Performance-based Optimisation of RC Frames with Friction Wall Dampers Using a Low-Cost Optimisation Method

Neda Nabid[1], Iman Hajirasouliha[2] and Mihail Petkovski[3]

Abstract Friction-based dampers can be considered as one of the suitable passive control systems for seismic strengthening and rehabilitation of existing substandard structures due to their high adjustability and good energy dissipation capability. One of the main issues in the design of these systems is to obtain the magnitude of the maximum slip force and the distribution of slip forces along the height of the building. In this study, a practical performance-based optimisation methodology is developed for seismic design of RC frame buildings with friction energy dissipation devices, which allows for an accurate solution at low computational cost. The proposed method aims at distributing the slip loads of the friction dampers to achieve a uniform distribution of damage along the height of the building. The efficiency of the method is evaluated through the optimum design of five different low to high-rise RC frames equipped with friction wall dampers under six natural and six synthetic spectrum-compatible earthquakes. Sensitivity analyses are performed to assess the reliability of the method using different initial height-wise slip load distributions, convergence parameters and earthquake records. The results indicate that optimum frames exhibit less maximum inter-storey drift (up to 43%) and global damage index (up to 75%), compared to uniform slip load distribution. The method is then developed to obtain the optimum design solution for a set of earthquakes representing a design spectrum. It is shown that the proposed method can provide an efficient tool for optimum seismic design of RC structures with friction energy dissipation devices for practical purposes.

Keywords Optimisation, Seismic performance, Structural damage, Friction damper, Energy dissipation

1 INTRODUCTION

Passive energy dissipation devices can be efficiently used for improving the seismic performance of new buildings or strengthening of existing substandard structures by dissipating the imparted seismic energy and reducing damage in structural elements. While yielding steel (typically in the form of buckling restrained braces) and fluid viscous dampers are currently the most widely used dampers in building structures, friction-based dampers can provide an alternative solution due to their high adjustability and good energy dissipation capability (Pall and Pall, 2004; Aiken, 1996; Sadek et al., 1996; Marsh, 2000). The first generation of friction dampers was introduced by Pall and Marsh (1982) for braced steel frames using series of steel plates clamped together designed to slip at a predetermined load. Fitzgerald (1989) utilised Slotted Bolted Connections (SBCs) in steel braced frames to dissipate earthquake input energy and control the axial loads in braced elements to avoid buckling. Experimental tests conducted by Grigorian et al. (1993) showed that SBCs with brass on steel frictional surfaces exhibit a nearly elastic-perfectly-plastic behaviour with a reasonably constant slip force under sinusoidal and natural seismic excitations. However, brass in contact with steel is susceptible to severe corrosion due to bimetallic contact (BSI, 1990), and therefore, their proposed system may not be suitable for practical applications. In a comprehensive study, Constantinou et al. (2007) investigated the short-term and long-term frictional properties of different bimetallic interfaces under service and high-speed seismic loading conditions. The results of their study indicate that bimetallic interfaces generally exhibit significant changes in friction force with time, and therefore, the friction force during a future earthquake event cannot be accurately predicted.

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[1]PhD Candidate, Department of Civil and Structural Engineering, The University of Sheffield, Sheffield, UK. E-mail: nnabid1@sheffield.ac.uk (Corresponding Author), ORCID ID: 0000-0003-2249-1413

[2]Senior Lecturer, Department of Civil and Structural Engineering, The University of Sheffield, Sheffield, UK. E-mail: i.hajirasouliha@sheffield.ac.uk, ORCID ID: 0000-0003-2597-8200

[3]Lecturer, Department of Civil and Structural Engineering, The University of Sheffield, Sheffield, UK. E-mail: m.petkovski@sheffield.ac.uk, ORCID ID:0000-0002-3788-0772

Several other friction-based dampers, mainly combined with steel bracing systems, were introduced over the last two decades, including Rotational Friction Damper (RFD) (Mualla, 2000; Shirkhani et al., 2015), Improved Pall Friction Damper (Wu et al., 2005) and Shear Slotted Bolted Connection (SSBC) (Nikoukalam et al., 2015). However, using brace-type control devices in RC frames may be accompanied by the risk of damaging the concrete at the connection zones due to high stress concentrations. To address this issue, wall-type systems can be used to provide adequate space to transfer forces to the surrounding elements.

In one of the early attempts, Sasani and Popov (1997, 2001) studied the behaviour of friction energy dissipaters combined with a lightweight concrete panel for different input displacements. The friction-based damper they proposed incorporates a precast concrete wall fixed to the lower floor beam using anchor bolts and to the upper floor beam using SBC friction energy dissipaters. The results of their study indicated that this type of damper can provide a stable hysteresis loop through friction mechanism between the sliding metal surfaces of the connectors. In another relevant study, Petkovski and Waldron (2003) showed the efficiency of SBC devices attached to concrete wall panels (with and without openings to enhance the seismic performance of multi-storey RC structures subjected to a set of natural earthquake records. More recently, Nabid et al. (2017) investigated the efficiency of friction-based wall dampers designed with different slip load distribution patterns in improving the seismic performance of substandard RC structures. Based on the results of their extensive analytical study, an empirical formula was proposed to obtain more efficient height-wise distribution of slip loads by considering different seismic performance parameters. However, they did not use any optimisation method to obtain the best design solutions.

Obtaining the optimum design solutions of energy dissipation devices can be a challenging task due to complexity and high nonlinearity of these systems under earthquake excitations (Whittle et al., 2012 and 2013). Over the past two decades, a number of different optimisation methods have been used for optimum design of energy dissipation devices, including Simulated Annealing (SA) (Milman and Chu, 1994), Linear Quadratic Regulator (LQR) (Gluck et al., 1996; Agrawal and Yang, 1999) Gradient-based Optimisation (Singh and Moreschi, 2001; Park et al. 2004; Lavan and Levy, 2006; Fujita et al., 2010), Fully Stressed Design Optimisation (Levy and Lavan, 2006), Genetic Algorithm (GA) techniques (Moreschi and Singh, 2003; Asahina et al., 2004; Lavan and Dargush, 2009; Honarparast and Mehmandoust, 2012; Hejazi et al., 2013) and a successive procedure using Sensitivity Analysis and Redesign (Takewaki, 2011; Adachi et al., 2013). While conventional structures are expected to exceed their elastic limits in severe earthquakes, for simplification purposes, most of these studies assumed a linear behaviour for the structural systems equipped with the supplemental energy dissipation devices.

The efficiency of friction-based dampers is strongly associated with the location of the dampers and the height-wise distribution of slip loads, which can be then tuned to obtain the required performance objectives. While most optimisation techniques developed for hysteretic dampers (e.g. Uetani et al., 2003; Murakami et al., 2013; Martinez et al. 2014) can be also adopted for friction-based devices, there are limited studies available on the optimum design of friction-based dampers. In one of the early attempts, a simplified seismic design procedure was developed by Filiatrault and Cherry (1990) which was capable to achieve the optimum slip load values while minimizing an energy performance index (RPI). Based on the results of their study, a design slip load spectrum was proposed to obtain the optimum distribution of the slip loads. It was also shown that the optimum slip load values are more affected by the amplitude and frequency of the input earthquakes (e.g. peak ground acceleration) rather than the characteristics of the structure. Patro and Sinha (2010) evaluated the seismic performance of shear building structures with dry-friction devices using a constant slip load distribution pattern subjected to a set of earthquake ground motions. They concluded that while the optimum slip load values can be considerably affected by the characteristics of the selected input earthquake, a suitable range of slip loads can be obtained which generally leads to a better seismic performance under a wide range of design earthquake ground motions. In another relevant study, the Genetic Algorithm (GA) optimisation method was used by Moreschi and Singh (2003) for optimum placement of friction and yielding metallic dampers in multi-storey steel braced frames to satisfy a prescribed performance objective. Using a similar approach, Miguel et al. (2014) adopted a GA methodology for multi-objective optimisation of friction dampers in shear building structures subjected to earthquake ground motions. In more recent studies, a Backtracking Search Algorithm (BSA) was adopted by Miguel et al. (2016a, 2016b) to optimise simultaneously the location and the slip force values of the friction devices in shear buildings subjected to seismic loads. It should be mentioned that most of the aforementioned optimisation methods are computationally expensive and/or require complex mathematical calculations, and therefore, may not be suitable for practical applications.

This study aims to develop a low-cost performance-based optimisation method for seismic design of non-linear friction-based wall dampers based on the concept of Uniform Damage Distribution. The proposed optimisation method can considerably improve the seismic behaviour of the structures in only a few steps by

controlling performance indices such as maximum inter-storey drift and maximum energy dissipation capacity. The efficiency of the method in improving the seismic behaviour of RC frames with friction wall dampers is demonstrated through several design examples under a set of natural and synthetic spectrum compatible earthquakes. It should be noted that the term "performance-based optimisation" in this study implies that the design of the proposed system is optimised directly based on the seismic performance parameters. Otherwise, unlike conventional performance-based design methods, only a single seismic hazard level is considered as will be discussed in the next sections.

2 MODELLING AND ASSUMPTIONS

2.1 Reference frames

Friction energy dissipative devices are commonly used in practice to reduce earthquake-induced response of structures for both new building designs and strengthening purposes. In this study, five different substandard moment-resisting RC frames with 3, 5, 10, 15 and 20 storeys are strengthened with wall-type friction energy dissipation devices in their middle span as shown in Fig. 1. To represent substandard structures in high-seismic regions, the reference frames were designed using the IBC-2015 (and ASCE/SEI 7-10 (2010)) proposed design spectrum with 0.2g peak ground acceleration (PGA), and 0.40g and 0.64g spectral response accelerations at short and 1-sec periods, respectively. The site soil profile was assumed to be type D of IBC (2015) soil category. The dead and live loads for interior storeys were considered to be 6 kN/m^2 and 2 kN/m^2, respectively, while the corresponding loads were reduced to 5 kN/m^2 and 1.5 kN/m^2 for the roof level. The RC frames were primarily designed to satisfy the minimum requirements of ACI 318-14 (2014) for intermediate ductility level. The yield strength of steel reinforcement (f_y) and the compressive strength of concrete (f_c') were selected to be 400 and 35 MPa, respectively.

Nonlinear time-history and pushover analyses were performed using DRAIN-2DX computer program (Parkash and Powel, 1993). The Rayleigh damping ratio of 0.05 was assigned to the first mode of vibration and to the mode at which the cumulative mass participation exceeds 95%. Nonlinear beam and column elements were respectively modelled using moment rotation (M-ϑ) and axial-moment interaction (P-M) plastic hinges at their both ends.


Fig. 1 Geometry of the reference RC frames equipped with friction wall dampers

2.2 Friction wall dampers

Fig. 2 shows the schematic view of the friction wall damper used in this study, which consists of a concrete wall panel attached to the surrounded beam and column elements by using a horizontal support at the bottom, two vertical supports in the lateral sides and a friction connection at the top. Panel-to-column connections with horizontal slots are used in the vertical supports to prevent transferring shear forces to the adjacent columns. The support between the wall panel and the lower floor beam is also fixed horizontally by using vertical slots to avoid transferring additional shear forces to the floor beam. By using this configuration, the lateral movement of the friction device connected to the top of each wall panel would be equal to the inter-storey drift at that level. The friction device is a typical Slotted Bolted Connection (SBC) with two external steel plates fixed to the top edge of the wall panel and a central T-shape slotted stainless steel plate sandwiched between the two external plates and anchored to the top floor beam. The over-sized holes located at the central stainless steel plate ensure the dry friction between the central plate and the two external brass plates (see Fig. 2).

In the analytical models developed in this study, an inelastic link element was used to simulate the Coulomb friction hysteretic behaviour of the friction device. It should be noted that the idealized Coulomb behaviour would be difficult to achieve in practice since the clamping force and the coefficient of friction may change with time (Constantinou et al., 2007; Symans et al., 2008). However, it will be shown in the following sections that a small variability in the friction force does not considerably affect the seismic performance of the optimum design dampers. An elastic panel element with 15 cm thickness was utilised to model the wall panel. The concrete wall panels were designed based on the maximum loads that could be transferred from the friction devices and therefore were assumed to remain in the elastic range.
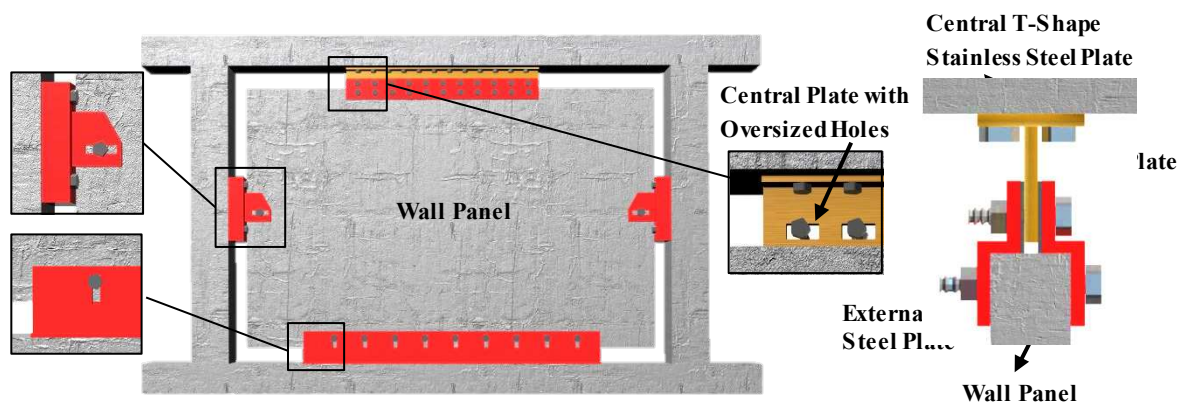
Fig. 2 Schematic view of the adopted friction-based wall damper and the utilised slotted bolted connection

2.3 Selected excitation records

To demonstrate the efficiency of the proposed optimisation framework, six medium-to-strong ground motions obtained from Pacific Earthquake Engineering Research Center online database (PEER, 2016) were selected including: Imperial Valley 1979, Superstition Hills 1987, Loma Prieta 1989, Cape Mendocino 1992, Northridge 1994 and Duzce 1999 (see Table 1 for more details). These ground motions have high local magnitudes (i.e. Ms>6.5) and were recorded on IBC-2015 soil class D profiles with less than 45 km distance from the epicentre. A set of six synthetic earthquakes were also generated using SIMQKE program (Vanmarke, 1976) to be well-matched with the IBC (2015) design response spectrum for the high seismicity regions (i.e. PGA=0.4g) with site class D. Fig. 3 illustrates the elastic acceleration response spectra of the six selected natural earthquakes, the IBC-2015 design spectrum and the mean spectrum of the generated synthetic earthquakes. For better comparison, the mean plus one standard deviation of the natural and synthetic earthquakes are also illustrated in this figure. It is observed that both the mean spectrum of the natural ground motions and the mean spectrum of the synthetic earthquakes provide a close approximation to the IBC response spectrum. However, the natural earthquakes show a considerably higher standard deviation (see Fig. 3). While on average both natural and synthetic records can be considered as good representatives of the selected design spectrum, using both datasets will help to investigate the sensitivity of the optimum design solutions to the frequency content of the individual design earthquakes.

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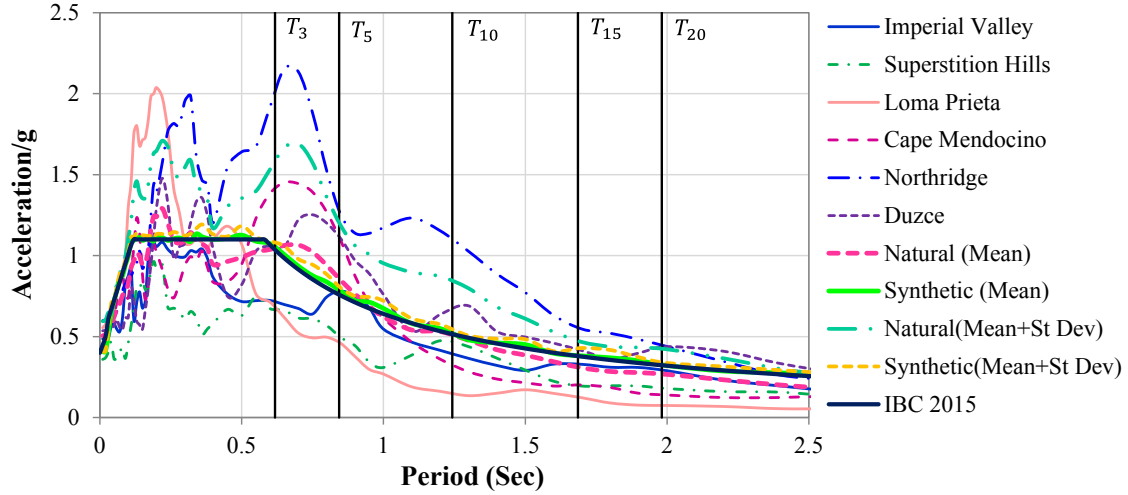
Fig. 3 Elastic spectral acceleration of natural and synthetic earthquakes and the IBC design spectrum for soil type D, 5% damping ratio

Table 1 Selected natural earthquake ground motion records

No.	Earthquake	Ms	Station/Component	Duration (s)	PGA (g)	PGV (Cm/s)	PGD (Cm)
1	1979 Imperial Valley	6.5	IMPVALL/H-E04140	39	0.485	37.4	20.23
2	1987 Superstition Hills (B)	6.7	SUPERST/B-ICC000	60	0.358	46.4	17.50
3	1989 Loma Prieta	6.9	LOMAP/G03000	40	0.555	35.7	8.21
4	1992 Cape Mendocino	6.9	CAPEMEND/PET000	36	0.590	48.4	21.74
5	1994 Northridge	6.7	NORTHR/NWH360	40	0.590	97.2	38.05
6	1999 Duzce, Turkey	7.2	DUZCE/DZC270	26	0.535	83.5	51.59

3   PRACTICAL DESIGN METHODOLOGY

One of the main features of the friction energy dissipation devices is their capability to adjust the slip forces (F_s) of the friction connections independently at different levels by regulating the clamping forces of the bolts. This offers the opportunity to use more efficient height-wise slip load distributions to improve the seismic performance of friction energy dissipation devices.

3.1 Performance parameters

In this study, maximum inter-storey drift and energy dissipation capacity of friction device were used as main performance parameters to obtain the best design solutions. Maximum inter-storey drift has been widely used in seismic design guidelines (e.g. ASCE/SEI 41-13 (2014)) as a simple and practical failure performance criterion to assess the damage in structural and non-structural elements. The energy dissipation capacity of the dampers is also a good measure to assess the efficiency of the passive control systems under seismic loads.

To assess the energy dissipation capacity of the friction devices, R_w is introduced as the ratio between the friction work of the friction device, W_{sf}, and the deformation work of the main structural elements (Petkovski and Waldron, 2003; Nabid et al., 2017).

$$R_w = \frac{W_{sf}}{W_{sb} + W_{sc}} \tag{1}$$

where W_{sb} and W_{sc} represent the static work of the beam and column elements, respectively.

Fig. 4 illustrates the variation of the mean of the maximum inter-storey drift ratios and the energy dissipation parameters, R_w, versus slip load ratio for the 3, 5, 10, 15 and 20-storey frames under the six selected natural earthquakes. The frames were designed using the conventional uniform height-wise slip load distribution. For better comparison, the maximum inter-storey drift ratios of the frames with friction wall dampers under different earthquakes were scaled to the maximum inter-storey drift ratios of the corresponding bare frames (i.e. no

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friction damper). The slip load ratio used in the figures also represents the ratio between the mean slip load and the mean storey shear strength at all storey levels.
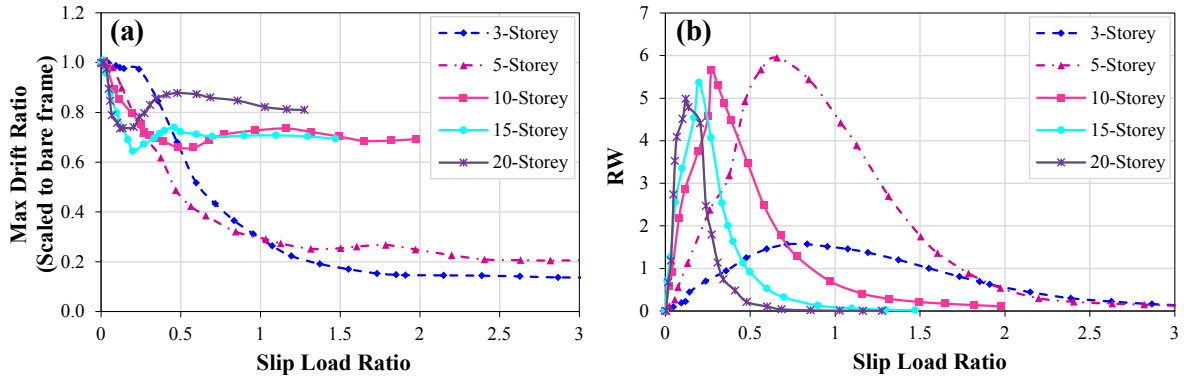
Fig. 4 Variation of (a) maximum drift ratio and (b) R_w of 3, 5, 10, 15 and 20-storey frames versus slip load ratio (ratio of mean slip load to mean storey strength), average of six natural earthquakes

As shown in Fig. 4, there is an optimum range of slip load ratios for all studied frames that on average leads to higher energy dissipation in the friction dampers and lower inter-storey drift ratios. This conclusion is in good agreement with the results reported by Petkovski and Waldron (2003) and Nabid et al. (2017). It can be noted from Fig. 4 (a) that using the friction wall dampers with uniform slip load distribution was more efficient for low to medium-rise frames (i.e. 3 and 5-storey frames) with almost 80% reduction in maximum inter-storey drift compared to the frames with no friction damper (i.e. slip load ratio of zero). This will be discussed in more details in the following sections.

3.2 Optimum slip load range

In a recent study, Nabid et al. (2017) compared the seismic performance of 3, 5, 10, 15 and 20-storey RC frames with friction-based wall dampers designed based on five different slip load distribution patterns. According to their results, the optimum range of the slip load ratio decreases by increasing the number of storeys, while in general it is not considerably affected by the selected slip load pattern. The following empirical equation was proposed to obtain the optimum slip load ratio for seismic design of multi-storey RC frames with friction-based wall dampers:

$$R = 1.12e^{-0.11n} \tag{2}$$

where R is the optimum design slip load ratio and n is the number of storeys. Considering Equation 2, the total slip load values can be calculated as follows:

$$\sum_1^n F_{s,i} = R \times \sum_1^n F_{y,i} \tag{3}$$

Where $F_{s,i}$ and $F_{y,i}$ are the slip load and the storey shear strength of the ith storey. The shear strength of each storey ($F_{y,i}$) can be easily calculated by conducting a non-linear pushover analysis (Hajirasouliha and Doostan, 2010). The total slip load values can be then distributed along the height of the structure using any prescribed distribution pattern. If the uniform height-wise slip load distribution is considered, the slip load values at each storey level ($F_{s,i}$) can be easily calculated using the equation below:

$$F_{s,i} = \frac{R}{n} \times \sum_1^n F_{y,i} = \frac{1.12e^{-0.11n}}{n} \times \sum_1^n F_{y,i} \tag{4}$$

It should be mentioned that for practical applications a uniform slip load distribution is commonly used for the seismic design of friction dampers. While this distribution pattern can simplify the design process, it does not necessarily lead to the best design solution under different input earthquakes (Nabid et al., 2017). Therefore, in the following section, an optimum design methodology is proposed to obtain more efficient slip load distribution patterns.

4   PROPOSED PERFORMANCE-BASED OPTIMISATION METHODOLOGY

In conventionally designed friction wall dampers (i.e. uniform slip load distribution), the deformation demand under strong earthquakes may not utilize the maximum level of seismic capacity in certain storeys while localised damage may be observed in the other storeys. If the slip loads at storeys with large inter-storey drifts are increased incrementally, while the slip loads are decreased at storeys with small drifts, it is expected to eventually obtain a status of uniform displacement demand. In such a case, the maximum capacity of each friction device to dissipate the earthquake input energy is utilised. It should be mentioned that a similar concept has been previously used by other researchers for optimum seismic design of different types of structural systems including shear-buildings (Moghaddam and Hajirasouliha, 2008; Hajirasouliha and Pilakoutas, 2012; Ganjavi et al., 2016), truss-like structures (Hajirasouliha et al., 2011), RC frames (Hajirasouliha et al., 2012), and viscous dampers (Levy and Lavan, 2006). However, this is the first time that this concept is adopted for seismic design of friction based energy dissipation devices to obtain the best height-wise distribution of the slip loads. On the other hand, a new iterative process is suggested for friction-based dampers to provide optimum design solutions at low computational cost. The proposed method can be used to optimise the seismic behaviour of the structures using different performance parameters such as inter-storey drift and energy dissipation capacity of the dampers.

Maximum inter-storey drift is widely accepted as an effective and practical response parameter to estimate the damage to both structural and non-structural components in building structures (e.g. ASCE 41-13 (2014)). By considering the maximum inter-storey drift as the main design parameter, the following optimisation algorithm is adopted in this study:

1. The friction wall dampers are initially designed with uniform slip load distribution using the slip load values obtained from Equation 4. It will be shown in the next section that the optimum design solution is independent of the initial slip load distribution.

2. The structure is subjected to the selected design earthquake and the corresponding performance parameters (here maximum inter-storey drift values) are obtained. The slip load values are then redistributed based on the ratio between the maximum inter-storey drift at each level and the mean of all storey drifts using the following equation:

$$\left(F_{s,i}\right)_{n+1} = \left(F_{s,i}\right)_n \times \left(\frac{\Delta_i}{\Delta_{Mean}}\right)_n^{\alpha} \tag{5}$$

where Δ_i and Δ_{Mean} are the maximum drift at i^{th} storey and the mean of all storey drifts, respectively, at n^{th} iteration. α is a convergence parameter ranging from 0 to 1. It will be shown in the following sections that this parameter can significantly affect the convergence and the computational cost of the nonlinear optimisation problem. In this study, α was set to be 0.2 for all the optimisation procedures. Using Equation 5 will eventually lead to a uniform height-wise inter-storey drift distribution.

3. To control the additional base shear and column axial forces imposed by the friction wall dampers, the new slip loads obtained from the previous step are scaled so as the mean of the slip loads in different storeys remains unchanged compared to the initial step ($F_{s,i}$ in Equation 4).

4. The coefficient of variation (COV) of inter-storey drifts (selected damage index) is calculated for each step using Equation 6 to control the dispersion of each storey drift relative to the mean value. The design procedure is then repeated from step 2 until COV_Δ is small enough (e.g. less than 10). Based on the concept of Uniform Damage Distribution, the design solution can be considered as practically optimum at this stage.

$$\left(COV_\Delta\right)_n = \left(\frac{\sqrt{(Var_\Delta)_n}}{(\Delta_{Mean})_n}\right) 100 \tag{6}$$

where Var_Δ is the variance of all storey drifts.

4.1 Optimum design for the selected natural earthquakes

The proposed optimisation method is adopted to obtain the optimum slip load distribution of the 3, 5, 10, 15 and 20-storey frames for the six selected natural earthquakes given in Table 1. Fig. 5 compares the height-wise inter-

storey drift distribution of optimum design frames and those designed with uniform slip load distribution under each design earthquake. The horizontal axis is the ratio between the maximum drift of the reference frame at each storey to the maximum drift of the corresponding bare frame, denoted as "maximum drift ratio". The results in Fig. 5 indicate that the optimum design frames exhibit a more uniform height-wise distribution of inter-storey drifts and a considerably lower maximum inter-storey drift ratio, which is consistent with the concept of Uniform Damage Distribution. Therefore, the proposed design method can efficiently prevent damage concentration and soft storey failure in multi-storey frames.
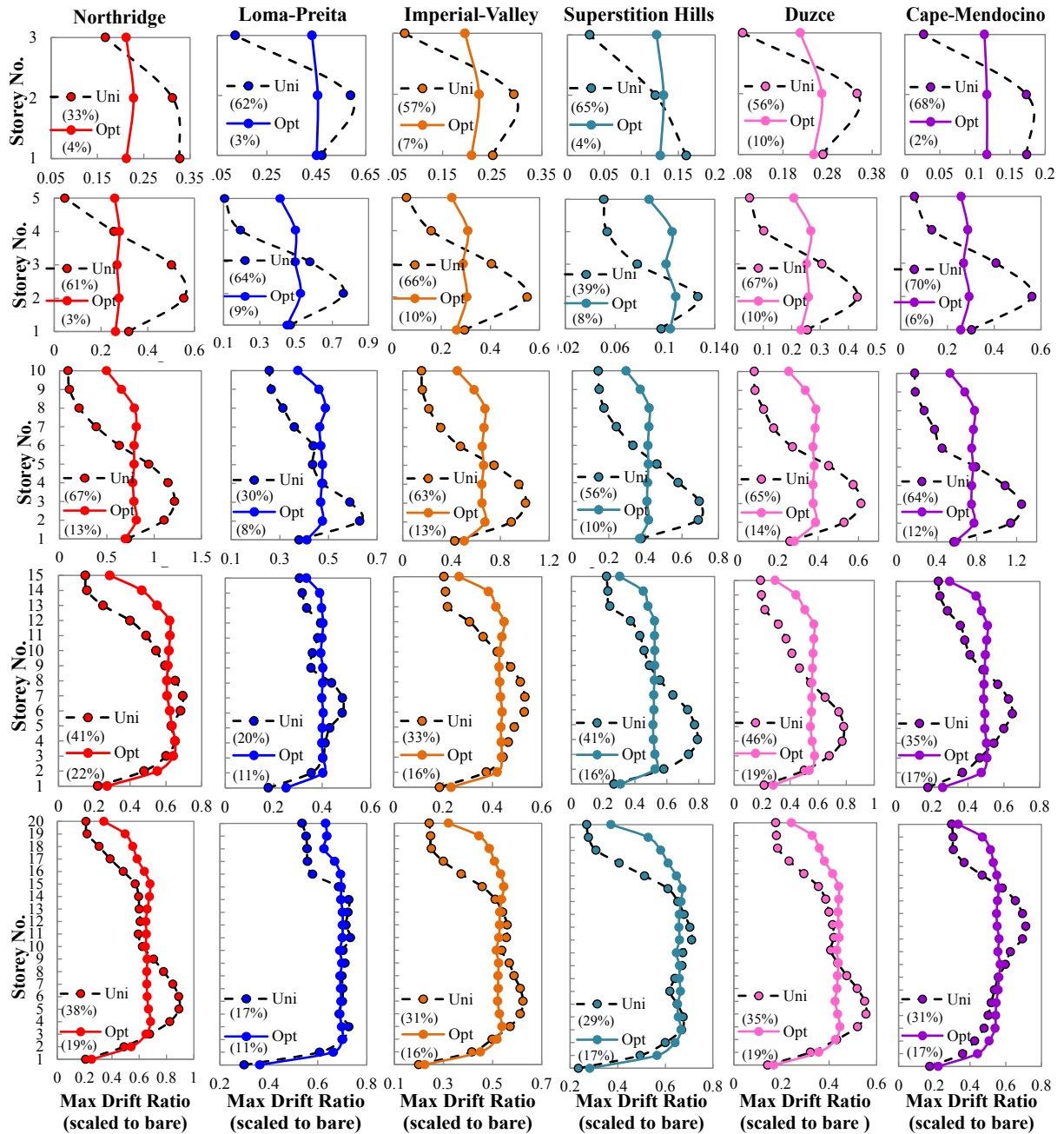
Fig. 5 Height-wise distribution and COV (%) of maximum inter-storey drift ratios (scaled to the maximum drift of the corresponding bare frame) for 3, 5, 10, 15 and 20-storey frames, six natural earthquakes

Table 2 summarizes the reductions in the maximum lateral drifts for the reference frames under the six natural earthquakes when designed using the proposed optimum design methodology compared to the conventional design using the uniform slip load distribution. It is shown that using the optimum distribution of the slip loads in the friction energy dissipation devices resulted in up to 33, 50, 39, 34, and 23% reductions in the maximum drift ratios for the 3, 5, 10, 15 and 20-storey frames, respectively. It should be mentioned that the efficiency of the proposed optimisation method is in general lower for high-rise buildings (i.e. 15 and 20-storey

frames), since the uniform slip load distributions led to a relatively more uniform distribution of maximum lateral drifts and therefore lower COV_Δ values as shown in Fig. 5.

Table 2. Reduction of the maximum drift ratios for the optimum design 3, 5, 10, 15 and 20-storey frames compared to conventionally designed frames, six natural earthquakes

| Earthquake	3-Storey	5-Storey	10-Storey	15-Storey	20-Storey
Northridge	30.5%	49.7%	33.2%	6.3%	23.3%
Loma Prieta	22.6%	31.1%	22.4%	16.4%	4.3%
Imperial Valley	23.9%	43.9%	32.9%	15.8%	12.5%
El Centro	19.1%	14.1%	39.4%	33.5%	5.6%
Duzce	22.9%	38.0%	36.6%	26.6%	19.7%
Cape Mendocino	32.6%	47.8%	37.6%	21.3%	20.1%
Average	**25.2%**	**37.4%**	**33.7%**	**20.0%**	**14.3%**

4.2 Optimum design for the synthetic spectrum-compatible earthquakes

To include the ground motion variability, the accuracy of the proposed optimum design method is also evaluated for the set of six synthetic spectrum-compatible earthquakes having the mean acceleration response spectrum close to the IBC-2015 design spectrum. As mentioned before, the obtained synthetic records can be considered as good representatives of the IBC design response spectrum. Fig. 6 (a) shows the variation of the maximum inter-storey drifts for the reference frames as the optimisation iterations proceed. A faster convergence was generally observed for the low-rise frames (i.e. 3 and 5-storey); however in all cases the optimum solution was obtained in less than 20 steps with almost no oscillation. It should be noted that to obtain the optimum design solutions using heuristic optimisation methods such as Genetic Algorithm (GA) and Particle Swarm Optimisation would require over 50,000 non-linear dynamic analyses (e.g. 100 samples, 500 generations). This clearly highlights the computational efficiency of the proposed optimisation method. As an example, the evolutionary change in the height-wise maximum drift distribution of the 10-storey frame is illustrated in Fig. 6 (b). It is shown that the maximum drift distribution is considerably more uniform in the optimum solution (i.e. step 20) compared to the conventional design based on uniform slip load distribution (i.e. step 0), while the maximum drift is also reduced.
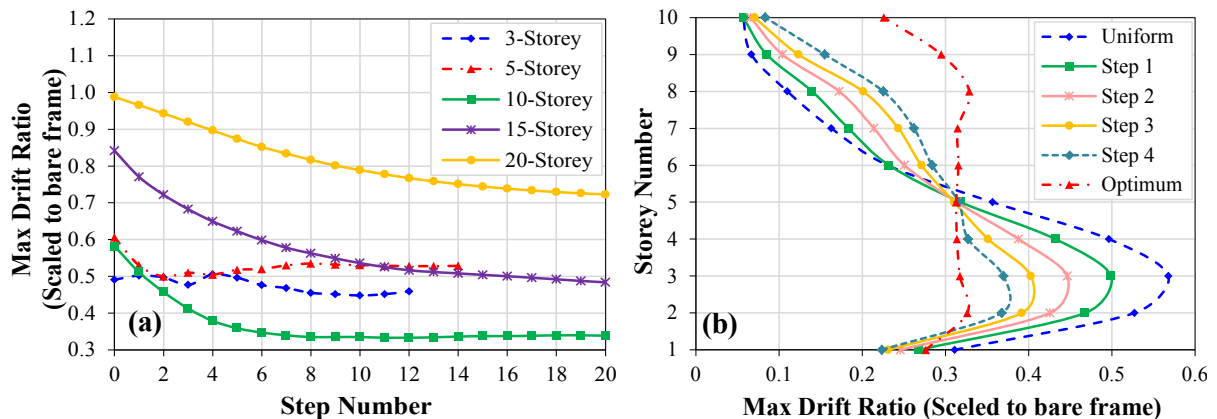

Fig. 6 Variation of (a) maximum inter-storey drift ratios (scaled to the maximum drift of the corresponding bare frame) for 3, 5, 10, 15 and 20-storey frames, and (b) height-wise distribution of maximum drift ratios for 10-storey frame, average of six synthetic earthquakes, α=0.2

Fig. 7 shows the height-wise distribution of the maximum inter-storey drift ratios (ratio of the maximum drift in the frame with friction damper to that of the corresponding bare frame) and the slip load ratios (ratio of the slip load to the mean of the storey shear strengths) for 3, 5, 10, 15, and 20-storey frames designed based on the conventional uniform and optimum slip load distributions (with the same average) subjected to the synthetic design earthquakes. Similar to the natural earthquake records, the results in Fig. 7 (a) indicate that the optimum design models always exhibit a considerably more uniform distribution of inter-storey drifts and a lower maximum inter-storey drift compared to the conventional design solutions. The proposed performance-based optimisation procedure could efficiently identify the storeys in which the friction wall damper is not required.
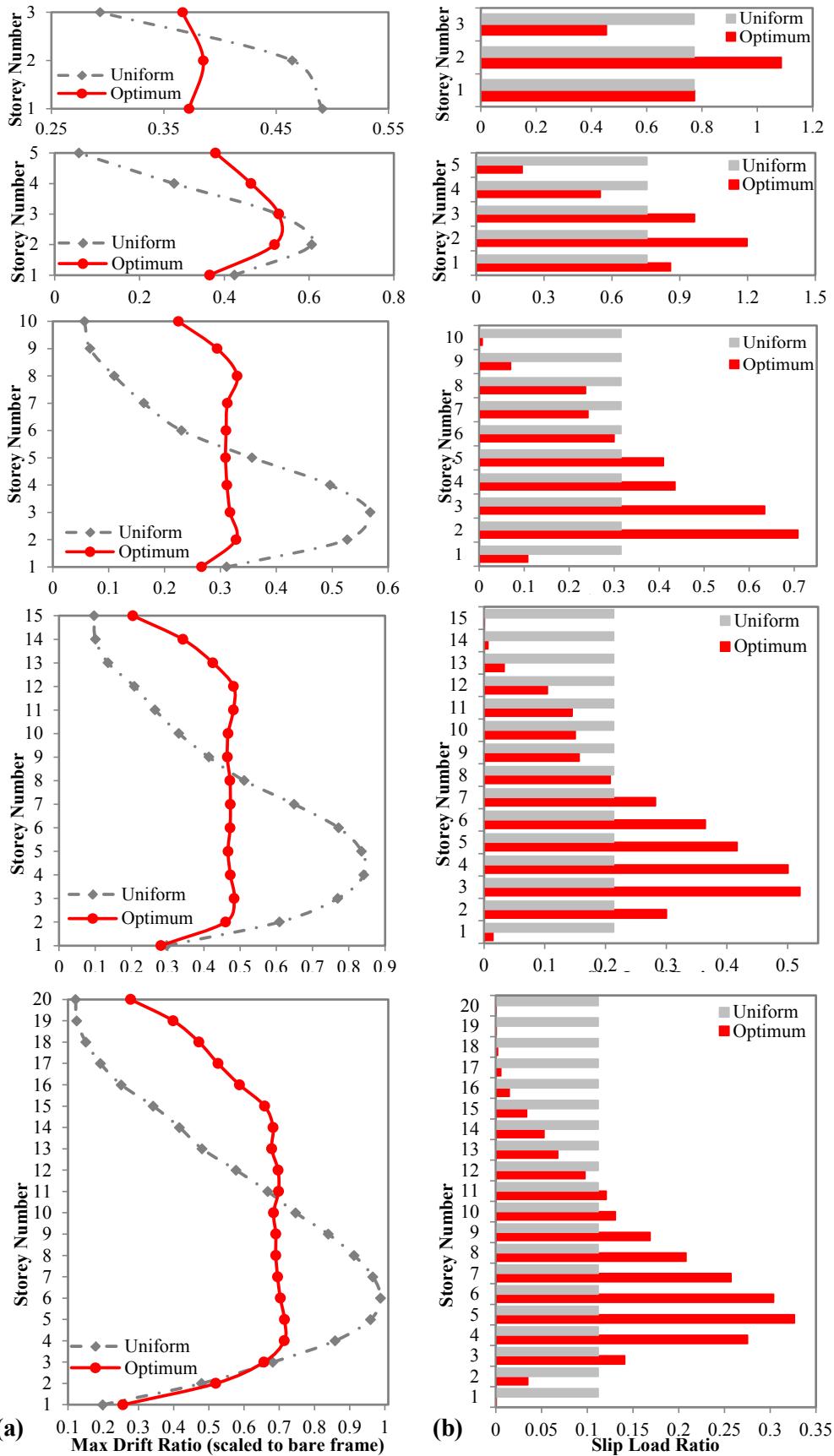
Fig. 7 Height-wise distribution of (a) maximum drift ratios (scaled to the maximum drift of the corresponding bare frame) and (b) slip load ratios (scaled to the mean storey shear strength) for 3, 5, 10, 15 and 20-storey frames designed with uniform and optimum slip load distributions, average of six synthetic earthquakes

For example, it is shown in Fig. 7 (b) that the slip load values at the first and the upper storey levels in medium to high-rise frames (10, 15 and 20-storey frames) tend to zero. This will in turn lead to more economical design solutions with less number of friction dampers. Fig 8 compares the maximum inter-storey drift, column axial load, base shear and energy dissipation capacity (R_w) of the optimum design frames with those designed using uniform slip load distributions. The results in Figs. 8 (a) to (c) are the ratio of the response of the optimum frame to that of the corresponding bare frame. The results in general indicate that, compared to conventional design solutions, using the proposed optimisation method could considerably increase the energy dissipation in the friction devices (up to 46%) and reduce the maximum inter-storey drifts (up to 43%) under synthetic spectrum-compatible earthquakes, while the maximum column axial load and the base shear ratios were almost unchanged (less than 4% difference). It can be noted form Fig. 8 (d) that the improvement in the energy dissipation capacity was less pronounced in the 20-storey frame; however, using the optimum slip load distribution could still reduce the maximum inter-storey drifts by 28%. The maximum reduction in the inter-storey drift was observed in the 10 and 15-storey frames, where the optimum design procedure could reduce the maximum inter-storey drifts by more than 40%.
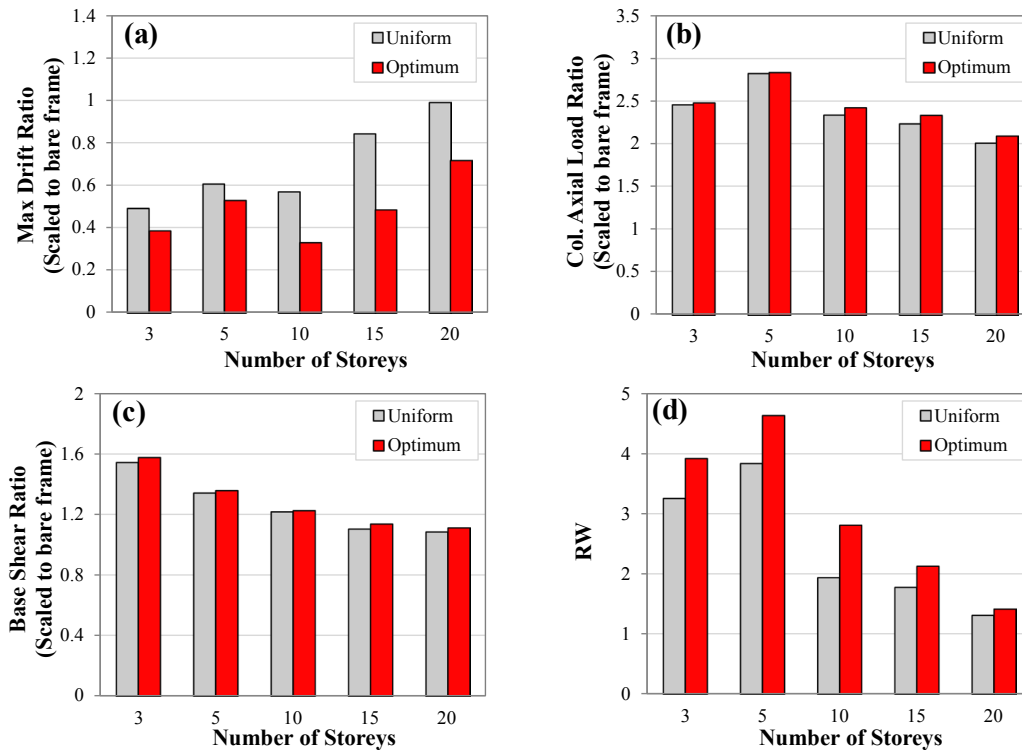

Fig. 8 (a) Inter-storey drift, (b) axial load and (c) base shear of 3, 5, 10, 15 and 20-storey frames with damper to those of the corresponding bare frames, and (d) energy dissipation capacity of the optimum and conventional design frames, average of six synthetic earthquakes

5   SENSITIVITY ANALYSES

5.1 Effect of initial slip load distribution

The effect of initial slip load distribution was investigated on the proposed optimisation method by considering three different distribution patterns including the conventional uniform, inverted triangular cumulative and uniform cumulative (shown in Fig. 9) for preliminarily design of the 5-storey frame under a synthetic earthquake. For better comparison, a similar mean slip load value was used in all preliminarily designed frames. Fig. 9 compares the corresponding variations of the slip load values for each individual storey as the iterations proceed. The graphs show that in the first few steps of the optimisation, there were considerable discrepancies between the slip load values using different initial slip load patterns. However, as the iterations proceed, depending on the floor level and the initial distribution pattern, the slip loads were either decreased or increased to converge to a certain value. This implies that the final optimum solution is independent from the initial slip load distribution considered in the optimisation process. However, using an appropriate initial design can result

in a faster convergence to the final optimum solution. In this example, the uniform cumulative pattern which was previously suggested by Nabid et al. (2017) as a more efficient slip load pattern is shown to converge to the optimum solution in a slightly smaller number of steps.
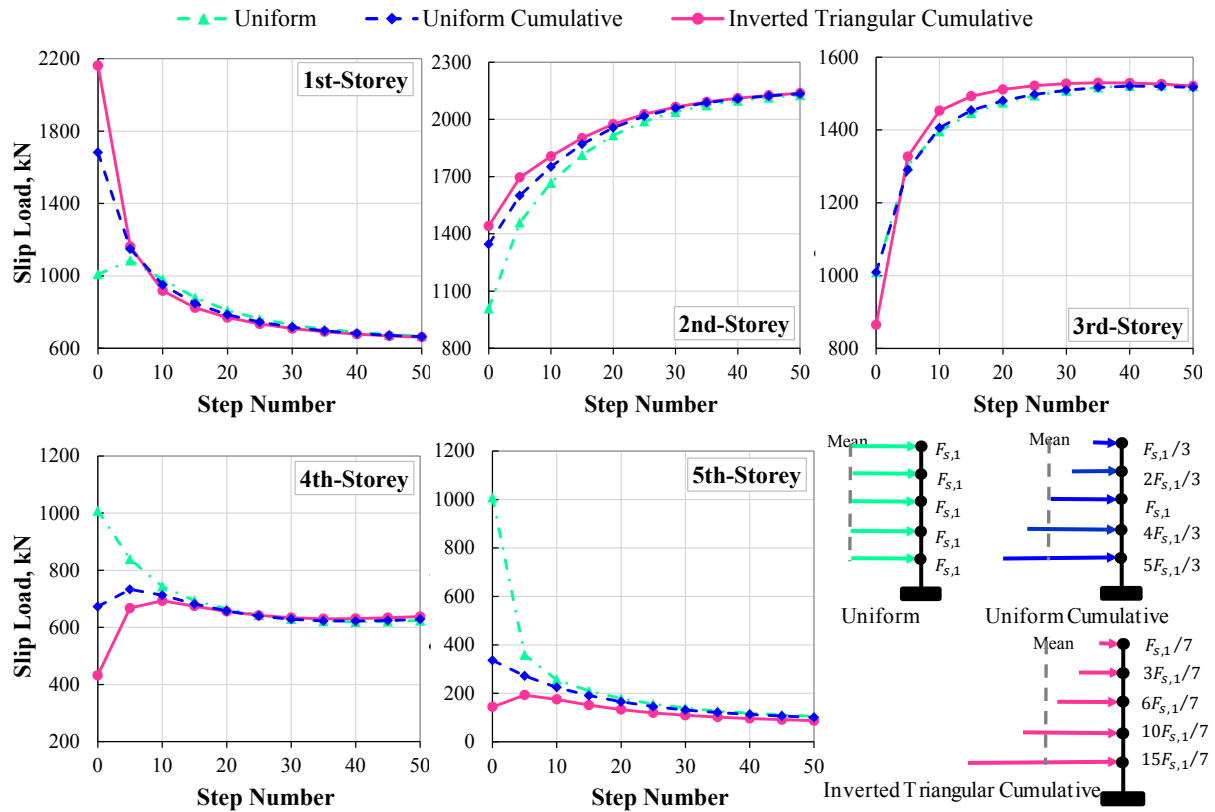
Fig. 9 Variation of slip load at each individual floor using different initial distribution patterns, 5-storey frame, α=0.2, synthetic earthquake
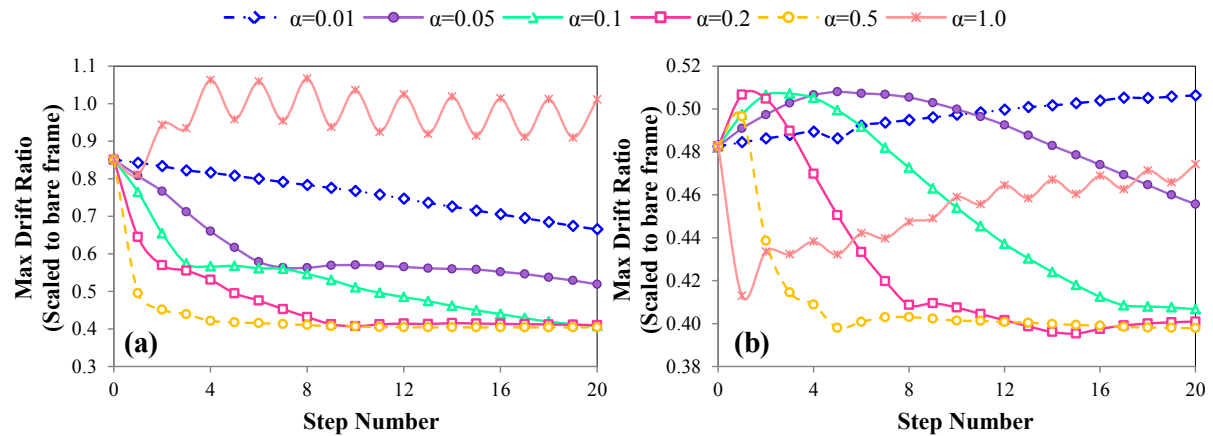
5.2 Effect of convergence parameter

The convergence parameter α plays an important role in the computational efficiency of the proposed optimisation method. Comprehensive sensitivity analyses were performed to obtain the most appropriate values of α for optimum design of RC frames with friction wall dampers. For example, Fig. 10 illustrates the variation of the maximum inter-storey drift ratios in 5 and 10-storey frames during optimisation process under a synthetic earthquake for α values of 0.01, 0.05, 0.1, 0.2, 0.5 and 1, starting from a reference frame designed with uniform cumulative slip load distribution. It is shown that in general as α increases from 0.01 to 0.5, the speed of convergence increases without any significant fluctuation. However, for α values greater than 0.5, the proposed method does not converge to the optimum design solution in both 5 and 10-storey frames. Also it can be noted that the convergence speed for the α values less than 0.2 is very slow, and therefore, a substantially higher number of steps is required to obtain the optimum solution. This indicates that an acceptable convergence is obtained by using α factor between 0.2 and 0.5. A similar conclusion was obtained from other frames and seismic excitations. All analyses in this study were carried out using α equal to 0.2.

5.3 Effect of design earthquake

The optimum distribution of the slip load pattern is influenced by the characteristics of the design earthquake and therefore varies from one earthquake to another. To study the effect of the design earthquake on the optimum solution, the reference frames were optimised for the six natural earthquake records listed in Table 1. Fig. 11 compares the optimum and uniform distributions of the slip load ratios under the selected earthquakes for the 5 and 10-storey frames. For better comparison, the results are scaled to the mean storey shear strength. It can be seen that while there are some discrepancies between the optimum slip load values at each floor for different earthquakes, the general distribution of the optimum slip loads follow a similar trend. While the spectral acceleration of the selected natural earthquakes showed a relatively high standard deviation, it is observed in Fig. 12 that the standard deviation of the optimum slip loads was insignificant for both 5 and 10-

storey frames. A similar conclusion was also obtained for the other frames. This can be explained as the selected spectrum compatible earthquakes are all strong ground motions recorded on a similar soil class profile, and therefore, have almost similar characteristics.

Fig. 10 Variation of maximum inter-storey drift ratios (scaled to the maximum drift of the corresponding bare frame) for (a) 5-storey and (b) 10-storey frames using different values of convergence parameter α

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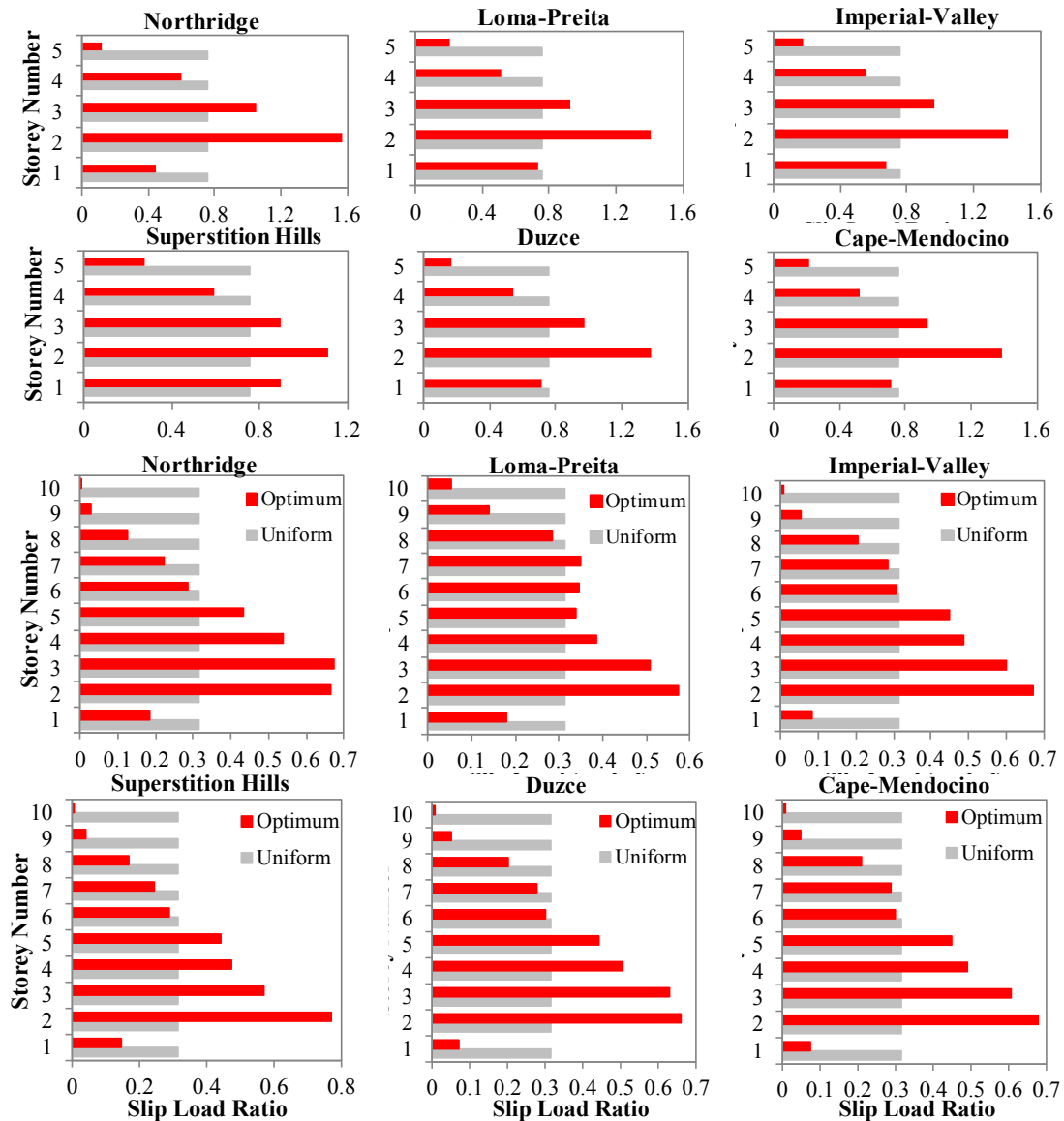
Fig. 11 Comparison of optimum and uniform distribution of slip load ratios (scaled to the mean storey shear strength) for 5 and 10-storey frames under six natural earthquakes
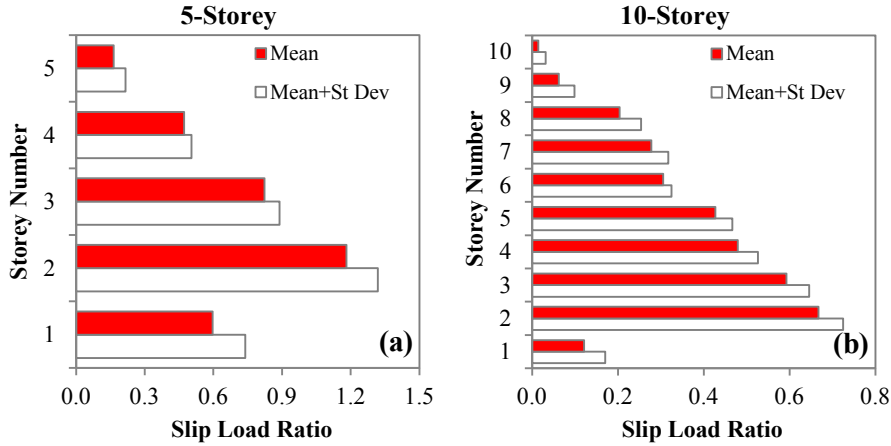
Fig. 12 Comparison of the mean and mean plus one standard deviation of optimum slip load ratios (scaled to the mean storey shear strength) for (a) 5 and (b) 10-storey frames under six natural earthquakes

6   GLOBAL DAMAGE INDEX

To assess the efficiency of the adopted optimisation methodology to reduce the overall structural damage under seismic excitations, a cumulative damage index is used based on a classical low-cycle fatigue approach (Krawinkler and Zohrei, 1983). Variations in the energy dissipation capacity of the structure were taken into account as a function of displacement demands in the adopted damage model (Miner, 1945; Teran-Gilmore and Jirsa, 2004). Independent damages are assumed to be caused by different plastic excursions which are identified by using a Rainbow Counting Method (Powell and Allahabadi, 1988). The cumulative damage index is calculated using the following equation:

$$DI_i = \sum_{j=1}^{N} \left(\frac{\Delta\delta_{pj}}{\delta_u} \right)^c \qquad (7)$$

where DI_i and N are defined as cumulative damage index at i^{th} storey and the total number of plastic excursions, respectively. The cumulative damage index ranges from 0 for intact to 1 for completely damaged storeys. $\Delta\delta_{pj}$ is considered as the plastic displacement of the jth excursion, δ_u is the ultimate plastic displacement, and the power factor c is the structural parameter accounting for the effect of plastic deformation magnitude. As suggested by Cosenza and Manfredi (1996), c is assumed to be 1.5 in this study.

To estimate the level of damage exhibited by the entire structure, the global damage index, DI_g, is defined as a weighted average of the damage indices at all storey levels, with the weighting function being the energy dissipated at each storey.

$$DI_g = \frac{\sum_{i=1}^{n} DI_i W_{pi}}{\sum_{i=1}^{n} W_{pi}} \qquad (8)$$

where n is the number of storey levels, DI_i is the damage index at i^{th} storey and W_{pi} is the dissipated energy at i^{th} storey. Fig. 13 (a) compares the global damage indices of the 3, 5, 10, 15 and 20-storey bare frames with the frames designed using the conventional uniform and the optimum slip load distributions under the spectrum compatible synthetic earthquakes. It is evident that the proposed friction wall dampers with the uniform slip load distribution could considerably reduce the global damage index of low to medium-rise bare frames (i.e. 3 to 10-storey), while they were not very efficient for tall buildings (i.e. 15 and 20-stroey). Using optimum design dampers, however, could efficiently reduce the global damage index in all cases. The results indicate that, compared to the conventionally designed frames with uniform slip load distribution, the proposed optimum

design methodology decreased the global damage index of the 3, 5, 10, 15 and 20-storey frames by 49%, 23%, 75%, 65% and 38%, respectively.
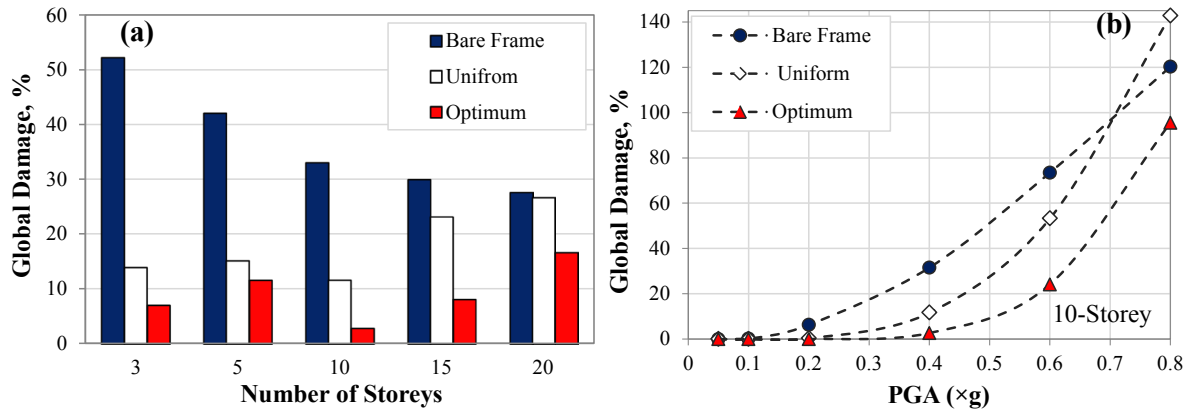
Fig. 13 Global damage index of the bare frames and the frames with friction dampers designed based on uniform and optimum slip load distributions: (a) 3, 5, 10, 15 and 20-storey frames, (b) Incremental dynamic analysis of 10-storey frames, average of six synthetic earthquakes

The efficiency of the proposed optimisation method is also evaluated for low-to-high earthquake intensity levels. Incremental dynamic analyses were conducted on the 10-storey bare frame and the frames with friction wall dampers designed based on uniform and optimum slip load distributions. Fig. 13 (b) compares the mean of the global damage indices (DI_g) of the frames under the set of six synthetic spectrum-compatible earthquakes. The PGA of the input earthquakes was ranged from 0.05g to 0.8g to cover small to large magnitude earthquakes. The results show that the frames with optimum design friction wall dampers experience considerably less global damage (up to 77%) at all intensity levels compared to those with conventional friction dampers. It is especially evident that the efficiency of the friction dampers with uniform slip load distribution is significantly reduced for earthquakes with PGA levels higher than 0.6 g. The main reason is that using identical slip load values at all storey levels leads to a high concentrated lateral displacement and localised damage at certain storeys, while the optimum slip load distribution obtained from the proposed design methodology results in a more uniform distribution of storey damage.

7   OPTIMUM DESIGN SOLUTION FOR A CODE DESIGN SPECTRUM

The seismic ground motion is the main source of uncertainty in the seismic design of structures. Therefore, there is a concern that this may influence the efficiency of the optimum structures that are designed based on a single earthquake event. Previous studies by Hajirasouliha et al. (2012) confirmed that, for performance-based design of RC structures, a better seismic design can be obtained by using a synthetic earthquake representing the mean spectrum of a set of natural earthquakes. In this study, the efficiency of this concept is evaluated for optimum design of RC frames with friction wall dampers for a specific code design spectrum.

Fig. 14 compares the mean optimum slip load ratio (ratio of the slip load to the mean storey shear strength) distributions obtained for the 10-storey frames with friction-based wall dampers subjected to the selected natural and synthetic earthquakes. As it was mentioned in Section 2.3, the mean spectrum of the six natural ground motions and the mean spectrum of the six synthetic earthquakes both have a very good agreement with the selected IBC-2015 design spectrum (see Fig. 3). It can be seen from Fig. 14 that the optimum slip load distributions corresponding to the natural and synthetic earthquakes are almost identical. This implies that there is a unique optimum design solution for each frame subjected to the design spectrum. Therefore, to manage the uncertainty of the design seismic loads, the frames can be designed based on the mean of the optimum slip load distributions corresponding to the set of earthquakes representing the design spectrum.

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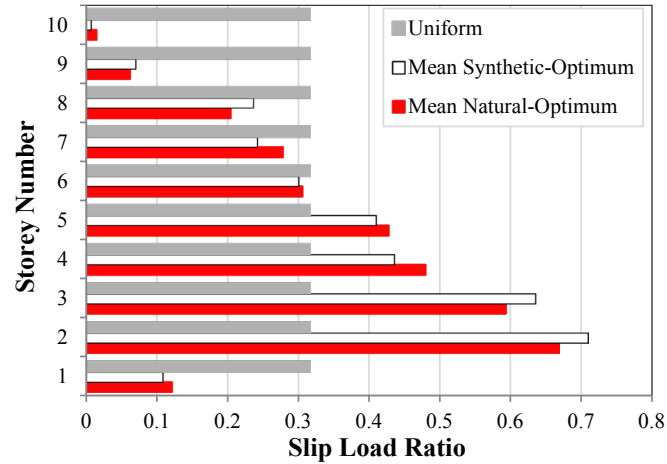
Fig. 14 Comparison of uniform and mean optimum slip load distributions for the six synthetic and six natural earthquakes (scaled to the mean storey shear strength).

Fig. 15 compares the maximum inter-storey drift ratios and global damage indices of the 10-storey frames designed based on (a) uniform slip load distribution, (b) mean of the optimum slip load distributions for the six synthetic spectrum compatible earthquakes, and (c) optimum slip load distribution corresponding to each individual natural earthquake. The results indicate that while using the mean of the optimum slip loads obtained for the synthetic earthquakes (i.e. Mean Synthetic-Optimum) is not as efficient as using the specific optimum slip load distribution obtained for each individual earthquake, it is still considerably more efficient than the conventional uniform slip load distribution. For the same mean slip load value, it can be seen from Fig. 15 (a) that the 10-storey frames designed with the Mean Synthetic-Optimum distribution exhibit on average 24% (by up to 34%) less maximum inter-storey drift compared to their conventionally designed counterparts using uniform slip load distribution. Similarly, Fig. 15 (b) shows that using the mean slip load distribution resulted in up to 72% lower global damage indices under the selected natural earthquakes. This can confirm the efficiency of using the mean of the optimum load distributions corresponding to a set of synthetic spectrum-compatible earthquakes (representatives of the design spectrum) to achieve better and more effective seismic design solutions for RC frames with friction-based wall dampers.
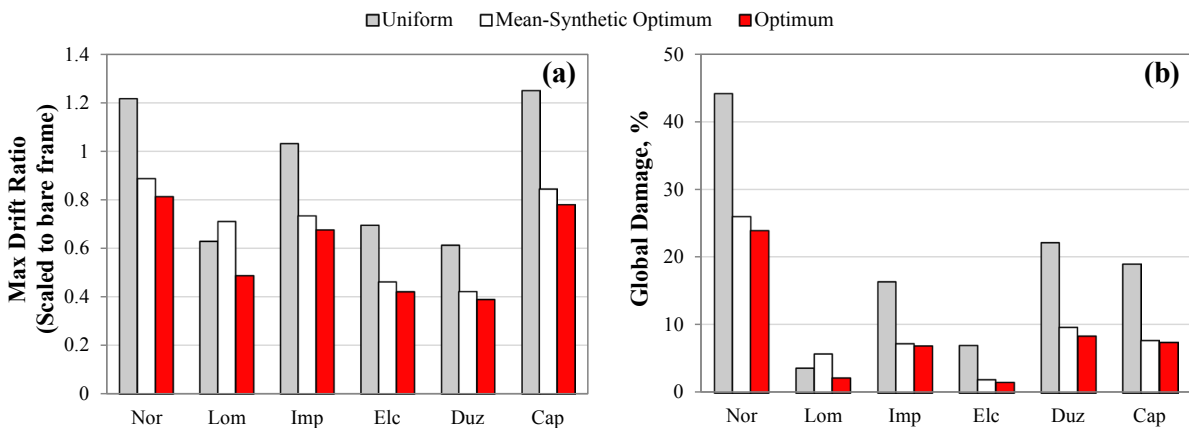
Fig. 15 (a) Maximum drift ratio (scaled to the maximum drift of the corresponding bare frame) and (b) Global damage index of 10-storey frames designed with uniform slip load distribution, mean of the optimum slip loads obtained for the synthetic earthquakes (Mean Synthetic-Optimum), and the optimum load distributions obtained for each individual earthquake (Optimum)

Overall, the results of this study demonstrate the efficiency and reliability of the proposed optimisation method, which can provide an efficient tool for optimum seismic design of friction energy dissipation devices for practical purposes. It should be noted that in general there are different sources of uncertainty in the value of the friction force in friction-based devices (Constantinou et al., 2007; ASCE 7-16, 2017). The preload applied to the friction interface may decrease over time due to the inherent creep in sliding interface materials or wear in the sliding interface (e.g. as a result of substantial motions). The friction coefficient may also change in the sliding interfaces within the lifetime of the structure. While these uncertainties should be considered in the

seismic design of friction-based devices, it was shown in section 3.1 that there is always an optimum range of slip load ratios for the proposed friction wall dampers, and therefore, the optimum design solution is not very sensitive to the small variations of friction forces. On the other hand, some of the above mentioned limitations may be addressed by adjusting the clamping forces in the friction dampers after a period of time. Finally, it should be mentioned that the results of this study can be directly used for optimum design of metallic yield dampers where the yield forces do not change over time.

8 CONCLUSIONS

A low-cost performance-based optimisation method was proposed to enhance the seismic performance of RC frames with friction-based wall dampers using the concept of uniform damage distribution. The efficiency of the method was demonstrated through the optimum design of 3, 5, 10, 15, and 20-storey RC frames with friction wall dampers under six natural and six synthetic spectrum-compatible earthquakes. The proposed method could efficiently converge to the best design solution by obtaining the optimum slip load distribution and removing less efficient dampers in only a few steps. The proposed method was then further developed to deal with the optimum design of friction dampers for a specific design spectrum. The following conclusions can be drawn based on the results of this study:

- It was shown that the convergence parameter, α, has a major effect on the computational cost and convergence of the optimisation process. The α values ranging from 0.2 to 0.5 were shown to be generally more efficient to converge to the optimum design solutions for friction-based wall dampers in only a few steps. The results of sensitivity analyses also indicated that the optimum solution is independent from the initial slip load distribution considered in the optimisation process. However, using a suitable initial design can result in a faster convergence.

- Compared to the conventionally designed friction wall dampers with uniform slip load distribution, the optimum design dampers exhibited up to 43% and 75% lower inter-storey drift ratios and global damage indices, respectively, when subjected to the synthetic spectrum-compatible earthquakes. It was shown that, for the same total friction force, using the proposed optimisation method can increase the energy dissipation capacity of the friction wall dampers by up to 46%. The improvement in the energy dissipation capacity was more pronounced in low to medium-rise buildings. The efficiency of the proposed method was also demonstrated under the set of six natural earthquakes, where using optimum design dampers in 3 to 20-storey frames resulted in up to 50% lower inter-storey drift ratios compared to their conventionally designed counterparts.

- By performing nonlinear incremental dynamic analyses, it was observed that the proposed optimum design method can significantly reduce (up to 77%) the global damage index of the conventionally designed frames over a wide range of earthquake PGA levels. The optimum design systems were shown to be efficient at all intensity levels, while the efficiency of the frames with conventionally designed friction dampers was significantly reduced at higher PGA levels.

- Although the final optimum solution is influenced by the characteristics of the input earthquake excitation, the results indicated that the distribution of the optimum slip loads for the set of spectrum compatible earthquakes follow a similar trend. It was shown that the seismic load uncertainty can be efficiently managed by using the mean of the optimum load distributions corresponding to the synthetic earthquakes representing the design spectrum. The friction wall dampers designed with mean optimum slip load distribution exhibited up to 34% less maximum inter-storey drift and 72% less cumulative damage. The proposed optimisation methodology is general and can be adopted for optimum design of other types of dampers and structural systems.

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