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A Comparison between a Shakedown Design Approach and the Analytical Design Approach in the UK for Flexible Road Pavements

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Abstract
Recently a shakedown approach has been proposed for structural design of flexible road pavements (Wang and Yu, 2013a). This new approach makes use of both elastic and plastic properties of materials, and therefore represents an advance from the existing analytical design approach in the UK where pavement life is related with elastic strains at critical locations using empirical equations. However, no direct comparison between designs using these two approaches has been made to date. In this paper, following a brief review of both approaches, the shakedown approach based on Wang and Yu (2013a) is used to design layer thicknesses for a typical asphalt pavement considered in the analytical approach TRRL Report LR1132. Typical values of plastic parameters are chosen for pavement materials at temperature 20°C, while stiffness moduli of materials are kept identical with the analytical design. The resulting shakedown designs are then compared with the thickness design chart using the analytical design approach. And the influence of temperature on the shakedown-based thickness design is also discussed in detail. It is found that if the shakedown design approach is conducted against the maximum wheel pressure at a relatively high temperature, the resulting pavement structure will probably not fail due to excessive rutting within the service life.

Keywords: shakedown design; analytical design; flexible road pavements; temperature

1 Introduction

Pavement design is a process aiming to find a combination of layer thicknesses and material types which can carry the designed load safely and economically during the service life. Current design methods for flexible pavements can be divided into two categories: one is empirical approach which utilizes design charts or empirical equations developed from experimental works and field tests, such

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as the standard design method in the UK; the other is mechanistic-empirical approach (also called analytical design approach in the UK), in which elastic stresses or strains at critical points are related to pavement life considering principle failure modes of pavements. The latter approach can maximize the whole life value by choosing different materials and layer thicknesses and therefore become increasing popular around the world. However, one major limitation of this analytical design approach is that strength properties of pavement materials are not well considered, especially for the rutting failure which is attributed to material plasticity.

In recent decades, an elastic-plastic shakedown concept has been widely recognized as a possible new basis for the design of flexible pavements. Shakedown is known as a phenomenon that an elastic-plastic structure, though deformed plastically in initial load cycles, respond purely elastically to subsequent load cycles if the applied load is above the yield limit but lower than a critical load (called 'shakedown limit'). For flexible pavements, shakedown is recognized as the pavement rutting depth ceases to grow with increasing pavement life, as reported by a number of researchers (e.g. Sharp and Booker, 1984; Ravindra and Small, 2008; Brown et al., 2008，2012). Most recently, Yu and Wang (2012) developed a method which can predict the shakedown limit of a three-dimensional half-space under moving loads. This method was further developed to design flexible pavements (Wang and Yu, 2013a,b; Wang and Yu, 2014). And a pavement structure designed in this way is supposed to remain a very small rutting depth throughout its service life or longer. In this paper, the shakedown approach will be directly compared with the analytical approach in the UK through a typical thickness design.

2 Analytical Design Approach in the UK

According to Design Manual for Roads and Bridges HD26/06 (Highways Agency, 2009), TRRL Report LR1132 (Powell, 1984) provides guidance that should be considered in the preparation of analytical flexible pavement design. Two principle failure modes are considered in this approach: fatigue cracking and excessive rutting. While the excessive horizontal tensile strain at the bottom of the bound layer ε_r leads to fatigue cracking, the excessive vertical compressive strain at the top of subgrade ε_z is related to pavement rutting. Empirical equations are then used to link the pavement life with the critical strains. Report LR1132 suggested the following empirical correlations for Dense Butumen Mecadum (DBM) and Hot Rolled Aspahlt (HRA) at 20°C:

Critertion against fatigue: $\log N = -9.38 - 4.16 \log \varepsilon_r$ for DBM (100pen), Eq. 1

$\log N = -9.78 - 4.32 \log \varepsilon_r$ for HRA (50pen), Eq. 2

Criterion against rutting: $\log N = -7.21 - 3.95 \log \varepsilon_z$, Eq. 3

where N is the number of standard axles (in millions).

3 Shakedown Design Approach

The shakedown design approach was developed based on the static shakedown theorem of Melan (1938). The theorem states that an elastic-perfectly plastic structure under cyclic or variable loads will shakedown if a self-equilibrated residual stress field exists such that its superposition with the load-induced elastic stress field does not exceed the yield criterion anywhere in the structure. In the shakedown design approach of Wang and Yu (2013a), a critical self-equilibrated residual stress field was introduced to the pavement shakedown problem. Then the pavement shakedown limit can be obtained by solving the following optimization problem:

$$\max \lambda,$$
$$\text{s.t.}\begin{cases} f\left(\sigma^{r}_{xx}\left(\lambda\sigma^{e}\right), \lambda\sigma^{e}\right) \le 0, \\ \sigma^{r}_{xx}\left(\lambda\sigma^{e}\right) = \min\limits_{z=j}\left(-M_i + \sqrt{-N_i}\right) \text{or } \sigma^{r}_{xx}\left(\lambda\sigma^{e}\right) = \max\limits_{z=j}\left(-M_i - \sqrt{-N_i}\right), \\ M = \lambda_n\sigma^{e}_{xx} - \lambda_n\sigma^{e}_{zz} + 2\tan\phi_n(c_n - \lambda_n\sigma^{e}_{zz}\tan\phi_n), \\ N = 4(1+\tan^2\phi_n)[(\lambda_n\sigma^{e}_{xz})^2 - (c_n - \lambda_n\sigma^{e}_{zz}\tan\phi_n)^2], \end{cases} \qquad \text{Eq. 4}$$

where σ_{ij} is elastic stress field due to an unit pressure, which can be obtained by using finite element method; c_n and ϕ_n are cohesion and friction angle of the material at the *n*th layer, respectively; λ_n is a scale parameter. The obtained maximum λ_n for each layer is denoted as λ^{n}_{sd} thus the shakedown limit of each layer denoted as $\lambda^{n}_{sd}p$. Finally, the shakedown limit of the whole pavement structure is obtained using Eq. 5. The shakedown limit then can be used as the maximum admissible load in the design against pavement rutting.

$$\lambda_{sd}p = \min\{\lambda^{1}_{sd}p, \lambda^{2}_{sd}p, \ldots, \lambda^{n}_{sd}p\}. \qquad \text{Eq. 5}$$

4 Comparison

4.1 A Typical Pavement Problem

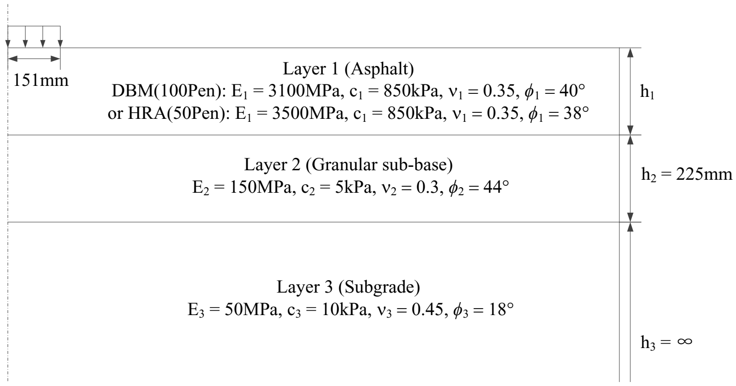

Figure 1: A flexible pavement structure and material properties

Figure 1 shows a typical flexible pavement structure which was used as an example in the report LR1132. E_n, ν_n, c_n, ϕ_n and h_n represent stiffness modulus, Poisson's ratio, cohesion, friction angle and thickness of materials at the *n*th layer, respectively. The first layer is either dense bitumen macadam (100 pen) or hot rolled asphalt (50 pen) with stiffness modulus 3100MPa or 3500MPa under a temperature of 20°C. CBR value of the subgrade soil is chosen as 5 percent and therefore its stiffness modulus is 50MPa and no capping layer is needed. Also, stiffness modulus of the subbase granular layer should be 150MPa with a maximum layer thickness 225mm. In the shakedown approach, friction angle and cohesion of each material are also required. Compared with abundant information for the strength of granular materials and soils, limited data can be found for standard hot mixture asphalt (summarized in Table 1). The strength properties of the asphalt mixture depend on various factors. For example, aggregate grading affects the friction angle, while the binder (bitumen) content and grade influence the material cohesion. This may explain the wide spread of values in Table 1. Considering the deformation resistance of DBM is usually higher than HRA (Thom, 2009), a slightly smaller friction angle is chosen for HRA while the same value of cohesion is used.

In both methods of design, it is also necessary to know the contact area between tire and pavement. It is usually assumed that each tire has a circular contact area. In the report LR1132, a contact radius of 0.151m and a standard wheel load of 40kN are used. Therefore, an average contact pressure 558kPa should be applied in the analytical design approach. It should be noted that the contact pressure is generally considered to be equal to the inflation pressure of tire, value of which can vary from 250kPa for a car to 3000kPa for aircraft (Huang, 2004; Thom, 2008). In spite of that, most pavements take the highest axle loads from track tires, the inflation pressure of which can be reach 860kPa for both single and dual configurations according to Michelin product specifications (e.g. XTE2). This means that the maximum contact pressure on most pavements could be 860kPa.

Reference	Type of asphalt mixture	T (°C)	c (kPa)	ϕ (°)
Airey and Prathapa (2013)	SMA	NA	NA	34.6
	DBM	NA	NA	41
Bindu and Beena (2013)	SMA	60	109	35
Chen et al. (2009)	SMA	25	420	43.3
		40	245	42.8
		60	204.4	38.6
Christensen et al. (2000)	NA	20	571-933	20.4-44.8
Fwa et al. (2004)	NA	28	1768.8	15.1
		40	616.4	33.4
		60	290.0	36
Zofka et al. (2014)	NA	25	760-1110	13.8-57.5

Table 1: Strength properties of asphalt mixture (NA = not available)

4.2 Thickness Design

Contour plots (Figure 2 and Figure 3) show the number of millions of standard axles that the pavement can afford (i.e. pavement life N) for various values of the contact pressure and asphalt thickness. In the analytical design approach, the contact pressure should be chosen as 558kPa which corresponds to the standard axle load 80kN. Figure 4 further exhibits the required asphalt thicknesses for various pavement lives when the design pressure is 558kPa. By the way, in the cases studied here, pavement rutting criterion is always more critical than the fatigue criterion according to Eqs. 1-3.

The shakedown limit (expressed as contact pressure) against the asphalt thickness is also displayed as dash lines in Figure 2 and Figure 3. The shakedown limit represents the maximum contact pressure that the pavement can withstand. Given the maximum possible pressure 860kPa, the corresponding asphalt thickness should be at least 315mm for DBM and 300mm for HRA. One should highlight that whether a pavement shakes down or not is controlled by the maximum applied load; therefore the contact pressure used here is 860kPa instead of 558kPa. In addition, it is interesting to notice that the shakedown design curve is very close to the analytical design curve when the pavement life is 3.5msa.

The shakedown-based thickness designs are also marked in Figure 4. It demonstrates that these designs (i.e. 315mm for DBM and 300mm for HRA) are identical with those from the analytical approach if the pavement life is 18msa. That is to say, in the case of 20°C, if the design life is at or below 18msa, the shakedown-based approach is safer; otherwise, the analytical design approach is more conservative.

By using the shakedown approach, it is also possible to identify which layer is more critical (i.e. more susceptive of rutting). It is found that the shakedown limit of the granular layer is always the minimum one among all layers in these cases (i.e. the granular layer is more critical in the current problem). However, one should bear in mind that for comparison purpose the temperature was kept as 20°C throughout the study. The real pavements should be subject to the change of air temperature which will alter material properties thus the capacity of pavements. For this reason, the effect of temperature on the shakedown-based designs will be discussed in the following subsection.

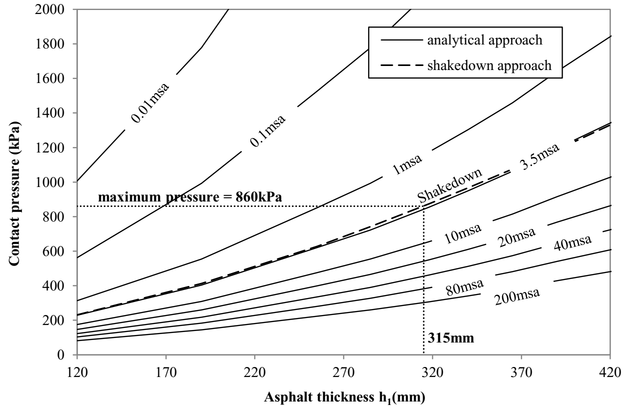

Figure 2: Comparison of DBM thickness designs

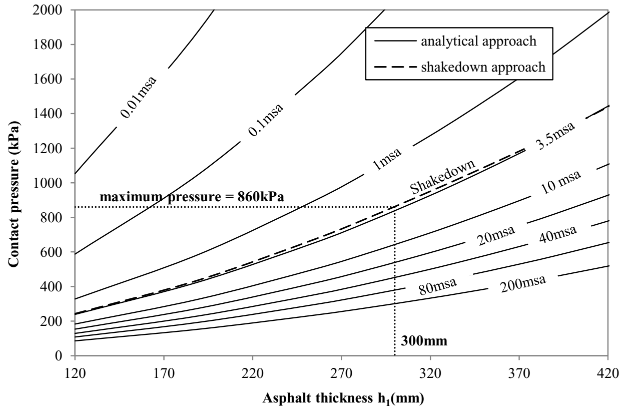

Figure 3: Comparison of HRA thickness designs

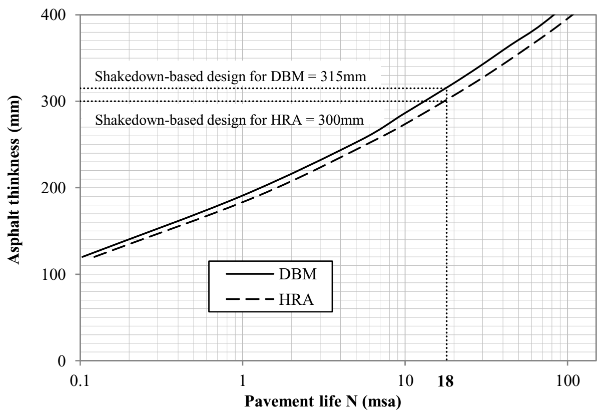

Figure 4: Comparison between analytical design curves and shakedown-based design

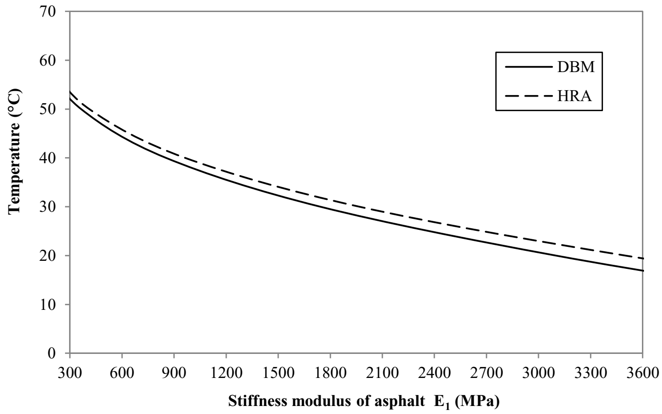

Figure 5: Influence of temperature on asphalt stiffness

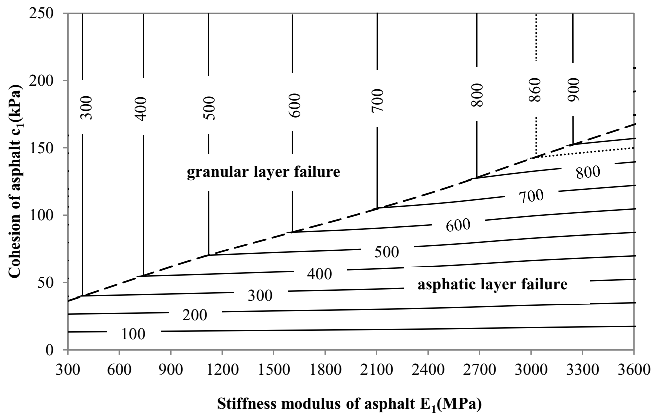

Figure 6: Pavement shakedown limits for various values of asphalt cohesion and stiffness (kPa)

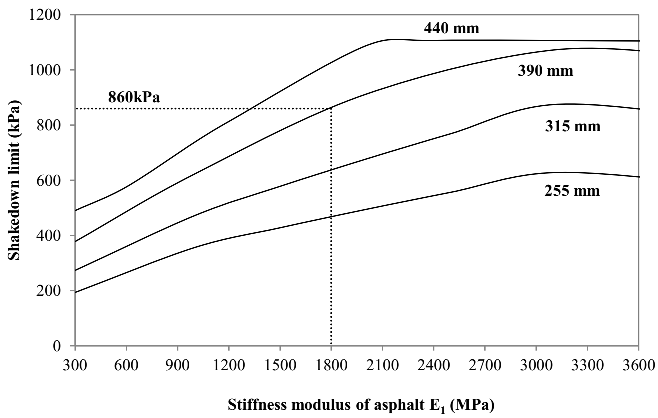

Figure 7: Influences of asphalt stiffness and layer thickness on the shakedown limit (c_1 = 150kPa)

4.3 Influence of Temperature

The change of air temperature will change the pavement responses to repeated moving loads. It is commonly known that pavements rut more under higher temperature. In order to guarantee a pavement will shakedown within its service life, shakedown-based designs must be undertaken by considering the most critical situation (i.e. at the highest temperatures).

The increase of temperature obviously changes the asphalt stiffness modulus and cohesion, while its effect on asphalt friction angle may be relatively small (Yang et al. 2009). In this study, the friction angle of the asphalt mixtures was decreased slightly to 35 degrees and the layer thickness is fixed to 315mm. Eq. 6 (Thom, 2008) was used to calculate stiffness modulus of asphalt at various temperature. Results are plotted in Figure 5 for both DBM and HRA.

$$\log(E_T) = \log(E_{20C}) - 0.0003 \times (20 - T)^2 + 0.022 \times (20 - T), \qquad \text{Eq. 6}$$

where E_T is the stiffness modulus at a specified temperature (T) and E_{20C} is the stiffness modulus at 20°C.

The interactive influences of asphalt cohesion and stiffness modulus on the pavement shakedown limit is exhibited in Figure 6. On the lower side of the dash line (i.e. asphalt cohesion is relatively low), the asphalt layer is more critical, and the shakedown limit drops obviously with reducing cohesion and increases slightly with decreasing stiffness. On the upper side of the dash line, the granular layer is more critical, and the pavement shakedown limit will not change with the asphaltic cohesion. If the maximum possible contact pressure is 860kPa, shakedown can only be reached when the cohesion is above 145kPa and the stiffness is above 3000MPa which means 21°C in DBM and 23°C in HRA.

The increase of the asphalt layer thickness can definitely increase the pavement shakedown limit as shown in Figure 7 for various values of asphalt stiffness modulus. Therefore at a relatively high temperature in the UK (say 30°C), the asphalt stiffness modulus is reduced to 1800MPa, so a minimum thickness of 390mm is required to support the maximum contact pressure 860kPa. According to Figure 4, this thickness can withstand around 80msa which is also the desired pavement life suggested in the report LR1132 for most flexible pavements.

5 Concluding Remarks

In this paper, thickness designs using both the analytical approach in the UK and the shakedown approach of Wang and Yu (2013a) are compared in details. It is found that if the standard temperature is 20°C, the analytical design approach is more conservative for a busy road (more than 18msa in the present study). If a relatively high temperature (e.g. 30°C) is used in the shakedown design, the designed asphaltic layer will be as thick as the one obtained by the analytical approach for a pavement life around 80msa. Further growth of temperature will require thicker asphalt which is even safer than the analytical approach. Therefore, the shakedown approach for flexible pavement design should be conducted considering the maximum contact pressure and a high air temperature (at least 30°C in the UK). Such a design then will be able to withstand long-term traffic loading without rutting failure.

Acknowledgements

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