26**	4.62	4.45	8.28	6.79	10.12	7.42	3.62	2.95	6.74	6.12	8.08	6.56	**3.54**	**3.90**	5.76	5.49	6.25	6.05
12 (24)	22.57	20.91	**20.79**	**18.34**	21.38	18.79	32.73	31.05	9.20	6.49	30.48	20.03	27.96	21.44	**13.15**	**10.48**	18.41	14.06	25.82	21.43
18 (18)	32.02	39.43	30.38	34.89	**28.51**	**28.57**	42.99	54.62	11.02	6.86	44.88	48.21	41.00	36.65	**12.07**	**10.48**	37.65	29.49	33.78	31.75
Pooled	17.06	12.03	**15.07**	**10.98**	16.80	11.87	23.22	17.63	7.49	4.73	21.64	14.56	20.49	13.31	**11.91**	**7.58**	16.27	10.69	18.42	12.74
UK											US									
1 (35)	1.15	0.75	1.25	0.97	**1.12**	**0.63**	1.82	1.72	1.04	0.94	1.13	0.61	1.08	0.73	0.91	0.81	11.09	5.67	0.78	0.83
2 (34)	1.27	1.47	1.47	1.38	1.39	1.36	2.92	2.60	1.49	1.16	1.48	1.44	1.94	1.40	1.64	1.02	11.85	7.16	1.83	1.28
3 (33)	2.30	1.84	2.14	1.77	**2.03**	**1.29**	2.71	2.57	1.88	1.54	2.55	2.17	3.13	2.04	**2.48**	**1.81**	13.73	8.23	2.39	2.11
4 (32)	2.71	2.40	2.56	1.85	**1.96**	**1.58**	4.22	3.20	1.97	1.77	3.67	2.94	3.36	2.42	**2.96**	**1.94**	13.21	6.80	3.35	2.49
12 (24)	10.04	8.79	4.58	3.94	4.74	3.67	22.51	21.34	6.80	6.78	8.58	8.43	9.04	4.76	7.12	5.70	28.82	22.74	7.41	6.74
18 (18)	17.92	18.72	**7.83**	**7.59**	10.89	9.27	42.06	43.99	9.59	10.26	18.02	17.57	16.16	11.51	**12.67**	**8.70**	43.35	40.67	12.02	10.93
Pooled	7.37	5.35	4.34	3.30	**4.00**	**3.10**	14.40	11.07	5.18	4.13	7.89	5.20	5.41	3.97	5.48	3.81	24.46	16.24	5.91	4.62

[a] Bold values indicate the steps for which the specified model accuracy measures (median and geometric mean) are the best ones among the four models.

[b] Starting with the 33-rd observation, we performed 35 one-month-ahead forecasts. (There are errors in the sample sizes of Table 2 in SGH). The number of data points in the estimation varied from 32 to 66.

[c] Similar to SGH (2015), we varied L from 1 to 18: the total number of absolute percentage errors (ape's) is equal to 477. The table reports a subset of the error measures.

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