ARTICLE TEMPLATE

The influence of frequency normalisation of FWD pavement measurements on backcalculated values of stiffness moduli

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ABSTRACT
Backcalculation is commonly used for determining stiffness moduli of pavement courses and its subgrade based on in-situ deflection tests. However, the in situ test results are dependent on the frequency domain, which in turn has a significant influence on the backcalculation results. In this paper, one of the methods for normalising the frequency of load and displacement functions obtained from a dynamic impulse generated by a Falling Weight Deflectometer (FWD) test is verified. The assessment of the method is carried out based on backcalculation results of pavement deflection tests induced by both the dynamic and static loads and an independent numerical experiment. The analysis carried out in this paper shows that the normalisation of vertical displacements and pavement loads to the values corresponding to the frequency of 0 Hz improves the accuracy of the backcalculation results. It can be concluded that the frequency normalisation procedure is an effective way for standardising the boundary conditions in the backcalculation of flexible pavements.

KEYWORDS
Pavement diagnostic; backcalculation; FWD; frequency normalisation; deflection; stiffness modulus

1. Introduction

It is a common practice to use backcalculation for determining stiffness moduli of pavement courses and its subgrade based on in-situ tests Hilmi Lav, Burak Goktepe, and Aysen Lav (2009); Irwin (2004); Lee and Kim (2011); Saltan, Terzi, and Kucuksille (2010); Sharma and Das (2008). The data usually come from the results of deflection test, pavement layers' recognition, the type and thickness of individual pavement courses. Backcalculation results are usually encumbered with a series of simplifications in pavement models. The application of models accurately rendering the real conditions in the pavement (thermodynamic models Graczyk (2010) or accurate dynamic models El-Ayadi, Picoux, Lefeuve-Mesgouez, Mesgouez, and Petit (2012); Picoux, Ayadi, and Petit (2009)) usually requires determining a larger number of calculation parameters (which proves difficult in practice), compared to less complicated models such as: MET (Method of Equivalent Thickness) El-Badawy and Kamel (2011); Subagio, Cahyanto, Rahman, and Mardiyah (2005), LET (Layers Elastic Theory model) Firlej (2007),

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and empirical models Wu and Yang (2012). Having in mind determination uncertainties of parameters such as temperature spread inside the pavement Graczyk (2010), subgrade moisture Carrera, Dawson, and Steger (2009), heat flow indices Graczyk (2010), damping parameters Qing-long, Xin, and Qing (2014); Qiu, Ling, and Fang (2008), indices defining the inter-layers connections Drenth (2010), it is still possible to find a place for consensual solutions with a minimum number of input parameters. Therefore, by referring to the conditions during one of the most common pavement in-situ test – a falling-weight deflectometer (FWD) or Integrated System for Precise Pavement Evaluation (ZiSPON) technology as it is described in Pożarycki, Górnaś, and Sztukiewicz (2017) - the user stands before the task of performing an analysis of the dynamic reaction of the tested structure. Considering the methods compromising between determination uncertainty of model's parameters and their number, the appealing choice seems to be the pavement model based on the elastic layer theory resting on the elastic half-space (LET model). However, choosing the static version of the LET model leads to the key issue of quantification errors, resulting from the application of the static calculation model to the analysis of data obtained from deflection tests of the pavement subjected to dynamic loads. A method of determining the parameters of pavement models based on LET that uses a method of transforming the dynamic displacements into their static substitutes is presented in Ruta, Krawczyk, and Szydło (2015); Ruta and Szydło (2005). Another procedure for considering the dynamic effect of the FWD test, which was examined by converting FWD dynamic surface deflection into pseudo-static surface deflection using pseudo-static correction factors, and a pseudo-static backcalculation, is described in Seo, Kim, Choi, and Park (2009). Using Mindlin plate theory and steady-state pavement response, Stolle and Guo (2005) also presented the method for pre-processing FWD data, which identifies the pseudo-static pavement response to surface loading. The analysis in the frequency domain was used in the method discussed by Guzina and Osburn (2002); Kang (1998); Westover and Guzina (2007). This method is governed by the functions of force and pavement deflections signals $F(t)_f$ and $u(t)_f$ respectively (t is time and f is signal frequency), which should be calculated to 0 Hz frequency, in order to obtain their values corresponding to the static conditions.

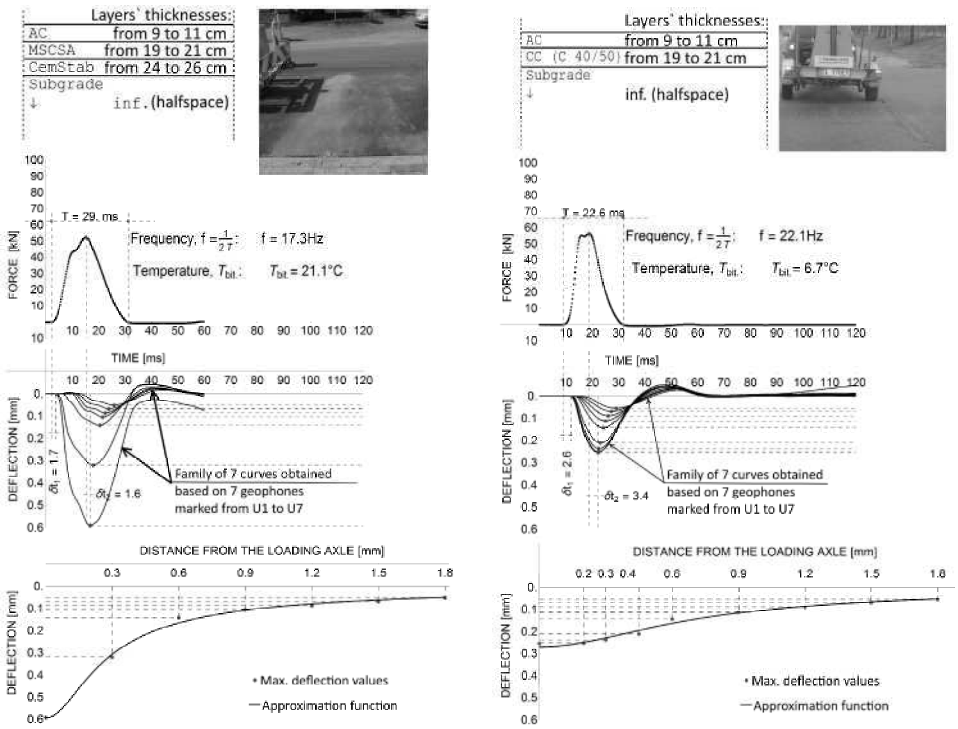
2. The purpose and scope of the paper

The purpose of this paper is to assess the influence of frequency normalisation of dynamic loads and deflections on the stiffness moduli of pavement layers. The assessment is carried out based on backcalculation results of pavement deflection tests induced by both the dynamic and static loads and an independent numerical experiment. The experiment was conducted to find the best match between the pavement deflections from the backcalculation simulations with impact dynamic loads and those calculated for the static models of loads. In any case, the study was focused on using a model with a small number of input parameters. Therefore the elastic model subjected to static loads was chosen for a back analysis.

3. Methods

3.1. *Pavement deflections curves - classic approach*

A deflection curve of the pavement, built by the use of maximum values of load and displacement functions in time, is the foundation of the classic backcalculation method. Selected examples of the analysis based on the measurements obtained from two different types of pavement are presented in Figure 1. The investigations carried out by the authors of this paper, let us notice, that despite the same load conditions assumed in both examples shown in Figure 1a and Figure 1b, the difference between the frequency of load function in time (the frequency was calculated following the simplified method discussed in the paper by Leiva-Villacorta (2012)) amounts to around 5 Hz (computed as the difference 22.1 - 17.3 = 5 Hz).



(a) Measurements of flexible pavement on a cement-stabilised subgrade.

(b) Behaviour of semi-rigid structure on a weak natural subgrade.

Figure 1. Classic set of the pavement deflection test results obtained by the same FWD device

3.2. *Numerical model of the layered structure*

Boundary conditions of the pavement calculation model were assumed adequately to the conditions corresponding with pavement in-situ tests conducted with an FWD-like device: 1) pavement load is uniformly distributed on the area of a circle with a radius equal to 15 cm, 2) shape of the deflection curve was estimated on the basis of calculations of vertical displacements on the surface of the uppermost course in the model, 3) values of vertical displacements were calculated at the locations separated from the load axis by the distance of: 0, 0.3, 0.6, 0.9, 1.2, 1.5 and 1.8 m (further in the paper displacements are marked with symbols from $U1$ to $U7$, respectively). A numerical pavement model was based on the finite element method (FEM). The scheme of the used model is shown in Figure 2.

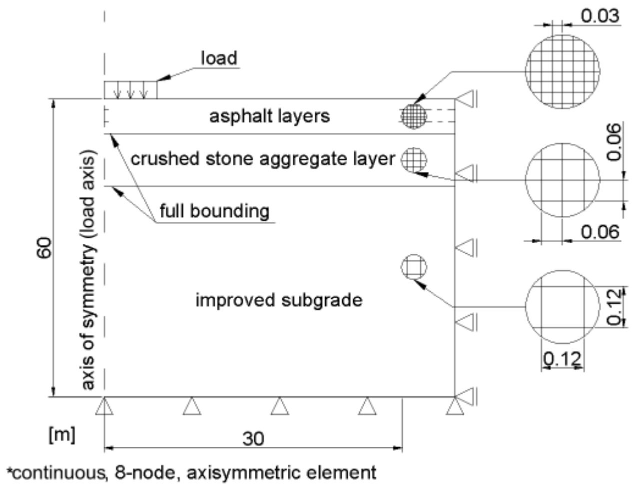


Figure 2. The general scheme of the FEM model used in calculations

An implicit dynamic analysis using direct integration in FEM was carried out in Matlab – Abaqus environement. Further basic parameters of the FEM model were adopted based on the principles described in Górnaś and Pożarycki (2014) taking into account in-situ characteristics of the pavement.

3.3. *Frequency normalisation*

The method to eliminate the influence of pavement dynamic load on the backcalculation results, was based on a static model and the ideas described by Guzina and Osburn (2002); Kang (1998); Westover and Guzina (2007). The idea was to use the values of pavement vertical displacements obtained after normalising the frequency of FWD measurement signals, that is $F(t)$ and $u_r(t)$ (loads in function of time and displacements in function of time, respectively), measured at distances r from the load axis. The Fourier transform, expressed with the formula (1), can be used for such a normalisation. The limits of the integral assumed in this paper range from -T = -∞ to +T = +∞.

$$\hat{f}(\xi) = \int_{-T}^{T} f(t) e^{-2\pi i t \xi} dt \tag{1}$$

When the procedure of signal normalisation $F(t)$ and $u_r(t)$, schematically described with expressions (2) and (3), is used in backcalculation, the only values that need to be considered are those corresponding to the frequency of 0 Hz.

$$FFT[F(t)] \rightarrow F(f) \rightarrow F(f = 0) \tag{2}$$

$$FFT[u_r(t)] \rightarrow u_r(f) \rightarrow u_r(f=0) \tag{3}$$

In theory, applying this normalisation allows one to assume that the obtained results are compliant with the measurement results of the pavement deflection tests subjected to static loads.

3.4. *Objective function for backcalculation*

Herein, the input data are comprised of a set of values of vertical displacements measured on the surface (or a set of values from the computer simulation of the experiment), the known system and the thickness of pavement courses, as well as mechanical calculations. The searched values are the elasticity moduli of the layers and calculations are iterative. The objective function was adopted as AVCF (Area Value with Correction Factor) criterion proposed by Sangghaleh et al. (2013), which is described with formula (5).

$$\delta_{AVCF} = \left\{ \frac{1}{n-1} \cdot \sum_{k=1}^{n-1} \left(\frac{A_k^c - A_k^m}{A_k^m} \right)^2 \right\}^{1/2} + \left| \frac{d_1^c - d_1^m}{d_1^m} \right|, \tag{4a}$$

$$A_k = \frac{\sum_{i=1}^{k-1} (d_i + d_{i+1}) \cdot (r_{i+1} - r_i)}{2 \cdot d_1}, (k \leq n), \tag{4b}$$

where: n - the number of locations where vertical displacements of the pavement were determined at one measuring station, A_k - area limited with a deflection curve for the first k measurement points, d_i - deflection value at i measurement point, r_i - distance of the i measurement point (which is the location of the displacement sensor, a geophone) from the load axis, c, m - indices of calculated/measured values of vertical displacements.
For minimisation, the authors used ,,Nelder Mead” optimising algorithm, more extensively discussed in Fuchang and Lixing (2012).

4. Synthetic uniform section analysis

The presented computer simulations are based on six uniform sections classified with respect to the thicknesses of pavement layers.

4.1. *Forward simulation issues*

4.1.1. Basic modelling data

The pavement models for each section A, B, C, D, E and F are shown in Table 1. Thicknesses are pseudorandom values randomized from the normal distribution.

Table 1. Modelling data set of six synthetic uniform sections.

Uniform section	Number of pavement model	Layer of the model						
		HMA			MSCSA			Subgrade
		h_1	Mean(h_1)	CoV_1	h_2	Mean(h_2)	CoV_2	h_3
A	1 ... 5	0.150 ... 0.150	0.150	0.00	0.200 ... 0.200	0.200	0.00	∞
B	6 ... 10	0.170 ... 0.170	0.170	0.00	0.200 ... 0.200	0.200	0.00	∞
C	11 ... 15	0.170 ... 0.170	0.170	0.00	0.200 ... 0.200	0.200	0.00	∞
D	16 17 18 19 20	0.158 0.152 0.151 0.154 0.161	0.155	2.71	0.181 0.245 0.266 0.250 0.234	0.235	13.78	∞
E	21 22 23 24 25	0.171 0.181 0.169 0.150 0.154	0.165	7.75	0.244 0.187 0.245 0.183 0.244	0.221	14.75	∞
F	26 27 28 29 30	0.171 0.158 0.150 0.161 0.181	0.164	7.33	0.181 0.182 0.198 0.201 0.252	0.203	14.28	∞

HMA – Hot Mixed Asphalt layer

MSCSA – Mechanically Stabilised Crushed Stone Aggregate layer

h - thickness of particular model‘s layers, [m]

Mean(h) - average thickness of model‘s layers for a given uniform section, [m]

CoV - coefficient of thickness variation of model‘s layers for a given uniform section, [%]

The courses in sections A, B and C have constant thicknesses. The differences in thicknesses of the models D to F are caused by the interference of the mean values by the white noise. White noise parameters were determined based on the range of layer thicknesses adopted from the analysis of the results obtained from in situ Ground Penetrating Radar (GPR) tests.

4.1.2. Sensitivity to different load frequencies

Under the impact loading, the real hot mixed asphalt (HMA) pavement surface deflections depend, among other factors, on inertia forces and viscosity. Therefore, in order to study the sensitivity of the flexible pavement structures to different load frequencies, the HMA layers were numerically described by using both the dynamic impact loads and the linear viscoelasticity (LVE). The LVE model description was based on the functions of relaxation (a generalised Maxwell model was assumed) discussed in more detail in Pożarycki and Górnaś (2014b, 2014c). The calculation values of the parameters used in the numerical simulations of the test deflections performed by FWD-like devices are presented in Table (2).

Table 2. The HMA parameters used for FWD simulations.

Layer	Material‘s model	Uniform section	
		A, B, D, E	C, F
HMA	Linear ViscoElastic	G = 2431, g = 0.5789, τ = 0.0166, ν = 0.34 ρ = 2500 kg/m^3	G = 2018, g = 0.5423 τ = 0.0221, ν = 0.34 ρ = 2500 kg/m^3
MSCSA	Elastic	E = 400, ν = 0.3, ρ = 2500 kg/m^3	
Subgrade	Elastic	E = 60, ν = 0.3, ρ = 2500 kg/m^3	

where:

E - elasticity (Young's) modulus [MPa],

G - shear (Kirchhoff's) modulus [MPa],

ν - Poisson's ratio [-],

g [-] and τ [s] - Prony series parameters.

In the forward calculations of viscoelastic material, the authors used the functions of relaxation in the frequency domain (5a) and (5b).

$$G' = G_0 \cdot \left(1 - \sum_{i=1}^{n}(g_i) + \sum_{i=1}^{n}\left(\frac{g_i \cdot \tau_i^2 \cdot \omega^2}{1 + \tau_i^2 \cdot \omega^2}\right)\right) \tag{5a}$$

$$G'' = G_0 \cdot \left(\sum_{i=1}^{n}\frac{g_i \cdot \tau_i \cdot \omega}{1 + \tau_i^2 \cdot \omega^2}\right) \tag{5b}$$

where:
G' – real part of the complex modulus [MPa],
G'' – imaginary part of the complex modulus [MPa],
G_0 – value of the temporary modulus (so-called static modulus) [MPa],
n – the number of Prony series terms needed to achieve the acceptable error value of the approximation of relaxation function (here n = 1 was used),
ω – angular frequency [rad/s] acc. to Benedetto, Olard, and Sauzéata (2004); Deigan (2007).
The LVE parameters values were based on the laboratory investigation. The parameters of sections A, B, D, and E were modelled based on the HMA designed for the typical subbase asphalt course. The parameters of sections C and F are shaped according to the HMA designed for the wearing course. Finally, the analysis of the laboratory results of the stiffness moduli dependency on loading frequencies in the range of 1 to 20 Hz, let us find out that only one term of the Prony series guaranteed the acceptable approximation error value ($\leq$ 10%) in the numerical simulations.

4.1.3. Modelling of dynamic impact loads

In order to obtain the set of the numerical functions of displacements $u_r(t)$ under dynamic loads of varying frequency, the load functions were defined based on typical signals obtained from the deflection tests of pavement tested by the use of FWD device (Figure 3).

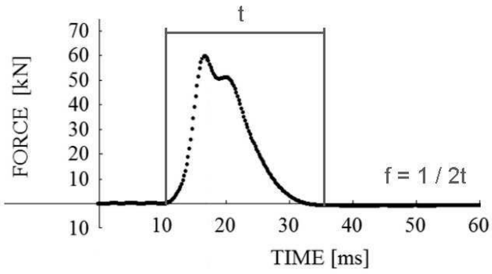


Figure 3. The scheme of determining the load frequency f

To simulate the load shapes, most similar to those generated in the FWD-like devices, the following boundary conditions for the in situ deflection tests were applied: 1) declared loading amplitude in FWD device was 50 kN, 2) three drops per one measurement place were applied, 3) the load shape from the third drop was used in the calculations. Finally, 30 loads shapes were randomized from the set of 125 real FWD measurements on the flexible pavement structure. For all the models in uniform sections, frequency f was calculated according to Leiva-Villacorta (2012). The results of those calculations are presented in Table 3.

Table 3. Frequencies and amplitudes of load functions for all analysed modelling cases

Uniform section	Number of pavement model[a]	Frequency[b] [Hz]	Amplitude [kN]	Uniform section	Number of pavement model[a]	Frequency[b] [Hz]	Amplitude [kN]
A	1	14.8	52.57	D	16	14.6	52.56
	2	14.7	52.54		17	12.2	53.11
	3	14.6	52.63		18	14.5	53.03
	4	14.6	52.63		19	14.5	52.66
	5	14.7	52.75		20	14.6	52.96
B	6	12.0	52.61	E	21	14.4	52.64
	7	14.6	52.49		22	14.6	52.73
	8	14.6	52.10		23	14.5	52.33
	9	13.0	53.03		24	14.4	53.04
	10	14.6	52.65		25	14.6	52.73
C	11	12.3	52.73	F	26	14.5	52.43
	12	14.6	52.91		27	14.5	52.82
	13	14.6	52.61		28	14.5	53.25
	14	12.1	52.56		29	14.5	52.91
	15	14.6	52.98		30	14.2	52.89

[a]Description in the form of ,,Number of pavement model” is equivalent to the ,,Location of the calculation point”.

[b]Based on these frequencies the average frequency was also used. This average quantity, equal to 14.2 Hz, was used in order to obtain the reference stiffness moduli for backcalculation purposes in the numerical experiment.

4.1.4. Vertical displacements from two different cases of dynamic loads

CASE No. 1: In the first step, a simple case of a sine function (6) was considered.

$$F(t, f) = A_1 \cdot sin(2\pi f \cdot t) \tag{6}$$

where: t - time [s], f - load frequency [Hz], A_1 - dynamic load amplitude [kN].

The vertical displacements obtained from the calculations are also described with a sine function. However, due to the viscoelastic HMA description, the displacements are shifted in time by the value of the phase angle compared to the load function, which is described by the equation (7).

$$u_r(t, f) = A_2 \cdot sin(2\pi f \cdot t + \varphi) \tag{7}$$

where: r - distance of the calculation point from the load axis in horizontal plane [m], φ - phase angle [^{o}], A_2 - displacement amplitude [μm].

Next, the maximum values of vertical displacement amplitudes for several different load frequencies, f = 0, 1, 5, 10, 15, 20 and 25 Hz, were calculated. The dynamic load amplitude was considered to be a constant value equal to 50 kN. As a result, displacements induced by the idealised set of dynamic load signals for all 30 pavement models were obtained.

CASE No. 2: In the second step, the loading models described in the previous subsection were used. Again, for all the 30 models, a set of results as a function of displacements in time for U1, U2, ..., U7 under the dynamic impact loads, were calculated and used to prepare:

- classic deflection curves (a curve based on the maximum displacement values referred to as max(U(t)) in this paper),
- deflection curves using the vertical displacement values after normalisation, that is, the values corresponding to the frequency of 0 Hz – the paper refers to this process as norm(U(t)).

The whole set of frequencies of the deflection functions $u_r(t)$ obtained for all the models in case no. 2 is presented in Table 4.

Table 4. Frequencies of the deflection functions under the dynamic impact load simulations

Section	Number of the model	Frequency, f [Hz]						
	(No. of data location point)	U1	U2	U3	U4	U5	U6	U7
A	1	18.35	18.96	19.34	19.46	19.53	19.70	20.15
	2	18.25	18.85	19.25	19.38	19.43	19.60	20.06
	3	18.15	18.75	19.14	19.28	19.34	19.52	19.97
	4	18.15	18.75	19.14	19.28	19.34	19.52	19.97
	5	18.24	18.85	19.25	19.38	19.45	19.60	20.06
B	6	15.04	15.66	16.11	16.33	16.44	16.60	16.91
	7	17.98	18.51	18.83	18.99	19.00	19.13	19.44
	8	17.97	18.50	18.83	18.99	19.00	19.13	19.44
	9	16.24	16.83	17.24	17.44	17.50	17.65	17.97
	10	17.97	18.50	18.84	18.99	19.00	19.13	19.44
C	11	15.40	16.07	16.51	16.73	16.85	17.02	17.42
	12	18.03	18.60	18.94	19.08	19.13	19.27	19.62
	13	18.03	18.60	18.92	19.08	19.13	19.27	19.65
	14	15.16	15.82	16.26	16.49	16.61	16.80	17.19
	15	18.03	18.60	18.94	19.08	19.13	19.27	19.65
D	16	18.15	18.75	19.16	19.30	19.38	19.53	20.00
	17	15.17	15.85	16.28	16.46	16.54	16.69	17.00
	18	17.67	18.25	18.56	18.66	18.71	18.80	19.05
	19	17.73	18.32	18.64	18.73	18.79	18.88	19.20
	20	17.88	18.43	18.76	18.88	18.91	19.02	19.30
E	21	17.54	18.08	18.38	18.49	18.52	18.60	18.82
	22	17.90	18.41	18.76	18.88	18.93	19.00	19.28
	23	17.66	18.18	18.48	18.61	18.62	18.70	18.92
	24	18.01	18.64	19.06	19.23	19.30	19.51	20.03
	25	17.87	18.45	18.78	18.90	18.93	19.04	19.31
F	26	18.03	18.59	18.95	19.10	19.15	19.34	19.74
	27	18.13	18.73	19.13	19.29	19.39	19.59	20.12
	28	18.12	18.75	19.14	19.27	19.36	19.58	20.10
	29	18.01	18.60	18.97	19.11	19.17	19.32	19.79
	30	17.27	17.80	18.08	18.20	18.25	18.32	18.53

4.1.5. The surface vertical displacement results

An example illustration of calculation results for both cases no. 1 and 2, of model no. 1 can be found in Figure 4. Symbols marked with U1, U2, ..., U7 show the displacements values calculated at the locations typically used in standard European FWD-like devices, which are: 0, 30, 60, 90, 120, 150, 180 cm from the load axis.

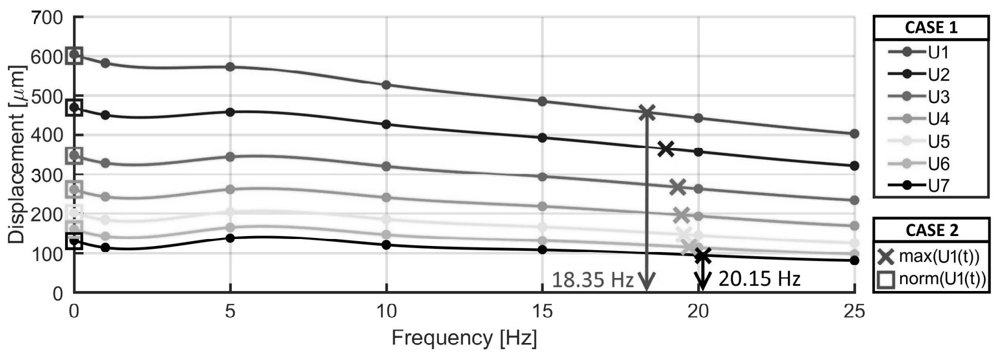


Figure 4. The values of vertical displacements plotted against the idealised (sinusoidal) displacement signals in the frequency function for model no. 1 (for the sake of clarity, only red colour symbols corresponding to the value U1 are indicated in the CASE 2 legend)

It can be seen from Figure 4 that each deflection value calculated using the CASE 1 method (i.e. based on the idealised sinusoidal loading) and the CASE 2 method (i.e. based on the real shapes of the loads functions generated in the FWD-like device) corresponds to the specific frequencies. As expected, the reference load frequencies, included in the calculations of CASE 1, result in the displacement functions whose frequencies correspond to the input load frequencies. This means that these functions are only offset from each other by the value of the phase angle. In the CASE 2 calculations, the situation becomes more complex. However, referring to the value of deflections calculated for model no. 1, shown in Figure 4, one can notice some regularity. The deflection values expressed in the frequency domain and calculated based on the normU(t) approach overlap with the deflection values calculated for the model loaded in the CASE 1. This proves that the normalization properly neutralizes the impact of the load frequency on the deflection values. Otherwise, the use of the maxU(t) approach would require that the calculated maximum deflection values marked with U1 to U7 are determined from the displacement functions of different frequencies. Moreover, the determined values of frequencies are not equal to the corresponding input load function frequencies used for the calculations, as one would expect. Again, with the reference to the example shown in Figure 4, these frequencies range from 18.35 Hz for U1 and 20.15 Hz for U7.

It is worth emphasising that the same regularity applies to all the 30 analysed models. In general, the values of displacements, triggered by the dynamic load and modelled after real FWD signals, overlapped with displacement values calculated for the idealised loads. However, a particularly good match can only be seen whenever the values corresponding to the load frequency of 0 Hz are compared. When the results of pavement tests performed with FWD device are identified as maximum displacement values in the pavement deflection curve, those values (from U1 to U7) correspond to various frequencies of the functions of displacement in time.

4.2. *Back analysis simulation issues*

The results presented in the preceding sections clearly illustrate that the backcalculation offers the possibility to: a) perform calculations with the objective function using the max values of $u_r(t)$ – a classic backcalculation, or b) take into consideration the values of vertical displacements calculated for the frequency of 0 Hz. Obviously, in relation to the purpose of the paper, the statically loaded models with elastic layers on the elastic halfspace were used in the both cases.

4.2.1. The reference values of stiffness moduli

The reference values of stiffness moduli of particular layers were determined for two different load frequencies. The first was equal to 0 Hz, corresponding to the reference value of the static loading conditions and the second was the mean frequency equal to 14.2 Hz – as a reference value of impact dynamic loads calculated based on the frequencies given in Table 3. In the context of the forward simulation issues stated above, only the reference values of stiffness moduli for MSCSA and subgrade courses can be directly adopted from Table 2. However, this only applies where the load frequency is equal to 0 Hz. Therefore, the remaining elastic reference values were calculated using idealised load models. The following procedure was applied:

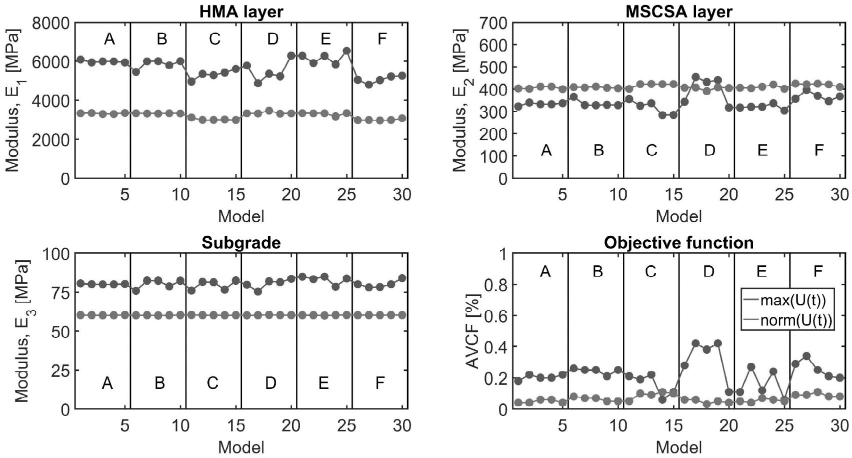
- based on the data in Table 2 the model was created

- based on the data in Table 3 the parameters for individual layers were adopted
- for the load frequency of 0 Hz the surface vertical displacements under the static load of 50 kN were calculated
- for the load frequency of 14.2 Hz the surface vertical displacements under the dynamic sinusoidal load function (equation 6) for $A_1 = 50$ kN and $f = 14.2$ Hz were calculated
- finally, for all known deflection curves the mean reference values of elastic properties $E_i^{ref}(0\text{ Hz})$ and $E_i^{ref}(14.2\text{ Hz})$ were returned by using the backcalculation.

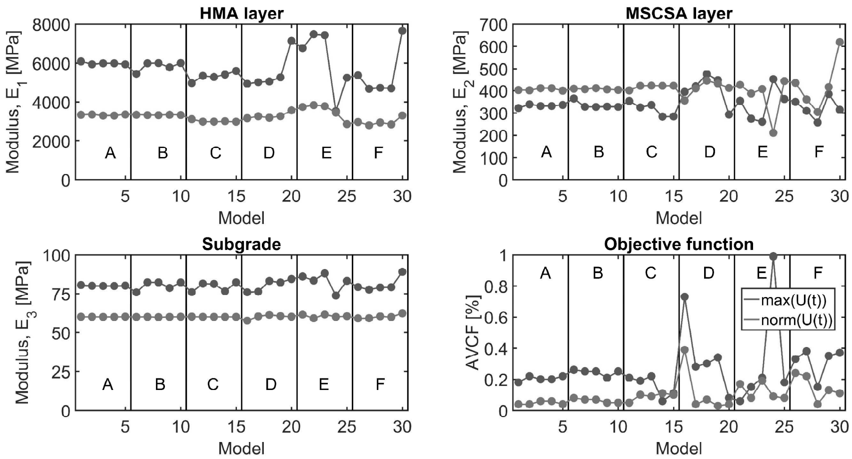
Furthermore, in order to get the elastic values of reference stiffness moduli, one must at least be aware of: I – the viscoelastic nature of pavement layers with bituminous binders, II – analysis is correct for the HMA temperature $\leq$ 20°C (the plasticity modelling can be abandoned), III – concept covers only the tests under relatively slow and short-term dynamic loads (a typical load time from 20 to 30 ms is considered) and IV – full interlayer bonding must be set up. Such constrains allow the elastic model to be considered in the back analysis. For the sake of clarity, all the calculated reference Young's moduli are listed in Table 5.

4.2.2. Backcalculation results based on the synthetic uniform sections data

In the current study, the backcalculation procedure was performed twice. In the first attempt, the layer thicknesses of each of the 30 models were equal to the assumed values shown in Table 1. In the second attempt, the simplified model with the mean layer thicknesses was assigned to each uniform section A, B, C, D, E and F. Depending on the approach the Figure 5 shows the obtained values.



(a) For each single section all of the declared layer thicknesses were used directly in backcalculation



(b) Only one model with the mean layer thicknesses for each single section was applied

Figure 5. Backcalculation results

In order to compare the backcalculated values with the reference ones, the measure in the form of relative difference error (8) was used. Critical indicators for assessing the impact of normalisation on the backcalculated values of the elastic properties are given in Table 5.

$$\Delta_i = \frac{E_i - E_i^{ref}}{E_i^{ref}} \cdot 100\% \tag{8}$$

where:
i – number of layer in the model,
Δ_i – relative difference [%],
E_i – backcalculated value of elastic modulus [MPa],
E_i^{ref} – reference value of elastic modulus [MPa].

Table 5. Backcalculated values of moduli compared to their reference elastic moduli

Method used to calculate the deflections	Modulus MPa E_1	E_2	E_3	Relative difference between moduli, % Δ_1	Δ_2	Δ_3	Coefficient of moduli variation, % CoV_1	CoV_2	CoV_3	Objective function, % AVCF
				Section A						
E_i^{ref} = 14.2 Hz	6463	296	71	–	–	–	–	–	–	–
E_i^{ref} = 0 Hz	3271	400	60	–	–	–	–	–	–	–
maxU(t)[a]	5986	332	80	7.97	-10.84	-11.25	1.0	1.9	0.3	0.016
normU(t)[a]	3324	405	60	-1.59	-1.23	0.00	0.8	1.4	0.1	0.011
maxU(t)[b]	5986	332	80	7.97	-10.84	-11.25	1.0	1.9	0.3	0.017
normU(t)[b]	3324	405	60	-1.59	-1.23	0.00	0.8	1.4	0.1	0.010
				Section B						
E_i^{ref} = 14.2 Hz	6080	296	71	–	–	–	–	–	–	–
E_i^{ref} = 0 Hz	3271	400	60	–	–	–	–	–	–	–
maxU(t)[a]	5844	336	80	4.04	-11.90	-11.25	4.1	4.9	3.7	0.019
normU(t)[a]	3326	407	60	-1.65	-1.72	0.00	0.2	0.7	0.1	0.013
maxU(t)[b]	5844	336	80	-4.04	-11.90	-11.25	4.1	4.9	3.7	0.019
normU(t)[b]	3326	407	60	-1.65	-1.72	0.00	0.2	0.7	0.1	0.013
				Section C						
E_i^{ref} = 14.2 Hz	5196	359	74	–	–	–	–	–	–	–
E_i^{ref} = 0 Hz	2923	400	60	–	–	–	–	–	–	–
maxU(t)[a]	5316	317	80	-2.26	13.25	-7.50	4.4	10.0	3.8	0.070
normU(t)[a]	3014	418	60	-3.02	-4.31	0.00	1.8	2.3	0.1	0.023
maxU(t)[b]	5316	317	80	-2.26	13.25	-7.50	4.4	10.0	3.8	0.070
normU(t)[b]	3014	418	60	-3.02	-4.31	0.00	1.8	2.3	0.1	0.023
				Section D						
E_i^{ref} = 14.2 Hz	7001	342	72	–	–	–	–	–	–	–
E_i^{ref} = 0 Hz	3271	400	60	–	–	–	–	–	–	–
maxU(t)[a]	5498	397	80	27.34	-13.85	-10.00	9.9	15.7	3.9	0.132
normU(t)[a]	3351	403	60	-2.93	-0.74	0.00	1.9	1.8	0.2	0.013
maxU(t)[b]	5477	406	80	27.83	-15.76	-10.00	17.1	17.3	5.0	0.237
normU(t)[b]	3287	411	60	-0.49	-2.68	0.00	4.9	8.3	2.4	0.155
				Section E						
E_i^{ref} = 14.2 Hz	6185	288	72	–	–	–	–	–	–	–
E_i^{ref} = 0 Hz	3271	400	60	–	–	–	–	–	–	–
maxU(t)[a]	6166	320	83	0.31	-10.00	-13.25	4.7	3.7	3.2	0.090
normU(t)[a]	3302	408	60	-0.94	-1.96	0.00	2.3	1.8	0.2	0.012
maxU(t)[b]	6089	341	83	1.58	-15.54	-13.25	27.8	22.7	6.6	0.380
normU(t)[b]	3534	375	61	-7.44	6.67	-1.64	11.4	25.0	1.7	0.054
				Section F						
E_i^{ref} = 14.2 Hz	5346	347	71	–	–	–	–	–	–	–
E_i^{ref} = 0 Hz	2923	400	60	–	–	–	–	–	–	–
maxU(t)[a]	5066	367	80	5.53	-5.45	-11.25	3.6	5.2	2.9	0.058
normU(t)[a]	2992	420	60	-2.31	-4.76	0.00	1.4	1.4	0.0	0.012
maxU(t)[b]	5240	323	81	2.02	7.43	-12.35	23.7	14.7	5.7	0.095
normU(t)[b]	2973	427	60	-1.68	-6.32	0.00	6.6	27.8	2.2	0.082

[a]Elastic moduli are the mean values of the backcalculated ones based on each of the 5 models defined in the single section

[b]Elastic moduli are the mean values of the backcalculated ones based on one model per section with the mean layer thicknesses

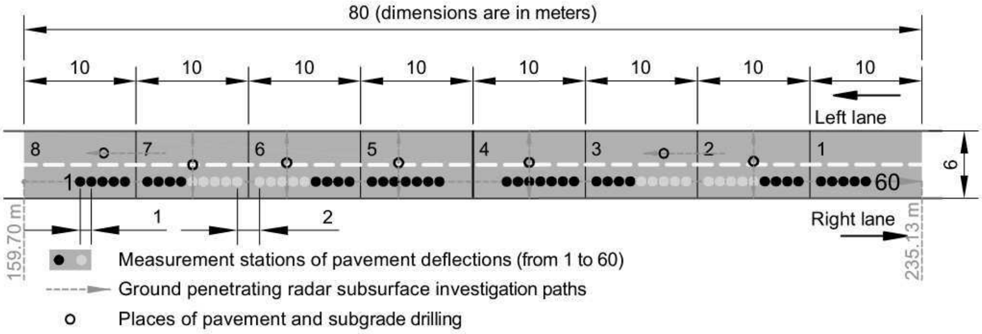
Compared to the calculations in which individual layer thicknesses at each measurement stations were used, thw assumption of the mean thicknesses of model layers in uniform sections results in a significantly larger spread of backcalculation data around the mean value. The spread depends on the layer for which the elastic modulus is searched for. The results are particularly noticeable when the relative difference Δ_i measure is used. Looking at the detailed results, only in cases of backcalculation that used the displacement value after normalisation (normU(t)), the value of $\Delta_i \leq 8\%$, regardless of the layers thicknesses assumed in calculations. Nevertheless, the backcalculation results obtained for each location where the layer thicknesses were known are much closer to the reference values and $\Delta_i \leq 5\%$. In the remaining cases, the values are higher, or even significantly higher than 10%. The values of $AVCF$ objective function in the calculations in which normalisation (maxU(t)) was not performed are approximately threefold higher compared to the calculations with normalisation. However, the objective function returns the acceptable results in both cases.

5. Backcalculation based on the in situ measurements

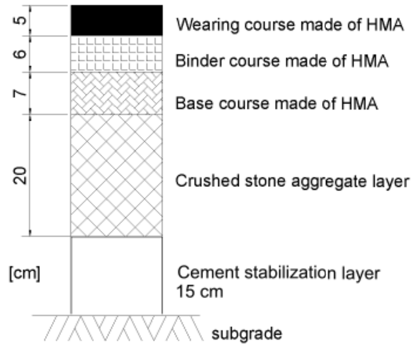
Using a static loading model for backcalculation offers two important benefits. Firstly, a model with a smaller number of necessary input parameters can be used (compared to a dynamic model); and secondly, the calculation results may be verified by the static tests results, e.g. Plate Bearing Test (PBT) method Pożarycki (2015). However, one must realise that such an assumption is after all still a simplified approximation and cannot be treated as an attempt to replace the standard tests performed under static loading conditions.

5.1. *Basic description of the test section*

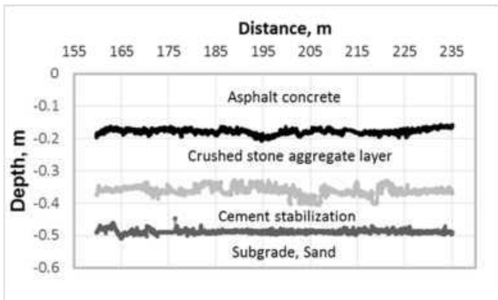
The backcalculation was performed for a result set from the in situ tests performed on a flexible pavement schematically shown in Figure 6a.



(a) The top view



(b) Designed layers thicknesses



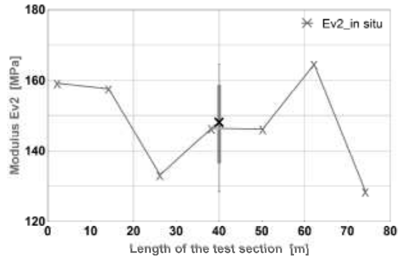
(c) Layers thicknesses measured by use of GPR device

Figure 6. A graphical description of the in situ test section of the length equal to 75 m

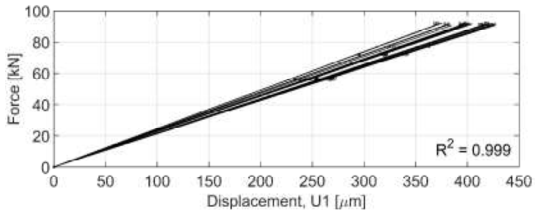
The roadway pavement was 5 years old (counting until the end of 2015); however, traffic loads at that time were considered to be sporadic (significantly below 7 ESAL‘s 115 kN/day). The thickness of each layer was determined using a GPR with an antenna of the frequency of 1 GHz. The main tests with the GPR were conducted mainly on the right traffic lane, along the measurement trace set for the purpose of the FWD test (from 159.70 m to 235.13 m). The GPR's resolution allowed to determine the combined thickness of all layers with the accuracy of ± 1 cm. For the purpose of calibrating the results of the GPR test of the layers, pavement samples, both cut from the asphalt layers and drilled from the subgrade, were collected. The highest value of the standard deviation computed for the combined thickness of asphalt layers in each 10-metre-long section was ≤ 0.4 cm and ≤ 3.5 for MSCSA layer and cement-stabilised, respectively. The measurements of pavement deflections were performed by using the 7 geophones located at 0, 30, 60, 90, 120, 150 and 180 cm from the FWD loading axis. The measurement temperature of asphalt layers was equal to 7.5°C.

5.2. *Subbase bearing capacity and linearity characteristic of analysed flexible structure*

For the sake of comparison, the bearing capacity of the half-space limited by the upper plane of the MSCSA layer was estimated using the standard pavement tests performed by PBT method. The results of the secondary deformation modulus (Ev2) measured on the surface of crushed stone aggregate layer of the in situ test section, are presented in Figure 7a. The box – whisker plot (also shown in Figure 7a) was obtained based on the PBT measurements, located every 10 m along the longitudinal axis of the test section. For Ev2 calculations, the stresses of 0.25 and 0.45 MPa were used. The principle axis of measurements was set out exactly in the middle of the lane. The geodetic measurement points were performed every 1 m, with the distance accuracy error of 2 mm. Furthermore, the FWD tests were performed on the surface of HMA base layer. In order to overlap the PBT measurement locations, the previously acquired principle axis was projected onto the new measurement surface, which was the surface of the HMA base course. To control the moving distance of the FWD device, an encoder with accuracy of 1 cm was used. The deflection analysis involved a semi-invasive verification, which was an assessment of the linearity of the characteristic of load-vertical displacement, using three different load values of 50, 70 and 90 kN (Figure 7b).



(a) Ev2 values determined on the surface of the MSCSA layer with the error box whisker



(b) Load-displacement characteristic for HMA wearing course measurements

Figure 7. In-situ measurements on the test section

Based on the obtained linearity characteristic, it was concluded that pavement degradation was in its primary development phase, and it was justified to use an elastic modulus for linear modelling. More examples for using this method can be found in paper Pożarycki and Górnaś (2014a).

5.3. *Backcalculated elastic moduli of test section layers*

To perform the backcalculation, the 3–layered model was used. All the HMA pavement layers were considered as a single asphalt concrete layer in the model. Its thickness was equal to the sum of the thicknesses of individual layers. The second course in the model reflected the pavement layer made of mechanically-stabilised crushed-stone aggregate, and the third one was a cement-stabilised subgrade of an infinite thickness. The vertical displacements used in backcalculation were obtained from deflection tests of a pavement subjected to impact dynamic loads of 50 kN, and the calculation value was assumed as the mean value of the three tests at a given station. In the case of HMA, a stiffness modulus E_1 was evaluated in the temperature of 7.5°C. The backcalculated values of elasticity moduli for the mechanically-stabilised crushed-stone aggregate layer and the improved subgrade are shown in Figure 8 and are marked as an E_2 and E_3, respectively. The calculated basic quality measures for analysed quantities are given in Table 6.

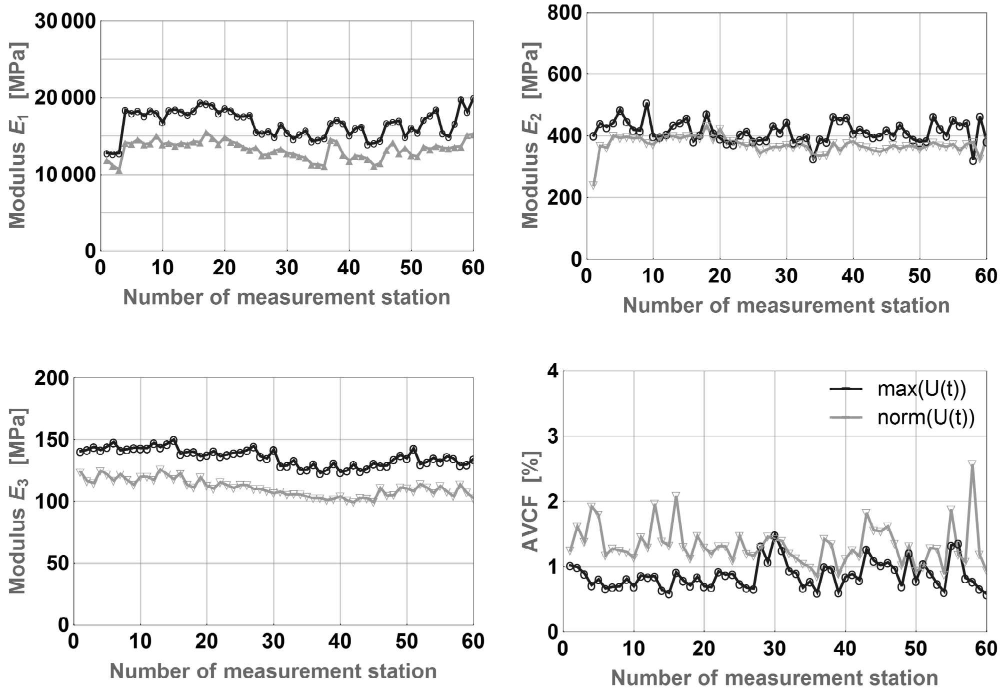


Figure 8. Backcalculated elastic modulus values in the 3–layered pavement model

Table 6. Basic quality measures

Method used to calculate	Mean moduli values, MPa			Coefficient of moduli variation, %			Objective function, %
deflections	$E_1(T = 7.5^oC)$	E_2	E_3	CoV_1	CoV_2	CoV_3	AVCF
max(U(t))	16 618	415	136	10.8	8.4	5.1	0.87
norm(U(t))	13 284	372	111	8.7	7.1	6.3	1.33
Abs[norm(U(t))-max(U(t))]	3 334	43	25	2.1	1.3	1.2	0.46

5.4. *Secondary deformation moduli against the backcalculation results*

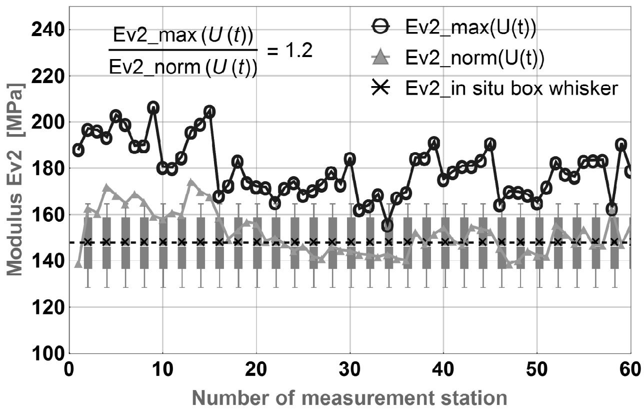
The simulations of the PBT method in conjunction with the results of the backcalculation, are inspired by the discussion introduced in Fengier, Pożarycki, and Garbowski (2013); Pożarycki (2015). However, using the general formula for calculating deflection values of a pavement transformed to the elastic half-space model Firlej (2007), provides an interesting addition to the discussion – the values of the secondary deformation modulus ($Ev2$), expressed approximately with the relation (9).

$$w = \frac{1+\nu}{E}[2-2\nu] \cdot \nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2} \quad \rightarrow \quad E = \frac{\pi}{4} \cdot \frac{q \cdot D \cdot (1-\nu^2)}{w} \approx Ev2 \tag{9}$$

where:
ν – Poisson‘s ratio [-], E – Young’s modulus [MPa], ∇^2 – the Laplace operator, Φ – stress function [MPa], z – height coordinate [m], q – circular loading [kN/m^2], D –

the plate diameter used in PBT method [m], w – vertical displacement value obtained by using a PBT method [m], $Ev2$ – secondary deformation modulus [MPa].

Finally, having at the disposal (1) a pavement model with the static load defined, (2) E_i values determined in backcalculation , (3) $Ev2$ values – which should correspond to measurement conditions of PBT on the surface of the MSCSA layer, secondary deformation modulus definition can be used for verifying the backcalculation results. By combining the results of in-situ pavement bearing tests using the PBT method (Figure 7) and the backcalculation results for a static model, the relationship between them based on both methods tested in the paper: maxU(t) and normU(t), could be described, as shown in Figure 9.



(a)

Figure 9. $Ev2$ values obtained by using the backcalculation vs $Ev2$ values achieved from the in situ measurements

In Figure 9, the range of Ev2 values obtained in situ, are represented by the box whisker plots, located at each calculation point where the FWD deflection tests were performed. In order to compare the secondary deformation modulus values, backcalculated based on the FWD deflection measurements on the surface of HMA base layer both maxU(t) and normU(t) approaches were applied. Finally it was concluded, that backcalculated values of Ev2 based on the normU(t) approach and the static load model, are comparable to those calculated based on the in situ measurements by using the PBT method.

6. Discussion

The calculation results obtained from the numerical experiment described in this paper unambiguously confirm the favourable influence of normalisation on the backcalculation results. Only in the cases where the normalised vertical displacement values were used (normU(t)), the calculation results were encumbered with an error lower than 8%. However, the error was less then 5% when in the individual uniform sections, backcalculation was carried out for all the locations with known layer thicknesses, and instead of using one model with an averaged layer thicknesses per section. The errors in determining the stiffness moduli of layers in backcalculation based on the maximum values of the functions of load and displacements were over 10 % in tested cases.

The assessment of the results based on in-situ tests is more complex. Stiffness moduli of the layer made of hot mixed asphalt depend on temperature, load frequency and stress conditions. Using the classic backcalculation method – maxU(t), the achieved value of mean stiffness modulus of asphalt layers of the test section pavement (determined in the temperature of 7.5°C and the load frequency of 18 Hz) equals 16 618 MPa. The application of normalisation – normU(t), results in the mean value of 13 284 MPa, which, according to the assumption, corresponds to a value determined in the temperature of 7.5°C as well, but in the frequency of 0 Hz. Therefore, the influence of normalisation in the perspective of results assessment for hot mixed asphalt layer is substantial, in contrast to the values obtained for MSCSA layers and improved subgrade. Consequently, it is beneficial to include the results of pavement tests from PBT method in the comprehensive verification of backcalculation, both in reference to the values obtained from backcalculation, as well as in-situ pavement tests, as shown in Figure 9. Knowing that the horizontal edges limiting the grey area of the rectangle mark the first quartile of the Ev2 result population from in-situ tests, one can conclude that 83% of Ev2 values calculated based on the normU(t) procedure constitutes a part of the group determined by the values of the 25th and 75th percentile of the $Ev2$ variable spread, determined in-situ.

7. Conclusions

The relationships of load and displacement in the function of time, obtained from pavement tests performed with dynamic deflectometers, are dependent on the frequency domain. Assuming the smallest possible number of parameters in the pavement models, the experiment was conducted to find the best match between the surface deflections from the backcalculation simulations with impact dynamic loads and those calculated for the static load model.

For flexible pavements analysed in this paper, the frequencies of the loads functions oscillate around 12 to 25 Hz. The corresponding frequency of vertical displacement functions, depending on the distance between measurement points and the FWD load axis, varies from 15 to 20 Hz. In backcalculation, all those factors have a significant influence on the calculation results, and the relative difference between values calculated at such boundary conditions can reach 20%. Even in reference to synthetic uniform sections, simplifications taking the form of omitting the influence of the frequency of force and displacement functions lead to miscalculations of stiffness moduli of the model pavement layers (errors are $> 10\%$). Based on the results presented in this paper, it can be concluded that the frequency normalisation procedure of those signals seems to be an effective way of standardising boundary conditions in the backcalculation of flexible pavements. Furthermore, the normalisation is verified by using the models with minimum input parameters – elastic moduli, Poisson's ratios and layer thicknesses. In the numerical experiment, the normalised vertical displacement values lead to backcalculated moduli encumbered with an error lower than 5%. However, the dependancy of the backcalculated stiffness values on the layer thicknesses should be more thoroughly investigated in the future.

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