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Effects of shear flows on the evolution of fluctuations in interchange turbulence

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We report a non-perturbative study of the effect of different types of shear flows on the evolution of vorticity and particle density fluctuations in interchange turbulence. For the same shear strength, the transport of density is less reduced by streamers than by zonal flows, with zonal flows leading to oscillation death. In the inviscid limit, vorticity (density) grows (decays) as a power law due to the streamer or zonal flow and exponentially due to the combined effect of the zonal flow and streamer with the same sign of shear. The zonal flow and streamer with the opposite sign of shear lead to oscillation at multiple frequencies. *Published by AIP Publishing.*
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Shear flows play a primary role in turbulence regulation and transport quenching in a variety of systems.[1–10] This effect is particularly important in magnetically confined fusion plasmas (see, e.g., Refs. 1 and 6–10) as shear flows tend to be stable. Since the discovery of the low-to-high (L-H) transition,[11] a great effort has been devoted to elucidating the detailed dynamics involved in the formation of transport barriers by different shear flows in different models.

In the case of coherent shearing by mean $\mathbf{E}\times\mathbf{B}$ flows, the transport of passive scalar fields was shown to be reduced as Ω^{-1}, where Ω is the mean shearing rate.[10] However, a significant reduction in particle (heat) transport was found in the simple interchange (ion-temperature gradient) turbulence model.[12] This result was based on the framework where the feedback of the velocity on density fluctuation was not included, but instead was treated as a source of free energy and thus as a part of a stochastic noise that drives the density fluctuation. The coupling between density and velocity fluctuations through this feedback however causes not only the excitation of waves (e.g., gravity waves) or instability, depending on the relative direction of the background density gradient and the effective gravity, but also very peculiar dynamics, as we will show.

In this Brief Communications (BC), we report the effects of shear flows on the evolution of the vorticity and particle density fluctuations in interchange turbulence by treating the coupling between the vorticity and particle density fluctuation consistently. To elucidate the effects of different types of shear flows, we consider (i) zonal flows, (ii) streamers, and (iii) combined zonal flows and streamers. Streamers[13] are radially elongated convective cells (i.e., poloidally localized, radial flows) and thus can directly contribute to radial transport by advection. Also, unlike zonal flows, streamers can be excited by a linear instability. On the other hand, streamers can lead to turbulence quenching due to shear, as zonal flows. While the generation of streamers has been studied in different (drift, interchange, RT, etc.) models,[13,14] much less work was done on the effect of streamer shear on turbulence suppression and time-variability. For instance, the effect of zonal flow and streamer shears on turbulence is simply assumed to be similar.[15] To highlight the different effects of shear flows, we focus on the initial value problem.

Model: In the interchange turbulence model with cold ions [which is similar to the classical Rayleigh-Bénard convection problem (e.g., Ref. 5)], we consider the quasi-linear evolution of flute-like perturbations of particle density ρ and vorticity $\delta\omega = \nabla\times\mathbf{v}$ in the two dimensional (2D) x–y plane.[12] Here, x and y represent the local radial and poloidal directions, respectively, perpendicular to a magnetic field $\mathbf{B} = B_0\hat{z}$; $\langle\mathbf{v}\rangle = \langle\delta\omega\rangle = \langle\rho\rangle = 0$, where the angular brackets denote the average. By taking the effective gravity in the radial direction as $\mathbf{g} = g\hat{x}$ (due to magnetic curvature), we have

$$\partial_t\delta\omega + \mathbf{U}\cdot\nabla\delta\omega = -g\partial_y\rho/\rho_m + \nu\nabla^2\delta\omega\,, \tag{1}$$

$$\partial_t\rho + \mathbf{U}\cdot\nabla\rho = -v_x\partial_x\rho_0 + D\nabla^2\rho\,. \tag{2}$$

Here, ρ_m is the constant background density, $\rho_0(x)$ is the mean background density, and ρ is the fluctuation. $\mathbf{u} = \mathbf{v} + \mathbf{U}$ is the total velocity, consisting of fluctuation $\mathbf{v} = -(c/B_0)\nabla\phi\times\hat{z}$ and the mean flow $\mathbf{U}$. We take $\mathbf{U}$ to be either zonal flows only $\mathbf{U} = (0, -xA_z)$, streamers only $\mathbf{U} = (-yA_s, 0)$, or combined zonal flows and streamers $\mathbf{U} = (-yA_s, -xA_z)$. D and ν capture the coherent nonlinear interaction (i.e., "eddy diffusivity and viscosity") and molecular dissipation. Unlike the previous work[12] where the source of free energy $v_x\partial_x\rho_0$ in Eq. (2) was treated as a part of the noise, we investigate the consequence of the coupling between $\delta\omega$ and ρ through the buoyancy by solving Eqs. (1) and (2) simultaneously. We recall that this coupling term determines the stability, giving rise to stable or unstable gravity waves depending on whether $\frac{\partial\rho_0}{\partial x} > 0$ (density increasing in the direction of gravity) or $\frac{\partial\rho_0}{\partial x} < 0$, respectively. In this BC, we focus on the stable case with $\nu = D$; similar results would follow in the unstable case.

In order to capture the effect of distortion of an eddy (i.e., wind-up) by $\mathbf{U}$ non-perturbatively, we employ the time-dependent wavenumber $\mathbf{k}$

$$\rho(\mathbf{x}, t) = \tilde{\rho}(\mathbf{k}, t)\exp\{i(k_x(t)x + k_y(t)y)\} \tag{3}$$

and similarly for $\mathbf{v}$ and $\delta\omega$. Plugging Eq. (3) in Eqs. (1) and (2), we find that we can eliminate the advection term by $\mathbf{U}$ if $(\partial_t \mathbf{k})\cdot\mathbf{x} + \mathbf{U}\cdot\mathbf{k} = 0$, which is

$$\partial_t k_x(t) = A_z k_y, \quad \partial_t k_y(t) = A_s k_x \tag{4}$$

for $\mathbf{U} = (-yA_s, -xA_z)$. By using Eqs. (3) and (4), we recast Eqs. (1) and (2) as follows:

$$\partial_t \hat{\omega} = k_y \hat{\varphi}, \tag{5}$$

$$\partial_t \hat{\varphi} = -\frac{k_y N^2}{k_x^2 + k_y^2}\hat{\omega}. \tag{6}$$

Here, $\widetilde{\varphi} = -\frac{ig}{\rho_m}\widetilde{\rho}$, $\hat{\omega}(t) = e^{\nu Q(t)}\widetilde{\delta\omega}$, $\hat{\varphi} = e^{\nu Q(t)}\widetilde{\varphi}$, and $Q(t) = \int_0^t dt_1\,[k_x(t_1)^2 + k_y(t_1)^2]$; $N^2 = \frac{g}{\rho_m}\frac{\partial \rho_0}{\partial x}$ is the square of the buoyancy frequency.

Zonal flow only: To understand the effect of the coupling between $\widetilde{\rho}$ and $\widetilde{\delta\omega}$, we start with the case of zonal flow only $\mathbf{U} = (0, -xA_z)$, in which case Eq. (4) becomes

$$k_x(t) = k_x(0) + k_y(0)A_z t, \quad k_y(t) = k_y(0). \tag{7}$$

In terms of $\tau = k_x/k_y = A_z t$ and $\alpha = |\frac{N}{A_z}|$, Eqs. (5) and (6) lead to

$$\partial_{\tau\tau}\hat{\omega} + \frac{\alpha^2}{1+\tau^2}\hat{\omega} = 0. \tag{8}$$

In the limit of $|N| \gg A_z \gg \nu k_y^2$ ($\alpha \gg 1$), we look for a WKB solution of the form $\hat{\omega}(\tau) \sim \exp[\frac{1}{\delta}\int d\tau_1(\psi_0 + \delta\psi_1 + \cdots)]$, where $\delta \ll 1$ is a small parameter. Plugging this in Eq. (8) gives us $\delta = \alpha^{-1} \ll 1$, $\psi_0(\tau_1) = \pm i(1+\tau_1^2)^{-1/2}$ and $\psi_1(\tau_1) = \frac{\tau_1}{2(1+\tau_1^2)}$, and thus,

$$\hat{\omega}(\tau) \sim (1+\tau^2)^{1/4} e^{\pm i\alpha\theta(t)} \tag{9}$$

to $O(\alpha^{-1})$. Here, $\theta(t) = \sinh^{-1}(\tau)$, and again $\tau = k_x(t)/k_y = A_z t$. By using Eq. (5), $\widetilde{\varphi} = -\frac{ig}{\rho_m}\widetilde{\rho}$, and $\hat{\varphi} = e^{\nu Q}\widetilde{\varphi}$ and by assuming the initial condition $\widetilde{\delta\omega}(t=0) = 0$, we find

$$\widetilde{\delta\omega}(t) = -i\frac{gk_y}{\rho_m|N|}[1+\tau_0]^{1/4}[1+\tau^2]^{1/4}\sin(H(t)) \times e^{-\nu Q_1(t)}\widetilde{\rho}(0), \tag{10}$$

$$\widetilde{\rho}(t) = \frac{[1+\tau_0^2]^{1/4}}{[1+\tau^2]^{3/4}}\left[\frac{\tau}{2\alpha}\sin(H(t)) + \sqrt{1+\tau^2}\cos(H(t))\right] \times e^{-\nu Q(t)}\widetilde{\rho}(0), \tag{11}$$

where $\tau_0 = \tau(t=0) = k_x(0)/k_y$, $H(t) = \alpha[\theta(t) - \theta(0)] = \alpha[\sinh^{-1}(\tau(t)) - \sinh^{-1}(\tau_0)]$ and $Q_1(t) = \frac{1}{3A_z k_y}(k_x(t)^3 - k_x(0)^3) + k_y^2 t$. The $Q_1(t)$ term dominates with leading order $-\frac{1}{3}\nu k_y^2 A_z^2 t^3$ for $\nu \neq 0$ as shearing enhances the dissipation over the molecular value. Consequences of this enhanced dissipation are discussed in Ref. 16. Furthermore, Eqs. (10) and (11) show that in the inviscid case, $\hat{\omega} \propto \tau^{\frac{1}{2}} \propto t^{\frac{1}{2}}$, while $\hat{\varphi} \propto t^{-\frac{1}{2}}$ for larger t. This tendency is seen in Figs. 1 and 2 obtained from numerical solutions of Eqs. (1) and (2). Figures 1 and 2 also show that the growth/decay increases with A_z, consistent with Eqs. (10) and (11).

The density fluctuations decay due to the fact that $\widetilde{v}_x = \frac{ik_y}{k_x^2+k_y^2}\widetilde{\delta\omega}$ decreases with time. For zonal flow, k_y is constant and $k_x^2 + k_y^2$ grows quadratically with time, faster than $\hat{\omega}^2 \propto t$, so buoyancy diminishes with time. This follows from the total fluctuating energy

$$E = \frac{|\hat{\omega}|^2}{k_x^2 + k_y^2} + \frac{|\hat{\varphi}|^2}{|N|^2} \tag{12}$$

being an adiabatic invariant.

Streamers only: For the streamer only $\mathbf{U} = (-yA_s, 0)$, Eq. (4) gives

$$k_x(t) = k_x(0), \quad k_y(t) = k_y(0) + k_x(0)A_s t. \tag{13}$$

Finding a WKB solution turned out to be tricky in this case. Briefly, we rewrite Eqs. (5) and (6) in terms of $y = \frac{1}{2}\ln(1+\tau^2)$ and look for a WKB solution in terms of y. The solution which satisfies $\hat{\omega}(t=0) = 0$ can be found to $O(\alpha^{-1})$ as

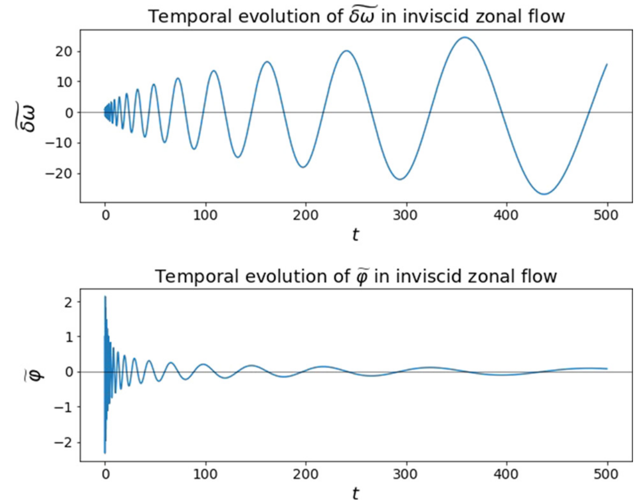

FIG. 1. For zonal flow, $\hat{\omega}$ grows in time, whilst buoyancy force is reduced and $\hat{\varphi}$ eventually dies out: $k_x(0) = 10$, $k_y(0) = 10$, $N^2 = 1000$, and $A_z = 2$.

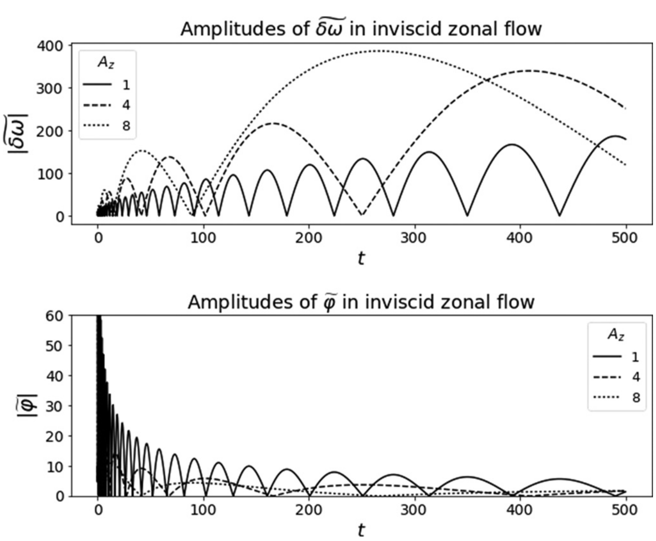

FIG. 2. For zonal flow, $\hat{\omega}$ grows as a power law in time. This growth is more exaggerated as the shearing factor A_z increases; $\hat{\varphi}$ decays as a power law in time, with the shearing factor amplifying this decay: $k_x(0) = 1$, $k_y(0) = 1$, and $N^2 = 200$.

$$\widetilde{\delta\omega}(t) = i\frac{gk_x}{\rho_m|N|}[1+\tau_0]^{1/4}[1+\tau^2]^{1/4}e^{-\nu Q_2(t)}\tilde{\rho}(0) \times\left[-\sin(R(t)) + \frac{\cos(R(t))}{2\alpha}W(t)\right], \quad (14)$$

$$\tilde{\rho}(t) = \frac{[1+\tau_0^2]^{1/4}}{[1+\tau^2]^{1/4}}\left[\cos(R(t)) + \frac{1}{2\alpha\sqrt{1+\tau_0^2}}\sin(R(t))\right] \times e^{-\nu Q_2(t)}\tilde{\rho}(0), \quad (15)$$

where $R(t) = \alpha[\sqrt{1+\tau^2} - \sqrt{1+\tau_0^2}]$, $W(t) = (1+\tau_0^2)^{-\frac{1}{2}} - (1+\tau^2)^{-\frac{1}{2}}$, and $Q_2(t) = \frac{1}{3A_s k_x}(k_y(t)^3 - k_y(0)^3) + k_x^2 t$.

In the inviscid case, Eqs. (14) and (15) show that $\hat{\omega} \propto \tau^{\frac{1}{2}} \propto t^{\frac{1}{2}}$, while $\hat{\varphi} \propto t^{-\frac{1}{2}}$ for large t, similar to the case of zonal flow only. We confirm this by numerical simulations shown in Figs. 3 and 4. However, the transport of particles $\langle \rho v_x \rangle$ in the zonal flow and streamer is different since $v_x = \frac{ik_y}{k_x^2+k_y^2} \propto t^{-2}$ and t^{-1} for large t in zonal flow and streamer cases, respectively. That is, the transport of particle is less reduced by streamers than zonal flows.

Another marked difference between the streamer and zonal flow cases is the frequency ω_f at which the fluctuations oscillate. For streamers, the frequency remains roughly constant, whilst it always decays in the zonal flow case. This is basically because $\omega_f = |N|\sqrt{\frac{k_y^2}{k_x^2+k_y^2}}$ when $\nu = D = 0$. For zonal flows, k_x grows, whilst k_y is constant, leading to the so-called oscillation death whereby ω_f decreases towards zero as time progresses. We can see clearly in Fig. 2 that the shearing rate A_z is responsible for the decrease in ω_f. In the streamer case, k_y grows instead, whilst k_x remains constant in time. Hence, ω_f approaches the constant value $|N|$, as observed in Figs. 3 and 4. This behavior can also be inferred from Eqs. (10), (11), (14), and (15).

Combined zonal flows and streamers: For $\mathbf{U} = (-yA_s, -xA_z)$, Eq. (4) gives us $\ddot{k}_x = A_s A_z k_x$, a solution depending on the sign of $A^2 = A_s A_z$. To elucidate their effect, we focus on the case where $|A_z| = |A_s| = |A|$.

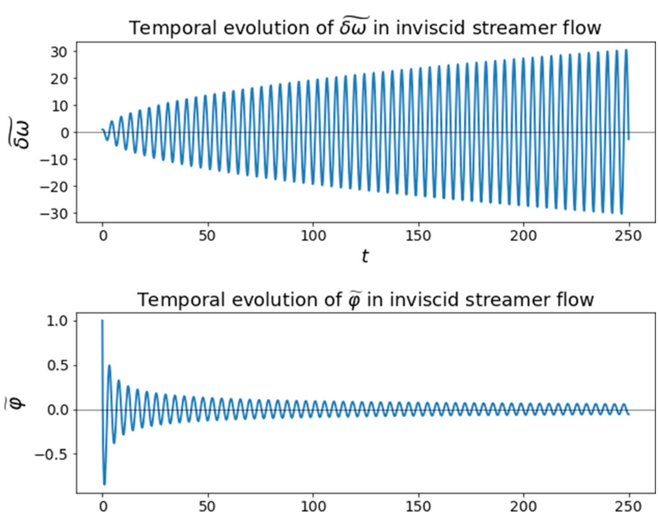

FIG. 3. For streamers, $\hat{\omega}$ grows in magnitude, whilst $\hat{\varphi}$ decays, similarly to the zonal flow case. ω_f approaches a constant value: $k_x(0) = 0.1$, $k_y(0) = 0.1$, $N^2 = 2$, and $A_s = 30$.

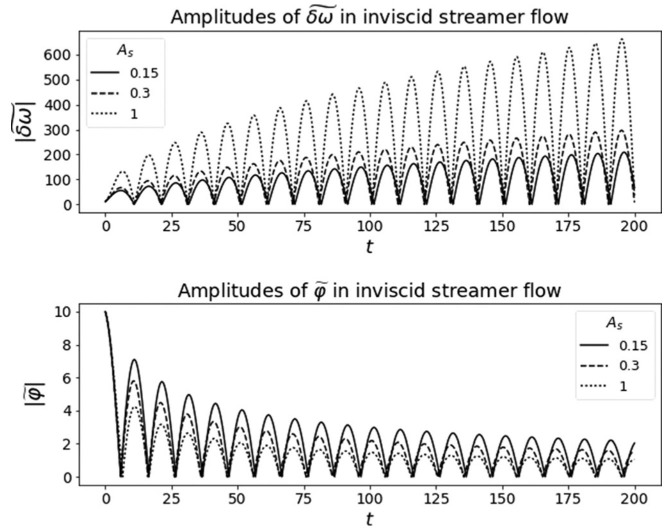

FIG. 4. For streamers, $\hat{\omega}$ and $\hat{\varphi}$ follow power laws in time, similar to the zonal flow case: $k_x(0) = 1$, $k_y(0) = 1$, and $N^2 = 0.15$.

When $A^2 < 0$, $\mathbf{k}$ rotates since

$$k_x(t) = K\cos(|A|t + \xi), \quad k_y(t) = K\sin(|A|t + \xi), \quad (16)$$

where $K^2 = k_x(t)^2 + k_y(t)^2 = k_x(0)^2 + k_y(0)^2$ and $\tan\xi = \frac{k_y(0)}{k_x(0)}$. In this case, we can show that E in Eq. (12) is now exactly conserved, enabling us to find an exact solution

$$\widetilde{\delta\omega}(t) = F\sin\left[\left|\frac{N}{A}\right|\sin(|A|t+\xi) + G\right]e^{-\nu K^2 t}, \quad (17)$$

where $F = \sqrt{\widetilde{\delta\omega}(0)^2 + \frac{K^2}{N^2}\tilde{\varphi}(0)^2}$ and $G = \sin^{-1}(\frac{\widetilde{\delta\omega}(0)}{F}) - |\frac{N}{A}|\sin(\xi)$. Comparing this with the previous cases, there is no net energy transfer between $\widetilde{\delta\omega}$ and $\tilde{\varphi}$ on a long time scale; both fluctuations oscillate with constant amplitudes as demonstrated in Fig. 5. Furthermore, Eq. (17)

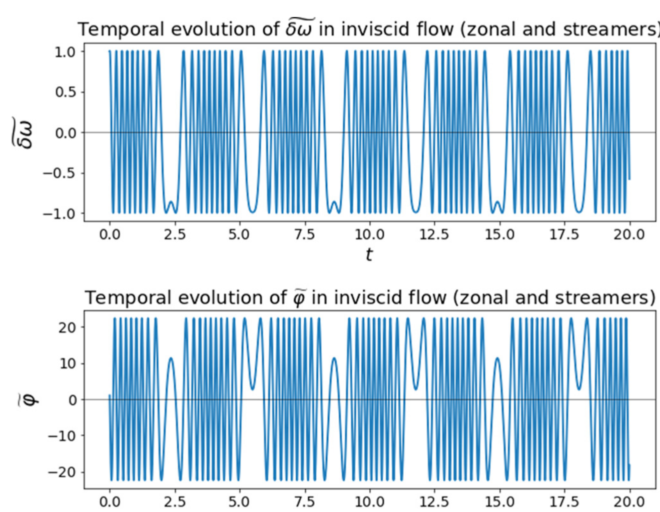

FIG. 5. For combined zonal flow and streamers with the opposite sign of shear, the amplitudes of both $\hat{\omega}$ and $\hat{\varphi}$ do not change: $k_x(0) = 1$, $k_y(0) = 1$, $N^2 = 1000$, and $A_s = 1 = -A_z$.

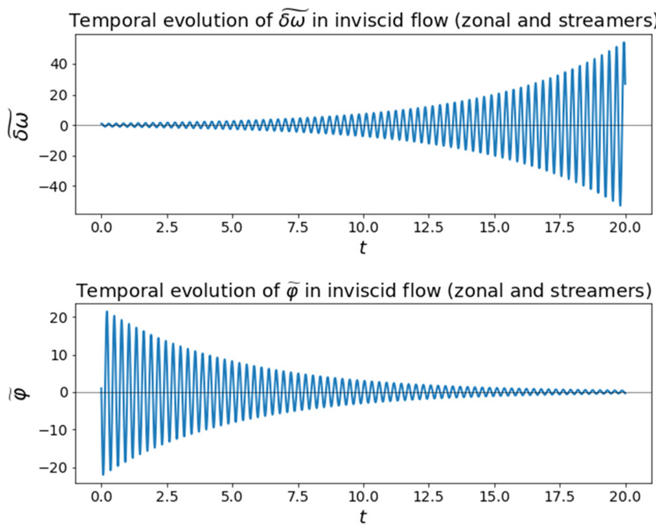

FIG. 6. For combined zonal flows and streamers with the same sign of shear, $\hat{\omega}$ and $\hat{\varphi}$ grow and decay exponentially, respectively: $k_x(0)=1$, $k_y(0)=1$, $N^2=1000$, and $A_s=A_z=0.4$.

manifests the oscillation frequency ω_f at the integer multiples of A; $\sin(\sin(At))$ involves the frequency nA for all integer n (see Fig. 5).

In comparison, for $A^2>0$, we have

$$k_x(t) = P\cosh(At+\chi), \quad k_y(t) = P\sinh(At+\chi), \tag{18}$$

where $A>0$, $P^2 = k_x(t)^2 - k_y(t)^2 = k_x(0)^2 - k_y(0)^2$, and $\tanh\chi = \frac{k_y(0)}{k_x(0)}$. Equation (18) leads to exponentially increasing and decreasing wave numbers in the two orthogonal directions (see Ref. 16). For $At \gg 1$, an inviscid solution is found to be $\hat{\omega} \propto e^{\gamma t}$, where $\gamma = \frac{A}{2}[1+\sqrt{1-\alpha^2}]$ and $\alpha = \frac{|N|}{A}$. Thus, $\hat{\omega}$ ($\hat{\varphi}$) grows (decays) exponentially in time for all α [see Eq. (12)], with the imaginary part of γ giving $\omega_f \to \frac{|N|}{2}$ for $\alpha \gg 1$ (consistent with the expectation from $\omega_f = |N|\frac{k_y}{\sqrt{k_x^2+k_y^2}} \to \frac{|N|}{2}$ as $k_x \sim k_y \propto e^{At}$). This prediction is confirmed by numerical solutions of Eqs. (1) and (2) shown in Fig. 6.

In summary, we elucidated the effects of zonal flows and streamers on the evolution of the vorticity and density fluctuations in interchange turbulence. In the inviscid limit, vorticity (density) grows (decays) as a power law $t^{\frac{1}{2}}$ $(t^{-\frac{1}{2}})$ due to streamers or zonal flows. However, due to the anisotropic stretching of wave numbers, the transport of density is less reduced by streamers than by zonal flows, with zonal flows leading to oscillation death (reduced oscillation frequency). This highlights different effects of zonal flows and streamers on turbulence regulation. Furthermore, the combined zonal flow and streamer induce oscillations at one frequency with exponentially growing (decaying) amplitude of vorticity (density) or at multiple integer frequencies with constant amplitude, depending on the relative sign of shear. Different effects of shear flows, in particular, oscillation death by zonal flows, are likely to persist in forced turbulence, which will be addressed in detail in a future extended work.

[1] K. H. Burrell, Phys. Plasmas **4**, 1499 (1997); T. S. Hahm, Plasma Phys. Controlled Fusion **44**, A87 (2002).
[2] F. H. Busse, Geophys. Astrophys. Fluid Dyn. **23**, 153 (1983).
[3] M. E. McIntyre, J. Atmos. Terr. Phys. **51**, 29 (1989).
[4] J. C. R. Hunt and P. A. Durbin, Fluid Dyn. Res. **24**, 375 (1999).
[5] E. Kim and N. Leprovost, Astron. Astrophys. **468**, 1025 (2006).
[6] M. G. Shats and W. M. Solomon, Phys. Rev. Lett. **88**, 045001 (2002).
[7] X. Garbet, Plasma Phys. Controlled Fusion **43**, A251 (2001).
[8] M. Dam, M. Brons, J. J. Rasmussen, V. Naulin, and J. S. Hesthaven, Phys. Plasmas **24**, 022310 (2017).
[9] C. S. Chang, S. Ku, G. R. Tynan, R. Hager, R. M. Churchill, I. Cziegler, M. Greenwald, A. E. Hubbard, and J. W. Hughes, Phys. Rev. Lett. **118**, 175001 (2017).
[10] E. Kim and P. H. Diamond, Phys. Rev. Lett. **91**, 075001 (2003); E. Kim, Mod. Phys. Lett. B **18**, 551 (2004).
[11] F. Wagner, G. Becker, K. Behringer, D. Campbell, A. Eberhagen, W. Engelhardt, G. Fussmann, O. Gehre, J. Gernhardt, G. V. Gierke, and ASDEX Team, Phys. Rev. Lett. **49**, 1408 (1982).
[12] E. Kim, P. H. Diamond, and T. S. Hahm, Phys. Plasmas **11**, 4554 (2004).
[13] P. H. Diamond, S. Champeaux, M. Malkov, A. Das, I. Gruzinov, M. N. Rosenbluth, C. Holland, B. Wecht, A. I. Smolyakov, and F. L. Hinton, Nucl. Fusion **41**, 1067 (2001).
[14] A. Das, A. Sen, S. Mahajan, and P. Kawet, Phys. Plasmas **8**, 5104 (2001).
[15] H. Zhu, S. C. Chapman, and R. O. Dendy, Phys. Plasmas **20**, 042302 (2013).
[16] I. Movahedi and E. Kim, "Effect of enhanced dissipation by shear flows on transient relaxation and probability density function," Phys. Plasmas (in press).