



Deposited via The University of Leeds.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/id/eprint/123965/>

Version: Accepted Version

---

**Proceedings Paper:**

Svärd, M, Markkula, G, Engström, J et al. (2017) A quantitative driver model of pre-crash brake onset and control. In: Proceedings of the Human Factors and Ergonomics Society 2017 Annual Meeting. HFES 2017, 09-13 Oct 2017, Austin, TX, USA. SAGE Publications, pp. 339-343. ISSN: 1071-1813.

<https://doi.org/10.1177/1541931213601565>

---

© 2017. This is an author produced version of a paper published in Proceedings of the Human Factors and Ergonomics Society Annual Meeting. Uploaded in accordance with the publisher's self-archiving policy.

**Reuse**

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.

# A quantitative driver model of pre-crash brake onset and control

Malin Svärd<sup>a</sup>, Gustav Markkula<sup>b</sup>, Johan Engström<sup>c</sup>, Jonas Bärghman<sup>d</sup>, Fredrik Granum<sup>a</sup>

<sup>a</sup> Volvo Cars Safety Centre, 41878 Göteborg, Sweden,

<sup>b</sup> Institute for Transport Studies, University of Leeds, LS2 9JT, Leeds, United Kingdom,

<sup>c</sup> Center for Truck and Bus Safety, Virginia Tech Transportation Institute, Blacksburg, VA 24061, United States

<sup>d</sup> Department of Mechanics and Maritime Sciences, Chalmers University of Technology, 419 96 Göteborg, Sweden

An existing modelling framework is leveraged to create a driver braking model for use in simulations of critical longitudinal scenarios with a slower or braking lead vehicle. The model applies intermittent brake adjustments to minimize accumulated looming prediction error. It is here applied to the simulation of a set of lead vehicle scenarios. The simulation results in terms of brake initiation timing and brake jerk are demonstrated to capture well the specific types of kinematics-dependencies that have been recently reported from naturalistic near-crashes and crashes.

## INTRODUCTION

With an increasing number of advanced driver assistance systems becoming standard in new vehicles, understanding driver response in relation to warnings and upcoming threats is essential to road safety benefit estimation (see e.g. Page et al., 2015). Traditionally, most mathematical models used to simulate driver braking behaviour in critical longitudinal scenarios have been based on probability distributions, determining a *reaction time* and a *predefined brake profile* (e.g. Green, 2000, and the review by Markkula, Benderius, Wolff & Wahde, 2012). In such models, the driver's reaction is typically initiated by what is assumed to be a sudden threat appearance, for example a brake light onset of the vehicle in front. However, recent analyses of real crashes and near-crashes in the second Strategic Highway Research Program (SHRP 2) dataset have shown a strong dependency on kinematics for both brake initiation and brake ramp-up, inconsistent with the existing situation-independent models (Markkula, Engström, Lodin, Bärghman & Victor, 2016).

A parallel modelling tradition, focusing more on non-critical braking, and building on ideas from ecological psychology, argues that driver braking is driven by perceptual cues such as visual *looming* (e.g. Lee 1976; Fajen, 2008). Such looming can be quantified for example using the measure

$$\tau^{-1} = \frac{\dot{\theta}}{\theta}, \quad (1)$$

where  $\theta$  is the optical size (width) of the lead vehicle on the driver's retina. Markkula et al. (2016) suggested that their findings from SHRP 2 naturalistic driving data could be explained if the driver's brake initiation is not solely related to the crossing of a looming threshold, but to noisy *evidence accumulation* of looming and other perceptual input over time, a type of mechanism for which there is much support from laboratory tasks in psychology and neuroscience (Gold & Shadlen, 2007). Combining this model mechanism with other well proven neuroscientific concepts, especially *motor primitives* (Giszter, 2015) and *prediction of sensory outcomes of motor actions* (Crape & Sommer, 2008), Markkula and colleagues (Markkula, 2014; Markkula, Boer Romano & Merat, 2017) have developed a computational framework for driver control behaviour. This paper describes, for the first

time, an application of this framework to braking behaviour, more specifically braking in critical scenarios.

The framework aligns well with the more general *predictive processing* theory, which has recently received much attention as a potential unifying account of brain function (Friston, 2010; Clark, 2016). The basic idea underlying predictive processing is that cognition and behaviour can be explained as prediction error minimization. Engström et al. (2017) provide a proposal on how predictive processing concepts can be applied to automobile driving, with the type of model presented in this paper as one example.

The proposed model is described in detail in the first section of the paper. The subsequent two sections illustrate that simulations utilizing the model successfully reproduce the dependencies on kinematics observed in naturalistic data. The paper concludes with an exploration of possibilities for extending the model to a wider variety of scenarios.

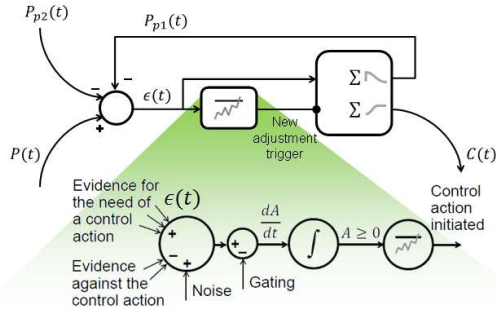
## THE MODEL

The computational framework adopted here (Markkula, 2014; Markkula et al. 2017) posits that:

- Driving control can be regarded as a series of intermittent, open loop control adjustments (motor primitives).
- The timing of new adjustments is determined by a process of evidence accumulation.
- The magnitude of adjustments is tuned to perceptual inputs.
- After each control adjustment, the driver makes a prediction of how this adjustment will affect future sensory inputs.

In the model proposed in the current paper, the driver collects evidence for the first brake onset by accumulating looming. Once the accumulated evidence exceeds a predefined threshold, the driver is assumed to press the brake pedal. The amplitude of braking will aim to resolve the situation at hand, that is, the driver will only use the brake force that is judged necessary to avoid collision and maintain a safe distance to the vehicle in front, given the currently available information and a prediction of the effect of the braking based on previous experience of similar conflict situations. That is, while issuing a brake adjustment, the driver makes a prediction of how the looming will gradually decay as a result of the braking.

Subsequently, the (new) predicted looming is compared to the actual looming and the model continues to operate on the prediction error (rather than the looming signal in itself) until either the situation is resolved, maximum braking is achieved or a collision occurs. Figure 1 provides an illustration of the model.  $P(t)$  is the perceptual input, that is, looming, and  $P_{p1}(t)$  is a looming prediction based on prior braking adjustments. The  $P_{p2}(t)$  prediction is a new addition here (compared to the framework by Markkula, 2014; Markkula et al. 2017) based on the predictive processing theory, and represents higher-level expectations of looming based on existing knowledge on how driving situations typically play out, as explained in further detail below.  $\epsilon(t)$  is the total looming prediction error and  $C(t)$  is the brake signal constructed from control adjustments issued as a result of prediction error accumulation.



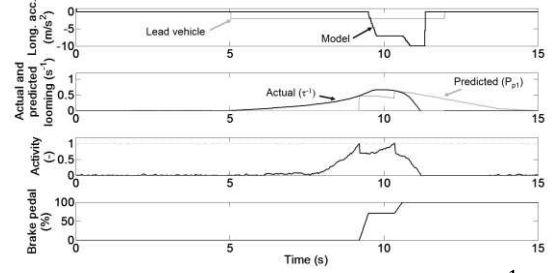
**Figure 1** Illustration of the fundamentals of the driver modelling framework, incorporating both brake modulation and higher level prediction to the framework presented by Markkula (2014).

### Kinematics dependent brake initiation

Mathematically, the accumulative part of the model, illustrated in the lower part of Figure 1, can be expressed as

$$\frac{dA(t)}{dt} = K \cdot \epsilon(t) - M + v(t), \quad (2)$$

where  $A(t)$  is the total accumulated prediction error, henceforth called the *activity*. When the activity reaches a certain threshold  $A_r$ , a brake adjustment is issued and the activity is reset to  $A_r$ .  $A_t$  and  $A_r$ , as well as  $K$  and  $M$  are free model parameters and  $v(t)$  is Gaussian zero-mean white noise with a standard deviation  $\sigma\sqrt{\Delta t}$  for a model simulation time step of  $\Delta t$ . The parameter  $K$  corresponds to the gain determining the impact of the prediction error on the accumulator, i.e. a higher  $K$  will lead to more rapid changes in activity level. Note that the prediction error  $\epsilon(t)$  may work both as evidence for ( $\epsilon(t) > 0$ ) and against ( $\epsilon(t) < 0$ ) braking, depending on its sign. The *gating*,  $M$ , can be interpreted as the sum of all non-looming evidence for and against the need of braking (again, see the lower part of Figure 1). For an example of activity and brake pedal signals in a rear-end scenario with a braking lead vehicle, see Figure 2.



**Figure 2** Longitudinal acceleration, predicted ( $P_{p1}$ ) and actual ( $\tau^{-1}$ ) looming, activity ( $A$ ) and brake pedal position ( $C$ ) from a straight rear-end scenario where the lead vehicle is braking with moderate deceleration at  $t = 5$  s.

### Kinematics dependent brake modulation

The magnitude of the  $i^{\text{th}}$  individual adjustment, issued at time  $t_i$ , is a linear scaling of the prediction error at brake onset and can be calculated by the heuristic

$$g_i = k \cdot \epsilon(t_i), \quad (3)$$

where  $k$  is a free model parameter. The shape of each brake adjustment is determined by a function  $G(t)$ , which needs to fulfil the requirements

$$G(t) = \begin{cases} 0, & \text{for } t \leq 0 \\ 1, & \text{for } t \geq \Delta T, \end{cases} \quad (4)$$

where  $\Delta T$  is a free model parameter representing the adjustment duration. The individual control adjustment  $G(t)$  will only affect the braking after time  $t_i$  and when complete, the adjustment will reach the magnitude  $g_i$  and stay at that level. For the sake of simulation simplicity, in this paper  $G(t)$  is chosen to be linearly increasing from  $G(0) = 0$  to  $G(\Delta T) = 1$ . Eventually, the total brake pedal signal  $C(t)$  is generated as the sum of all brake adjustments,

$$C(t) = \sum_{i=1}^N g_i G(t - t_i), \quad (5)$$

where  $N$  is the number of brake adjustments with  $t_i < t$ .

### Low level perception prediction

At each brake adjustment, a prediction of the resulting looming will be added to the signal  $P_{p1}(t)$ . Each prediction will be scaled with the magnitude of the prediction error at time  $t_i$ , that is at start of the brake adjustment, and take the shape of a function  $H(t)$  fulfilling the requirements

$$H(t) = \begin{cases} = 0, & \text{for } t \leq 0 \text{ and } t \geq \Delta T_p \\ \rightarrow 1, & \text{for } t \rightarrow 0^+ \\ \rightarrow 0, & \text{for } t \rightarrow \Delta T_p \end{cases}, \quad (6)$$

where  $\Delta T_p$  is a free model parameter.  $P_{p1}(t)$  can be generated as

$$P_{p1}(t) = \sum_{i=1}^N \epsilon(t_i) H(t - t_i), \quad (7)$$

where  $N$  is the number of brake adjustments with  $t_i < t$ . For simulation simplicity, a piecewise linear function  $H(t)$  was used in this paper, constant at 1 for a duration  $\Delta T_{p0}$  (i.e.  $H(t \leq \Delta T_{p0}) = 1$ ) before falling linearly to zero during a second duration  $\Delta T_{p1}$  such that  $\Delta T_{p0} + \Delta T_{p1} = \Delta T_p$ .

The  $H(t)$  definition (eqs. 6 and 7) assures that the overall looming prediction  $P_{p1}(t) + P_{p2}(t)$  just after each new brake adjustment immediately becomes equal to the actual observed looming  $P(t)$  (see the example in Figure 2). Moreover, the shape of  $H(t)$  reflects the assumption that control adjustments aim to resolve the conflict that triggered them, that is after each brake adjustment the driver predicts that the looming will decay to the high-level prediction  $P_{p2}(t)$ .

### High level perception prediction

While the lower level prediction  $P_{p1}(t)$  predicts how the looming is affected by each new brake adjustment, there is also a higher level prediction  $P_{p2}(t)$ , which can be viewed as the expected looming or as a looming target, a level of looming that the driver aims to achieve when braking. This can be illustrated with the following example:

When approaching a signalled intersection with a vehicle in front standing still at a traffic light, the driver in the host vehicle may expect and accept a certain amount of looming, if also predicting (or already observing) that the light will soon shift to green and the lead vehicle accelerate away. In this case,  $P_{p2}(t)$  will initially increase over time, permitting the amount of looming to become relatively large without increasing the activity level in the model (i.e.  $\varepsilon(t) = P(t) - P_{p2}(t) = 0$ ). However, if the prediction of lead vehicle acceleration, and the associated quickly reduced looming, is not met, the driver will suddenly be exposed to a high amount of unexpected looming, triggering an immediate and large, but at this point not necessarily sufficient, brake adjustment. This suggests a mechanism for how expectations may lead to late reactions in critical scenarios (see Engström et al., 2017, for a more extensive discussion of this point).

Hence, including the higher level prediction in the model makes it possible to use for a wider range of scenarios where the driver expectation may play an essential role. However, the mathematical details in how to set up the  $P_{p2}(t)$  for different scenarios is outside the scope of this paper and warrants further investigation.

## MODEL APPLICATION

To illustrate that the driver model is able to qualitatively reproduce driver behaviour observed in naturalistic data, it was applied to a set of (artificially created) straight lead vehicle scenarios assuming a certain off-road glance behaviour.

### Model tuning and scenario description

The driver model parameters were hand tuned to demonstrate that the model is able to capture the kinematics-dependence seen in naturalistic data. The parameters are presented in Table 1. However, before future use of the model for real safety benefit analysis, it is crucial to perform thorough parameterization and validation against empirical data, indeed the example simulation in Figure 2 suggests that

the present manually parameterized model may be responding overly late.

**Table 1** The parameter values used in the simulations.

Parameter	Value	Parameter	Value	Parameter	Value
K	3	$A_t$	1	$\Delta T$	0.5
M	0.3	$A_r$	0.7	$\Delta T_{p0}$	0.5
$\sigma$	0.007	k	1.5	$\Delta T_{p1}$	4

The driver model was applied to a set of lead vehicle scenarios set up by the European New Car Assessment Programme (Euro NCAP), forming the standard scenario set used for consumer rating tests of forward collision warning and advanced emergency brake systems (Euro NCAP, 2015). The set consists of 26 straight lead vehicle scenarios divided into three categories:

*Car to Car Rear Stationary.* 11 scenarios where the host vehicle drives with a moderate speed of 30-80 km/h towards a vehicle that is standing still.

*Car to Car Rear Moving.* 11 scenarios where the host vehicle drives with moderate speed of 30-80 km/h towards a slower vehicle, cruising at 20km/h.

*Car to Car Rear Braking.* 4 scenarios where both vehicles drive with an initial speed of 50km/h. The distance between the vehicles is 12 m or 40 m, until the lead vehicle starts braking with a deceleration of -2 or -6 m/s<sup>2</sup>.

### Driver glance behaviour

Applying the driver model to the Euro NCAP scenarios assuming a completely attentive driver results in a very low amount of collisions. This is expected since inattention is shown to be a cause for a large proportion of the rear-end collisions observed in real life (see e.g. Neale, Dingus, Klauer, Sudweeks & Goodman, 2005; Victor et al., 2015). To create a realistic driver glance behaviour in all scenarios, a glance distribution was added to each event. For this purpose, the baseline distribution extracted by Bårgman, Lisovskaja, Victor, Flannagan, and Dozza (2015) from SHRP 2 lead vehicle scenarios for normal driving was chosen, sampled into bins of 0.2 s.

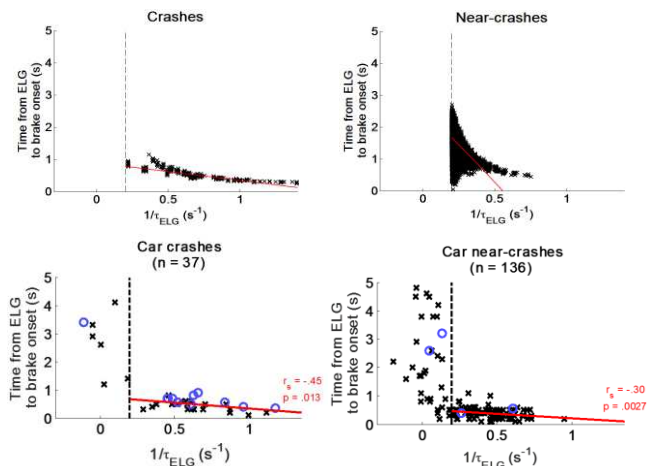
Previous studies have shown that drivers are not likely to start looking away from the road unless the velocity relative to the lead vehicle is close to zero (Tijerina, Barickman and Mazzae, 2004). Based on the further assumption that this is judged by the driver in terms of looming, we here defined the *anchor point* for the glance distribution as  $\tau^{-1} = 0.2 \text{ s}^{-1}$ . Therefore, the simulations were set up in a manner so that the glances first started at the anchor point, and for each simulation run the glance starting point was incrementally moved 0.2 s backwards in time until the glance no longer overlapped the anchor point (i.e., assuming equal probability of glance initiation at these times). Since the aim is to simulate the last glance off-road, scenarios with glances ending before the anchor point are here treated as equivalent to scenarios where the driver looks ahead for the entire scenario (called "eyes-on-threat" scenarios by Markkula et al., 2016) and are out of scope for the current simulations.

## SIMULATION RESULTS AND COMPARISON TO NATURALISTIC DATA

The model in this paper differs from other state of the art driver models in its ability to capture the kinematic dependence of both brake onset timing and brake ramp-up in a way that reflects the behaviour of human drivers. In this section, this will be illustrated by reproducing two figures from Markkula et al. (2016), comparing results from the simulated Euro NCAP scenarios to the original figures, which are based on analysis of empirical data reported from the SHRP 2 naturalistic study. The data are divided into crash events, defined as the vehicles touching each other, and near-crash events, defined as events where the driver model brakes with a maximum deceleration exceeding 0.5 g. The latter is similar to the SHRP 2 near-crash definition, although not an exact match since the SHRP 2 data were also manually annotated (see Victor et al., 2015).

### Deceleration onset timing

The top panes of Figure 3 show the results from the simulation for crashes and near crashes respectively, which are compared to the corresponding SHRP 2 data in the bottom panels. The plots represent the time it takes from the end of the last off-road glance (ELG) until the driver initiates braking as a function of the kinematic urgency of the situation, expressed as  $\tau^{-1}$  at ELG. Positioning the glance anchor at  $\tau^{-1} = 0.2 \text{ s}^{-1}$  implies that the simulation results will only reflect events described as eyes-off-threat by Markkula et al. (2016).



**Figure 3** Time between the end of last off-road glance and brake onset as a function of  $\tau^{-1}$  at the end of the glance. Top: simulated data. Bottom: Naturalistic data; reprinted from (Markkula et al., 2016, Fig. 4), with permission from Elsevier.

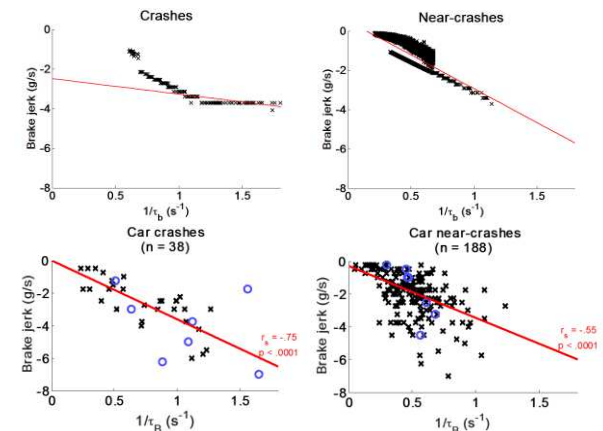
For the crash events (left panels), the behaviour in the simulated eyes-off-threat events is remarkably similar to the behaviour reported from the SHRP 2 data set, despite the limited parameter tuning carried out. The main difference for the crashes is the tendency of a longer time to brake onset for low looming levels, close to the anchor point, in the simulated

scenarios. In the naturalistic data there were very few crashes in this region.

For the near-crash cases (right panels), although again the qualitative pattern of reduced response times with increasing severity is captured, the difference between simulated and naturalistic data is more pronounced. The time between looking back to the road and brake initiation is longer in the simulated cases than in the naturalistic data, in particular for less urgent situations (with a low  $\tau_{ELG}^{-1}$ ). This could partly be a result of the assumption that no evidence accumulation at all occurs while glancing off-road. This might also explain the lack of real crashes with last glances at a low looming level close to the anchor point. In a real situation, the driver would most likely perceive a certain amount of accumulation in the peripheral view while glancing (Lamble, Laakso & Summala, 1999). This peripheral accumulation may also make the driver look back earlier in urgent situations. However, it is also possible that simply a more thorough parameterization on empirical data could lead to lower simulated reaction times for the cases with ELG close to the anchor point.

### Deceleration ramp-up behaviour.

The model in this paper is not only able to adjust the brake onset timing to the kinematic urgency of the situation, but it also adapts the ramp-up of braking accordingly. The top panels in Figure 4 shows how the brake ramp-up for the simulated events is more severe when the looming is high at the time of brake initiation, while starting to brake at a low looming level permits a slower ramp-up of the brake force. These observations agree with the results obtained from SHRP 2 data in Fig. 8 in Markkula et al. (2016), here reprinted in the bottom panels of Figure 4. However, while the general shape of the simulated data in the top panels of Figure 4 is similar to what is found in SHRP 2 data, there are also a few differences.



**Figure 4** Average brake jerk as a function of  $\tau^{-1}$  at brake onset. Top: Simulated data. Bottom: Naturalistic data; reprinted from (Markkula et al., 2016, Fig. 8), with permission from Elsevier. (Note that the original publication mistakenly reported the y-axis unit as  $\text{m/s}^3$ ).

One of the most evident differences is that the brake ramp-up of the simulated scenarios saturate at a minimum brake jerk of  $-4.07 \text{ g/s}$ . This is a result of limitations in the brake capacity of the virtual vehicle model used in the

simulations. In the naturalistic data a wider variety of vehicles with different brake capacities, as well as drivers with different characteristics, were included.

The second difference is that there are no crashes with a brake onset at low looming levels. It is reasonable to believe that a crash occurring even though the driver starts braking at a low looming level either (i) has a very rapid course of events, making the looming increase rapidly, or (ii) results from the driver initially expecting the scenario to become less critical than it actually did (and thus not requiring hard braking). The first reason will not be reflected in the simulated data set, since the Euro NCAP scenarios do not develop in the rapid way described. The latter reason might be possible to capture with the current model if adding a higher level prediction signal,  $P_{p2}$ , accounting for the initially expected looming. A non-zero  $P_{p2}$  would result both in a smaller initial brake adjustment and later subsequent brake adjustments.

A final interesting difference between the simulated scenarios and the naturalistic data set in Figure 4 is the division of the near-crash brake jerk plot into two regions: one cluster of data points seemingly following a straight line and one wider cluster positioned slightly above. The two clusters correspond to the number of brake adjustments done by the driver model, with the lower cluster being for single adjustment responses. In reality, the number of brake adjustments used by drivers may differ as well, but the outcome in terms of brake jerk is likely to be noisier. If adding more parameter variations and, for example, motor system noise, the clusters may become indistinguishable in simulation results as well, making the near-crash plot in the upper part of Figure 4 more similar to the corresponding plot in the lower part of the figure.

## DISCUSSION AND FUTURE WORK

The model proposed in this paper demonstrates how intermittent minimization of accumulated looming prediction error results in qualitatively realistic, kinematics-dependent brake initiation and brake ramp-up in critical longitudinal scenarios. One important item for future work is a thorough parameterization of the model on naturalistic data. Another is to properly account for crashes without off-road-glances, for example by incorporating higher-level expectancy ( $P_{p2}$ ).

The modelling framework also allows extension to additional perceptual cues as sensory input. For example, driver reaction to a warning or brake light could be accounted for by adding an instantaneous increase in the activity level. This increase in activity, to be parameterized on data from naturalistic or controlled studies, will result in a shorter time to reach the reaction threshold, that is, shorten the time to brake initiation.

The development of driver behavioural models to be used in simulation is essential for performing realistic road safety benefit analysis and estimating the real life impact of advanced driver assistance systems. Due to the generic framework of the current model, it should be possible to adapt it for other use cases, as for example run off road or intersection crashes. Since the higher level prediction part in

the model is a tool to model driver expectancy, the framework also forms a solid base to build models describing the driver's behaviour in relation to a failing automated driving system.

## ACKNOWLEDGMENT

The work was partially funded by Vinnova, Swedish governmental agency for innovation, as part of the project Quantitative Driver Behaviour Modelling for Active Safety Assessment Expansion (QUADRAE).

## REFERENCES

- Bärgman, J., Lisovskaja, V., Victor, T., Flannagan, C., & Dozza, M. (2015). How does glance behavior influence crash and injury risk? A 'what-if' counterfactual simulation using crashes and near-crashes from SHRP2. *Transportation Research Part F: Traffic Psychology and Behaviour*, 35, 152-169. doi: 10.1016/j.trf.2015.10.011
- Clark, A. (2016). *Surfing Uncertainty*. Oxford: Oxford University Press
- Crapse, T. B., Sommer, M. A. (2008). Corollary discharge across the animal kingdom. *Nature Reviews Neuroscience*, 9(8), 587-600.
- Engström, J., Bärgman, J., Nilsson, D., Bianchi Piccinini, G.F., Seppelt, B., Markkula, G., Victor, T. (2017). Great expectations: A predictive processing account for automobile driving. *Theoretical Issues in Ergonomics Science*, in press. doi: 10.1080/1463922X.2017.1306148
- Euro NCAP. (2015). Test protocol – AEB systems, Version 1.1. Retrieved from: <http://www.euroncap.com/>
- Fajen, B. R. (2008). Perceptual learning and the visual control of braking. *Perception & Psychophysics*, 70(6), 1117-1129.
- Friston, K.J. (2010). The Free-energy Principle: A Unified Brain Theory? *Nature Reviews Neuroscience*, 11(2), pp. 127-38
- Gold, J. I., Shadlen, M. N. (2007). The neural basis of decision making. *Annual Review of Neuroscience*, 30, pp. 535-574.
- Giszter, S. F. (2015). Motor primitives—new data and future questions. *Current opinion in neurobiology*, 33, 156-165.
- Lamble, D., Laakso, M., Summala, H. (1999). Detection thresholds in car following situations and peripheral vision: Implications for positioning of visually demanding in-car displays. *Ergonomics*, 42(6), 807-815.
- Lee, D. (1976). A theory of visual control of braking based on information about time-to-collision. *Perception*, 5(4), pp. 437-459.
- Markkula, G., Benderius, O., Wolff, K., Wahde, M. (2012). A review of near-collision driver behavior models. *Human Factors*, 54(6), 1117-1143.
- Markkula, G. (2014). Modeling driver control behavior in both routine and near-accident driving. In *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*, Vol 58, pp. 879-883. Chicago, IL.
- Markkula, G., Engström, J., Lodin, J., Bärgman, J., Victor, T. (2016). A farewell to brake reaction times? Kinematics-dependent brake response in naturalistic rear-end emergencies. *Accident Analysis & Prevention*, 95, pp. 209-226.
- Markkula, G., Boer, E., Romano, R., Merat, N. (2017). Sustained sensorimotor control as intermittent decisions about prediction errors: Computational framework and application to ground vehicle steering. Submitted for publication. <https://arxiv.org/abs/1703.03030>
- Neale, V.L., Dingus, T.A., Klauer, S.G., Sudweeks, J., Goodman, M. (2005). An overview of the 100-car naturalistic study and findings. *National Highway Traffic Safety Administration, Paper*, (05-0400).
- Page, Y., Fahrenkrog, F., Fiorentino, A., Gwehenberger, J., Helmer, T., Lindman, M., op den Camp, O., van Rooij, L., Puch, S., Fränze, M., Sander, U., Wimmer, P. (2015). A comprehensive and harmonized method for assessing the effectiveness of advanced driver assistance systems by virtual simulation: The P.E.A.R.S. initiative. In *Proceedings of the 24th International Technical Conference on the Enhanced Safety of Vehicles (ESV)*, Paper nr. 15-0370. Göteborg, Sweden
- Tijerina, L., F. S. Barickman, and E. N. Mazzae. 2004. Driver Eye Glance Behavior During Car Following. DOT HS 809 723. National Highway Traffic Safety Administration.
- Victor, T., Bärgman, J., Boda, C-N., Dozza, M., Engström, J., Flannagan, C., Lee, J.D., Markkula, G. (2014). Analysis of Naturalistic Driving Study data: Safer glances, driver inattention, and crash risk, *Transportation Research Board, SHRP2 Report S2-08A-RW-1*.