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# Semiclassical approach to atomic decoherence by gravitational waves

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## Abstract

A new heuristic model of interaction of an atomic system with a gravitational wave is proposed. In it, the gravitational wave alters the local electromagnetic field of the atomic nucleus, as perceived by the electron, changing the state of the system. The spectral decomposition of the wave function is calculated, from which the energy is obtained. The results suggest a shift in the difference of the atomic energy levels, which will induce a small detuning to a resonant transition. The detuning increases with the quantum numbers of the levels, making the effect more prominent for Rydberg states. We performed calculations on the Rabi oscillations of atomic transitions, estimating how they would vary as a result of the proposed effect.

## 1   Introduction

In general relativity, gravity is the curvature of the space-time continuum produced by the mass of objects [41]. Similar to how accelerating electrical charges produce electromagnetic waves, accelerating masses will produce ripples in the fabric of space-time [16, 17], which are called gravitational waves (GWs). They are predicted to exist in a very wide range of frequencies, depending on the source that generate them [2, 11, 14, 27, 29, 30, 37, 39, 40, 44], but even the most energetic ones (like the ones produced by rotating neutron stars [3]) will only produce a very small distortion of space, making them very hard to detect.

Although GWs have been previously observed indirectly [43], it was only recently that a direct detection was performed by analyzing the signal of very big interferometers [18–20]. These experiments not only open the window to a new way to observe the universe but also showed that gravitational waves have physical effects that can be detected on Earth using currently available technology.

Some theories have calculated the effect that gravitational waves have on particles, where decoherence is expected to arise [4, 34]. Our approach is to obtain the change in the energy levels of an hydrogen-like atom [35, 48] and

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analyze how the properties of the atom can be altered in order to make the shift significant enough to appear in the spectroscopic signal of said atoms. In our model, a gravitational wave passing by an atom will curve the space, deforming the electromagnetic potential from the nucleus as felt by the electron, which will change the energy of the system. The new energy is derived by estimating the wave function of the electron after the interaction. The results of these calculations suggest that the atom will suffer a small shift in the energy of its transitions, being this more prominent for high-energy levels. By maximizing the order of the perturbation, it could be possible to apply our model for the detection and characterization of gravitational waves. Because the only constrain we impose to the model is for the interacting gravitational wave to have a wavelength considerably bigger than the size of the atom, the proposed detection scheme could be applied to the detection of relic gravitational waves and the high-frequency range of the stochastic gravitational wave background [1, 8, 9, 22, 26, 47], which current experiments cannot observe, providing a powerful tool for the study of the universe.

## 2   Wave-Function shift

Let us consider an hydrogen-like atom in an excited state. The potential felt by the valence electron will be given by

$$V_n = -\frac{k_B \ e^2}{r_n} \ , \tag{1}$$

where $k_B$ is the Boltzmann constant, $e$ is the elementary charge and $r_n$ is the distance that depends on the principal quantum number $n$ and contains the correction due to the screening of the nucleus (quantum defect) [28]. An incoming gravitational wave will compress the space along one of the axis of the plane transversal to its propagation direction and expands the space along the perpendicular direction within the plane (see Fig. 1), producing a change in the coordinates

$$\vec{r} \to \vec{r}'(r, \theta, \phi) \ . \tag{2}$$

This will result in the electron perceiving the nucleus at a different distance, altering the effective potential,

$$V_n(\vec{r}) \to V'(\vec{r}') \ . \tag{3}$$

To better understand the potential shift, picture the GW modifying the wavelength of the force-carrying photons between the electron and nucleus: Their wavelength will be compress along a certain angular direction and expanded along the perpendicular angle, changing accordingly the momentum
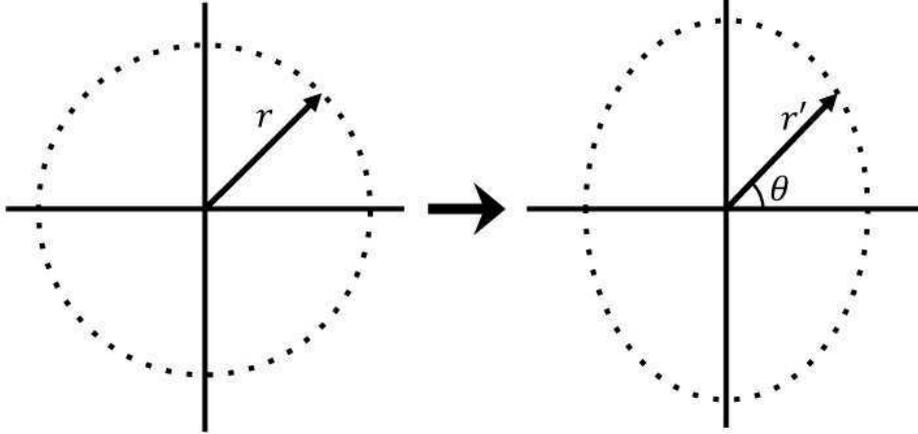
Figure 1: Representation of the space strain resulting from an incoming gravitational wave, in the plane transversal to the direction of propagation.

exchange between the charges.

The potential is expected to change smoothly, as the GW is a gradual distortion of space (adiabatic approximation). This assumption should specially hold for GWs with wavelength bigger than the size of the atom, which is typically the case. The adiabatic change will make the wave function of the atom to evolve into the appropriate eigenstate of the new potential

$$\psi(V_n) \to \psi' = \psi'(V') \ . \tag{4}$$

Rather than finding the eigenstates related the new perceived potential, we estimate the wave function as evolving into the distorted space coordinates,

$$\psi(\vec{r}) \to \psi' = \psi(\vec{r} \to \vec{r}') \ . \tag{5}$$

This transformation gives back the original wave function in the absence of the perturbation ($\vec{r}' = \vec{r}$), which is required for the adiabatic approximation. In the classical picture, Eq. (5) can be interpreted as the stable orbital becoming an ellipse as result of the loss of radial symmetry in the electric potential in order to valance it with the kinetic energy, which is roughly the meaning of the Schrödinger equation.

In order to obtain the energy of the system, we calculate the resulting wave function $\psi'$ in terms of the eigenstates of the unperturbed potential,

$$\psi' = \sum_{n,l,m} C_{n,l,m} \psi_{n,l,m}(r, \theta, \phi) \ , \tag{6}$$

where $n$, $l$ and $m$ are the quantum numbers. We are not suggesting that the wave function is projected into the eigenstates (as this will imply an non-adiabatic process), but just using an equivalent representation of the

transformed wave function in the original base. The coefficients $C_{n,l,m}$ in Eq. 6) can be calculated with

$$C_{n,l,m} = \int_0^\infty r^2 dr \int_0^\pi sin\theta \ d\theta \ \int_0^{2\pi} d\phi \ \psi_{n,l,m}^* \psi' \ . \tag{7}$$

Solving numerically Eq. (7) for an arbitrary change requires a lot of computational time, even for the most simple atomic system. To simplify calculations, we use the following approximation: First consider an hydrogen-like atom in an excited state with principal quantum number $n_0$, and no angular momentum (azimuthal quantum number $l = 0$). The initial wave function of the system is

$$\psi_{n_0,0} = \sqrt{\left(\frac{2}{n_0 a_0}\right)^3 \frac{(n_0 - 1)!}{8\pi n_0 (n_0!)^3}} e^{-\frac{r}{n_0 a_0}} L_{n_0-1}^1 \left(\frac{2r}{n_0 a_0}\right) \ , \tag{8}$$

where $a_0$ is the Bohr radius and $L_{n_0-1}^1$ is the corresponding associated Laguerre polynomial [25]. In the plane transversal to the GW's direction, we model the distortion of the space as

$$r \to r' = r \frac{1 - S_p}{\sqrt{cos^2\theta + \left(\frac{1-S_p}{1+S_p}\right)^2 sin^2\theta}} \equiv r A_\theta \ , \tag{9}$$

where $S_p$ represents the strain on space in said plane. For GWs with wavelength significantly bigger than the expected size of the atom, we can consider the strain $S_p$ a constant. This allows us to substitute Eq. (9) directly into Eq. (8), such that the wave function is transformed as

$$\psi' = \sqrt{\left(\frac{2}{n_0 a_0}\right)^3 \frac{(n_0 - 1)!}{8\pi n_0 (n_0!)^3}} e^{-\frac{r A_\theta}{n_0 a_0}} L_{n_0-1}^1 \left(\frac{2r A_\theta}{n_0 a_0}\right) \ . \tag{10}$$

In order to find an analytical solution to Eq. (7), we use the mathematical identity

$$L_{n_0-1}^1 \left(\frac{2r A_\theta}{n_0 a_0}\right) = e^{-\frac{2r}{n_0 a_0}(1-A_\theta)} \sum_{k=0} \frac{(1 - A_\theta)^k}{k!} \left(\frac{2r}{n_0 a_0}\right)^k L_{n_0-1}^{1+k} \left(\frac{2r}{n_0 a_0}\right), \tag{11}$$

and the approximations

$$e^{-\frac{r}{n_0 a_0}(3-2A_\theta)} \approx e^{-\frac{r}{n_0 a_0}} \ , \tag{12}$$

$$(1 - A_\theta)^k \approx [S_p cos(2\theta)]^k \ , \tag{13}$$

which are valid for very small values of the strain constant ($S_p \ll 1$)[18]. Using Eq. (11) to Eq. (13) with the identity in Eq. (10) allows us to

Table 1: Value of the component $\Theta_{k,l}$ by solving Eq. (18) for different values of $k$ and $l$.

| | l=0 | l=2 | l=4 | l=6 |
|---|---|---|---|---|
| **k=0** | 1 | 0 | 0 | 0 |
| **k=1** | -1/3 | 4/15 | 0 | 0 |
| **k=2** | 7/15 | -8/105 | 32/315 | 0 |
| **k=3** | -9/15 | 4/21 | -32/1155 | 128/3003 |

separate the wave function after the interaction into a radial and an angular part

$$\psi' = \sum_{k=0} \frac{S_p{}^k}{k!} R'_{n_0,k}(r) \, Y'_k(\theta) \, , \tag{14}$$

$$R'_{n_0,k}(r) = \sqrt{\left(\frac{2}{n_0 a_0}\right)^3 \frac{(n_0-1)!}{2n_0(n_0!)^3}} e^{-\frac{r}{n_0 a_0}} \left(\frac{2r}{n_0 a_0}\right)^k L_{n_0-1}^{1+k}\left(\frac{2r}{n_0 a_0}\right) \, , \tag{15}$$

$$Y'_k(\theta) = \frac{1}{\sqrt{4\pi}} \cos^k(2\theta) L_{n_0-1}^{1+k}\left(\frac{2r}{n_0 a_0}\right) \, , \tag{16}$$

which is similar to the solution of the Schrödinger equation for hydrogen-like atoms ($\psi_{n,l,m} = R_{n,l}(r) \, Y_l^m(\theta, \phi)$). The integral of the radial component is equal to

$$\int_0^\infty R'_{n_0,k}(r) \, R_{n,l}(r) \, dr = \sqrt{\frac{(n_0-1)!(n_0-l-1)!}{[n_0!(n_0+l)!]^3} \frac{[(n_0+k)!]^3}{(n_0-l-1)!}} \delta_{n_0,n} \, , \tag{17}$$

where $\delta_{n_0,n}$ is the Kronecker delta. Solving the integral for the angular component yields to

$$\int_0^\pi \int_0^{2\pi} \sin\theta \, Y'_k(\theta) \, Y_l^m(\theta, \phi) \, d\phi \, d\theta = \begin{cases} \sqrt{2l+1} \, \Theta_{k,l} & \text{for } l = 0, 2, 4, ... \\ 0 & \text{for } l = 1, 3, 5, ... \end{cases} \tag{18}$$

This equation indicates that after the gravitational interaction, the atom will be perceived as a superposition of states with even azimuthal number. Some of the non-trivial values of $\Theta_{k,l}$ are shown in Table (1). With the product of Eq. (17) and Eq. (18) we obtain the coefficients $C_{n,l,m}$ of Eq. (14). Because the terms of the summation are proportional to $S_p{}^k$, which is expected to be extremely small ($\sim 10^{-20}$) [4], we disregard all the terms in Eq. (14) for $k \geq 2$, obtaining

$$\psi'_{n_0,0,0} \approx C_0(n_0)\psi_{n_0,0,0} + C_2(n_0)\psi_{n_0,2,0} \, , \tag{19}$$

with

$$C_0(n_0) = 1 - \frac{S_p}{3}(n_0+1)^3 \, , \tag{20}$$

$$C_2(n_0) = S_p \frac{4(n_0+1)}{3(n_0+2)^2} \sqrt{(n_0{}^2-1)(n_0{}^2-4)/5} \, . \tag{21}$$

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The coefficient $C_0$ will be several orders of magnitude bigger than $C_2$, meaning the change in the system is expected to be very small. Eq. (20) can be interpreted as the GW coupling the energy levels associated with $\psi_{n_0,0,0}$ and $\psi_{n_0,2,0}$, similar to the expected quadrupole interaction [17]. A transition between these energy levels can be attributed to an interaction with the graviton (with $l = 2$), which will occur with probability $|C_2|^2$.

For the general case, following the same calculations (Eq. (7) to Eq. (13)) for an atom in an initial state with principal quantum number $n_0 \geq 2$ and azimuthal quantum number $l_0 \geq 1$ give us that the wave functions evolves into an state that can be expressed as a superposition of eigenfunctions with azimuthal number equal to the initial one plus or minus factors of two ($l = l_0 \pm 2, 4, 6, ...$). With similar arguments to the case of initial angular momentum $l_0 = 0$, the resulting wave function can be approximated to

$$\psi' \approx C_0\psi_{n_0,l_0} + C_{+2}\psi_{n_0,l_0+2} + C_{-2}\psi_{n_0,l_0-2} \ , \tag{22}$$

where the coefficients $C_i$ are given by

$$C_0 = 1 - S_p\frac{(n_0 + l_0 + 1)^3}{(2l_0 - 1)(2l_0 + 3)}$$

$$C_{+2} = 2S_p\frac{(l_0 + 1)(l_0 + 2)}{2l_0 + 3}\sqrt{\left(\frac{n_0 + l_0 + 1}{n_0 + l_0 + 2}\right)^3\frac{(n_0 - l_0 - 1)(n_0 - l_0 - 2)}{(2l_0 + 1)(2l_0 + 5)}}$$

$$C_{-2} = 2S_p\frac{l_0(l_0 - 1)(n_0 + l_0 + 1)^3}{2l_0 - 1}\sqrt{\frac{(n_0 + l_0)^3(n_0 + l_0 - 1)^3}{(n_0 - l_0)(n_0 - l_0 + 1)(2l_0 + 1)(2l_0 - 3)}}$$

$$\tag{23}$$

Because the coefficients are non-linear functions of the quantum numbers, each energy level will be displaced by a different degree. This will result in a small shift in the energy of the atomic transitions.

# 3   Transition detuning

An atom interacting with a gravitational wave experiences a change in its energy levels

$$E_{n,l} \to E'_{n,l} \ , \tag{24}$$

which depends on the coefficients $C_0$, $C_{+2}$ and $C_{-2}$,

$$E'_{n,l} = C_0{}^2 E_{n,l} + C_{+2}{}^2 E_{n,l+2} + C_{-2}{}^2 E_{n,l-2} \ . \tag{25}$$

Therefore, the difference in energy between two distinct energy levels,

$$\Delta E = E_2 - E_1 \ , \tag{26}$$

is shifted by a factor $\delta$,

$$\Delta E' = E_2' - E_1' = \Delta E + \delta \ , \tag{27}$$

which will be the detuning for light used to drive a transition between states with energy $E_1$ and $E_2$. From Eq. (25) we have that

$$E_i' - E_i = -E_i \left( 1 - C_0{}^2 - C_{+2}{}^2 \frac{E_{i+2}}{E_i} - C_{-2}{}^2 \frac{E_{i-2}}{E_i} \right) \ . \tag{28}$$

Using the values of $C_j$ indicated in Eq. (23) yields

$$E_i' - E_i = -E_i \left[ 2S_p \frac{(n_i + l_i + 1)^3}{(2l_i - 1)(2l_i + 3)} + \kappa(S_p{}^2) \right] \ , \tag{29}$$

where $\kappa$ is a factor proportional to $S_p{}^2$. Because this term is very small, we can approximate

$$E_i' - E_i \approx -2S_p \frac{E_i(n_i + l_i + 1)^3}{(2l_i - 1)(2l_i + 3)} \ . \tag{30}$$

With this, we finally arrive to the relation

$$\delta = -2S_p \left[ \frac{E_2(n_2 + l_2 + 1)^3}{(2l_2 - 1)(2l_2 + 3)} - \frac{E_1(n_1 + l_1 + 1)^3}{(2l_1 - 1)(2l_1 + 3)} \right] \ . \tag{31}$$

Light with energy $\Delta E$ will be therefore detuned by $\delta \sim S_p \Delta E$ to the atom's transition. Eq.(31) indicates that the detuning can be increased by using transitions between states with high quantum numbers *i.e.* Rydberg states, which will be easier to detect [38]. For states with $n \sim 50$ [21, 42], the detuning $\delta$ can increase by a factor of $10^5$, even for transitions of states with close quantum numbers. Using Rydberg atoms for gravitational wave detection has been proposed by previous studies [10, 23, 36], further supporting our results.

The experimental observation of the proposed effect could be very difficult, even using the highly excited states previously mentioned. The analysis of the spectrum of extraplanetary Rydberg atoms could provide an advantage given the extremely high energies at which they can be found [5, 24]. As an example, the H110$\alpha$ emission of the Carina nebula (4.8 GHz)[5] would experience a change in its wavelength of $5.6 \times 10^{-16}$m, which could be measured using extremely accurate interferometry, like the one currently applied in gravitation wave detectors [18, 19]. Other studies have suggested a shift in the spectrum for excited atoms in regions with high spacetime curvature, but to achieve a shift of the order of the previous example it would require a characteristic radius of curvature of $\sim 100$ Km, involving being in the vicinity of a superdense object such a neutron star [35].

The detuning could also have an observable effect in Rabi oscillations [31]

for high energy states. For a two-level system, the probability of finding it in the excited state will evolve as

$$P_e(\Delta, t) = \frac{\omega^2}{\Delta^2 + \omega^2} \sin^2\left(\frac{\sqrt{\Delta^2 + \omega^2}}{2}t\right) , \tag{32}$$

where $\omega$ is the Rabi frequency and $\Delta$ is the frequency detuning. The Rabi cycle will deviate from its resonant dynamics $P_e(\Delta = 0, t)$ by

$$\delta P = P_e(0, t) - P_e(\Delta, t) = \sin^2\left(\frac{\omega}{2}t\right) - \frac{\omega^2}{\Delta^2 + \omega^2} \sin^2\left(\frac{\sqrt{\Delta^2 + \omega^2}}{2}t\right) \tag{33}$$

When the detuning is much smaller than the Rabi frequency ($\Delta \ll \omega$), we can use the approximation

$$\sqrt{\Delta^2 + \omega^2} \approx \omega + \Delta^2/2\omega , \tag{34}$$

which can be substituted in Eq. (33) to obtain the approximation

$$\delta P \approx \sin^2\left(\frac{\omega}{2}t\right)\left[1 - \frac{\omega^2}{\Delta^2 + \omega^2}\cos^2\left(\frac{\Delta^2}{4\omega}t\right)\right] . \tag{35}$$

Using Eq. (35) for short times (compared to the period of the gravitational wave), we get that the deviation can be approximated as

$$\delta P \approx \left(\frac{\Delta^2}{4\omega}t\right)^2 . \tag{36}$$

This equation indicates that the deviation increases with the detuning and decreases with the Rabi frequency, making again the effect more prominent in transitions of states with high quantum numbers [15]. For comparison, the deviation in the Rabi cycle $\delta P$ for the transition 50S - 51P of a Rubidium atom [7] will be $10^4$ higher than the one for the transition 1S - 2P in the same atom.

For longer times ($t > \pi\omega/\Delta^2$), the deviation will be mostly due to the increased frequency of the cycle. In this case, according to Eq. (31), the ideal transition will be obtained by maximizing the energy difference between the levels [13].

If a system undergoing Rabi oscillations is measured at times corresponding to N completed cycles ($t = 2N\pi/\omega$), the expected deviation will be

$$\delta P(t = 2N\pi/\omega) \approx \left(\frac{N\pi\Delta^2}{2\omega^2}\right)^2 . \tag{37}$$

In Fig. (2) we show the expected measured deviation for completed cycles using the mentioned 50S - 51P transition. In this figure it can be seen that
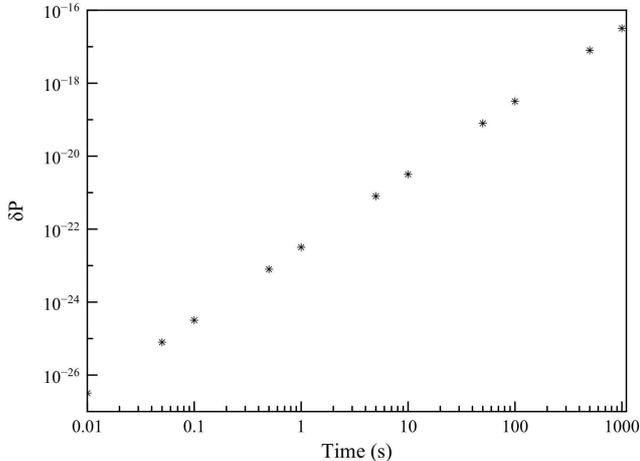
Figure 2: Deviation in the Rabi cycle for a 50S - 51P transition with a Rabi frequency of 47 kHz [7], which arises from the interaction with a gravitational wave with peak strain of $Sp = 10^{-20}$.

the effect becomes significant for very long times, which may require experiments to implement a high-Q cavity in order to extend the coherence time of the atoms by suppressing most of the undesired transitions [45, 46]; this could also help to inhibit the interaction of surrounding electromagnetic radiation, which otherwise may conceal the proposed effect. For the excitation levels used in our example, a high-Q cavity has been measured to increase the coherence times of the cavity-atom system to 0.02 s [12], limiting the capability of observing the proposed effect by the achievable resolution of the state population within this timeframe. Although a big ensemble of Rydberg atoms can be prepared to improve the probability of detection [13], collective effects should be taken into consideration [10] especially for such highly excited atoms, as they exhibit long range correlations [6, 32, 33].

## 4   Conclusion

We have modeled the effect of a gravitational wave passing through an atom as a distortion of the electromagnetic potential perceived by the electron. We calculated the resulting wave function as a superposition of the initial state and eigenstates with azimuthal quantum number that differ by two from the initial one. Our calculations indicate that the energy of the transitions in the atom will be shifted as a result of the interaction. Light with frequency equal to a atomic transition will then become slightly off-resonance, inducing a small deviation in the Rabi cycle of the atom. The deviation will increase drastically for transitions involving Rydberg states, which can

make the proposed effect easier to detect. The proposed effect presents the possibility of using atomic spectroscopy for detection of gravitational waves. Because the model only assumes the gravitational wave as having a wavelength bigger than the atomic scale, observations could be done in frequency ranges different from those of large scale interferometry detectors. The capability of the experiments to distinguish between the different gravitational wave frequencies, and therefore their sources, will depend on the state shift resolution within the period of the analyzed gravitational waves.

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