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Local and global granular mechanical characteristics of grain-structure interactions

Abstract

The focus of this work is on systematically understanding the effects of packing density of the sand grains on both the internal and bulk mechanical properties for strip footing interacting with granular soil. The studies are based on particle image velocimetry (PIV) method, coupled with a high resolution imaging camera. This provides valuable new insights on the evolution of slip planes at grain-scale under different fractions of the ultimate load. Furthermore, the PIV based results are compared with Finite Element Method (FEM) simulations in which the experimentally characterised parameters and constitutive behaviour are fed as an input, and a good level of agreements are obtained. The reported results would serve to the practicing engineers, researchers and graduate students in unravelling the mechanics of granular soil at both local and global levels when they interact with structures. The outcomes would be beneficial not only to the geotechnical engineering community, but also to related disciplines dealing with granular materials such as materials processing, minerals and space exploration.

Introduction

Cohesionless sands comprise of discrete grains of varying size and packing density. Their mechanical behaviour is different from that of conventional solid, liquid and gaseous state of matter [1]. Numerous researchers have studied the micromechanical characteristics of granular materials using experiments, theoretical descriptions and computer simulations [2]. From the micromechanical perspective [3], some studies have attributed the origin of shear strength of granular media to the anisotropy of strong force chains [4-6]. Their dilation characteristics are attributed to the displacement network of granular media [7].

In foundation engineering, ultimate bearing capacity (q_{ult}) and allowable settlement (S) are used as key design parameters [8-9]. In sand, settlement controls the design criteria of footing [10-11] which is independent of the loading rate [9]. Also, the settlement of footings could depend on their width for a given soil [11], but ultimate bearing capacity of sand is less dependent on footing width (B) when less than 1m as reported by Terzaghi and Peck [12]. In soil-structure interaction analysis [13], engineers use constant vertical displacement profiles for rigid footings interacting with sand at the level of the footing. However, the settlement in sand could vary significantly below the level of the footing-sand interface within the influence zone of depth (z) about 2-4 times the width of the footing (B) in homogenous sand [14-15]. The previous research discussed above on the settlement profiles

70   along the footing central axis do not vary linearly with depth. However, detailed information on how the

71   displacement field evolves within the sand bed under mechanical loading is still not well established. Historically,

72   bi-linear model (simple triangle approximation) is used to describe the variation of elastic displacement in sands

73   [14] and others use nonlinear variation [16]. At the micro scale, grain displacements are non-uniform [17].

74   However experimental results on the role of relative density of sand for all three major types, loose, medium-

75   dense and dense sand on their geomechanical characteristics using particle image velocimetry (PIV) is not yet

76   probed systematically. This is addressed here using two-dimensional particle image velocimetry (PIV). The

77   Dynamic Studio Software Platform (DSSP) helps to display the large amounts of PIV-based experimental data in

78   pictorial forms [18]. Recently PIV was applied to understand the flow properties of granular materials [19]. Here,

79   the authors focus on the local deformation and velocity fields and bulk strength for different relative densities of

80   sand when a strip shallow footing interacts with sand under quasi-static axial loading (P). Detailed experimental

81   characterisation of the sand material is made using a range of experiments. The aim is at first to compare the

82   variation of displacement fields measured in sand packing using PIV with FEM analysis. Thereafter, the variation

83   of fundamental mechanical features at both local and global scales are studied in detail using PIV for strip footing

84   interacting with sand packing of different relative densities in a systematic manner.

85

86   **Digital Particle Image Velocimetry (DPIV) Analysis**

87

88   DPIV pertains to the digital platform of Particle Image Velocimetry (PIV), is often used in the field of fluid

89   mechanics to track the motion of fluid flow using tracer particles [20]. It has been also used to study the

90   displacement and(/or) strain distribution in some cases of granular materials [21-22]. Recently, PIV has been

91   applied to get measurements of soil deformation in geotechnical engineering problems [21-25]. In this study,

92   DSSP is used to analyse the digital images acquired during test using PIV. The DSSP platform provides a range

93   of techniques for characterising particle motions, making it the most convenient for making advanced scientific

94   imaging-based measurements [18]. The algorithms provided within DSSP are used to analyse the PIV

95   measurements further. This functionality built in the DSSP was used to analyse the digital frames of the grains,

96   and to calculate two velocity components vectors of the grains and their evolution during load application within

97   the sand layer between two successive images. In this study, the area of interest or the target area (full image) was

98   specified before being divided into sub-sections called interrogation areas (IA) of 16×16 pixels each covering a

99   zone of soil approximately 2.2 mm^2. Each of these interrogation area was tracked using an adaptive PIV method

100 [18, 19, 26] to identify the movement of soil based on particle images (here 30 images per second) obtained from

101 the front of the Perspex test rig. The interrogation areas from each successive images are cross-correlated with

102 each other, pixel by pixel [18]. The correlation produces a peak signal detection, identifying the common grains

103 movement and thus also the velocity vector output is computed with sub-pixel interpolation. A velocity vector

104 plot over field of view (target area) is acquired by repeating the cross-correlation for each interrogation area over
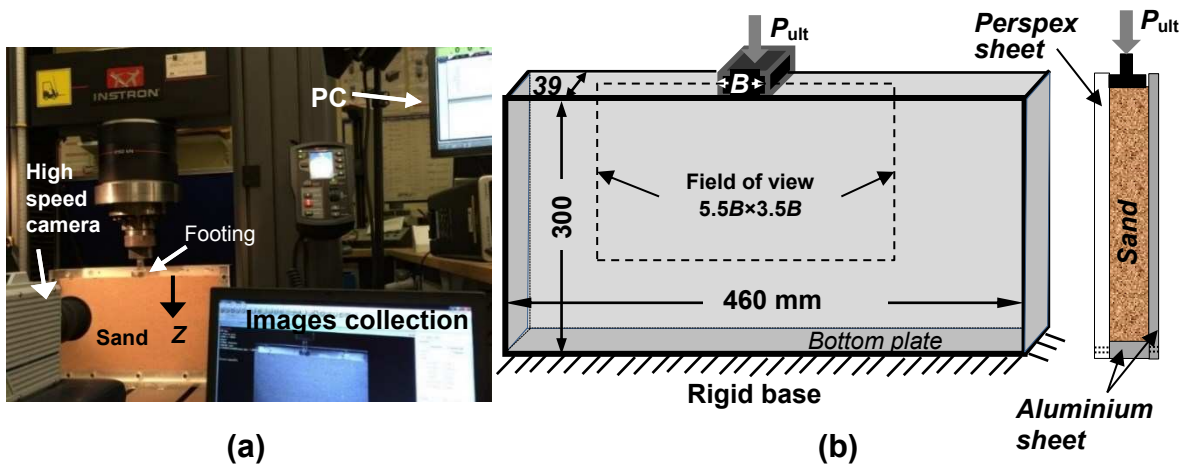
105 the two images [18].

106

Materials and Experiments

107

108

109 The samples used here are disturbed dry silica sand samples obtained in UK.   Sand properties were characterised

110 (Table 1) according to the American Society for Testing and Materials [27-28]. Their experimentally measured

111 material properties and size distribution showed the following properties: maximum dry density ($\gamma_{d\ max.}$) =16.50

112 kN/m^3 and minimum dry density ($\gamma_{d\ min.}$) =14.23 kN/m^3. In addition, using the sieve analysis as shown in Online

113 Resource OR1, the following properties of sand were obtained from the grain size distribution curve: D_{10}=0.25

114 mm; D_{30}=0.31 mm; D_{60}=0.40 mm (10%, 30% and 60% of the particles are finer than these particular particle sizes

115 respectively); D_{50}= 0.37 (Mean grain size of soil particle); uniformity coefficient C_U=1.55; and the coefficient of

116 curvature C_C =0.93. These data revealed that the soil chosen is a poorly graded sand [17, 29]. The roundness of

117 the grain was mostly spherical to sub-prismoidal and the angularity of the grains are characterised as angular and

118 sub-angular [28]. For this, digital microscopy images of the grain samples were used as shown in the Online

119 Resource OR1.

120 Bearing capacity of the footing was tested using an aluminium box of 460 mm in length, 300 mm in height and

121 39 mm in thickness, filled with dry silica sand. The box had smooth and transparent Perspex walls of 15 mm

122 thickness also to eliminate any bending effects during the test (Fig. 1). The authors also verified during all tests

123 that under the ultimate loads (P_{ult}) of the dense sand packing did not lead to any remarkable out of plane movement

124 of the container's face. This was checked using a dial gauge (0.01 mm resolution) mounted to the side walls from

125 a magnetic base (though the picture of this arrangement is not included here). The surface roughness of the footing

126 in contact with sand, and the Perspex walls of the experimental box was measured using 3D optical microscopy

127 based on white light interferometry [30] from which the mean roughness value R_a was obtained as 3.204 μm and

128 0.99 μm respectively. The rigid foundation base was relatively rough (ratio between the angle of interfacial friction

129 of the footing (δ) and angle of internal friction of the sand (ϕ), (δ / ϕ) is 0.25). Two cases of footing width are

4

130   considered in this study, i.e., smaller and larger footing width with dimensions $38 \times 38 \times 15$ mm^3 and $76 \times 38 \times$

131   15 mm^3 respectively (footing width $B/D_{50} = 102$ and 204 respectively to avoid any grain size effect, and

132   height=15 mm) was used here. It is recognised that the scale effects of the footing model could affect the

133   estimations of their strength characteristics [31]. For example, a footing with relatively small width would require

134   a relatively low stress level in the laboratory experiments, as if it were on a denser "state" of soil than a larger

135   footing, even if they were tested on sand with the same void ratio [31]. To minimize the scaling effect, it is

136   suggested that the model testing for studying the effect of packing density should not be too close to the limits of

137   void ratio (e_{max} and e_{min}, [32]). Taking this into account in the present study, the packing densities are kept away

138   from these limits (Table 1). The value of B/D_{50} used here is within the permissible limit of testing strip footing in

139   the lab although footing sizes used in real practice could be higher [33]. Such model dimensions have been used

140   in previous experimental studies in this field [8, 34]. To minimise any frictional effects of the footing with the

141   wall, a small gap of 1 mm is allowed between the footing and the back wall, so that they do not affect the

142   deformation of the soil recorded by PIV at the front of the box. It is also worth noting that about 12.5% of particles

143   were in the size range of 0.5-0.9 mm as shown in the Online Resource OR1, which helped to avoid any noticeable

144   leakage of grains from behind the footing. These measures ensure that the observed movement from the images

145   is due to the inner movement in the grains under mechanical loading [30].

(a)

(b)

147   **Fig. 1** (a) Experimental setup using PIV with a live image of footing in contact with sand (b) schematic diagram
148   of the experimental setup (dimensions are in mm)

150   The degree of compaction of granular soil is normally characterised according to the relative density D_r, defined
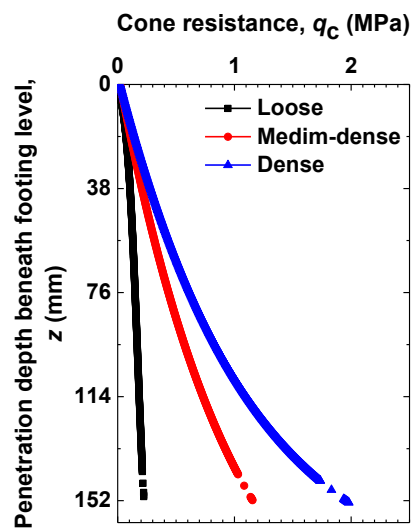
151   as [35]:

152    $$D_r(\%) = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100 \qquad (1)$$

153 Where e_{max} and e_{min} are void ratio of the soil in loosest and densest conditions respectively and e is in-place void

154 ratio of the tested soil (Table 1). Three cases of relative densities (D_r) loose (L), medium-dense (M) and dense

155 (D) were used in this study. The loose granular packing (γ=14.7 kN/m^3, D_r =24 ±2%, e= 0.76) was prepared by

156 pouring the grains mass uniformly across the width of the box in small layers using pluviation technique method

157 [36] so that any segregation of the grains was avoided during the construction process. The top surface of the sand

158 layer was gently levelled off using a hand scraper. This researchers also took care not to disturb the constructed

159 loose sample in any way before applying the axial loading in our experiments. The mass of sand grains laid in

160 the box to the required height pertains to the density of the loose sample. The medium-dense packing (γ=15.30

161 kN/m^3, D_r =53 ±2%, e= 0.7) was hand compacted in three layers, using 50 blows per layer in 0.035 m lifts each

162 with a 16 cm^2 compaction hammer of 0.92 kg weight designed for this purpose [31]. The dense sand (γ=15.80

163 kN/m^3, D_r =72 ±2%, e= 0.64) was achieved in five layers, 60 blows per layer. The footing was placed

164 symmetrically on the top surface of the sand bed.

165 An axial compression loading (q) was applied slowly on the footing (0.05 mm/s penetration velocity) using an

166 Instron loading machine with 0.1N resolution (Fig. 1). The loading machine also had an inbuilt dial gauge (linear

167 variable differential transformer, LVDT) to record the vertical displacement of the indenting footing on the sand

168 packing. The macroscopic load and vertical displacement of the footing were also measured from the tests. The

169 PIV camera with an allowable frame speed up to 100000 frames per second (fps) was fixed in front of the box

170 and two light sources were used to illuminate the rig. However, as the loading condition is quasi-static in this

171 study, the recording at 30 fps was found to be adequate until soil failure was reached, although higher frame

172 speeds were considered in the early stages of the experimental programme. We had verified that the recording at

173 greater than 30 fps did not affect the result noticeably. The resolution of the images was 1920 × 1080 pixels.

174 Initially, a number of trials were conducted to determine the suitable acquisition rate of the recorded PIV images

175 for the analysis. It was found that, for the current experiments, an acquisition of 1 frame/s of the recoded images

176 is adequate in which images were captured at displacement increments of 0.0017 mm. DSSP was used to analyse

177 the images using an adaptive PIV [18]. The adaptive PIV iteratively adjust the size of the individual interrogation

178 areas (IA) in order to adapt to local seeding densities and flow gradients [18]. This is suitable to study granular

179 systems even under different flow conditions [19] and bearing capacity of layered system [26]. Here, the

180 distribution of velocity vectors of the grains was examined in the image analysis using a minimum interrogation

181 area (IA) of size 16× 16 pixels and maximum IA size of 64 × 64 with a measurement resolution of sub- pixel [18].

182 The space-pixel dimension of the measurement was calibrated by printing a known scale on the test box along the

183   horizontal and vertical directions. White et al. [24] have shown that the precision of the measurement (i.e., the

184   random difference between multiple measurements of the same quantity) improves with larger PIV patches and it

185   is inversely proportional to the amount of the measurement resolution. This size of the mesh patch used here

186   reveals a standard error better than 0.01 pixel [18, 24]. In the experiments, two illumination lights were positioned

187   above the testing box to avoid reflection and glare on the measurement side of the Perspex wall. It was verified

188   that the variation in image scale in both horizontal and vertical direction were not significantly different. The PIV

189   camera lens was focused normal to plane the footing-soil interface region where the measurements are most

190   important to make. Therefore, the dimension of target area was $\sim 5.5B \times 3.5B$ (Fig.1). The displacement measures

191   i.e. resultant displacement (S_R), vertical displacement (S_v) and horizontal displacement (S_h) were evaluated under

192   a given load in total (i.e., between the reference image at zero load (q=0) and the image at the required fractions

193   of the ultimate load level, such as $0.34q_{ult}$ and q_{ult}. A typical mean size of sand grain ($D_{50} = 0.37$ mm) was

194   represented by about 3×3 pixels. Hence the PIV experimental measurements made here are at the local-scale.

195   Following the footing tests, two standard cone penetration test (CPTs) tests were also conducted for each soil

196   density to characterise the samples using a 10 mm diameter model CPT [25, 29]. The CPT was inserted at a

197   penetration rate of 1 mm/s in the current experiments as shown in Online Resource OR2, but using the identical

198   filling procedure of the grains used in the footing–sand indentation experiments presented earlier. Figure 2 shows

199   the CPT penetration profiles for the soil for all sand packings. The penetration resistance (Cone resistance $=q_c$)

200   profiles are plotted against the penetration depth from the bottom level of the footing.

201

202   **Fig. 2** CPT data for the sand packing

203 As expected, the penetration resistance of dense sand is higher than medium-dense and loose sand. The penetration

204 resistance of loose sand remains almost constant with depth after z/B =2.5, but penetration resistance for medium-

205 dense and dense sand increase with depth at an increasing rate. The rate of the penetration resistance of dense

206 sand is larger than that of the medium-dense. Again, the differences in the penetration resistance for different

207 relative densities are primarily accounted for the relatively larger volumetric compressibility in loose, medium-

208 dense sand than the dense sand. The CPTs results for all the densities show the average response of the two results
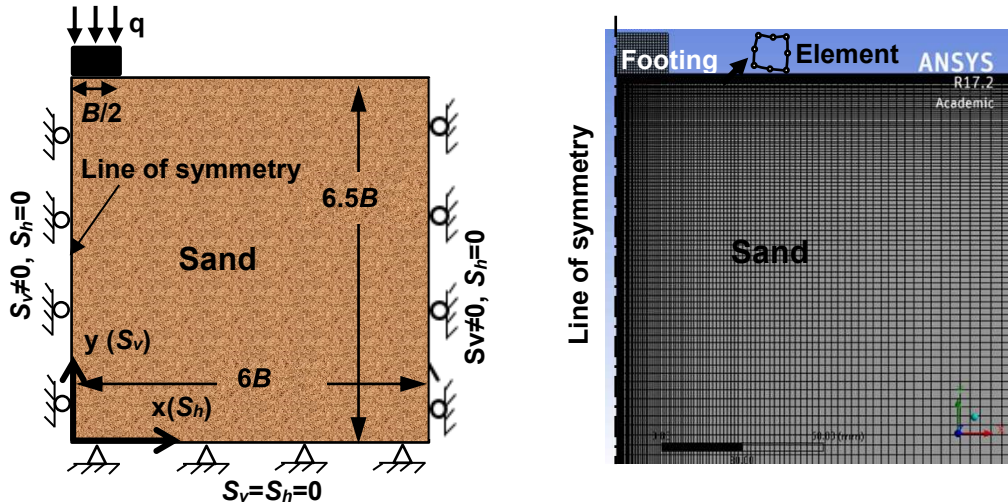
209 (error within 5%).

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211 **FEM Simulations**

212

213 Non-linear elastic finite element simulations have been made for the cases of a single footing indenting on the

214 loose, medium-dense and dense sand packing using ANSYS workbench 17.2 version [37]. The ANSYS program

215 is a broad purpose finite element modelling (FEM) package for numerically solving a wide variety of mechanical

216 interactions [37].

217 In the present study, ANSYS is used to create a two-dimensional solid geometry. The chosen domain along with

218 applied boundary conditions is shown in Fig. 3. The simulations were held under identical boundary conditions

219 for footing indenting with different types of sand packing as in the case of physical experiments. In the

220 simulations, the bottom most nodes were fully constrained in both the horizontal and vertical directions ($S_v = S_h$

221 = 0). A line of symmetry is used along the footing centre line ($S_v \neq 0$, $S_h = 0$). The far side of the assembly was

222 fully constrained in the horizontal direction ($S_h = 0$) and free to move in the vertical direction ($S_v \neq 0$) [38-39]. The

223 contact regions between the rigid footing and the sand were modelled as a relatively rough surface (interface

224 friction coefficient=0.25) corresponding to the experimental study [40]. An adaptive FE mesh was generated at

225 the footing-soil interface where the largest stresses and strains would be expected. It should be mentioned that

226 Skewness mesh metric (a measure of mesh quality) of 0.132 maximum value was obtained which is acceptable

227 [41]. The size of the elemental geometry is shown in Fig. 3. The nodes and element numbers in the soil body are

228 equal to 44000 and 14360 respectively.

229 Material model for soil describes the nonlinear plasticity behaviour which depends on the engineering soil

230 properties in the current ANSYS simulations. For this, the experimentally characterised bulk stress-strain

231 relationship corresponding to the load-displacement curves of different packing densities presented in Figure 4

232 are discretised into a large number of linear segments and fed as user defined digital input [37, 41].

8

Fig. 3 (left) Chosen domain and boundary conditions, half of domain analysis not to scale (right) finite element mesh, and element enlarged. S_v is vertical displacement component and S_h is horizontal displacement component. B is the footing width

Furthermore, the experimentally characterised material physical properties were used i.e. bulk density, initial modulus of elasticity (E) and typical value of Poisson's ratio (v) for sand (E = 25 MPa, 35 MPa and 50 MPa whereas v=0.2, 0.25 and 0.35 for the loose, medium-dense and dense sands respectively [11]). In the present analysis, ANSYS used the multilinear isotropic hardening of the stress-strain relation [41]. The width of the loading area is 0.5B. The loading is applied vertically in increments of constant displacement of 0.15B, uniformly across the width of the footing within the time step of 0.01 second (~300 cumulative iteration). The evolution of different displacement components in the solid geometry (depicting the sand packing) is tracked for different loading levels and compared with corresponding PIV measures later.

Results and Discussions

The load–settlement relationship for a typical footing (B=38mm) interacting with sand is presented in Fig. 4. It is worth mentioning that these PIV curves loads are simultaneously measured using the corresponding load outputs from the Instron. The settlements obtained using the PIV and LVDT gauge agree well. This justifies applying PIV to examine the displacement measures in sand layer later. Using the load-settlement data, the tangent intersection method [42] was applied to obtain the value of the ultimate bearing capacity (Fig. 4). The ratio of ultimate vertical settlement under ultimate load (S_u) to footing breadth (B), S_u/B is ~ 5.0, 7.1 and 11.7 % for the loose, medium-

9

256 dense and dense sand respectively. These measures and the nature of bulk load-settlement curves are consistent

257 [11] with punching (without a well-defined peak), local shear failure (moderate peak) and general shear failure

258 (well-defined peak) for sand described by Liu and Iskander [17], Dijkstra et al. [29], Vesic [43].

260 **Fig. 4** Load-settlement curves of footing (B=38mm) interacting with loose, medium-dense and dense sand. The
261 guide arrows show the ultimate load level (P_{ult}) of the sand packing

263 Though not presented here, we also obtained a good level of comparison with De Beer's study [44] for the

264 variation of N_γ (Bearing capacity factor) with γB for different sand packing. Both the bearing pressure and the

265 failure strain increase with the packing density of sand. The authors wish to point out that, in the case of strip

266 footings used in practice, 3D condition could exist around the ends of the strip footings even if the footing is long.

267 However, for most parts of long strip footings, plane-strain condition could exist [8, 25, 30] as assumed in the

268 current 2D plane-strain experiments [34].

269 The peak angle of internal resistance (ϕ_{peak}) for all cases of the packing density was also determined from triaxial

270 compression test at different confining pressures 100, 200, and 300 kPa. For sands, the angle of internal friction

271 typically ranges from 26° to 45°, increasing with the relative density. Three cases of relative densities were used

272 as that in the experiment tests: loose, medium-dense and dense. The height of the sand samples was typically 76

273 mm, and the diameter was 38 mm. Subsequently, the plots of deviator stress (σ_d) against axial strain (ε_a) were

274 made. The peak angle of friction of the soil is obtained according to the stress state at peak strength. The measured

275 angles of internal friction are 32°, 39.5°, and 44.3° for loose, medium-dense and dense sand respectively. Using

276 these, the peak angle of shearing resistance of the samples was evaluated and plotted against the relative density.

277 This variation is described in (2) as:

278 $$\phi_{peak} = 24.7 + 0.267\, D_r \tag{2}$$

10

279   Also the (ϕ_{peak}) determined from the standard direct shear test (ASTM D3080) under three different normal stresses

280   50, 100 and 200 kPa. The peak angle of shearing resistance of the samples was evaluated and plotted against the

281   relative density. This variation is described in a mathematical form in (3) as follows:

282 $$\phi_{peak} = 25.2 + 0.277\, D_r \tag{3}$$
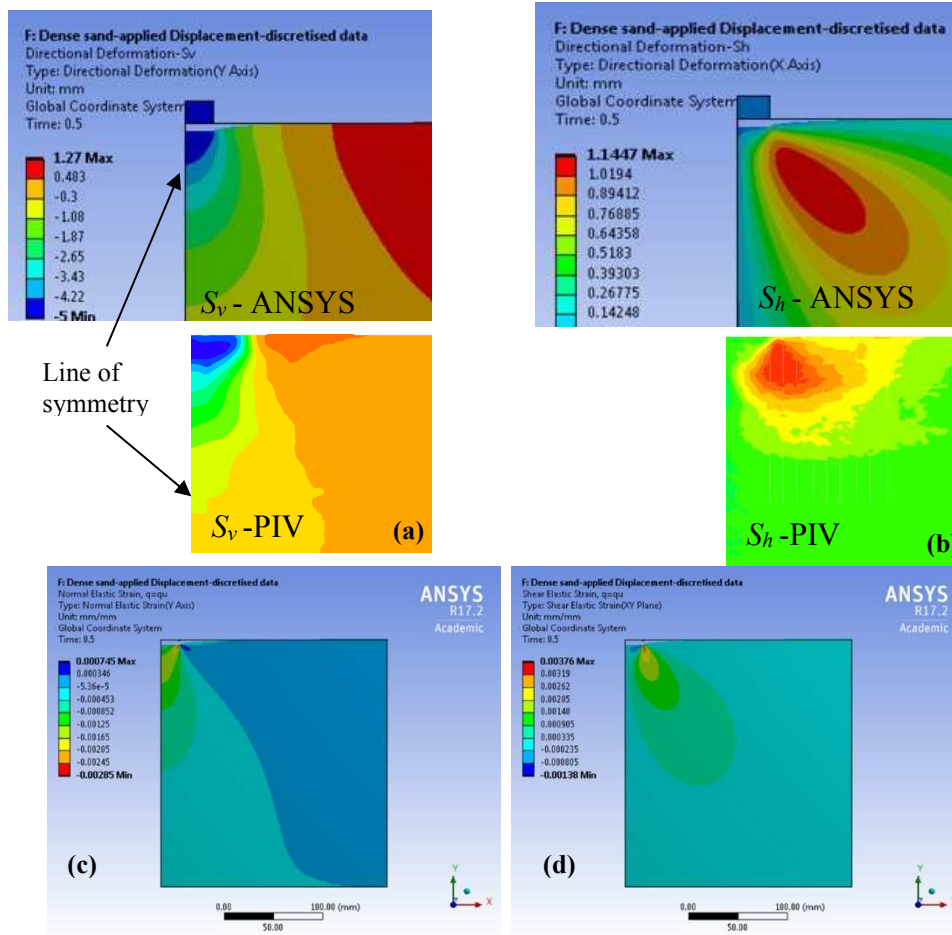
283   This macroscopic relations obtained from the above said characterisation experiments are in agreement with other

284   literature [8]. The authors wish to point out that, some literature suggest that the shear friction angle measured

285   using direct shear tests may not pertain to that of sand under the plane strain experiments [36]. Jewell [45]

286   suggested that symmetrical direct shear test could provide a more reliable measure of the plane strain angle of

287   friction and the angle of dilation for sand than conventional direct shear test. In some studies, the angle of internal

288   friction of sand obtained from the conventional direct shear test also correlated well with the experimental results

289   of plane–strain condition [46]. However, in analysing the footing-soil interactions using theoretical and

290   computational methods by idealising soil as an elastic media [47], friction angle does not form as a direct input in

291   the analysis.

292

293 **Comparison of the PIV measurements with FEM analysis**
294

295   Here the typical results are presented below for the case of footing interacting with the dense sand packing. Figure

296   5(a, b) shows the variation of PIV-based vertical displacement component and horizontal displacement component

297   profiles in the dense sand at ultimate load and compared with the FEM (ANSYS) analysis. It is evident that a

298   good level of agreement between the PIV and FEM approaches are obtained both qualitatively and quantitatively

299   up to 2.5B from the footing edges. Furthermore, quantitative comparison of variation of the normalised vertical

300   displacement component S_v and the normalised horizontal displacement component S_h along a horizontal section

301   at a depth of 0.5B below the level of footing under the ultimate load is provided for different packing conditions

302   of sand in the Online Resource OR3. A good level of agreement is obtained between them. Figure 5(c, d) shows

303   the variation of normal and shear elastic strain contours at ultimate load for the case of dense sand using FEM. It

304   can be seen from the normal and shear elastic strain map that there is a strain concentration around the corner of

305   the footing. It can be noticed from the contours of shear strain in the dense media that the soil is sheared in the

306   area below the edge of the footing. Though not presented here, similar observations were made for the loose and

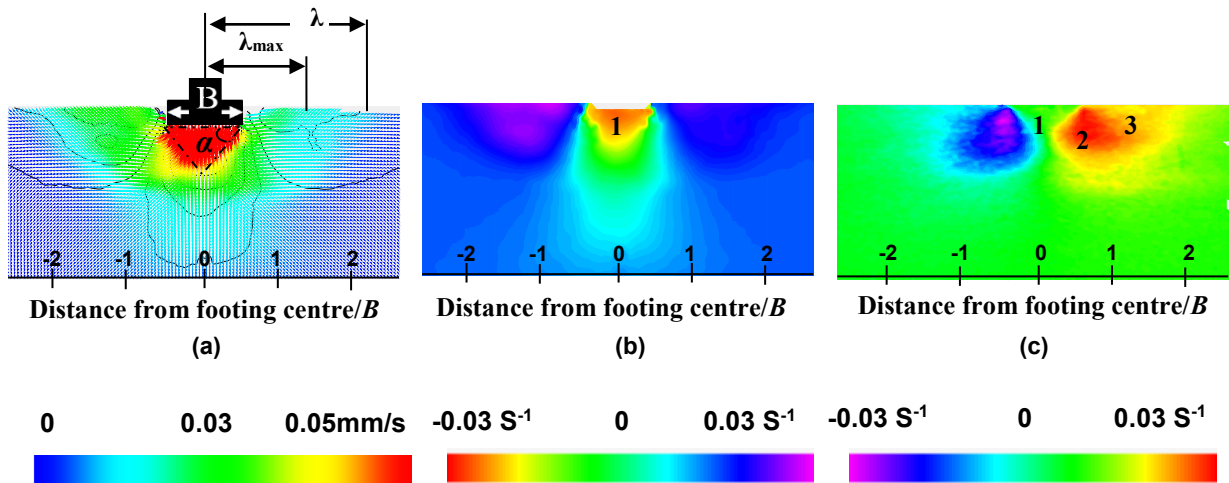307   medium-dense sand packing used in this study.

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Fig. 5 Comparison of PIV-based vertical displacement profile in dense sand at ultimate load below the footing with FEM analysis (identical colour codes are used): (a) vertical displacement component (b) horizontal displacement component below the footing. Taking advantage of FEM, the strain distributions are presented: (c) normal and (d) shear elastic strain

314

PIV based analysis of footing-sand interactions

316

Figures 6-8 present typically the evolution of resultant velocity vectors in the sand packing under a typical load

of q=q_{ult} (*B*= 38mm). Also the corresponding evolution of vertical and horizontal strain rate ($\dot{\varepsilon}_v$ and $\dot{\varepsilon}_h$

respectively) in the sand packing are provided which help to identify the difference regions of granular flow in

the sand packing such as dead, active and passive zones as discussed below. The authors had also verified that

these generic observations were similar in the case of the larger footing width (*B*=76 mm). A detailed evolution

of this for three more stages of loading is provided in the Online Resource OR4-OR6. In this plot the contours of

the vertical velocity are also obtained from the DSSP and superimposed for information.

12

Fig. 6 (a) Resultant velocity vectors at a typical load of q=q_{ult} in loose sand and the scalar contours of the vertical velocity using PIV (b) vertical strain rate $\dot{\varepsilon}_v$ (c) horizontal strain rate $\dot{\varepsilon}_h$. Zones: 1- dead zone, 2-active zone, 3- passive zone. B= 38 mm

Fig. 7 (a) Resultant velocity vectors at a typical load of q=q_{ult} in medium-dense sand and the scalar contours of the vertical velocity using PIV (b) vertical strain rate $\dot{\varepsilon}_v$ (c) horizontal strain rate $\dot{\varepsilon}_h$. Zones: 1- dead zone, 2- active zone, 3- passive zone. B= 38 mm
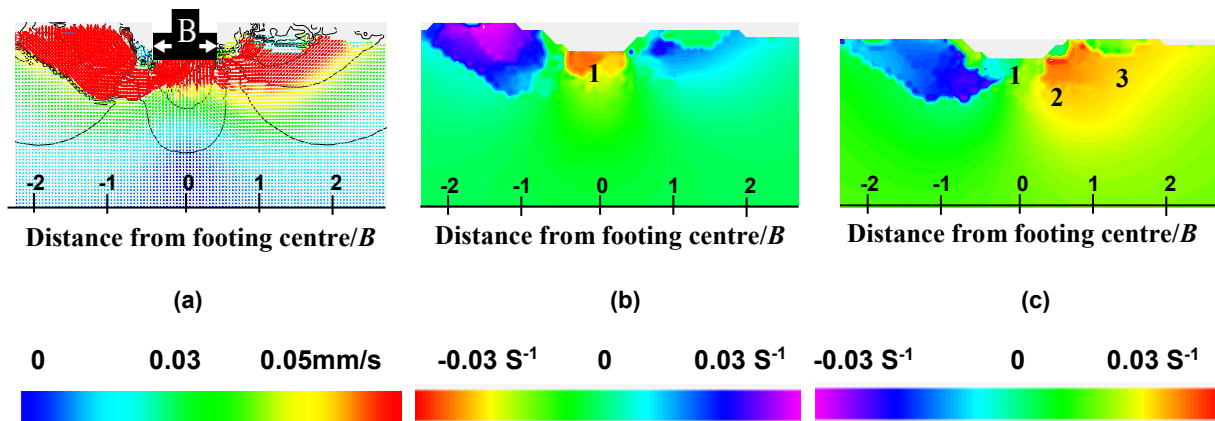

Fig. 8 (a) Resultant velocity vectors at a typical load of q=q_{ult} in dense sand and the scalar contours of the vertical velocity using PIV (b) vertical strain rate $\dot{\varepsilon}_v$ (c) horizontal strain rate $\dot{\varepsilon}_h$. Zones: 1- dead zone, 2-active zone, 3- passive zone. B= 38 mm
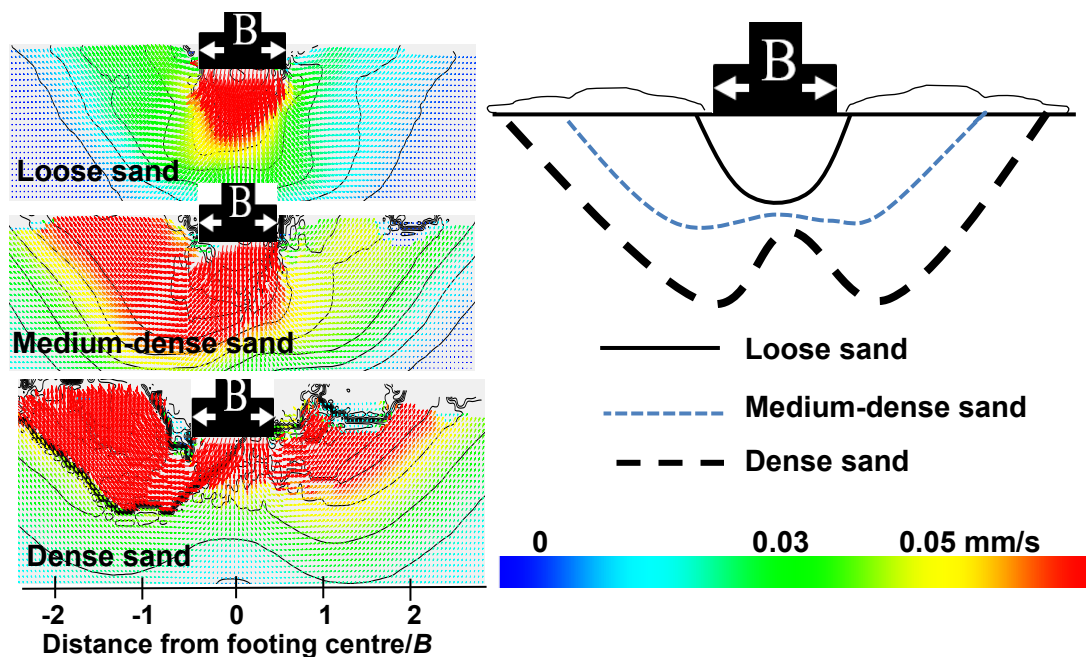
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339 At the early stages of the loading (c.a. $q<0.5q_{ult}$), approximately a triangular wedge of dead zone (region 1 in Figs.

340 6-8 with a constant amount of resultant velocity of the grains but has the highest vertical velocity) is formed

341 beneath the footing in all cases of packing densities [48]. The authors point out that the dead zone does not mean

342 that the grains are not moving at all, but move as a block of grains with almost the same velocity. In granular

343 mechanics, the dead-zone is characterised by the block of materials beneath the indenting objects with the granular

344 materials, and moving as if they are continuous extension of the indenter, i.e., no slip at the indenter-granular

345 interface [22]. Noticeably, outside this zone the particles tended to move downward and sideward symmetrically

346 until the ultimate bearing capacity is reached in the sand packing. Similar trends were noticed in other studies, for

347 example in sand [22], different soil types [48] and soft metal [49].

348 The depth of this wedge at the ultimate bearing load is equal to about B, whose vertices (slip planes) intersect the

349 horizontal at an angle (α= Angle of dead zone wedge/active zone 1) of about 62° ±2°. These are consistent with

350 Prandtl's assumption [48] for smooth footing (α = 45+ ϕ/2), which have not been confirmed using microscopic

351 experiments, but using PIV here. Furthermore, Kumar and Kouzer [38] have reported similar measures for smooth

352 footing using plasticity limit analysis with the help of finite element method (FEM). For a further increase in the

353 load, the grains in the dead zone tend to punch the neighbouring grains in zone 2 radially outwards (Figs.6-8). A

354 failure pattern consistent with Vesic [43] at the ultimate failure load is visualised (Figs. 6-8, and as in Online

355 Resource OR4-OR6) for all cases of sand considered here. By and large, the grains flow symmetrically with

356 respect to the central axis of the footing until reaching the ultimate load ($q=q_{ult}$) then, unsymmetrical flows occur

357 beyond the ultimate load. This is consistent with the classical literature, e.g. Vesic [43] for medium and dense

358 sand, but the current study observes that this could happen in the case of loose sand as well. At ultimate load, the

359 dense soil failed suddenly corresponding to the pronounced peak in the bulk strength curve presented in Fig.4,

360 and the unsymmetrically strong velocity distribution presented in Fig. 8 at the localised level. So, in the higher

361 relative density tests, the horizontal displacement (Fig. 8c) seems to be highly non-symmetric under ultimate load.

362 In reality local structural non-homogeneities could exist, and this triggers the non-symmetrical flow of grain (post-

363 failure) even under the symmetric loading conditions. At this stage, the grains flow like a fluid. The grains beneath

364 this flow region are solid-like and almost stagnant. The shear failure occurs progressively from the dead zone and

365 extending radially outwards.

366 The sand surface forms a heap spreading up to about 2.7B, 2.6B and 2.4B away from the footing centre (λ=

367 Distance of sand heap from the centre of the footing, Fig. 6) for the loose, medium-dense and dense packing

368 respectively (Figs. 6-8). The height of the heap attains maximum at distances of about 1.7B, 1.55B and 1.31B for

14

369 loose, medium and dense sand respectively (λ_{max}= Location of sand heap attaining maximum height from the

370 centre of the footing). The slope of sand heap at the ultimate load is 31°, 33° and 38 ° for the loose, medium-dense

371 and dense sand respectively. The average value of the slop of the heap for the different sand packing is (34°)

372 identical to the angle of repose of the sand. These angles are close to the residual angle of internal friction of the

373 sand (ϕ_{cr}) about 31°, 33°, 37° for the loose, medium-dense and dense sand samples respectively. These residual

374 angles are consistent with the previous literature [50].

375 The effect of sand packings at ultimate load on the failure mechanism are summarised both quantitatively and

376 systematically in Fig. 9 for a typical case of footing (B=38mm). This shows the resultant velocity vectors of soil

377 movement at ultimate bearing capacity and the schematic diagram of the failure mechanism underneath the footing

378 of different packing densities. It is evident that the footing load test in the loose packing corresponds to a punching

379 failure and local shear mechanism, consistent with the literature [29]. On the other hand, tests in the medium-

380 dense and dense packings exhibit a general failure mode with relatively larger horizontal displacement and soil

381 heap next to the footing, typical of a conventional rigid plastic bearing capacity, and Prandtl's wedge-shaped zone

382 mechanism [48]. Hence, the formations of velocity discontinuities are linked to the density of the sand packing.

383

384 **Fig. 9** (left) Resultant velocity vectors of soil movement under ultimate bearing capacity, B=38 mm (right) sketch
385 of general schematic failure mechanism underneath footing of different packing densities

386

387 Interestingly, the boundaries of the zone of plastic flow in sand at failure load sketched using PIV here (in terms

388 of the resultant velocity vector, and the components of strain rate presented in Figs.6-8) are remarkably similar to

15

389 such intuitive diagrams suggested by Fröhlich in the 1930's [48]. However, at or beyond the ultimate load, the

390 PIV experiments have shown non-symmetric flow of grains even under the symmetric loading conditions on the

391 footing. They could be attributed to the potential existence of (even minor level) non-symmetrical structural

392 arrangements of the grains at local scale in reality. These could amplify the non-symmetrical velocity patterns

393 under the above said condition.

394 Figure 10 shows the typical normalised vertical displacement component in dense sand at different horizontal

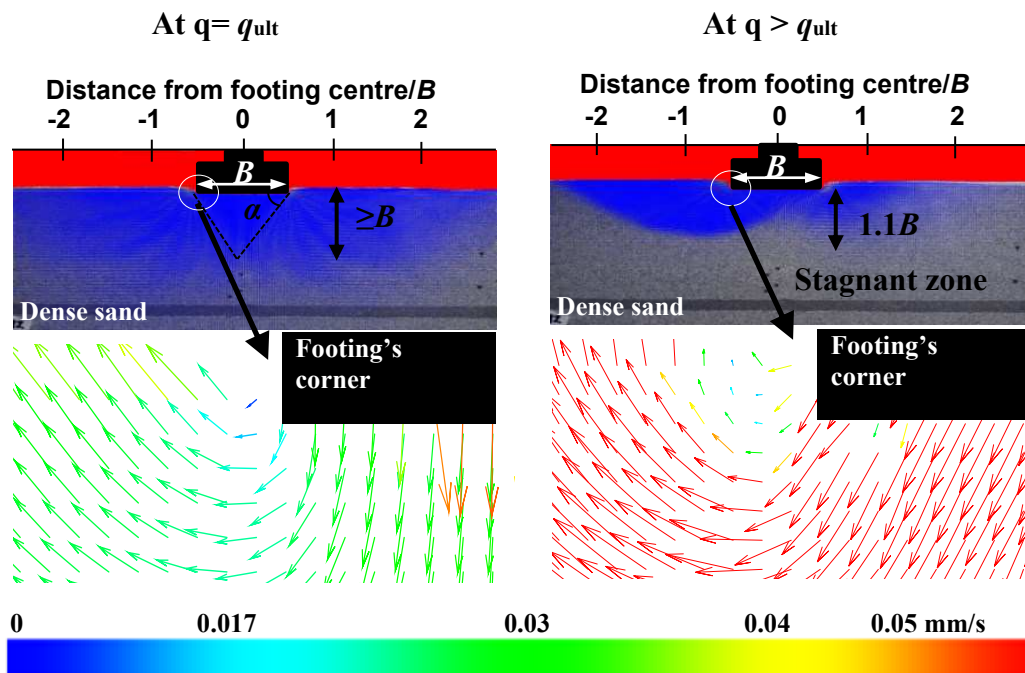395 sections below the footing-soil interface under the ultimate load (q_{ult}). The plots of this for loose and medium-

396 dense sand are provided in the Online Resources OR7. It is evident that at a depth of z/B=2.5 the vertical

397 displacement is practically negligible in all cases of footing width and packing densities. Furthermore, for a given

398 packing density, the normalised vertical displacement in the soil at a given depth (e.g. z/B= 0.1) decreases for an

399 increase in the width of the footing under the ultimate load. However, here it is found that the absolute value of

400 the vertical displacement in the soil for a given depth increases for increase in the width of the footing as also

401 referred in other studies [51]. The heap close to the free surface (secondary peaks) increases with increasing

402 relative density but decreases with z/B (Fig. 10). Also, the heap height decreases with footing width as shown in

403 Online Resource OR7. This is related to potentially particles interlocking and rolling over of the grains. This has

404 a significant effect in the development of the vortex map adjoining the footing sides [22]. The discontinuities in

405 the velocity measures directly beneath the footing and the edges of the footing could result some error in the

406 measurement [25]. However, these measurements are taken at and beyond a depth z/B= 0.1 after the ultimate

407 loading is applied. Therefore, any such potential errors are expected to be minimal here.

(a) *B*=38 mm   (b) *B*=76 mm

408
409 **Fig. 10** PIV-based normalised vertical displacement component profiles in dense sand under the ultimate load
410 (q=q_{ult}) along different horizontal sections at different depths (z/B) from the footing-soil interface
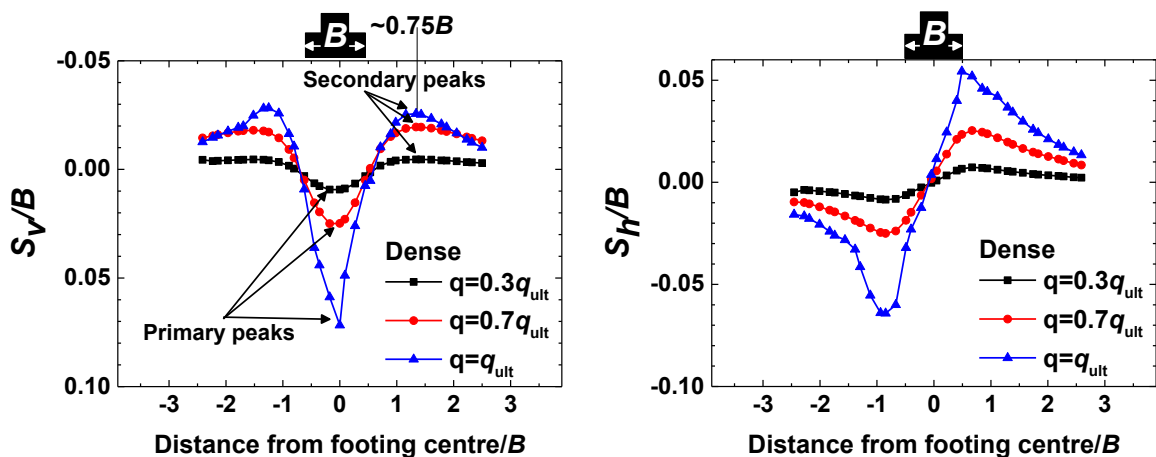411
412

413   The images of flow of grains near the corner of the footing (B=38 mm) for a typical case of dense sand is

414   superimposed on their resultant velocity distribution plots and presented in Fig.11 for the pre-failure and post-

415   failure stages. These plots for the cases of loose and medium-dense sand are provided in the Online Resource

416   OR8. The grains flow radially outwards and non-uniformly in all cases of the sand packing. The grains resting

417   beyond the influence zone of deformation (or velocity) do not move in the post-failure stage, which is beyond

418   about 1.25B, 1.2B, and 1.1B for loose, medium-dense and dense sand respectively for both cases of footing width.

419   This trend is in agreement with Liu and Iskander [17] who stated that this influence zone is limited within a depth

420   of about 1B of the footing at post-failure stage in the case of loose sand (D_r is limited to 21% in their experiments).

421   From the PIV results, the authors observe that the depth of this influence zone is lower than that of pre-failure

422   stages possibly due to relatively radially outwards movement from the edge of footing as a block, which is

423   consistent with conventional Terzaghi's bearing capacity analysis (Fig.11) [48]. The vortex in the total velocity

424   distribution profiles is clearly seen nearer the corner side of the footing for all sand densities. It started when the

425   vertical displacement (S_v) ratio $S_v/B \sim 0.1$. This is a result of gravitational movement of sand from the surface heap

426   while the subsurface grains move upward with loading. Eventually the grains fall back from the passive zone

427   toward the edge of the footing and form a loop-like resultant velocity profile. The spatial nature of the vortex is

428   consistent with Murthy et al. [22] where the sand medium was dense. The present study confirms the presence of

429   such vortex maps in loose and medium-dense sand as well.

Fig. 11 Vortex formation of resultant velocity vectors for footing (B=38) interacting with dense sand. Enlarged
view of the corner of the footing is also presented here

17

433 Computational studies based on finite element method, for example Griffiths et al. [52] and Kumar and Kouzer

434 [38] have reported velocity discontinuities near the footing edge, but the current study has shown their evolution

435 in the sand packing using PIV experiments. The presence of such vortices around the corner at q > q_{ult} is likely to

436 be related to the local density at the corner rather than the bulk density underneath the footing. Hence, even in a

437 loose sand under compression loading, once the materials close to the footing corner compact to a certain density

438 that is close to the maximum density of the tested soil then vortex will be observed around the edge of the footing.

439 Since features like this are really local features and not bulk features, so it is the density in the local zone around

440 the corner that would be important than the whole soil density for characterise of the vortex formation.

441 Figure 12 quantifies the normalised vertical displacement component S_v and horizontal displacement component

442 S_h of dense sand (for a typical case of B= 38mm) at different loading levels (q ≤ q_{ult}). These plots for loose and

443 medium-dense sand are provided in the Online Resource OR9. S_v presents an inverted triangle-like profile that

444 becomes deeper and narrower with increasing load level. The maximum value of S_v occurs along the footing

445 centre, then decreases gradually towards the footing edge [17]. S_v decreases to zero within a distance of $0.25B$

446 from the footing edge. This behaviour is due to the lack of confinement in the soil [11]. The secondary peaks in

447 the distribution of S_v diminish with decrease in the density of sand.   Such patterns, at times non-symmetric, are

448 seen mostly at or beyond the ultimate load even under the symmetric loading conditions on the footing as

449 discussed earlier. The secondary peaks increase with increasing density as shown in Online Resource OR9. This

450 could be due to the particles interlocking, jamming and dilation that increase with the relative density of sand.

451

452 **Fig. 12** For a typical case of dense sand: (left) Normalised vertical displacement component (right) normalised
453 horizontal displacement component at a horizontal cross section $0.5B$ below footing using PIV at different loading
454 levels. Signs: vertical displacement (positive down, negative up), horizontal displacement (Negative toward left,
455 positive toward right from the central axis). B = 38 mm

456

457 The profile of S_h component presents S-like shape with a neutral point (zero value) occurring along the axis of

458 symmetry of the footing. The soil along the vertical axis of symmetry is confined by the maximum vertical

459 displacement and therefore $S_h \sim 0$. It is worth mentioning that, though not presented here, the variation of resultant

460 displacement ($S_R = \sqrt{S_v^2 + S_h^2}$) at the footing-soil interface was uniform along the footing width.

461

462 Previous classical approaches have estimated the elastic settlement of footings using influence factors, which

463 could vary along the depth of sand [8, 53]. Such variations are also observed from numerical solutions, for

464 example using finite element method [53], elastic theory [16] and simple triangular profile using in situ cone

465 penetration tests [14]. However, they show different types of profiles. Using PIV here, the variation of S_v/B along

466 the centre line of the footing is examined, and S_h/B along the edge of the footing with depth for a typical case of

467 dense sand (B=38 mm) is presented in Figure 13. These results for the cases of loose and medium-dense sand are

468 provided in the Online Resource OR10.

469

470 **Fig. 13** Settlement profiles with depth z from the bottom surface of the footing at different loading levels for a
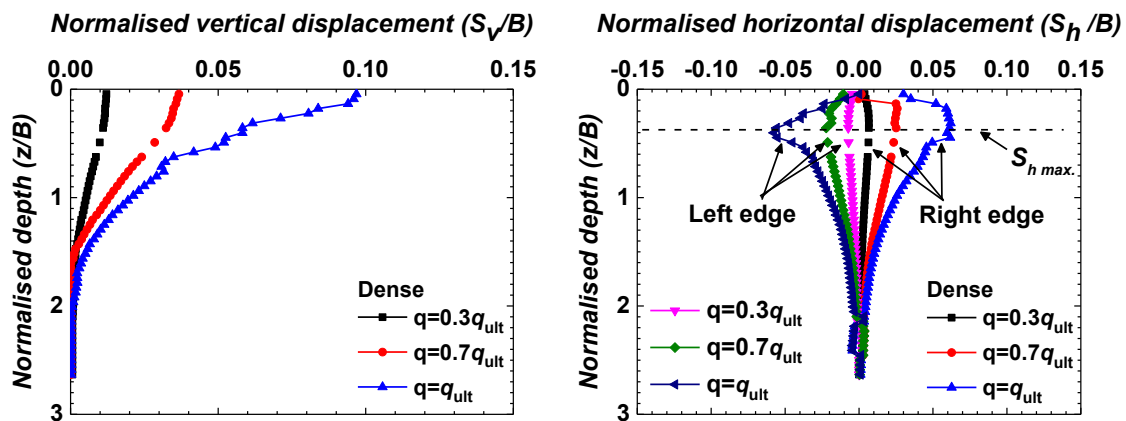471 typical case of dense sand: (left) normalised vertical displacement component (right) normalised horizontal
472 displacement for the sand packing. B= 38 mm

473

474 They show a nonlinear response for all cases of sand packing. They gradually decrease to a negligible value

475 beyond $\sim z/B$= 2.5, similar results have been reported for loose sand by Liu and Iskander [17], however this

476 distance decreases for an increase in the relative density of sand. The normalised vertical displacement (S_v/B)

477 attains the peak at a depth of about $0.1B$ for all cases of sand packing and footing widths, which are almost

478 independent of the loading stages.

19

479   Similarly, the normalised horizontal displacement component (S_h/B) attains maximum at a depth of about $0.25B$

480   from the surface of the footing. At $q \leq q_{ult}$, the maximum value of normalised vertical displacement for smaller

481   width (B=38mm) is: $S_{v\,max}/B$ =0.070, 0.086 and 0.0096 and $S_{h\,max}/B$= 0.02, 0.03 and 0.07 for loose, medium-dense

482   and dense sand respectively. These values increase with the relative density and load level. But, these values for

483   the larger width of footing (B=76 mm) is: $S_{v\,max}/B$ =0.045, 0.052 and 0.074 for loose, medium-dense and dense

484   sand respectively. The authors observed that the values of $S_{v\,max}/B$ at ultimate load are close to the measured values

485   of S_u/B presented earlier.   Interestingly, the values of $S_{v\,max}/B$ agree with the common assumption of using S/B

486   between $0.05B$ -$0.1B$ for estimating q_{ult} from the load-settlement plots in foundation engineering designs [25, 36,

487   53]. Overall, the displacement measures reported here could be used to derive more realistic description of

488   displacement profiles in soil media in future.

489 **Conclusions**

490

491   PIV is shown to be effective and promising in understanding the local and global geomechanical characteristics

492   of footing interacting with sand media of different relative densities in a coherent manner. Where possible, the

493   displacement measures and generic characteristics of velocity fields in the sand are compared with existing

494   literature and FEM analysis and a good level of agreement is obtained.   PIV clearly shows detailed descriptions

495   of the stages of velocity discontinuities for the sand media. The velocity profiles of the medium-dense and dense

496   sand are consistent with Vesic [43] but the advanced measurements reported here detect their evolutions more

497   precisely. For the loose sand, the velocity discontinuities could reach the free surface. Significant vortex zones

498   are existent near the footing corner at and beyond the ultimate bearing capacity of sand of all relative densities

499   studied here. The boundaries of the zone of plastic flow in sand at failure load profiled using the advanced PIV

500   here are remarkably similar to such intuitive diagrams suggested by the forefathers of soil mechanics, for example

501   Fröhlich in the 1930's and Terzaghi's in 1940's [48].   The depth at which the settlement vanishes in the sand

502   decreases for increase in the relative density of sand. The present study provides both the spatial and temporal

503   distribution of displacements in soils of different packing densities under key stages of loading elegantly. PIV

504   could be applied in future to develop robust failure surfaces for more complex soil profiles and foundation types

505   encountered in geotechnical engineering applications. Further analysis is required for evaluating the scaling and

506   size effects of footing-sand interactions in a detailed manner with a better resolution of the digital measurements

507   (e.g. using multiple grids per grain). Also simulations using discrete element method (DEM) could be more

508   suitable to model the grain-scale movements of granular assemblies under mechanical loading, but they would

509 require extensive level of computing resources to study the cases considered in this work. However, as shown in
510 the current manuscript, FEM-based displacement fields can match to the level of local-scale grain displacements
511 of corresponding experimental systems based on PIV. In particular, experiments-based, user-defined constitutive
512 relation was used as input in the FEM simulations here which is found to be useful. Studies are underway in our
513 group to extend the present research strategy to account for more complex conditions and realistic sand profiles
514 in footing-sand interactions – for example layered soil systems, interference effects and roughness effects of
515 footings.

516

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Figure Captions

610 **Fig. 1** (a) Experimental setup using PIV with a live image of footing in contact with sand (b) schematic diagram
611 of the experimental setup (dimensions are in mm)

612

613 **Fig. 2** CPT data for the sand packing

614 **Fig. 3** (left) Chosen domain and boundary conditions, half of domain analysis not to scale (right) finite element
615 mesh, and element enlarged. S_v is vertical displacement component and S_h is horizontal displacement component.
616 B is the footing width

617

618 **Fig. 4** Load-settlement curves of footing (B=38mm) interacting with loose, medium-dense and dense sand. The
619 guide arrows show the ultimate load level (P_{ult}) of the sand packing

620

621 **Fig. 5** Comparison of PIV-based vertical displacement profile in dense sand at ultimate load below the footing
622 with FEM analysis (identical colour codes are used): (a) vertical displacement component (b) horizontal
623 displacement component below the footing. Taking advantage of FEM, the strain distributions are presented: (c)
624 normal and (d) shear elastic strain

625 **Fig. 6** (a) Resultant velocity vectors at a typical load of q=q_{ult} in loose sand and the scalar contours of the vertical
626 velocity using PIV (b) vertical strain rate $\dot{\varepsilon}_v$ (c) horizontal strain rate $\dot{\varepsilon}_h$. Zones: 1- dead zone, 2-active zone, 3-
627 passive zone. B= 38 mm

628

629 **Fig. 7** (a) Resultant velocity vectors at a typical load of q=q_{ult} in medium-dense sand and the scalar contours of
630 the vertical velocity using PIV (b) vertical strain rate $\dot{\varepsilon}_v$ (c) horizontal strain rate $\dot{\varepsilon}_h$. Zones: 1- dead zone, 2-
631 active zone, 3- passive zone. B= 38 mm

632

633 **Fig. 8** (a) Resultant velocity vectors at a typical load of q=q_{ult} in dense sand and the scalar contours of the
634 vertical velocity using PIV (b) vertical strain rate $\dot{\varepsilon}_v$ (c) horizontal strain rate $\dot{\varepsilon}_h$. Zones: 1- dead zone, 2-active
635 zone, 3- passive zone. B= 38 mm

636

Fig. 9 (left) Resultant velocity vectors of soil movement under ultimate bearing capacity, B=38 mm (right) sketch of general schematic failure mechanism underneath footing of different packing densities

Fig. 10 PIV-based normalised vertical displacement component profiles in dense sand under the ultimate load ($q=q_{ult}$) along different horizontal sections at different depths (z/B) from the footing-soil interface

Fig. 11 Vortex formation of resultant velocity vectors for footing (B=38) interacting with dense sand. Enlarged view of the corner of the footing is also presented here

Fig. 12 For a typical case of dense sand: (left) Normalised vertical displacement component (right) normalised horizontal displacement component at a horizontal cross section $0.5B$ below footing using PIV at different loading levels. Signs: vertical displacement (positive down, negative up), horizontal displacement (Negative toward left, positive toward right from the central axis). B = 38 mm

Fig. 13 Settlement profiles with depth z from the bottom surface of the footing at different loading levels for a typical case of dense sand: (left) normalised vertical displacement component (right) normalised horizontal displacement for the sand packing. B= 38 mm

Table 1. Experimentally measured physical properties of the sand used

Type of sand	Loose (L)	Medium-dense (M)	Dense (D)	Standards
Dry density (γ_d): (kN/m^3)	14.70	15.30	15.80	ASTM C29/C29M
Void ratio (e_o)	0.76	0.70	0.64	
Relative density, D_r : %	24	53	72	ASTM C128
Peak angle of internal friction, ϕ_{peak}: °	32(32.4)[*]	39(39.5)*	44.3(46.1)*	
Residual angle of internal friction, ϕ_{cr}: °	30	32	36.3	ASTM D3080
Maximum dry density (γ_{dmax}): kN/m^3		16.50		ASTM D698
Minimum dry density (γ_{dmin}): kN/m^3		14.23		ASTM D4254 method C
Maximum void ratio (e_{max})		0.83		ASTM C29/C29M
Minimum void ratio (e_{min})		0.58		ASTM C29/C29M
D_{10} : mm		0.25		
D_{30} : mm		0.31		ASTM D421
D_{50}: mm		0.37		ASTM D422
D_{60} : mm		0.40		
Uniformity coefficient, C_U		1.55		
Coefficient of curvature, C_C		0.93		ASTM D2487
Mineralogy		Silica		
Grain shape		Mostly spherical to sub-prismoidal		Head (2006)
Angularity of grains		Angular and sub angular		
Angle of repose of the sand		34°		Head (2006)

()* Direct shear test results