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Circular geodesic radiation in Schwarzschild spacetime: A semiclassical approach

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Extreme curvature settings and non-trivial causal structure of curved spacetimes may have interesting theoretical and practical implications for quantum field theories. Radiation emission in black hole spacetimes is one such scenario in which the semiclassical approach, i.e. quantum fields propagating in a non-dynamical background spacetime, adds a very simple conceptual point of view and allows us to compute the emitted power in a straightforward way. Within this context, we re-examine sources in circular orbit around a Schwarzschild black hole, investigating the emission of scalar, electromagnetic and gravitational radiation. The analysis of the differences and similarities between these cases provide an excellent overview of the powerful conceptual and computational tool that is quantum field theory in curved spacetime.

Keywords: graviton two-point function; static de Sitter spacetime; infrared behavior.

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1. Introduction

The detection of gravitational waves[1,2] propelled science into what can be a new age of observations and theoretical predictions. In this context, the study of gravitational waves in astrophysical systems and cosmological settings is of particular importance. In an attempt to understand what was then thought to be observed gravitational waves,[3] C. W. Misner interpreted this gravitational wave signal of Weber's not as coming from an quadrupole radiation at the center of our galaxy, but from a localized source radiating synchrotron modes of gravitational waves.[4] From the analysis of the emitted scalar radiation by a source in circular orbit around a Schwarzschild black hole,[5] it was claimed that a similar behavior for the gravitational radiation was to be expected, namely, radiation concentrated in the plane of orbit and spectrum with a high frequency peak. If the gravitational radiation Weber claimed to have detected were of the synchrotron type, and if our Solar System happened to be approximately on the same plane as the source's orbit, then the radiated power claimed to have been measured would be explained more easily than by the quadrupole formula. If Weber's signal was coming from a quadrupole source, the estimated power from the observations would be deemed high enough to consume almost all the galaxy's mass in a small fraction of its age.

However, the extrapolation of the scalar radiation results to gravitational radiation are not as straightforward as it seemed.[6–8] Although scalar, electromagnetic and gravitational radiation emitted by a source in circular orbit around a black hole can be treated in a unified manner,[9] their polarization states lead to different spectra.

Quantum field theory (QFT) in curved spacetime provides a natural framework to treat these radiation processes systematically. The simplest case, the scalar radiation from a source in geodesic circular orbit around a Schwarzschild black hole, was analyzed in Ref. 10 (see also Ref. 11) and started this program. The scalar emitted power was computed by considering the one-particle-emission amplitude, when a massless scalar field is excited by a scalar source in a stable geodesic orbit around a Schwarzschild black hole. For unstable orbits, the scalar radiation indeed exhibits a synchrotron behavior.[12] The mass of the scalar field was found not to qualitatively change the results, at least for stable orbits.[13] A more realistic scenario was presented in Ref. 14, where the scalar source was replaced by an electric charge and the emission of photons by this charge was analyzed, together with the absorption of the electromagnetic radiation by the black hole. The case for gravitational radiation, where the source is now a particle in a geodesic circular orbit, was considered in Ref.15. It has been shown that there is an enhancement of the high multipole modes for unstable orbits, which would be characterized as synchrotron radiation. However, the low multipoles, especially the quadrupole one, still have relevant contributions to the emitted power, even for unstable orbits. Thus, the gravitational radiation is not as concentrated in the plane of orbit as in the scalar radiation case.

In this context, the purpose of this paper is to present the analysis of the emitted radiation by a source in circular orbit around a Schwarzschild black hole, in a unified framework, treating the radiation (scalar, electromagnetic, or gravitational) as a quantum field propagating in the curved background. A perturbative approach is taken to compute the one-particle-emission amplitudes at tree level. We then compute the emitted power of radiation for each case. Although equivalent to the classical methods, the use of the QFT framework provides a very simple conceptual point of view as well as tools for possible extensions.

2. Quantization of bosonic fields in Schwarschild spacetime

We will consider a source, in circular orbit around the black hole, that can emit radiation of three different spins: 0 (scalar), 1 (electromagnetic) and 2 (gravitational), depending on the associated current. Each radiation is described by a different quantum field: a scalar field $\hat{\phi}(x)$, an electromagnetic field $\hat{A}_\mu(x)$ and a gravitational field $\hat{h}_{\mu\nu}(x)$, respectively.

The quantization of the fields is done as follows: we start with the free (quadratic) Lagrangian density $\mathcal{L}[\psi]$ of each field, where ψ represents one of the three fields. The Lagrangian depends only on the fields and their first order covariant derivatives. The conjugate momentum current is defined by

$$p_\psi^\lambda \equiv \frac{1}{\sqrt{-g}} \frac{\partial \mathcal{L}[\psi]}{\partial(\nabla_\lambda \psi)}. \tag{1}$$

Note that, since ψ may have spacetime indices, the conjugate momentum current can possibly be a vector (scalar case), a 2-tensor (electromagnetic case) or a 3-tensor (gravitational case). With the conjugate momentum current defined, we introduce the following symplectic product:

$$\Omega(\psi, \psi') \equiv -\int_\Sigma \mathrm{d}\Sigma n_\lambda (\psi p_{\psi'}^\lambda - \psi' p_\psi^\lambda), \tag{2}$$

where Σ is a Cauchy hypersurface with future-directed unit normal vector n_λ and $\psi p_{\psi'}^\lambda$ is to be understood as the contraction of the possible indices of ψ with those in $p_{\psi'}^\lambda$ (e.g. $\psi p_{\psi'}^\lambda = h_{\mu\nu} p_{h'}^{\lambda\mu\nu}$ if $\psi = h_{\mu\nu}$). We assume that this symplectic product is non-degenerate. A possible basis for the solutions of the free field equation is the set of positive-frequency solutions, characterized by a time-dependence of the form $e^{-i\omega t}$, $\omega > 0$, together with their complex conjugates. We expand the field $\psi(x)$ in terms of this basis, namely

$$\hat{\psi}(x) = \sum_n [\hat{a}_n \psi_n(x) + \hat{a}_n^\dagger \psi_n^*(x)], \tag{3}$$

where n represents all possible labels of the solutions. Note that, since the Schwarzschild solution represents a static spacetime, the notion of positive-frequency solutions is well defined. We require the positive-frequency solutions to be normalized according to the following inner product:

$$\langle \psi_n, \psi_{n'} \rangle = i\Omega(\psi_n^*, \psi_{n'}) = \delta_{nn'}. \tag{4}$$

Here $\delta_{nn'}$ may involve Dirac delta functions. The operators $\hat{a}_n$ and $\hat{a}_n^\dagger$ have the interpretation of annihilation and creation operators of modes labeled by n, respectively.

This quantization method presents no difficulties for the scalar field. In this case, the symplectic product is closely related to the usual Klein-Gordon inner product for the scalar field. For the electromagnetic and gravitational fields, due to the fact that both theories have gauge invariance, the symplectic product over the whole space of solutions is degenerate: pure gauge solutions (i.e. $A_\mu = \partial_\mu \alpha$ or $h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$) are orthogonal to all solutions including themselves. Hence we have to restrict the space of solutions by imposing some gauge conditions such that the symplectic product is nondegenerate on the space of solutions satisfying them. Hence, from now on, we assume that gauge conditions have been chosen such that the symplectic product is nondegenerate in all three cases (see Ref. 16 for the details in the gravitational case).

We can expand each field in terms of harmonic tensors on the two-sphere. The scalar field will be expanded in terms of scalar spherical harmonics $Y^{(lm)}(\theta, \varphi)$. The electromagnetic field and the gravitational field are expanded in terms of the scalar spherical harmonics, their covariant derivatives and vector spherical harmonics $Y_i^{(lm)}(\theta, \varphi)$. We can treat the scalar-type part of the fields separately from the vector-type part[a]. The decomposition in spherical harmonics allows us to rewrite the field equations in terms of a equivalent set of equations on the spacetime spanned by the t and r coordinates. This is done in details for the scalar case in Refs. 10, 11. The electromagnetic field was given this treatment in Ref. 17. The gravitational case is more involved. Nevertheless, a covariant decomposition in spacetimes with a certain symmetry group was given in Refs. 18, 19, for arbitrary dimensions. In any case, for all the three fields, the positive-frequency solutions can be expressed in terms of radial functions $u_n^s(r)$, which satisfy

$$-f(r)\frac{\mathrm{d}}{\mathrm{d}r}\left[f(r)\frac{\mathrm{d}}{\mathrm{d}r} + V_n^s(r)\right]u_n^s(r) = \omega^2 u_n^s(r), \quad f(r) = 1 - \frac{2M}{r}, \tag{5}$$

where $V_n^s(r)$ is an effective potential and M is the black hole mass. The additional label, given by s, indicates the dependence with the spin of the field. In the spacetime of a Schwarzschild black hole, we need to choose appropriate initial data for the wave equation (5) on past null infinity $\mathcal{I}^-$ and on the past horizon $\mathcal{H}^-$. For this particular problem, we choose the independent modes which are purely incoming from $\mathcal{I}^-$, the so-called *in*-modes, and the modes purely incoming from $\mathcal{H}^-$, the so-called *up*-modes. These modes are normalized with respect to the symplectic product in Eq. (2). A pictorial representation of these modes is given in Fig. 1.

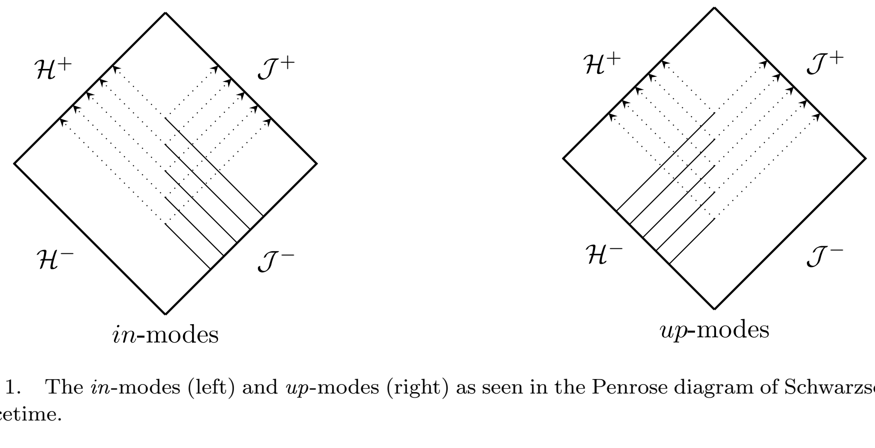

Fig. 1. The *in*-modes (left) and *up*-modes (right) as seen in the Penrose diagram of Schwarzschild spacetime.

[a]Obviously, the scalar field has no part depending on the vector spherical harmonics.

Thus, the set of labels n can be written as $n = (\omega; P, \lambda; l, m)$, where $\lambda = in, up$ denotes the particular independent mode of Eq. (5) and (l, m) are the angular quantum numbers. The label $P = S, V$ stands for the type of harmonic tensor used in the expansion.

To first order in perturbation theory, the amplitude for the source to emit a quantum particle, with quantum numbers represented by n, can be written as

$$\mathcal{A}_n[\psi] = i\langle n| \int d^4x\sqrt{-g}\psi j[\psi]|0\rangle, \tag{6}$$

where $j[\psi]$ is the current associated to each of the three cases and $\psi j[\psi]$ denotes the (possible) contraction of indices in ψ with those in $j[\psi]$. The vacuum state in this case is the Boulware vacuum, i.e. the one annihilated by all the $\hat{a}_n$. The current can be linked to the energy-momentum tensor of the source (for the gravitational case, the current is the energy-momentum tensor). For a source in circular geodesic orbit around the black hole and from (6), the structure of the currents implies that the amplitude $\mathcal{A}_n$ is proportional to $\delta(\omega - m\Omega)$, where Ω is the angular velocity of the source, and hence only particles with $\omega = m\Omega$ have a nonvanishing probability of being emitted.

The power of emitted particles, associated to the field ψ and with quantum numbers P, λ, l and m, is given by

$$W_{P,\lambda;l,m}[\psi] = \int_0^\infty \mathrm{d}\omega\ \omega \frac{|\mathcal{A}_n[\psi]|^2}{T}, \tag{7}$$

where $T = \int_{-\infty}^{\infty} \mathrm{d}t = 2\pi\delta(0)$ is the total time as measured by an asymptotic static observer. Several other quantities associated to the radiation can be computed such as the spectral distribution and the angular distribution of energy.

The solutions to Eq. (5) can be obtained using numerical methods and used to compute the emitted power and related quantities. In the QFT framework, this was done for the scalar case in Refs. 10, 11. The electromagnetic and gravitational cases were treated in Ref. 14 and Ref. 15, respectively. We exhibit here the spectral distribution for the three cases in Fig. 2. We see that only the scalar case exhibits a high frequency peak.[6–8] Although an enhancement of the high multipole modes is present in the electromagnetic and gravitational cases, the low multipoles still give an important contribution to the emitted power. We note here that the gravitational radiation lacks the $l = 1$ dipole mode, which is excited by the source's orbit in the scalar and electromagnetic cases.

3. Discussion

The framework of quantum field theory in curved spacetimes (QFTCS) proves to be very useful in the radiation setting presented here, allowing us to present all three situations in a unified way. Their comparison is then straightforward. The presence of a black hole allows the source particle to be in a (geodesic) circular orbit and the

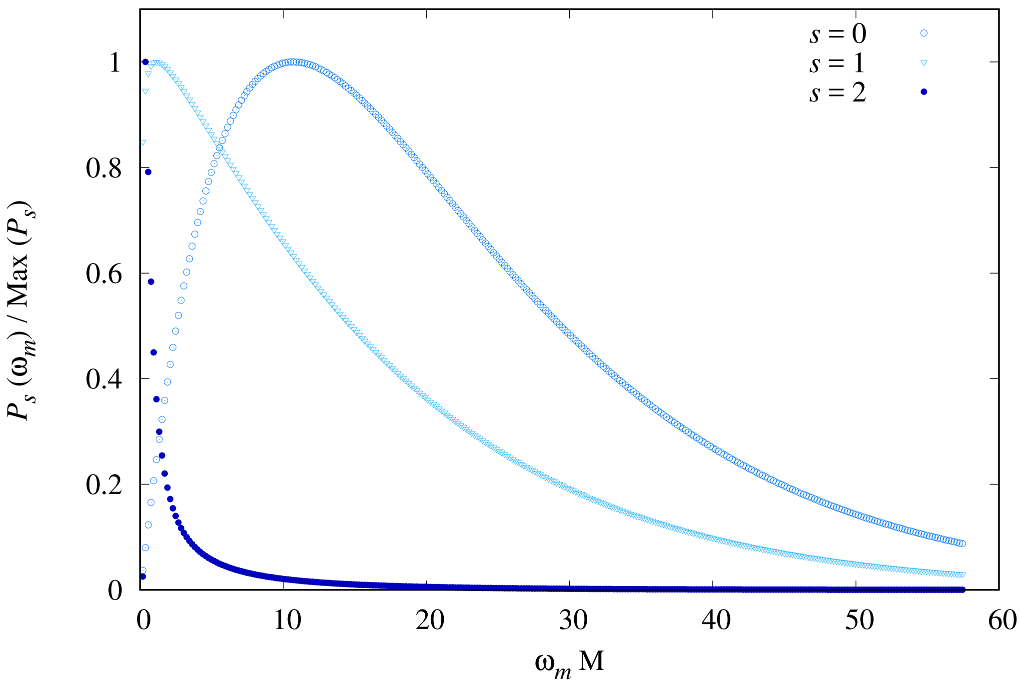

Fig. 2. Normalized spectral distributions for the three cases: scalar ($s = 0$), electromagnetic ($s = 1$) and gravitational ($s = 2$) radiations. The source, in each case, is located at $r/M = 3 + \epsilon$, with $\epsilon = 10^{-2}$. All the functions are normalized by their maximum achieved value.

radiation emitted in this setting can be readily computed in the quantum framework at tree level.

The recent progress in the gravitational-wave research presents an exciting prospect for testing existing physical theories and possibly discovering new physics. The framework of quantum field theory in curved spacetime provides an alternative picture of gravitational radiation as a tree-level quantum emission.

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