*God over All: Divine Aseity and the Challenge of Platonism*, by William Lane Craig. Oxford University Press, 2016. Pp. 241. $80 (hardcover).

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Platonism in the philosophy of mathematics is the view that there are abstract mathematical objects, such as the number 2. “Abstract,” here, is most often characterized by the “way of negation” (John P. Burgess and Gideon Rosen, *A Subject with no Object: Strategies for Nominalistic Interpretation of Mathematics* (Clarendon Press, 1997), 17), as applying to non-spatiotemporal, acausal, mind- and language-independent objects. The main motivation for thinking that there are mathematical objects is that many mathematical claims that we take to be literally true seem to imply the existence of such things (for example, the claim that “There is an even prime number” seems to imply that there is at least one mathematical object). The main motivation for thinking that these objects are *abstract*, as negatively characterized above, is that no account in terms of the obvious positive features seems to work. It is hard to see how the number 2 could be thought of as a spatiotemporally located object entering into causal relations, and, given the finiteness of human minds and the plausible thought that even before humans were on the scene to think or talk about such things, “when two dinosaurs met two dinosaurs there were four dinosaurs” (Martin Gardner, “Is mathematics for real?,” *New York Review of Books* 28 (1981), 13), it is also hard to see how mathematical objects, such as numbers, could be dependent for their existence on human minds or languages. Mathematical truths are standardly held to be true of necessity and eternally, so the objects they concern should likewise exist of necessity and eternally.

However, the Platonist account of mathematical objects faces a serious obstacle. Precisely because of their negative characterization, it is hard for the Platonist to give a positive account of how we can come to know truths about abstract objects. Given their acausal, nonspatiotemporal, mind- and language-independent nature, what reason have we for thinking that our beliefs about abstract mathematical objects get things right about their nature? This is the key epistemological challenge to Platonism, first raised by Paul Benacerraf (“Mathematical Truth,” *Journal of Philosophy* 70 (1973): 661-680), with focus on the presumed *acausal* nature of mathematical objects as abstracta in light of the then-dominant causal theory of knowledge, and then refined by Hartry Field (*Realism, Mathematics and Modality* (Blackwell, 1989)) as the challenge to explain how, given the Platonist’s account of the nature of abstract objects, it is reasonable for them to expect our mathematical beliefs reliably to reflect how things are with such objects.

In his extremely thought-provoking discussion of God and abstract objects, *God Over All*, William Lane Craig is concerned with another problem raised by the Platonist account of mathematical objects, along with other purported abstracta such as properties and propositions, by virtue of their status as abstract objects: the challenge to divine aseity. (The book, an expansion of Craig’s 2015 Cadbury Lectures, is a taster for his forthcoming *God and Abstract Objects* (Springer-Verlag), which Craig explains offers “a more extensive, in-depth discussion of the questions and views treated” in *God over All* (viii).) According to the Judaeo-Christian concept of God, God depends on no other being for His existence, whereas all other things that exist depend for their existence on God. If there are abstract objects, such as numbers, then they present a challenge to this account of the nature of reality. For, as Craig points out, “insofar as these abstract objects are taken to be uncreated, necessary, and eternal” (3), this puts them in conflict with the doctrine of divine aseity. As such, mathematical objects, as abstracta present a prima facie challenge to Judaeo-Christian theism, as do other purported abstracta (such as, for example, propositions or properties).

Is there room, then, for philosophers of religion concerned with the challenge to divine aseity, and philosophers of mathematics concerned with the problem of explaining our knowledge of mathematical objects as abstracta, to join forces in rejecting these problematic objects? That is Craig’s hope. In particular, in *God over all*, Craig looks to contemporary defences of *nominalism* in the philosophy of mathematics to argue for the plausibility of avoiding the challenge to divine aseity by denying the existence of abstract objects. We will consider Craig’s attempt to put so-called “easy road” nominalist challenges to mathematical Platonism to the service of the theist in a moment. But first, some thoughts about the alternative options for the theist, which Craig considers and sets aside in his book.

In outlining the motivation for mathematical Platonism, above, I said that the characterization of mathematical objects as abstract via the “way of negation” had come about through the implausibility of characterizing such objects via any of the positive characteristics listed: spatiotemporal, causal, mental, linguistic. Consider in particular the thought that mathematical objects could be accounted for as mental objects. Constructivists in the philosophy of mathematics, such as L. E. J. Brouwer (“Consciousness, philosophy, and mathematics,” (1948), extracted in Paul Benacerraf and Hilary Putnam, *Philosophy of Mathematics: Selected Readings* (1983), 90-96), attempted to account for mathematical objects as human mental constructions, but although the constructivist programme brought with it some interesting new branches of mathematics, it places strong limitations on what is to count as acceptable mathematics, based on its account of what is constructible-in-principle by the human intellect. More broadly, Gottlob Frege (*The Foundations of Arithmetic* (1888), trans. J. L. Austin, 2nd ed (Blackwell, 1953), 38) mocked as “weird and wonderful…the results of taking seriously the suggestion that number is an idea,” noting that if the number 2 is an idea in someone’s head then there would be no unique number 2, but just my 2, your 2, and ever more number 2s coming into existence with each new generation of thinkers. But these objections depend on taking mathematical objects to be ideas in human minds. As Craig notes, if we bring a theistic outlook to the question of the nature of mathematical objects, the idea of mathematical objects as mental objects has more plausibility (72). If mathematical objects are ideas in *God’s* mind, as suggested by the doctrine of *divine conceptualism*, then they are not subject to the same limitations of human finite and temporally constrained existence. And with only one God, we can make sense of their uniqueness too. Furthermore, a theistic outlook may help to answer the epistemological objection to Platonism: perhaps God is the origin of our ideas of abstract objects such as numbers (something of this thought is present in Mark Steiner’s discussion of the puzzle of the applicability of mathematics (*The Applicability of Mathematics as a Philosophical Problem* (Harvard University Press, 1998))). The combination of realism about mathematical objects with theism (either by accounting for mathematical objects, as the divine conceptualist does, as *ideas* in God’s mind, or alternatively, as proposed by the *abstract creationist*, accounting for themas created *abstracta*), looks at least initially promising

In Chapters 4 and 5, Craig outlines a number of objections to abstract creationism and divine conceptualism, but in each case the most significant challenge seems to be a form of the *bootstrapping objection*. This objection arises because it is assumed that, amongst the purported abstract objects created by God, are *properties*. But, since “in order to create properties, God would already have to possess certain properties” (60), we appear to be stuck in an explanatory circle. To avoid this objection, Craig suggests that the only live option is to reject what he calls “the Platonistic ontological assay of things” (68), according to which there are properties and substances which exemplify those properties. But to drop the Platonic account of properties as “fundamental ontological constituents of things” (68) would be, Craig claims, to lose “any rationale for positing the existence—in a metaphysically heavy sense—of such objects” (68), thus undermining the motivation for accounting for abstracta as God’s creations in the first place. And without this motivation in place, neither divine conceptualism nor abstract creationism are required – we can instead move straight to nominalist attempts to do without abstracta altogether.

This move of Craig’s is, however, altogether too quick, and an apparent result of lumping together *all* purported abstracta (in this case, specifically, mathematical objects as abstract particulars, and properties as abstract universals) under one umbrella. The motivation for a Platonic account of *properties* as abstracta may well be the “Platonistic ontological assay of things,” according to which properties must exist metaphysically prior to being exemplified in substances. But the primary motivation for *mathematical* Platonism is entirely separate from the motivation for Platonism over nominalism as an account of universals, residing in an account of the semantics of claims that we take to be uncontroversially true. Abandon a Platonistic account of properties as *universals*, and the problem of the apparent necessity of positing mathematical objects such as numbers as abstract *particulars* remains. But if we drop the Platonistic account of universals, the bootstrapping objection no longer threatens an account of mathematical objects as either created abstracta or divine concepts. Furthermore, the abandonment of the so-called “Platonistic ontological assay of things” as provided by the Platonistic account of universals may actually *require* that we hold on to mathematical objects as abstract particulars. Thus, for example, Gonzalo Rodriguez-Pereyra’s defence of *resemblance nominalism* (*Resemblance Nominalism: A solution to the Problem of Universals* (Clarendon Press, 2002)) offers a nominalist account of universals as *sets of resembling particulars*, and so, at least on the face of it, requires acceptance of the existence of *sets* as abstract particulars. A rejection of the traditional Platonist’s abstract universals, coupled with the acceptance of realism about mathematical objects (as either abstract particulars or divine ideas) is perhaps better motivated than Craig suggests.

So it seems that, if we reject the Platonistic account of universals, realism about mathematical objects as God’s creations may be pursued by the theist either via a defence of divine conceptualism or of absolute creationism. But *ought* we be realists about mathematical objects? In the second half of his book Craig considers a version of the standard philosophical argument in favour of realism about mathematical objects – the so-called *indispensability argument* for mathematical Platonism – and (appealing to a number of recent anti-Platonist responses to this argument) finds it wanting. It is here where I must declare an interest, as a defender of one such response (Leng, *Mathematics and Reality* (Oxford University Press, 2010)). So although as the above considerations indicate, I suspect that, *if* we had reason to accept the existence of mathematical objects, then this would not be fatal to the standard doctrine of divine aseity (since divine conceptualism and abstract creationism remain in play), nevertheless, I am broadly in agreement with Craig that the indispensability argument for the existence of mathematical objects can be resisted, so that we do not have reason to accept the existence of mathematical objects. The core of the response I favour is that offered by those who have come to be known (following Colyvan, “There is no easy road to nominalism,” *Mind* 119 (2010): 285-306) as “*easy road”* nominalists. Such nominalists argue that, while quantification over mathematical objects may turn out to be indispensable to our best scientific theories, nevertheless our continued reliance on such theories does not commit us to believe in the existence of mathematical objects, since it is plausible that what accounts for the success of our scientific theories is not that those theories are literally true, but that they are *nominalistically adequate*. If we think of an empirical scientific theory as a body of claims apparently relating “abstract” mathematical objects to “concrete” physical things, the claim that empirical science is nominalistically adequateis roughly, as Mark Balaguer puts is, the claim that “the physical world holds up *its end* of the ‘empirical science bargain’” (*Platonism and Anti-Platonism in Mathematics* (Oxford University Press, 1998), 134) – that is, in all physical respects things are the way they would have to be for our theory to be true.

Craig’s discussion of such nominalist responses to the indispensability argument is at once intriguing and frustrating. Intriguing because, as a relative outsider to this debate, his focus is on issues that are often brushed aside in the philosophy of mathematics literature. Chapter 8, for example, includes an extended discussion of the question of whether fictionalism is self-defeating, raising the question of what attitude fictionalists should take to “useful fictions” themselves (an interesting issue, though one that I think the fictionalist has the resources to solve). Frustrating because, again as a relative outsider to the debate, Craig mischaracterizes a number of the key issues in a way that skews his response to some of the standard *easy road* positions. Most problematic in this regard is Craig’s presentation of the indispensability argument itself, which he takes from Mark Balaguer’s (2009) *Stanford Encyclopedia of Philosophy* entry on “Platonism in Metaphysics.” The argument quoted is certainly not what is standardly taken to be the indispensability argument for Platonism, and a quick check of the *Stanford* entry shows why: the argument quoted is what Balaguer calls “the singular term argument,” which he says originates with Plato but is first clearly formulated by Frege. The argument is roughly the one given at the start of this review: that there are literally true sentences whose truth would require the existence of abstract (mathematical) objects, so we should conclude that such objects would exist.

Craig singles Balaguer’s “singular term argument” out for discussion apparently because to his mind it presents the key issues at work in the indispensability argument for Platonism “free of Quine’s more controversial theses” (45). *But this isn’t the indispensability argument.* In particular, the indispensability argument contains two key features that are not present here. First is a justification for taking some claims about mathematical objects as literally true: this is because some such claims are indispensable in formulating our best empirical-scientific theories, and (according to the Quinean picture) we are committed to accepting those theories as true at least until something better comes along. Second is a focus on indispensable *existential quantification* over mathematical objects in our theories, rather than on the use of singular terms apparently referring to mathematical objects. Here, Quine is well aware that we may sometimes use singular terms to speak “as if” they refer to objects, without really being committed to the existence of such objects. The test, for Quine, for genuine commitment is whether we are willing – in our most serious, literal, moments, i.e., in our tidied up versions of our best scientific theories – to “quantify in,” moving from, e.g., Fa to ∃xFx. For Quine, the use of the existential quantifier in contexts where we take ourselves to be speaking literally is the indicator of ontological commitment. If science (whose claims must be taken seriously), in its most careful moments requires existential quantification over mathematical objects, then we ought to accept the existence of such objects.

The reason I bring this up is that much of Craig’s discussion of “the indispensability argument” is focussed on rejecting the Quinean account of ontological commitment, and here much attention is paid to examples of uses of non-denoting singular terms in sentences that are plausibly taken to be literally true. Craig suggests that, rather than taking singular terms as automatically referring, we could adopt a positive free logic and allow for claims using mathematical singular terms to be literally true. This is the approach Craig favours, as he thinks it is implausible to claim (as I have done, much to Craig’s amusement (149)!) that, since “2” and “4” do not refer, “2 + 2 = 4” is not literally true. But, even if we allow Craig the use of a positive free logic and thus the literal truth of unquantified claims of arithmetic such as that “2 + 2 = 4,” there remains the issue that *in empirical science* we do not restrict ourselves to unquantified arithmetic; we also *quantify in*. In positive free logic, the use of the existential quantifier is taken to indicate ontological commitment, so even if the unquantified claims of elementary arithmetic can be literally true without requiring the existence of mathematical objects, the mathematical claims used in empirical science still does seemingly require the existence of such objects.

A further move, considered by Craig, would be to follow Jody Azzouni (*Talking about Nothing: Numbers, Hallucinations, and Fictions* (Oxford University Press, 2010)) in denying that even the existential quantifier is indicative of ontological commitment. Craig has some sympathies towards this position, but ultimately pulls back from endorsing Azzouni’s view wholesale. Instead, following Theodore Sider (*Writing the Book of the World* (Oxford University Press, 2011)), he claims that, while most ordinary uses of the existential quantifier are not ontologically committing, we can introduce a new language, *Ontologese*, whose quantifiers are stipulated as indicative of ontological commitments, and argues that we should use *this* language when we are presenting our most serious metaphysical claims about the nature of reality. Craig thus endorses a positive free logic as the proper logic of *Ontologese*, suggesting that when we are presenting our most serious metaphysical account of reality existential quantifiers, but not singular terms, should be read as indicative of commitment to the existence of objects.

Again, this is all very well and good, but, in light of the Quinean theses mentioned above, this makes clear how close Craig’s position on ontological commitment is, in fact, to Quine’s. In both cases, it is existential quantification, rather than the mere use of singular terms, that is indicative of genuine ontological commitment, and, in both cases, even existential quantifiers are only indicative of genuine ontological commitment *when we are speaking seriously and literally about ontological matters*. All Quine adds to this is the claim that *empirical science* (rather than, e.g., the metaphysics classroom) provides the context where we speak seriously and literally about ontological matters: if we are aiming to carve nature at its joints, Quine says we can do no better than trust what our scientific theories say about how to do so.

Despite, then, Craig’s efforts to challenge the Quinean account of ontological commitment assumed by myself and others in the debate over the indispensability argument, what is *really* required for his response to Quine to work is for him either to show that we can *dispense* with quantification over mathematical objects in our best scientific theories (this is Field’s *hard road* to nominalism; *Science without Numbers* (Princeton, 1980)), or to make plausible the claim that we need not – and ought not – take seriously all the claims of even our best scientific theories (this is the *easy road* offered, in various ways, by myself, Jody Azzouni (*Deflating Existential Consequence: A Case for Nominalism* (Oxford University Press, 2004), Mark Balaguer, (*Platonism and Anti-Platonism in Mathematics*); Stephen Yablo (“The Myth of the Seven,” in Mark Eli Kalderon, ed., *Fictionalism in Metaphysics* (Oxford University Press, 2005), 88-115), and others). Craig throws himself in with the easy road camp, and of course I am with him in thinking that this approach can be made to work, but again lack of engagement with the Quinean case for taking empirical science as the arena in which ontological matters are settled means that some of Craig’s defences of the “easy road” approaches seem beside the point. Thus, for example, Chapter 10 includes a very nice discussion of features of pure mathematics that make it apt for a ‘make-believe’ account, but the Quinean could concede all of that and argue that nevertheless, once mathematical theories make their way into empirical science, they get confirmed as literally true. Even as a reader whose sympathies lie firmly with easy road nominalism, then, I was left with the feeling that, while Craig’s thinking is along the right lines in endorsing a nominalist approach in the philosophy of mathematics, nevertheless he has not quite appreciated the strength of the case against.

In relation to the case against nominalism, I would like finally to note a feature of nominalist approaches in the philosophy of mathematics, both in the “easy road” and the “hard road” camp, that is overlooked by Craig, and that has led some to question how much is actually gained by the nominalist rejection of the Platonist’s abstract objects. In almost all cases (with, perhaps, Weir (*Truth through Proof* (Oxford University Press, 2010)) as a notable exception), nominalists in the philosophy of mathematics propose to do without abstract mathematical objects by means of a trade of ontology for modality. Thus, for fictionalists both of the easy road and hard road stripe, what is *fictional* in a given pure mathematical theory is its axioms and their logical consequences. And again, in the case of applied mathematics, the nominalistic adequacy of a mathematically-stated empirical theory is a matter of it being *consistent with* any nominalistic facts, and thus *logically implying* no false nominalistic claims. Indeed, Craig’s concern about the potential “self-defeating” nature of fictionalism is answerable once we introduce a modal element (Craig touches on this when he suggests that “reference to stories, theories, models, and the like” can be dispensed with in the fictionalist account in favour of “counterfactual claims about what mathematical entities would be like were they to exist” (165)). Acceptance of primitive modal facts about logical consequence allows the fictionalist to make good on such “counterfactuals”: they are, strictly, claims about what follows logically from the assumption that there are objects satisfying a given theory’s axioms. Fictionalist (and related nominalist) accounts in the philosophy of mathematics are heavily reliant on the assumption that there are primitive (unreduced) modal facts about *what follows from what*, and that we can come to know some such facts.

In the context of the philosophy of mathematics, where the standard challenge for Platonism is the “knowledge problem,” those in the Platonist camp may reasonably question whether anything has been achieved by this attempt to do away with a problematic ontology by trading ontology for modality. By introducing primitive modal facts, perhaps the nominalist has just replaced one mystery (how we can come to know abstract objects) with another (how we can come to know facts about what follows from what)? On the other hand, the Platonist will claim, if we equip ourselves with abstract mathematical objects, then we can do away with mysterious primitive modal facts. Once we have the set theoretic universe, then model theory provides us with a reductive account of modal truths as truths about set theoretic models (though see Shalkowski (“The Ontological Ground of the Alethic Modality,” *The Philosophical Review*103 (1994): 669-688) for a dissenting voice about the prospects of reductive accounts of modality). If we shift this debate to the philosophy of religion, analogous worries may appear relating to Craig’s attempt to bypass the challenge to divine aseity by doing away with abstract objects. If the rejection of abstract objects requires the acceptance of primitive modal facts, then one wonders whether the challenge of divine aseity is avoided only at the cost of reintroducing the challenge of divine *sovereignty*.

Craig takes pains in Chapter 4 to distinguish between Platonism’s challenge to divine aseity and its challenge to divine sovereignty, while noting that the two are often run together. Craig’s interest is in the challenge to divine aseity, so he largely sets the sovereignty issue aside. But the substantial modal assumptions made by nominalists in the hope of avoiding abstracta bring the challenge of divine sovereignty to front of stage. If the rejection of abstract objects requires the acceptance of primitive modal facts governing what would have to be true were there objects satisfying our mathematical axioms, then the question arises as to whether God has sovereignty over these facts. This is not to say that the challenge of modal facts to divine sovereignty is as pressing as the challenge of abstract objects to divine aseity, but just to note that the nominalist proposal to do away with abstract objects in the philosophy of mathematics does not come for free, and that the resulting modal commitments may bring with them problems of their own. Despite Craig’s best efforts to avoid the issue of divine sovereignty in this book, then, the completion of his proposed solution to the problem of divine aseity may well require some with substantial further work to be done on this topic.