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Large Scale International Testing Of Railway Ground Vibrations Across Europe

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1.1 Abstract

This paper provides new insights into the characteristics and uncertainties in railway ground-borne vibration prediction. It analyses over 1500 ground-borne vibration records, at 17 high speed rail sites, across 7 European countries. Error quantification tests reveal that existing scoping models, for at-grade tracks, are subject to a mean error of approximately ± 4.5 VdB. Furthermore, it is found that seemingly identical train passages are subject to a standard deviation of ± 2 VdB, thus providing an indicator of the minimum error potentially achievable in detailed prediction studies. Existing vibration attenuation relationships are also benchmarked and potential new relationships proposed. Furthermore, it is found that soil material properties are the most influential parameter that effect vibration levels while the effect of train speed is low. Additionally, sites with train speeds close to the 'critical velocity' are examined and it is found that their vibration characteristics differ vastly from non-critical velocity sites.

The study presents one of the most comprehensive publications of experimental ground-borne railway vibration data and comprises of datasets from Belgium, France, Spain, Portugal, Sweden, England and Italy. First, several international metrics are used to analyse the data statistically. Then the effect of train speed is investigated, with train speeds ranging from 72 to 314km/h being considered. Next the effect of train type is analysed, with correlations presented for TGV, Eurostar, Thalys, Pendolino, InterCity, X2000, Alfa Pendular, AVE-S100 and Alitalia trains. Then, vibration frequency spectrums are considered and critical speed effects analysed. Finally, an investigation into the typical standard deviation encountered in vibration prediction is undertaken.

Keywords:

Ground-borne vibration; Critical Velocity; Field-Experimental Data; Scoping Study; High Speed Railway; Environmental Impact Assessment (EIA)

1.2 Introduction

Railway ground vibrations are a growing environmental challenge. This is partly due to a rapid global growth in railway infrastructure and an increasing desire to place new lines within urban environments. It is also partly due to more aggressive railway scheduling (i.e. more frequent passages) and both heavier and faster trains.

Before the construction of a new line or the upgrading of an existing line, it is usually necessary to undertake a ground-borne noise and vibration assessment. These assessments are often expensive because the complex interactions between train, track and soil potentially require

rigorous analysis. Despite this, if vibration levels are not accurately predicted, unexpected remediation measures may be required post-construction [1], [2], [3].

To obtain a high accuracy estimate of vibration levels (i.e. frequency curves), in practice a commonly used method is [4], [5], [6]. This requires the use of physical tests performed at the proposed track construction site to determine the transfer function of the surrounding soil. Then the transfer function is combined with similar track transfer functions for the train and track, resulting in an overall estimate of the ground-borne vibration characteristics.

Under certain conditions, it is impractical to use this procedure and instead an analytical or numerical approach is preferred. A large body of research is ongoing in this area, with early work being undertaken by [7] and [8], to derive analytical expressions for vibration levels. More advanced analytical [9] and semi-analytical [10] models have recently been proposed, particularly for predicting vibrations from underground lines [11], however there is an increasing trend for the utilisation of numerical techniques. In particular, time domain and frequency domain finite element method (FEM) ([12], [13], [14], [15], [16], [17]) approaches have been widely developed. A shortcoming of the FEM is that it becomes computationally expensive for large domains and requires the use of an absorbing boundary to truncate the modelling space [18]. To reduce run-times, the computational domain has been reduced to 2.5 dimensions by assuming that the track is invariant in the direction of train passage ([19], [20]). Although this considerably reduces computational times, the invariant track assumption makes it challenging to model discrete components (e.g. sleepers), thus leading to the incorporation of Floquet transforms ([21], [22]), or non-isotropic material properties ([12], [23])

Another alternative solution is to couple the FEM with the boundary element method (BEM), either using 2.5D ([24], [25]) or 3D formulation ([26]). This FE/BE approach allows for large offsets to be computed more efficiently than using only the FEM. Despite this, the contrasting nature of the FE and BE methods can be computationally challenging and fully 3D models still require long run times.

Although numerical railway modelling has advanced significantly, a persistent challenge is the acquisition of high accuracy soil material properties, for use as modelling inputs. Soil is a non-engineered material that forms naturally and thus is highly inhomogeneous. This makes it difficult to quantify its material characteristics, even using time consuming and expensive in-situ tests (e.g. Multi-channel analysis of surface waves analysis).

To overcome this, at the early stages of a vibration assessment, it is common to forego rigorous analysis in preference of a 'scoping' approach ([4], [27], [28]) using very limited site data (i.e. soil properties typically ignored). This allows for the rapid approximation of vibration levels to determine the sites where ground-borne vibration levels might exceed national limits. Then the aforementioned numerical modelling or physical testing approaches can be used to calculate the potential vibration levels at these locations with greater accuracy. To minimise project cost it is

important that only the locations where vibration levels will exceed national limits are analysed in greater detail. Each site where vibration levels are over-predicted (i.e. a 'false positive'), will result in unnecessary additional project costs. Similarly, each site where vibration levels are under-predicted will result in unexpected additional project costs from abatement installations post-construction. Therefore it is imperative that the accuracy of scoping assessments is maximised.

In an attempt to perform scoping predictions of railway vibration, [29] presented a mathematical model to quickly approximate velocity levels. Results were compared to results from [30] and a positive correlation was found. Another approach was proposed by [4] which used empirical factors to adjust an experimentally defined vibration curve. This approach was built upon by [31], [27] who included soil parameter information to increase prediction accuracy.

Alternatively, [32] presented an empirical model where a basic vibration value was multiplied by factors account for conditions such as train speed, track quality and building factors. It was also able to predict more complex frequency curves in a similar manner to that proposed by [33].

To perform a scoping assessment, it is common to use a combination of historical vibration results and empirical relationships to estimate vibration levels. Therefore, to improve scoping accuracy, it is important to better understand the underlying characteristics of railway vibration. One approach to this is to analyse existing experimental results. Despite this, due to a recent surge of interest in numerical modelling, little attention has been given to the analysis of historical experimental data.

Another potential stumbling block for experimental analysis is that freely available experimental data is scarce. In an attempt to overcome this, this current work documents the combined efforts of several railway research institutions to analyse a large body of experimental results. To the author's knowledge, although such efforts have been made in the field of acoustics [34], this research is one of the most comprehensive analyses into the statistical characteristics of railway vibration. Therefore it presents a highly original and commercially valuable analysis.

This paper aims to quantify the level of error that can be expected when using scoping and detailed assessment methods, while also investigating the effect of train speed, critical velocity and train type on ground-borne vibration propagation. There is a focus on vibrations from at-grade high speed lines, due to their current popularity, however several lower speed lines are also considered.

1.3 Test site information

Experimental data from a total of 17 test locations, across 7 countries was examined (Figure 1). All sites consisted of ballasted track and key details regarding each test location are

provided in Table 1 and Table 2. Ground wave velocity profiles are shown in Figure 3. It should be noted that some datasets contained a mix of ground vibration and track vibration data. For the purposes of this (far-field) study the track vibration signals were removed. A more detailed description of each of the test sites and experimental setups, please refer to: [35], [36], [37], [38], [39], [40], [41], [42], [35], [25], [43] and [44].

Further considerations included:

- At some sites, three component vibration signals were recorded, however to maximise compatibility this study only considers vertical component vibrations.
- Although the datasets were recorded by several different research institutions and using different types of recording equipment, all methodologies were broadly in-line with the recommendations detailed in [45]. A selection of the measurement sites are shown in Figure 2.
- Vibration velocities were solely analysed in this investigation. Therefore, where necessary, acceleration time histories were converted into their equivalent velocity components.
- The majority of datasets included full time history vibration records. Despite this, only instantaneous vibration data (velocity decibels – equation (1)) was available for the test sites described by [35]

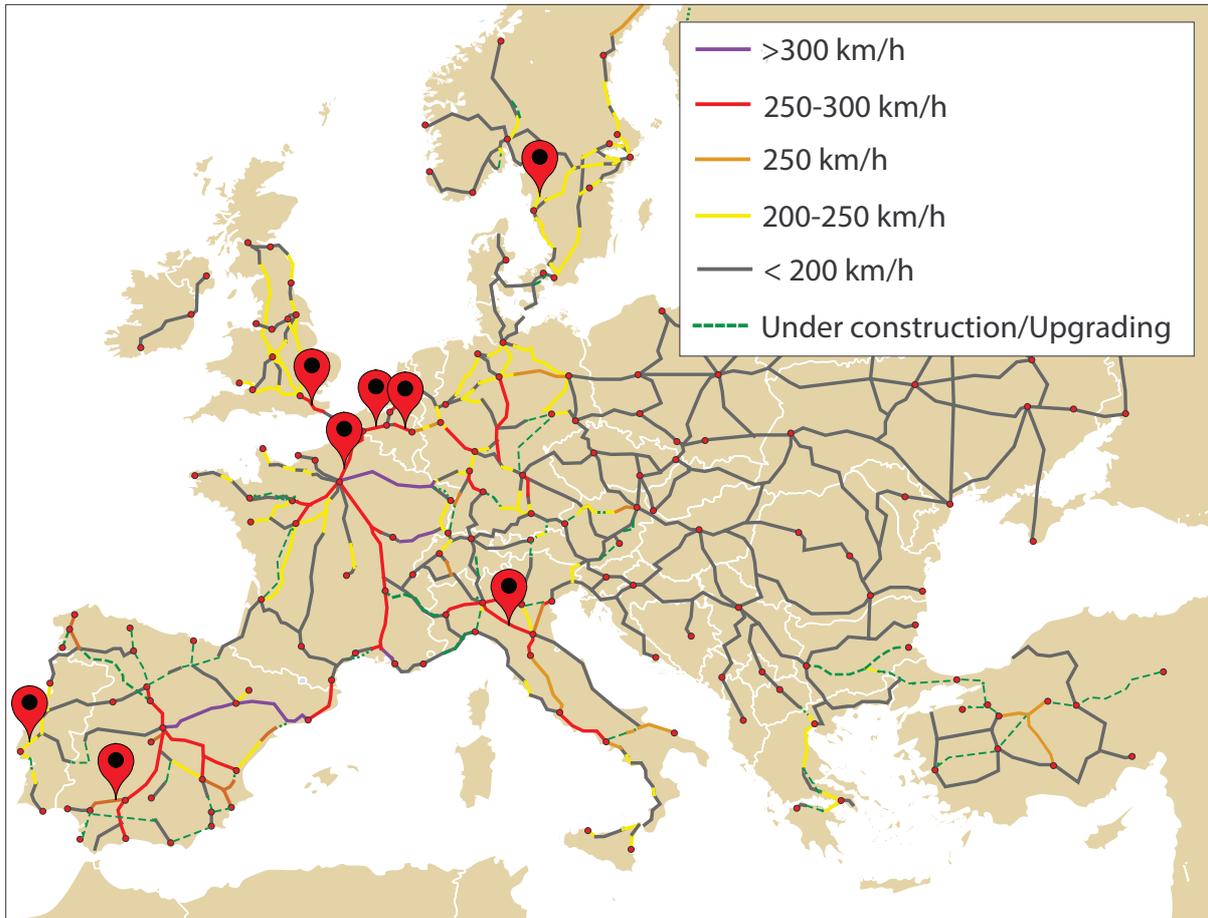
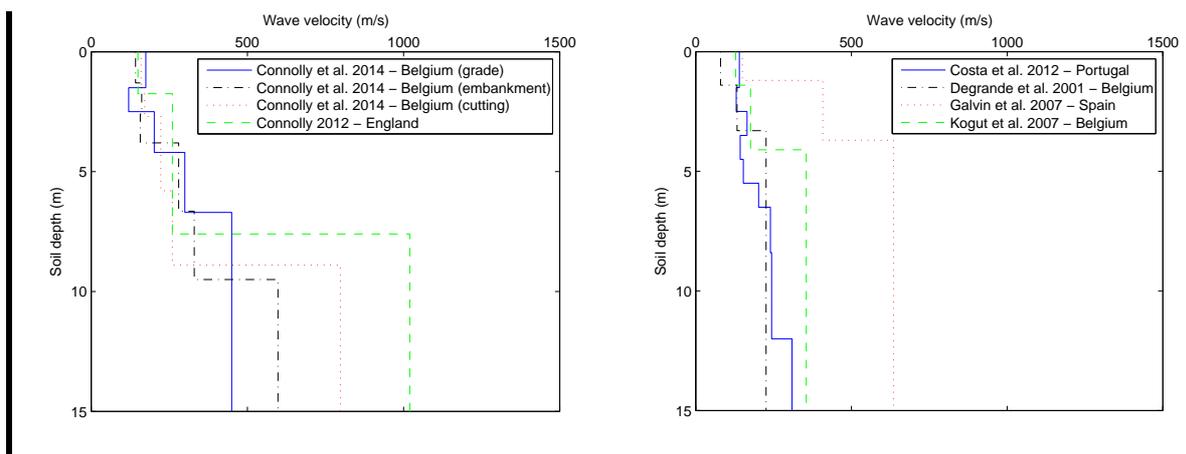


Figure 1 – Geographical map showing test site locations and a selection of the European railway network (note: some markers removed due to close proximity to others)



Figure 2 – Photographs from test sites, (a) Top left: Connolly et al. 2014, test site, (b) Top right: Connolly 2013, test site, (c) Bottom Left: Costa et al. 2012 test site, (d) Bottom right: Galvin et al. 2007



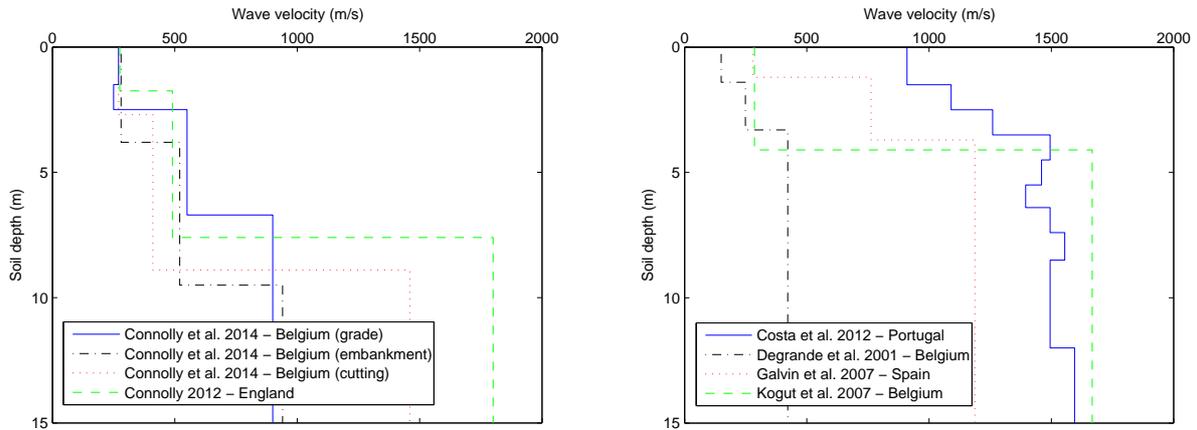


Figure 3 – Wave speed profiles, (a) Top left: Vs profiles part 1, (b) Top right: Vs profiles part 2, (c) Bottom left: Vp profiles part 1, (d) Bottom right: Vp profiles part 2

Site number	Country	Recording year	Reference
1	Belgium	2012	Connolly et al., 2014
2	Belgium	2012	Connolly et al., 2014
3	Belgium	2012	Connolly et al., 2014
4	Belgium	2012	Connolly et al., 2014
5	England	2012	Connolly, 2013
6	Portugal	2012	Costa et al., 2012
7	Belgium	1997	Degrande et al., 2001
8	Spain	2006	Galvin, 2007
9	Belgium	2002	Kogut et al., 2002
10	France	1996	Harris Miller Miller and Hanson, 1996
11	France	1996	Harris Miller Miller and Hanson, 1996
12	Italy	1996	Harris Miller Miller and Hanson, 1996
13	Italy	1996	Harris Miller Miller and Hanson, 1996
14	Italy	1996	Harris Miller Miller and Hanson, 1996
15	Sweden	1996	Harris Miller Miller and Hanson, 1996
16	Sweden	1996	Harris Miller Miller and Hanson, 1996
17	Sweden	1996	Harris Miller Miller and Hanson, 1996

Table 1 - Test site details – general description

Site number	Latitude	Longitude	Track arrangement	Train types recorded	Number of train passages recorded	Total data points
1	50.5575	3.600882	At grade	Eurostar, Thalys, TGV	15	200
2	50.56104	3.625706	Embankment (5.5m)	Eurostar, Thalys, TGV	15	200
3	50.5555	3.569042	Cutting (7.2m)	Eurostar, Thalys, TGV	19	296
4	50.56091	3.624199	Over-pass (embankment 5.5m)	Eurostar, Thalys, TGV	6	144
5	51.26214	0.619494	At grade	Eurostar, Javelin 395	9	72
6	39.04162	- 8.9190906	At grade	Alfa Pendular	5	45
7	50.574605	3.731634	At grade	Thalys	9	90
8	37.6772	-4.989409	At grade	AVE S100, Alitalia	7	70
9	50.7161	5.050412	At grade	InterCity, Thalys	4	32
10	49.17422	2.752382	At grade	Eurostar, TGV	25	150
11	49.03528	2.514819	At grade	Eurostar, TGV	15	30
12	44.73881	10.529849	At grade	Pendolino	3	12
13	44.71292	10.626525	Curve (at grade)	Pendolino	4	16
14	44.73602	10.545299	At grade	Pendolino	10	70
15	57.96383	12.599003	At grade	X2000	11	77
16	58.01647	12.722296	Curve (embankment - 2m)	X2000	5	10
17	58.06962	12.980293	Embankment - 1.5m	X2000	6	24
Total					168	1538

Table 2 - Test site details – Global coordinates and passage details

1.4 Vibration metrics

Three internationally used metrics were used to assess vibration levels. As the aim of this research was to analyse a wide range of vibration signals for scoping assessment purposes, absolute vibration measurements were desirable, rather than frequency curves. The most commonly used metric for scoping assessment is VdB, as described by [4]. VdB is calculated using a logarithmic scale as:

$$VdB = 20 \times \log_{10} \left(\frac{v_{rms}}{v_0} \right) \quad (1)$$

Where v_{rms} is the moving root mean square amplitude (rms slow, 1 second) and v_0 is a reference level for background vibration (chosen as 2.54×10^{-6} m/s).

In addition to VdB, peak particle velocity (PPV) and KB_{fmax} were also used to assess vibration levels. PPV [46] was calculated as:

$$PPV = \max|v(t)| \quad (2)$$

where $v(t)$ is the velocity time history. Similarly, KB_{fmax} [47] was calculated by taking the maximum amplitude of:

$$KB_f(t) = \sqrt{\frac{1}{\tau} \int_0^t KB^2(\xi) e^{-\frac{t-\xi}{\tau}} d\xi} \quad (3)$$

where $\tau = 0.125$ seconds (rms fast), and $KB(\xi)$ was the velocity time history. It should be noted that $KB(\xi)$ was first transferred into the frequency domain, giving $KB(f)$, filtered according to equation (4), and then transferred back into the time domain.

$$KB(f) = \frac{1}{\sqrt{1 + \left(\frac{5.6}{f}\right)^2}} \quad (4)$$

1.5 Experimental analysis

1.5.1 Distance effects

Figure 4 shows the dependence of VdB on distance from the track, using a logarithmic scale. For data recorded on the 'far' track (in the case of double track configurations), an offset factor was used to adjust the distance values accordingly. It was observed that the datasets collected by [35] contained vibration signals at the upper range of values, whereas the results collected by [44], were in the lower range. This lower range may have been because testing was performed during homologation tests without passenger mass contributing to the axle loads. It should be noted that a best fit line is not shown due to readability issues, however a variety of potential best fit relationships are plotted in Figure 6.

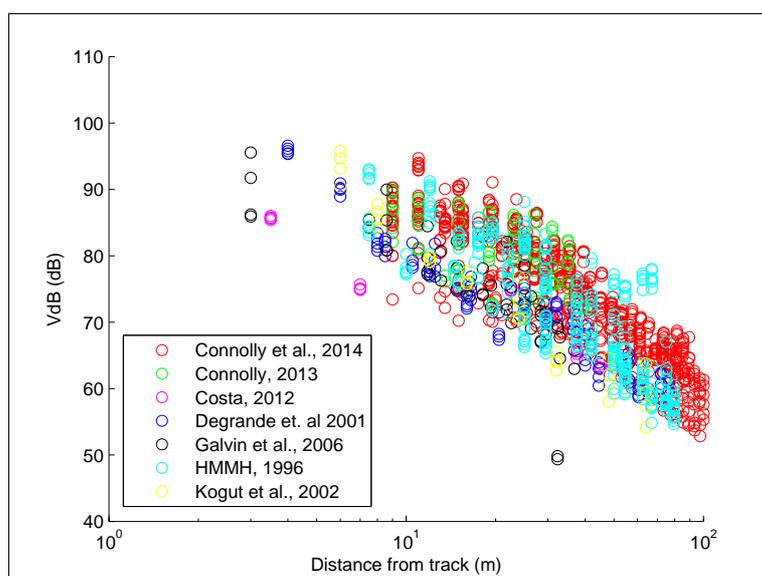


Figure 4 – VdB versus distance

Figure 5 shows the relationship between vibration levels and distance from the track, using the alternative PPV and $KB_{f_{max}}$ metrics. As time histories were not available for the data collected by [35], PPV and $KB_{f_{max}}$ could not be calculated and were not plotted. As expected, there was a strong relationship between vibration levels and distance. This can be seen in Table 3 where best fit equations are presented along with Pearson's 'R' correlation coefficient. The best fit curves are shown in Figure 5 and the 'R' coefficient for each was found to be similar.

Metric	Best fit equation (m/s)	R coefficient
PPV	$0.0032 * e^{(-0.056 * d)}$	-0.064
$Kb_{f_{max}}$	$0.00078 * e^{(-0.046 * d)}$	-0.0582

Table 3 – Correlation data for PPV and $KB_{f_{max}}$ metrics

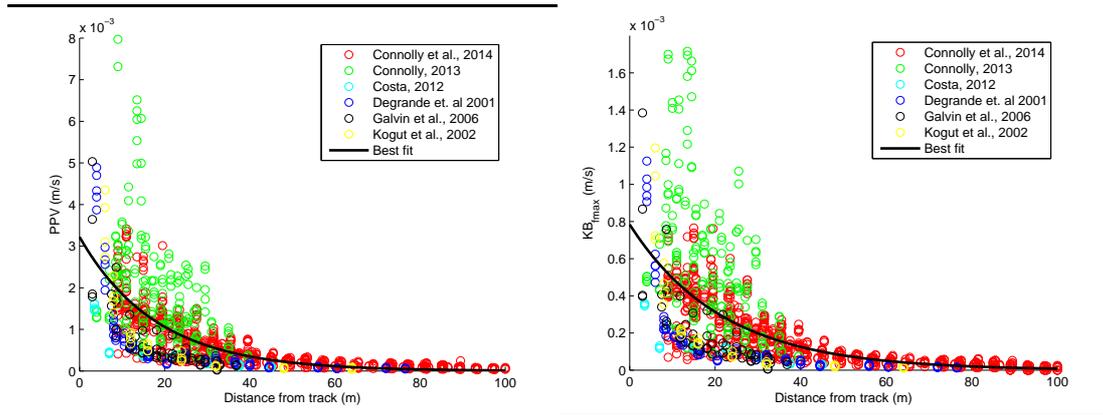


Figure 5 – Variation in vibration levels with track offset, (a) Left: PPV, (b) Right: KB_{fmax}

1.5.2 Analysis of existing mathematical VdB attenuation relationships

Ground-borne vibration levels are a function of both the propagation path (i.e. soil material properties) and the initial excitation/source (i.e. train-track interaction forces). Soil material properties are complex due to the highly variable, non-homogenous properties associated with in-situ soil. To circumvent these modelling challenges during vibration scoping assessments, approximate attenuation relationships are commonly used. A variety of previously proposed relationships are shown in equation (5) - equation (8). To test the suitability of these relationships for modelling railway vibration, they were fitted to the experimental field results and new best fit relationships proposed for VdB calculation.

The existing relationships tested were:

$$VdB(d) = -20 \times \log_{10}(d) + c \quad (5)$$

$$VdB(d) = -10 \times \log_{10}(d) + (a \times d) + c \quad (6)$$

$$VdB(d) = a \times d^b \quad (7)$$

$$VdB(d) = -20 \times \log_{10}(1 + d) + (a \times d) + c \quad (8)$$

where d was distance from the track and a , b and c were correlation factors.

Firstly, the relationship proposed by [48] was tested (equation (5)). This logarithmic based relationship was originally proposed to predict ground-borne noise in the ground floor rooms of residential structures in close proximity to underground lines. It provides a straightforward approach that only relies on the amplitude of excitation for calculation. Vibration attenuation is assumed to vary on a constant logarithmic scale with distance.

Next, an alternative logarithmic relationship (equation (6)) proposed by [49] was analysed. In a similar manner to [48], a measure of the source strength is used, however it is combined with both spreading source and dissipative factors. Originally [49] proposed additional factors for speed correction, however these were only for speeds lower than 60 km/h and have thus been ignored. During testing it was found that the coefficient 'a' had a negligible effect on curve fitting accuracy and was therefore removed.

An alternative power based approach (equation (7)) was also explored. This relationship has been attributed to [50], but has also been analysed by [51], and more recently by [52] who investigated power relationships for different types of excitation.

The final relationship analysed was that proposed by [4]. Although a graphical representation for the attenuation law was chosen by [4], a non-linear regression analysis revealed that the mathematical relationship was similar to that proposed by [49]. Again, it is shown in (equation (8)).

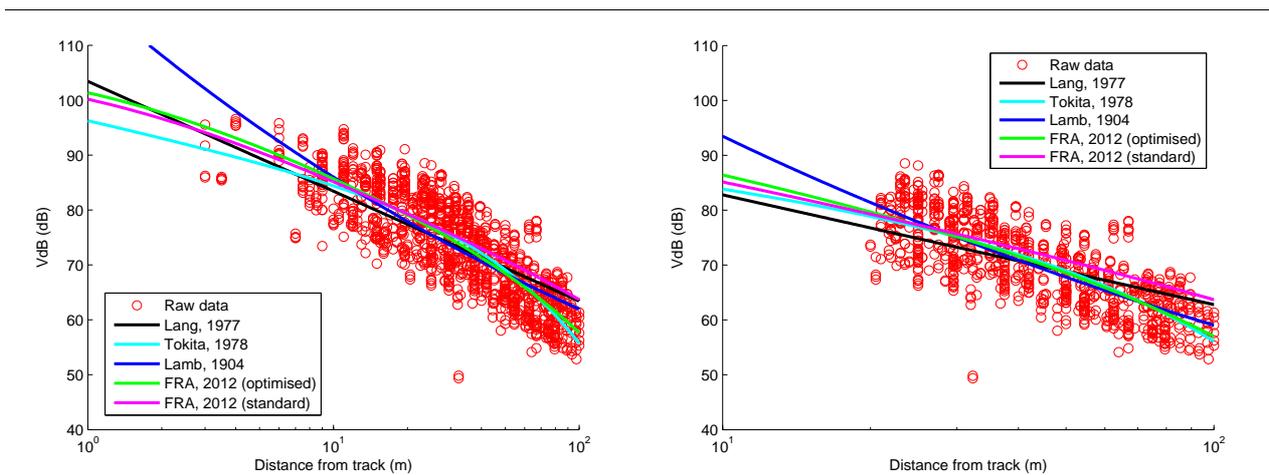


Figure 6 – Best fit relationships for VdB data, Left: all data, Right: offsets >20m

From Figure 6 it was found that each optimised relationship performed adequately in predicting vibration VdB levels. All curves gave similar predictions at a range of approximately 30-40m from the track, however at other distances they exhibited greater variability. Overall, the optimised relationship proposed by [49] yielded the highest Pearson's R coefficient, performing slightly better than the non-optimised relationship proposed by [4] (Table 4). Similarly, an optimised version of [4] was calculated and found also to slightly outperform the non-optimised relationship proposed by [4]. Using this optimised version of [49] the mean error across all locations was ± 4.5 dB and the maximum error (ignoring the two distinct outliers) was 13.75dB.

Railway vibrations at small offsets close to at-grade sections are typically only a concern from a track dynamics standpoint rather than from an environmental standpoint. One reason is that existing structures situated close to new lines are typically purchased by the railway operator during the construction/planning phase. Therefore ground-borne vibration levels between the edge of the compulsory purchase zone and 100m [4], are of greatest interest for environmental consultants. To account for this, the correlation coefficient was recomputed for various increasing distances between 0m and 100m, however no significant increase in correlation was found.

Reference	Equation	Best fit values (all distances)		
		a	b	c
Lang, 1977	$VdB(d) = -20 \times \log_{10}(d) + c$	-	-	103.42
Tokita, 1978	$VdB(d) = -10 \times \log_{10}(d) + (a \times d) + c$	-	0.208	96.47
Lamb, 1904	$VdB(d) = a \times d^b$	0.142	-	119.37
FRA, 2012 (optimised)	$VdB(d) = -20 \times \log_{10}(1 + d) + (a \times d) + c$	0.097	-	107.47
FRA, 2012 (standard)	$VdB(d) = -20 \times \log_{10}(1 + d) + (a \times d) + c$	0.024	-	106.21

Table 4 - Best fit relationships and correlation coefficients

1.5.3 Speed effects

Train speed versus VdB vibration levels was plotted to investigate whether train speed had an effect on VdB levels. As the overall dataset was composed of vibration records taken at inconsistent distances from the track, for every train passage a best fit curve was fitted to the data. Then each best fit curve was used to approximate the train speed at any particular location irrespective of experimental methodology. Figure 7 shows how VdB varied with train speed at 2 individual track offsets (1m and 80m respectively). On each plot, the approximated VdB level (as calculated using [49]) is shown. The black line represents the FRA base curve which is non-speed adjusted, whereas the blue line represents the same FRA curve but with the FRA speed adjustment factors included. This speed adjustment factor was computed using equation (9) [4], where the 'reference speed' was 241.403 km/h (150 mph).

$$\begin{aligned}
 VdB_{\text{speed adjusted}} &= VdB_{\text{base curve}} \\
 &+ 20 \log_{10} \left(\frac{\text{train speed (km/h)}}{\text{reference speed}} \right)
 \end{aligned} \tag{9}$$

Considering Figure 7, for the location nearer the track, the scatter of data was lower than for further away. This led to standard deviations of 6.5 dB and 12.1 dB respectively. In general, even at minimal offsets, it was found that the correlation between train speed and VdB level was low. For example, at 1m from the track, the vibration level for the lowest train speed (72 km/h) was higher than for some of the highest train speeds (e.g. 300 km/h).

Although the spread of data was high, a tentative best fit curve was calculated for each location. Closest to the track the best fit curve showed a minimal positive correlation between train speed and VdB. In contrast, at the large offset there was a negative correlation. Due to the high scatter of data, it was unlikely that the best fit curves were representations of the true correlation, however they did act as an indicator to confirm the low dependence of VdB on train speed. Therefore, it was concluded that for the datasets analysed, removing train speed adjustment factors from scoping prediction processes would not have impacted prediction accuracy.

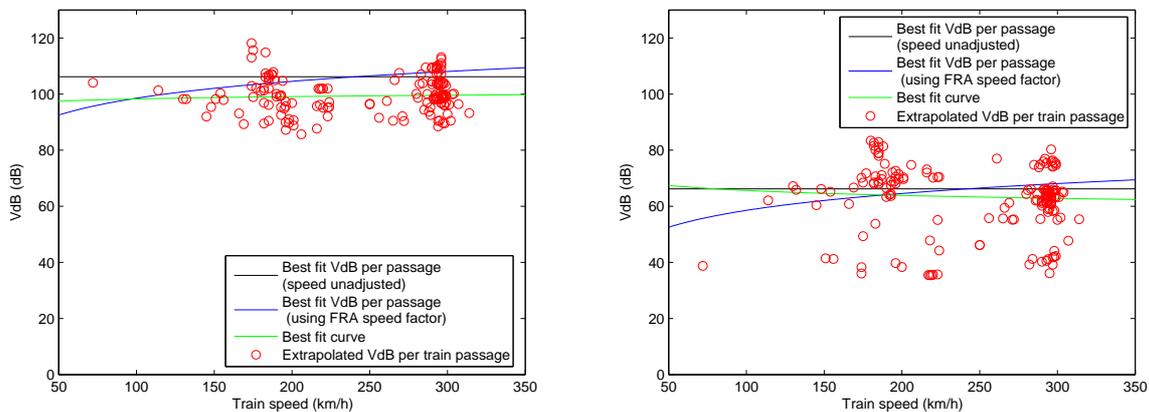


Figure 7 – The effect of train speed on VdB, Left: 1m from track, Right: 80m from track

1.5.4 Train type comparison

Figure 8 shows best fit curves for the passage of each type of train at each measurement site. It was found that the vibration levels generated close to the track were in the range 100-110 VdB. Despite this, as the distance from the track was increased, the discrepancy also increased, to approximately 50dB. This increase in discrepancy was likely due to soil material properties rather than train type. This finding was consistent with those of [53], and showed that the deviations introduced by different soil properties are much larger than those introduced by uncertainties in the train type or track configuration [54], [55], [56].

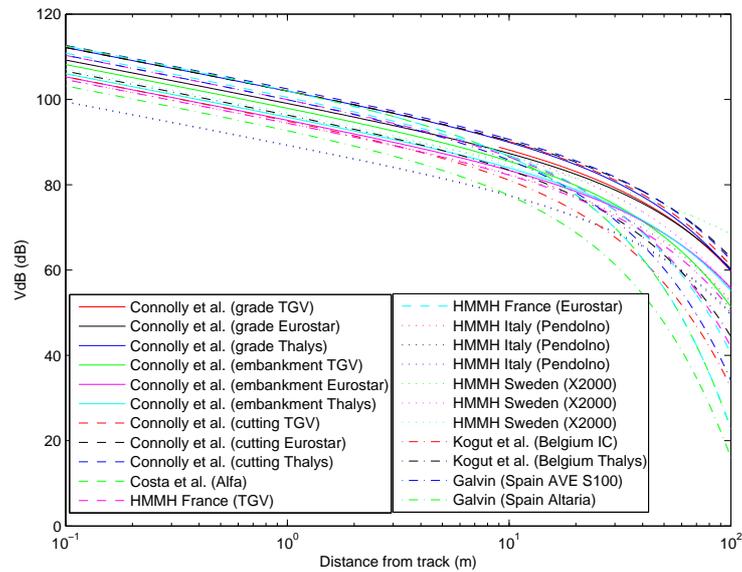


Figure 8 - Best fit relationships for all test sites

The effect of train type was also analysed by grouping each type of train (irrespective of test site) and calculating the resulting best fit curve. Again, it was found that vibration levels were similar at distances close to the track and discrepancies increased at greater offsets. In addition to soil effects, this may also have been because although the quasi-static excitation component (i.e. total mass) of each individual train was relatively similar, the dynamic excitation component (i.e. unsprung mass and track unevenness) varied considerably. Dynamic excitation dominates the vibration levels at greater offsets meaning this is where discrepancies between train type become more evident.

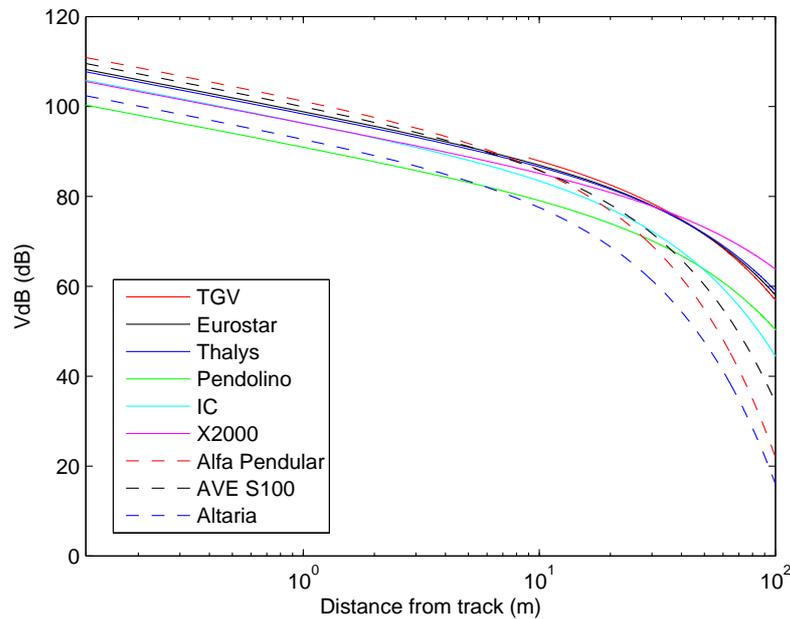


Figure 9 – Best fit relationships for each train type

1.5.5 Frequency content and critical velocity effects

The frequency content of vibration generated due to train passage is complex and arises from numerous excitation sources ([55], [57], [58]). At distances close to the track, the vibration response is dominated by quasi-static excitation (i.e. train weight), whereas at distances further from the track, the response is dominated by dynamic excitation (e.g. rail unevenness). Therefore, as this study was focused on vibration levels outwith the track structure, dynamic excitation was most likely the dominant cause of vibrations.

Dynamic excitation generated within the track is both filtered and dampened by the soil as it propagates. This makes it challenging to predict and is one reason why railway scoping models typically do not attempt to make frequency content predictions, instead opting for an instantaneous vibration metric. Therefore to analyse the characteristics of dynamic excitation in greater detail, the mean frequency spectrum (in 1/3 octave bands) at each test site was calculated at distances of 15m and 25 from the track as shown in Figure 10. It should be noted that due to inconsistencies between the field experiment techniques, not all datasets had vibration records at identical offsets. Despite this, most datasets did have data points close to the 15m and 25m locations, thus justifying their selection.

Figure 10 shows that there were discrepancies between the entire range of 1/3 octave bands for each test site. The best fit curve shows that at both locations, the dominant frequency components were located approximately in the 20-40Hz range. Figure 11 also shows the standard

deviation for each 1/3 octave, at both track offsets. Comparing the 15m and 25m cases, the standard deviation increases with greater distance from the track. Furthermore, there is an increase in standard deviation at higher frequencies.

An interesting finding was that results presented by [36] (site 7) had elevated vibration energy at low frequencies (in the range 2-10Hz), in comparison to the other results. Excitations in this range are usually governed by vehicle response (e.g. bogie and axle passage frequencies) and are quasi-static in nature. Usually quasi-static response is confined to locations close to the track structure ([59]) and if observed in the free field, can be indicative of critical velocity effects. Therefore the critical velocity of each site was analysed. Before doing so, it should be noted that sites 2 and 3 were removed from the analysis due to the presence of the embankment and cutting earthworks profiles respectively, which made the critical velocity calculation challenging. Despite this, for the cutting, as soil properties typically increase with depth, it was likely that the critical velocity of this site was high in comparison to the alternative sites.

For the remaining sites the critical speed was approximated by identifying the intersection between the dispersion curves of the track and soil (using an uncoupled analytical solution - [60]). In this procedure, the critical speed of the track-ground system is computed from the P-SV dispersion relationship of the ground and from the bending wave dispersion relationship of the free track (Figure 13). The critical speed of the system is given by the inverse of the slope of the line connecting the origin of the frequency-wavenumber axes, and the intersection point of the curves. To calculate the track dispersion curve, it was assumed that all sites consisted of a generic ballasted track (with UIC60 rails connected to concrete sleepers at 0.65m centres), as described in Figure 12.

The resulting critical velocity approximations are shown in Table 5 and plotted in Figure 14. For test site 7 ([36]), the critical velocity was calculated to be 418 km/h, while the mean train speed was 276 km/h and the maximum train speed was 314 km/h. These train speeds were thus much closer to the critical velocity than for the alternative test sites (66% and 75% for the mean and max train speeds respectively). As critical velocity effects have been shown to influence vibration levels when the train speed is greater than 50% the critical velocity ([61]), then it may have been possible that this amplified the quasi-static train excitation at low frequencies.

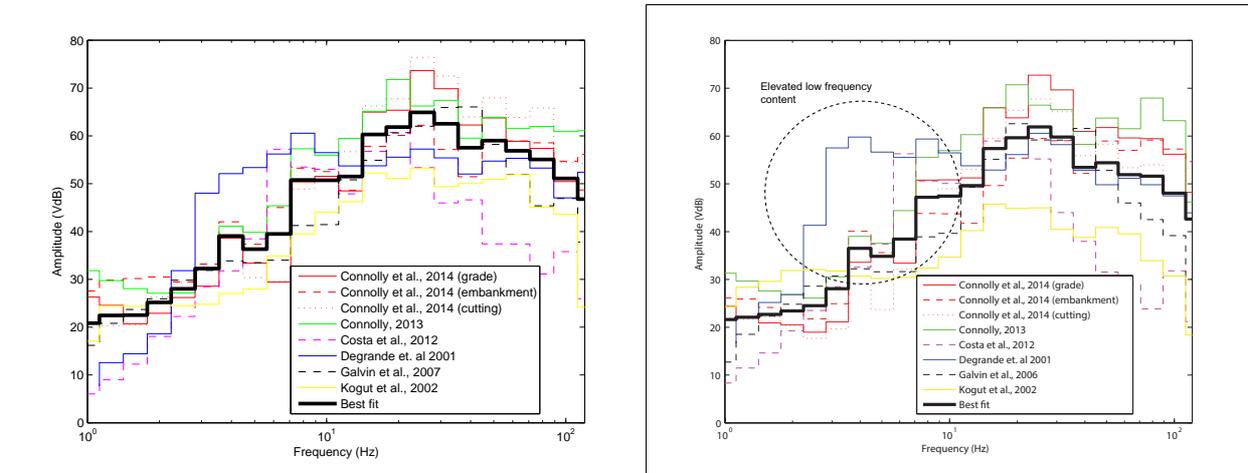


Figure 10 - One third octave bands, (a) Left: 15m offset from track, (b) Right: 25m offset from track

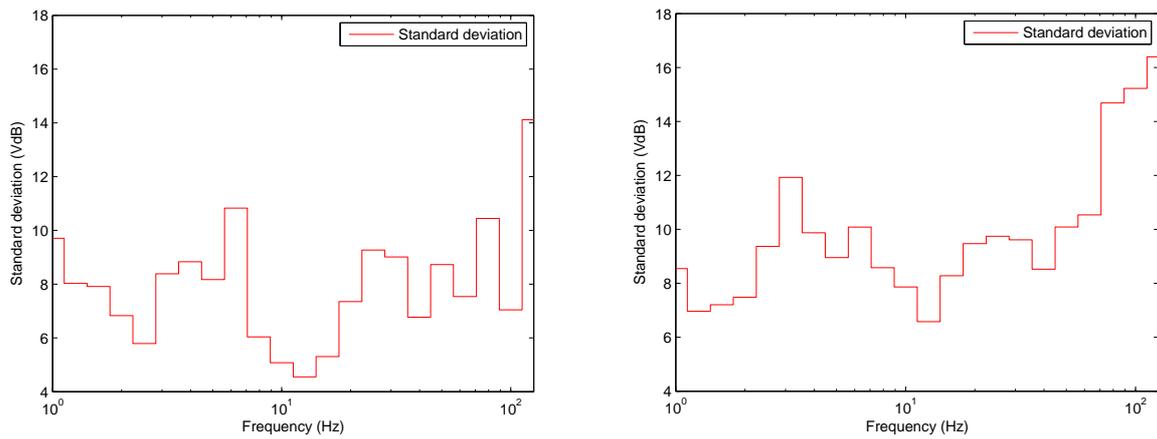


Figure 11 - Standard deviation in one third octave bands, (a) Left: 15m offset from track, (b) Right: 25m offset from track

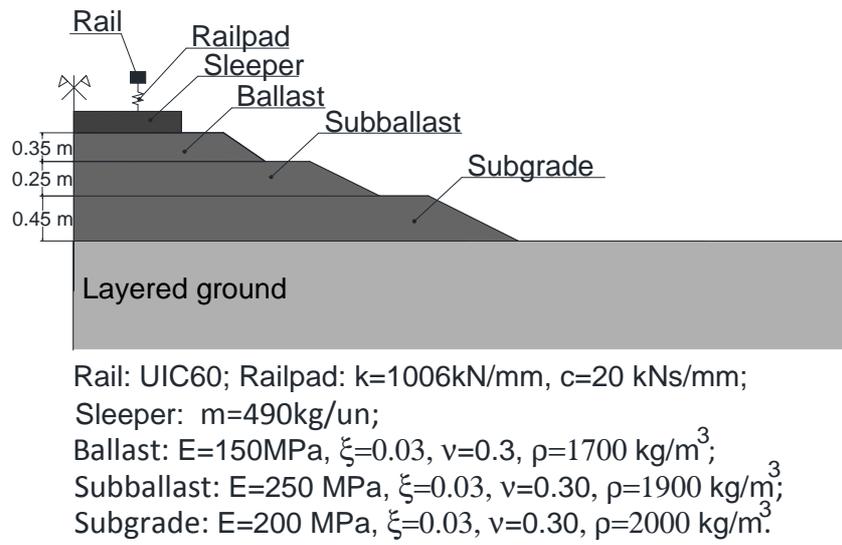


Figure 12 – Analytical track model used for track dispersion curve calculation

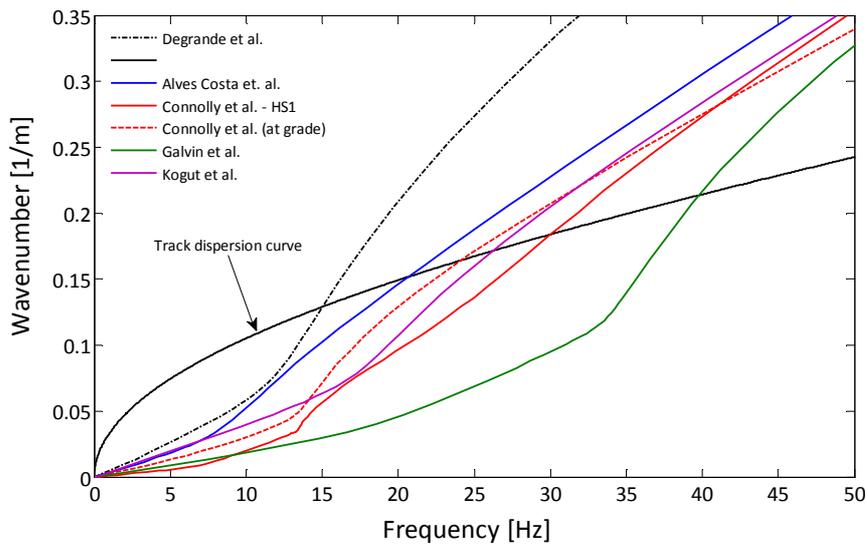


Figure 13 – Primary track and soil dispersion curves for critical velocity calculation

Site number	Site name	Critical velocity (m/s)	Max train speed (m/s)
1	Connolly et al (Belgium grade)	147	84
5	Connolly et al (HS1 grade)	163	83
6	Costa et al (Portugal grade)	136	62
7	Degrande et al (Belgium grade)	116	87
8	Galvin et al (Spain grade)	186	83
9	Kogut et al (Belgium grade)	150	85

Table 5 – Critical velocities of each test site

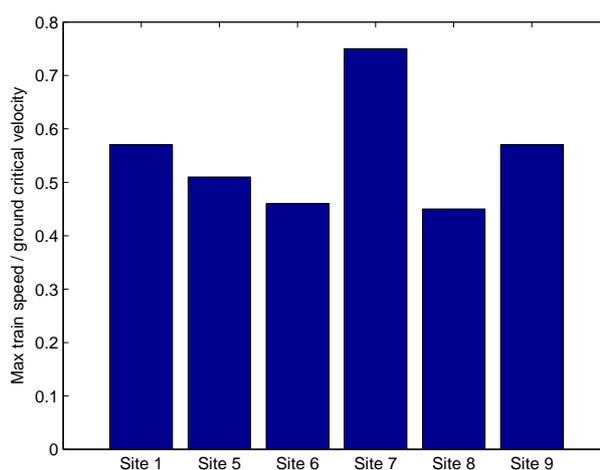


Figure 14 – Train speed as a function of ground critical velocities

1.5.6 Sources of discrepancy

The large number of variables that can potentially affect railway vibration levels means that numerical modelling can be challenging. Prediction errors arise from inaccuracies in modelling site specific variables and non-site specific variables. One of the most relevant site specific parameters, as previously discussed, is soil material properties ([53], [54]). Alternatively, non-site specific parameters include factors such as variances in train weight (i.e passenger numbers), wheel defects and train speed.

Numerical models have previously been used to investigate expected error levels from the detailed analysis of underground tunnels [62]. In this study it was concluded that $\pm 10\text{dB}$ should be expected, which arises from input/modelling inaccuracies of both site specific variables and non-site specific variables. Despite this, few investigations have investigated the potential errors/discrepancy using results from full scale physical tests.

Site-specific parameters are unlikely to change during the undertaking of experimental testing, meaning that all discrepancies between successive train passages are generated by non-site specific factors. To quantify the repeatability of measurements and the effect of non-site specific variables on vibration levels, the standard deviation was calculated for each measurement location, at each test site, for each type of train passage. The results for each test site are shown in Table 6 where the overall mean standard deviation was found to be 1.996.

Reference	Track arrangement	Train type	Mean standard deviation
Connolly et al., 2014	At grade	Eurostar	0.7
Connolly et al., 2014	At grade	Thalys	0.8
Connolly et al., 2014	At grade	TGV	2.7
Connolly et al., 2014	Embankment (5.5m)	Eurostar	7.4
Connolly et al., 2014	Embankment (5.5m)	Thalys	3.4
Connolly et al., 2014	Embankment (5.5m)	TGV	2.4
Connolly et al., 2014	Cutting (7.2m)	Eurostar	1.6
Connolly et al., 2014	Cutting (7.2m)	Thalys	0.7
Connolly et al., 2014	Cutting (7.2m)	TGV	0.4
Connolly 2013	At grade	Eurostar	3.8
Connolly 2014	At grade	Javelin 395	0.3
Kogut et al., 2002	At grade	InterCity	1.2
Kogut et al., 2002	At grade	Thalys	1.9
Galvin, 2007	At grade	AVE S100	2.6
Galvin, 2007	At grade	Altaría	0.2
Costa et al., 2014	At grade	Alfa Pendular	3
Degrande et al., 2001	At grade	Thalys	3
Harris Miller Miller and Hanson, 1996	At grade	TGV	2
Harris Miller Miller and Hanson, 1996	At grade	Eurostar	1.9
Harris Miller Miller and Hanson, 1996	At grade	TGV	1.3
Harris Miller Miller and Hanson, 1996	At grade	Eurostar	1.7
Harris Miller Miller and Hanson, 1996	At grade	Pendolino	0.7
Harris Miller Miller and Hanson, 1996	At grade	X2000	1.7
Harris Miller Miller and Hanson, 1996	Curve (embankment - 2m)	X2001	3
Harris Miller Miller and Hanson, 1996	Embankment - 1.5m	X2002	1.5

Overall mean standard deviation	1.996
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Table 6 – VdB standard deviations for all test sites

The quantification of standard deviation is significant because, as shown earlier, train speed has only a minimal effect on vibration levels, meaning that train weight (i.e. passenger numbers) and wheel defects are the most likely contributors towards the non-site specific discrepancies. These contributions are challenging to model because they can vary randomly between successive train passages. Therefore the key finding is that even if current vibration prediction approaches (experimental and numerical) are able to model the exact properties of a given test site, then a standard deviation of ± 2 VdB will likely remain.

Additionally, Figure 15 shows how standard deviation varies with distance from the track, at each location for a variety of train passages. It was found that there was a relatively constant relationship with distance with the majority of points being lower than 5 VdB. Despite this, for distances closest to the track (<20m) there were several points with elevated standard deviations, with a maximum at 13.3 VdB. Therefore it was postulated that the discrepancies close to the track were due to differences in train weight (i.e. passenger numbers) affecting the quasi-static excitation mechanism, which dominates the near-field response. Similarly, it was postulated that the discrepancies further from the track were due to changes in the dynamic excitation (i.e. wheel defects).

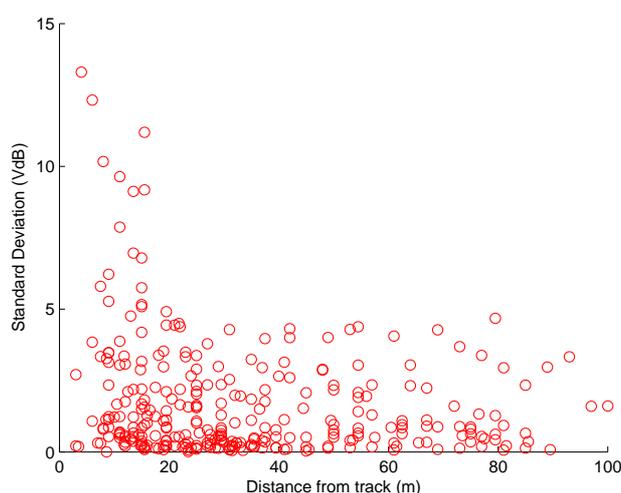


Figure 15 – Standard deviation versus distance

1.6 Conclusions

This paper analysed over 1500 vibration records, at 17 (approximately at-grade) high speed rail sites, across 7 European countries to gain new insights into the prediction of ground-borne vibrations. The prediction of 3 international metrics (VdB, $KB_{f_{max}}$ and PPV) was considered and new best fit relationships were proposed for each. Similarly, several VdB best fit relationships were benchmarked to determine their suitability. In addition, investigations were performed to determine the typical standard deviations of ground vibration, typical frequency contents, the effect of train speed, and the effect of train type. The key findings were:

- The mean error found between existing vibration prediction curves and experimental results was ± 4.5 VdB. Furthermore, the largest error encountered was 13.75dB.
- The mean standard deviation between similar, yet independent train passages, at the same test site is ± 2 VdB. This measure of repeatability indicates that even high accuracy prediction models, as a minimum, can expect a similar level of error.
- The standard deviation in vibration levels, for similar trains running on identical track, is relatively constant with distance (typically < 5 VdB). Despite this, at distances very close to the track this can increase and is most likely due to changes in quasi-static excitation.
- All train types (TGV, Eurostar, Thalys, Pendolino, InterCity, X2000, Alfa Pendular, AVE S100 and Alitalia) generated similar levels of ground-borne vibration close to the track, however discrepancy increased with distance.
- Increases in train speed caused an almost insignificant increase in vibration levels. Previously proposed speed vs vibration relationships were found to have a low correlation with the experimental data.
- The discrepancy between measured vibration levels (both VdB and frequency content) was found to increase with distance. This highlights the challenges and uncertainty faced when attempting to predict vibration levels at large offsets.
- Critical velocity effects have a strong influence on the low frequency (quasi-static) excitation mechanisms generated due to bogie and axles passages. When running close to critical speed, these low frequency components will propagate to greater offsets in comparison to trains running at lower velocities.
- The effect of soil material properties appeared to be a significant factor on vibration levels. Therefore, when possible, soil material properties should be included in vibration assessment calculations.

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