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# Model based Kinematic & Dynamic Simulation of 6-DOF Upper-limb Rehabilitation Robot

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**Abstract**—Globally, a large population is suffering from motor disabilities caused by acute lesions to brain nervous system. One example is stroke, which is the third largest killer in New Zealand and the United States. Traditional manual therapy usually requires cooperative and intensive efforts from therapists and patients. Robot-assisted upper-limb rehabilitation techniques have been actively researched in the past few decades. However, limitations still exist such as inappropriate robotic modelling, mechanical design or limited range of motion (ROM). This paper proposes a mathematical model for a 6-Degree of Freedom (DOF) Universal Robot to be used in a rehabilitation system. This study focuses on the kinematics and dynamic analysis by using the Denavit–Hartenberg (D-H) parameters method with coordinate transformation theory. In order to simplify the computation process, Kane equation method is introduced in this paper. Simulation results show that the proposed model is correct although the fluctuation is possible to be reduced further. It concludes that the mathematical model can provide an intuitive and effective environment for designing the rehabilitation robot and planning the clinical trials.

**Keywords**—kinematic; dynamic; rehabilitation system; simulation

## I. INTRODUCTION

Human's upper-limbs have a more complex nervous system and need to complete the more complicated movements than lower-limb's, which means it is more difficult to be recovered in a short time [1, 2]. The Stroke Foundation of New Zealand reported that after three months of treatment, there were still 55% to 75% of stroke patients with upper-limb disorders [3]. Being both common and severe urges researchers to develop rehabilitation strategies to deal with upper-limb disorders.

Timely rehabilitation training is an efficient way to treat the stroke patients [4]. However, there are some limitations on the conventional manual therapy: the shortage of therapists; cooperative and intensive efforts from therapists and patients over prolonged sessions; the subject evaluation methods; the expensive cost of rehabilitation training and so on [5]. Furthermore, assistance provided by therapists is experience based, which means optimal movement for upper-limb cannot always be achieved.

As means to improve the manual rehabilitation, robot-assisted rehabilitation therapies have been researched for a long time. In recent years, some novel robotic devices have

been developed for upper-limb rehabilitation. In terms of hardware, end-effector and exoskeleton types of robots are the two major trends of research [6-8]. The training strategies of robotic devices are reflected through the control algorithms applied. Initially, trajectory tracking control is implemented to various robots to replicate the movements of upper-limb [9]. Further development of training strategies have been targeted to assistance-as-needed (AAN) concept [10]. In current AAN approaches, the assessments are mainly through kinematic data.

The aim and the importance of dynamic analyses in rehabilitation robotics is that some control strategies are based on the dynamic models and feedbacks such as force control and impedance control. So the more precise model and feedback can give birth to the more accurate control, which is very important for clinical training. For now, Lagrangian mechanics equations [11], Newton-Euler equations [12] and Gauss equations [13] are the main methods for the dynamic analysis. However, the establishment and solution of these equations become more and more difficult due to the increasing number of links, the connection state and the complexity of constraint equation [14]. The rehabilitation system in this research is based on a 6-DOF robot, which means the system is too complex to do the dynamic analysis by using the methods above. In order to solve the problem, this paper simplifies the structure of rehabilitation system based on the D-H parameters method [15]. The use of Kane equations [16] allows for the dynamic analysis of complex system. This study focuses on the analysis and simulation rather than the modelling. However, the accuracy of simulation is not high. Several solutions have been introduced to reduce or wipe out the fluctuation to improve the accuracy.

## II. STRUCTURAL ANALYSIS AND MODELING

### A. Structural Analysis

The rehabilitation system in this research is based on Universal Robot 10 (UR10, Universal Robots A/S, Denmark), which consists of seven revolute joints: base, shoulder, elbow, wrist-1, wrist-2 and tool mounting bracket. Each joint is driven by one motor separately to realize the relative motion of adjacent links, and the whole system is a typical series structure device. Moreover, all joints of UR10 are rotation joints, it means that only  $\theta$  is variable parameter when describing each joint. So for this rehabilitation system,

only six joint variables (rotation variables, except joint7 which is on the top of link6) are needed to figure the changes of whole system out, and other eighteen parameters are used to represent the fixed part of kinematics. The information of joint parameters of UR10 is shown in Table 1.

TABLE I. JOINT PARAMETERS FOR UR10 [1]

Joint <sub>i</sub>	$\theta_i$ (deg/sec)	$a_i$ (mm)	$\alpha_i$ (deg)	$d_i$ (mm)
1	2.0	0	90	127.3
2	3.0	-612	0	0
3	3.0	-572.3	0	0
4	7.0	0	90	163.941
5	23.0	0	90	115.7
6	4.0	0	0	92.2

Here,  $\theta_i$  is the velocity-time function (inputs) and represents the rotation angle of Z axis,  $d_i$  represents the distance between two adjacent common perpendiculars of Z axis,  $a_i$  represents the length of each common perpendicular and  $\alpha_i$  represents the joint torsion angle between two adjacent Z axes.

### B. The D-H parameters method

The D-H parameters method [15] is used to establish the mapping relationship between the joint coordinate system and the Cartesian coordinate system of UR10, so the position relationship between tool mounting bracket and other joints can be extracted in real time, which is the critical part in control and optimal trajectory-planning.

The D-H parameters method [15] is a kind of simple method for modelling the link and joint of robotic device, this method can be used for any mechanical configuration, regardless of the structural order or the complexity. The establishment principle of the Cartesian coordinate system and the definition of joint variables are as follows.

1) All joints are represented by Z axis. If the joint is a rotating one, Z axis follows the right-hand rule; if the joint is a sliding one, Z axis follows the direction of linear movement. And,  $Z_n$  represents  $n+1$ th Z axis.

2) If  $a_n$  represents the common perpendicular between  $Z_{n-1}$  and  $Z_n$ , the direction of  $X_n$  is along the direction from  $a_n$  to  $Z_n$ .

3) The direction of  $Y_n$  is according to the principle of right-hand rule, which will not be showed in Fig.1 for the purpose of simplification.

The Fig.1 shows the Definition of UR10's centre of mass parameters. Here, Fig.1 is the 3-D sketch of D-H parameter system.

Table 2 shows the information of D-H parameters for UR10.

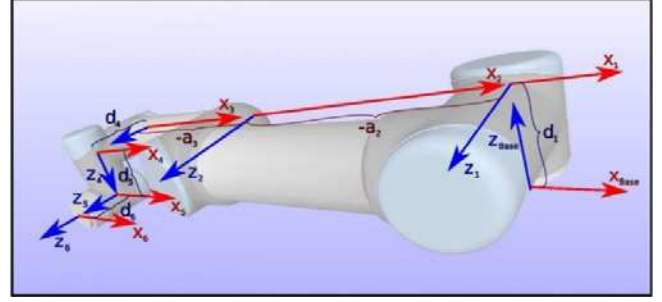


Figure 1. The Definition for centre of mass parameters [1]

TABLE II. THE D-H PARAMETERS FOR UR10 [1]

Link i	Name	Mass(g)	L(mm)
1	Base	7100	3.42
2	Upper Arm	12700	41.154
3	Low arm	4270	24.945
4	Wrist1	2000	1.931
5	Wrist2	2000	1.931
6	Wrist3	365	2.6

Here, L represents the Length from gravity centre to rotation centre.

## III. KINEMATIC & DYNAMIC SIMULATION

### A. Kinematic analysis

Kinematic analysis of upper-limb rehabilitation robot is the base of control and implementation for clinical training. In order to make sure that robot can accomplish the task, it is necessary to know the top position of robot. The transformation matrix  $T$  of coordinate system from link  $\{i\}$  to link  $\{i-1\}$  can be called as link transformation matrix, which is related to the four parameters in Table 1, and the common transform equation is as follows.

$${}^{i-1}T_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & a_{i-1} \\ \sin \theta_i \cos \alpha_{i-1} & \cos \theta_i \cos \alpha_{i-1} & -\sin \alpha_{i-1} & -d_i \sin \alpha_{i-1} \\ \sin \theta_i \sin \alpha_{i-1} & \cos \theta_i \sin \alpha_{i-1} & \cos \alpha_{i-1} & d_i \cos \alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

So for  $n$  links, the transformation matrix  $T$  can be written as:

$${}^0_nT = {}^0_1T {}^1_2T \cdots {}^{n-1}_nT \quad (2)$$

According to the joint parameters listed in Table 1, equation 1 and equation 2, the transformation coordinates of the tool mounting bracket (Joint7) which is relative to the base can be obtained. So based on the actual mathematical model of UR10 built above, adding the appropriate driver function to each joint to drive patients' arms to complete the task of drinking water in six seconds. The motion trajectory of tool mounting bracket can be simulated by the MATLAB Symbolic Math Toolbox (MUPAD), the results are as follows.

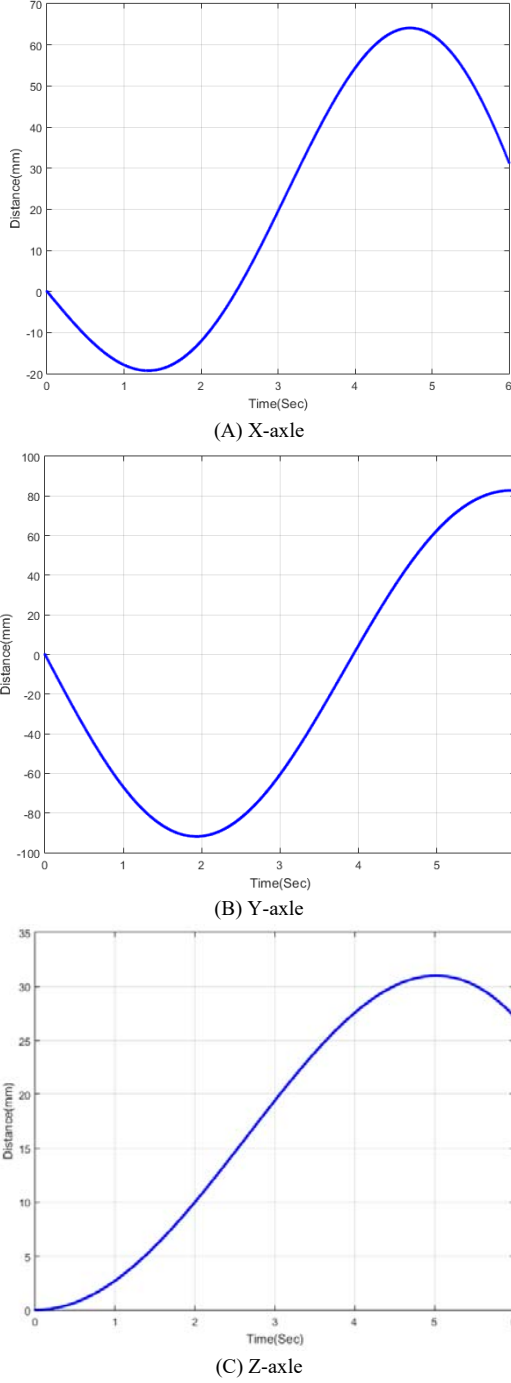


Figure 2. Trajectory of tool mounting bracket (Joint 7)

### B. Dynamic analysis

The upper-limb rehabilitation robot is a complex dynamic system, which consists of many joints and links with multiple inputs and outputs, it means that the system owns a perplexing coupling relationship and non-linearity relationship. Moreover, during the training, the movement of patients' upper-limb is not following a fixed trajectory, so the joint torque requirements should be calculated in real

time. As a result, efforts are made to develop an algorithm to count joint torques of UR10. For now, several robot dynamic analysis methods have been researched, such as Lagrangian mechanics equations, Newton-Euler equations, Kane equations, Gauss equations, Roberson-Wittenburg equations and so on. In general, Lagrangian mechanics equations and Newton-Euler equations are the two most commonly used methods for robot dynamic modelling. However, the computing processes of these two methods are very complex since they have to perform the second derivative of each parameter, which means that the computational efficiency is low and cannot guarantee the accuracy, so they are not suitable for the robot with multi-DOF. While Kane equations [16] is a simple method based on analytical mechanics and inspired by Lagrangian equations, and it is appropriate for multi-DOF robot system. The Kane method adopts the generalized velocity as the independent variable, and combines with some concepts, such as partial velocity, generalized active force and generalized inertia force, to establish the dynamic equation. The angular velocity, the angular acceleration, the joint velocity, the joint acceleration, the center of mass's velocity and acceleration of link  $i$  can be represented by the kinematic recurrence equations [17] as follows.

$$\omega_i = R_{i-1}^i \omega_{i-1} + \dot{\theta}_i k_i \quad (3)$$

$$\dot{\omega}_i = R_{i-1}^i \dot{\omega}_{i-1} + R_{i-1}^i \omega_{i-1} \times \dot{\theta}_i k_i + \ddot{\theta}_i k_i \quad (4)$$

$$v_i = R_{i-1}^i (\omega_{i-1} \times l_{i-1} + v_{i-1}) \quad (5)$$

$$\dot{v}_i = R_{i-1}^i [\dot{v}_{i-1} + \dot{\omega}_{i-1} \times l_{i-1} + \omega_{i-1} \times (\omega_{i-1} \times l_{i-1})] \quad (6)$$

$$v_{ci} = v_i + \omega_i \times l_{ci} \quad (7)$$

$$\dot{v}_{ci} = \dot{v}_i + \dot{\omega}_i \times l_{ci} + \omega_i \times (\omega_i \times l_{ci}) \quad (8)$$

Here,  $R_{i-1}^i$  is the rotational transformation matrix from the coordinate system  $i$  to the coordinate system  $i-1$  and  $R_{i-1}^i = (R_{i-1}^{i-1})^T$ ;  $\theta_i$  and  $\omega_i$  is the rotational velocity and the angular velocity of joint  $i$  separately;  $k_i$  is a  $3 \times 1$  rotational axis vector, and  $k_i = [0 \ 0 \ 1]^T$ ;  $l_{ci}$  is the position vector of link  $i$ 's center of mass in the coordinate system, and  $l_{ci} = [l_{ci} \ 0 \ 0]^T$ .

Besides, the torque equation of link is as follows.

$$T_j = \sum_{i=1}^n M_{i-\dot{q}_j} \quad (9)$$

$$M_{i-\dot{q}_j} = m \dot{v}_{ci} g v_{ci-\dot{q}_j} + N_i \omega_{i-\dot{q}_j} \quad (10)$$

$$N_i = I_i \dot{\omega}_i + \omega_i \times (I_i \omega_i) \quad (11)$$

$$I_i = \begin{bmatrix} I_{i_{xx}} & 0 & 0 \\ 0 & I_{i_{yy}} & 0 \\ 0 & 0 & I_{i_{zz}} \end{bmatrix} \quad (12)$$

Here,  $I_i$  is the inertia tensor.

So the angular velocity, the angular acceleration, the joint velocity and the joint acceleration of base can be got as follows where  $g$  represents the gravitational acceleration.

$$\omega_0 = [0 \ 0 \ 0]^T \quad (13)$$

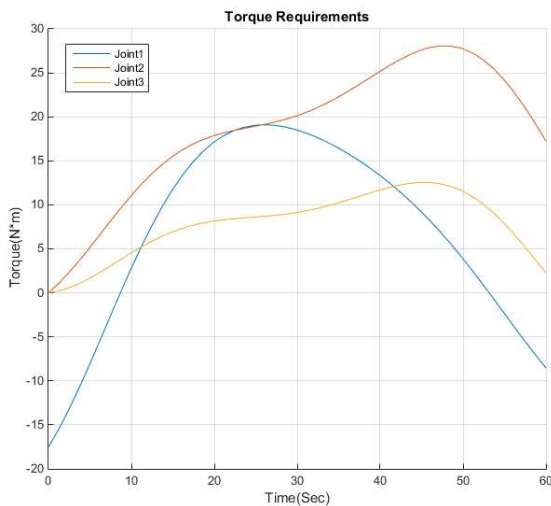
$$\dot{\omega}_0 = [0 \ 0 \ 0]^T \quad (14)$$

$$v_0 = [0 \ 0 \ 0]^T \quad (15)$$

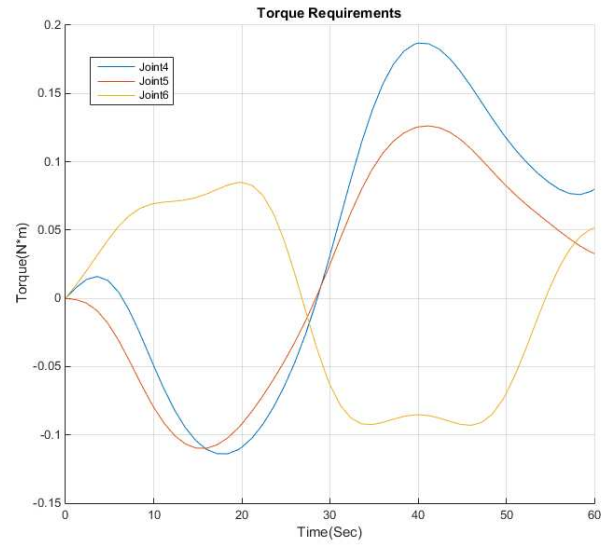
$$\dot{v}_0 = [0 \ g \ 0]^T \quad (16)$$

The actual mathematical model is implemented based on MATLAB (SimMechanics) as the kinematic analysis above. Through the calculation, a MATLAB function is generated to calculate joint torque requirements from joint space trajectories in real time. Only fundamental algebra and trigonometry calculations are involved in the MATLAB function. This means the computational power requirement is relatively small and will benefit future implementation.

The Fig.3 shows the whole dynamic analysis results: Fig.3 (A) describes the torque requirements of joint 1-3 when moving the hand up and down, Fig.3 (B) describes the torque requirements of joint 4-6 when doing the same action.



(A) Simulation results of joint 1-3



(B) Simulation results of joint 4-6

Figure 3. The dynamic analysis results

Moreover, another dynamics simulation is to figure out the stability of each joint, which is another way to verify the correctness of Dynamic analysis. In this section, torques for joints are the inputs: 20 N\*m for Joint1, 20 N\*m for Joint2, 15 N\*m for Joint3, 10 N\*m for Joint4, 5 N\*m for Joint5 and 5 N\*m for Joint6. Fig.4 shows the dynamic model of UR10.

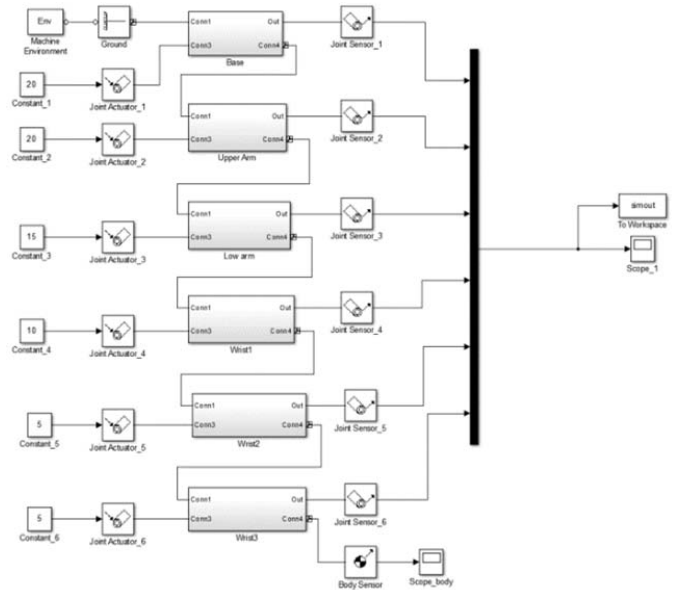


Figure 4. Dynamic model of UR10

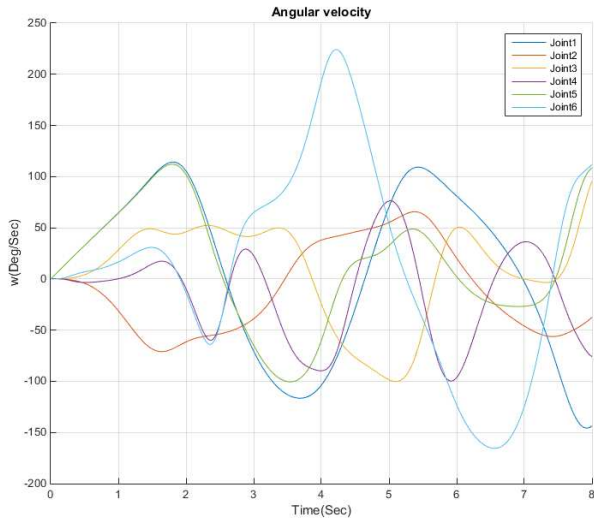


Figure 5. Simulation results of joint 1-6

As can be seen from Fig.5, angular velocity curves of joint 1-5 are stable except joint6's. However, due to the small range of joint6's motion, the angular velocity has little effect on its movement. The simulation results show that robot moves with a nearly constant velocity, which means this dynamic analysis is correct and the mathematical model is stable, safe and reliable.

#### IV. DISCUSSION AND CONCLUSION

This model based kinematic and dynamic simulation addresses the aforementioned issues of existing rehabilitation robots. The simulation results shows that the establishment of model is correct, and can provide an intuitive and effective environment for designing the rehabilitation robot and planning the clinical trials. However, the results should be noted here. First, in kinematic analysis, the trajectory of joint 7 (Fig.2) is within the designed route, although the ductility of Z axis is not verified in this simulation. Second, in dynamic analysis, there are some fluctuations in angular velocity (Fig.5) which will make the patients uncomfortable during training. The possible causes of the fluctuation in angular velocity curve are analyzed and explained as follows.

1) As described previously, this model only contains rigid segments and rotational joints. Each component is simplified as a trunk segment. Over-simplifying has effects on the model.

2) In calculation, even though the Kane method is more suitable for multi-DOF robot, the calculation accuracy is below Lagrangian mechanics method since the choice of generalized speed needs some experience and skills, which requires some time to find a better one.

Up to now, the model is only in a preliminary stage. Further improvement will involve 1) revising the model without over-simplifying the components; 2) improving the Kane equations and finding the better parameters to do the dynamic analysis to figure out the better one.

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