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# Compatibility of the chameleon-field model with fifth-force experiments, cosmology, and PVLAS and CAST results

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We analyse the PVLAS results using a chameleon field whose properties depend on the environment. We find that, assuming a runaway bare potential  $V(\phi)$  and a universal coupling to matter, the chameleon potential is such that the scalar field can act as dark energy. Moreover the chameleon field model is compatible with the CAST results, fifth force experiments and cosmology.

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One plausible explanation for the observed accelerated expansion of the universe is the presence of a pervading scalar field whose dynamics lead to an approximately constant energy density today [1]. As a result, the mass of this scalar field turns out to be extremely small, i.e. of the order of the present Hubble rate. Such an almost massless scalar field is in direct conflict with gravitational experiments when its coupling to matter is of order of gravitational strength. Indeed fifth force experiments give stringent bounds on the gravitational coupling. One must therefore either decouple almost massless scalar fields from ordinary matter or shield macroscopic bodies. The former mechanism has been used to argue that the dilaton of string theory in the strong coupling regime does not lead to gravitational problems and can drive the acceleration of the expansion [2]. The latter possibility is at play when scalar fields behave as chameleon fields. Such fields couple to matter strongly and non-linear effects can reduce the interaction range of the force mediated by the chameleon field created by a massive body [3]. This is all due to the presence of a thin-shell effect whereby the scalar field is essentially constant inside the massive body, except for a thin shell whose width governs the strength of the scalar interaction with other massive bodies. For lighter bodies, the thin shell effect does not appear and chameleon fields can become invisible provided their environment dependent mass is large enough.

The coupling of a chameleon field to the electromagnetic sector could lead to variations of the fine structure

constant. In a cosmological setting, the chameleon field settles down at the bottom of its time-dependent (effective) potential very early in the universe, thus preventing large mass variations during Big Bang Nucleosynthesis. As the minimum of the potential evolves adiabatically with the time variation of the matter energy density, a variation of the fine structure constant could be induced, albeit negligible for an order one gravitational coupling [4].

The coupling of a scalar field to photons has been invoked in order to explain the original PVLAS dichroism detection [5], recently superseded by measurements showing upper bounds on both the dichroism and the birefringence [6]. The scalar field coupling strength must be suppressed by a scale bounded by  $M \geq 10^6$  GeV for masses which are typically  $m \leq 10^{-3}$  eV. It is tantalising that such a scalar field mass is within the ball park of the energy density of the universe. Moreover the coupling to photons

$$-\frac{1}{4} \int d^4x e^{\phi/M} F_{\mu\nu} F^{\mu\nu} \quad (1)$$

with  $\phi \ll M$  is reminiscent of the coupling of the dilaton to photons. The former results obtained by the PVLAS collaboration are in conflict with astrophysical bounds such as CAST [7], which for the same mass for the scalar field, require much smaller couplings ( $M > 10^{10}$  GeV). Recently, a lot of work has been done in order to explain the discrepancy theoretically (see e.g. [8]). If a future PVLAS type measurement indicated a coupling in the range  $10^6 \text{ GeV} \leq M \leq 10^{10} \text{ GeV}$ , the question of whether this could be made compatible with the CAST result would be raised. We will address this question in the following.

In this letter we point out that the PVLAS results in the interesting range  $10^6 \text{ GeV} \leq M \leq 10^{10} \text{ GeV}$  with a

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mass  $m \approx 10^{-3}$  eV are not in conflict with astrophysical bounds such as CAST if the particle concerned were a chameleon. Moreover, low values of  $M \approx 10^6$  GeV are favoured in order to be compatible with dark energy. We require that the scalar field couples to all matter forms and, in the following, we will assume that all the couplings of  $\phi$  to matter are universal and are given by the one suggested by the analysis of the PVLAS experiment. Our model is of the scalar-tensor type

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa_4^2} R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{e^{\phi/M}}{4} F^2 \right) + S_m(e^{\phi/M} g_{\mu\nu}, \psi_m) \quad (2)$$

where  $S_m$  is the matter action and the fields  $\psi_m$  are the matter fields. As a consequence, particle masses in the Einstein frame become

$$m(\phi) = e^{\phi/M} m_0 \quad (3)$$

where  $m_0$  is the bare mass as appearing in  $S_m$ . The effective gravitational coupling is given by

$$\beta = \frac{m_{\text{Pl}}}{M}, \quad (4)$$

and therefore very large ( $\beta \leq 10^{13}$ ) when assuming the results from the PVLAS experiment ( $M \geq 10^6$  GeV) [6]. To prevent large deviations from Newton's law one must envisage non-linear effects shielding massive bodies from the scalar field. One natural possibility is that the scalar field  $\phi$  coupled to photons has a runaway (quintessence)-potential leading to the chameleon effect. For exponential couplings, this is realised when

$$V(\phi) = \Lambda^4 \exp(\Lambda^n / \phi^n) \approx \Lambda^4 + \frac{\Lambda^{4+n}}{\phi^n} \quad (5)$$

The first term corresponds to an effective cosmological constant while the second term is a Ratra-Peebles inverse power law potential. Acceleration of the universe is obtained provided  $\Lambda \approx 10^{-12}$  GeV, which of course assumes that the scalar field  $\phi$  is responsible for the acceleration of the universe.

In the presence of matter, the dynamics of the scalar field is determined by an effective potential

$$V_{\text{eff}}(\phi) = \Lambda^4 \exp(\Lambda^n / \phi^n) + e^{\phi/M} \rho - \frac{e^{\phi/M}}{2} (\mathbf{E}^2 - \mathbf{B}^2) \quad (6)$$

where  $\rho$  is the energy density of non-relativistic matter and we have used the fact that  $(1/4)F_{\mu\nu}F^{\mu\nu} = -(\mathbf{E}^2 - \mathbf{B}^2)/2$ , with  $\mathbf{E}$  and  $\mathbf{B}$  being the electric and magnetic field, respectively. The origin of the last terms is due to the coupling between matter, electromagnetism and the scalar field. Since we will consider the PVLAS experiment in the following, we will set  $\mathbf{E} = 0$ .

The effective potential leads to a stabilisation of the scalar field for

$$\phi = \left( \frac{n\Lambda^{4+n}M}{\rho_{\text{tot}}} \right)^{1/(n+1)}, \quad (7)$$

where  $\rho_{\text{tot}} = \rho + \mathbf{B}^2/2$  is the effective energy density with contributions from both matter and the magnetic field. The mass at the bottom of the potential is given by

$$m^2 = n(n+1) \frac{\Lambda^{n+4}}{\phi^{n+2}} \quad (8)$$

Let us now consider the case of the vacuum chamber used in the PVLAS experiment such that the energy density inside the cavity is  $\rho_{\text{tot}}$  and the mass of the scalar field  $m_{\text{lab}}$ , then

$$\Lambda^{4+n} = \frac{(n+1)^{n+1}}{n} \rho_{\text{tot}}^{n+2} M^{-n-2} m_{\text{lab}}^{-2n-2} \quad (9)$$

Taking  $m_{\text{lab}} = 10^{-3}$  eV, considering the fact that the density has contributions from the gas ( $\rho_{\text{lab,gas}} \approx 2 \times 10^{-14}$  g/cm<sup>3</sup>) and the magnetic field ( $B = 5$  T, corresponding to  $\rho_{\text{lab,field}} \approx 7 \times 10^{-14}$  g/cm<sup>3</sup>), and the lower bound  $M = 10^6$  GeV, determines  $\Lambda$

$$\Lambda^{4+n} \approx \frac{(n+1)^{n+1}}{n} 10^{-12n-48} \quad (10)$$

For  $n = \mathcal{O}(1)$  we find that

$$\Lambda \approx 10^{-12} \text{ GeV} \quad (11)$$

as required to generate the acceleration of the universe. Hence we find that the cosmological constant  $\Lambda^4$  is compatible with the laboratory experiments. Higher values of the coupling scale  $M$  and the mass  $m_{\text{lab}}$  would lead to a smaller value of  $\Lambda$ , incompatible with cosmology. Hence we only consider the lower bound  $M = 10^6$  GeV in the following.

As already mentioned, the lower bound given by the PVLAS experiment is in conflict with the CAST experiment on the detection of scalar particles emanating from the sun, as it requires  $M \geq 10^{10}$  GeV. However, this bound does not apply when the mass of the scalar field in the sun exceeds  $10^{-5}$  GeV. Let us evaluate the mass of the chameleon field inside the sun. Using eqn. (9) one obtains

$$m_{\text{sun}} = m_{\text{lab}} \left( \frac{\rho_{\text{sun}}}{\rho_{\text{lab}}} \right)^{(n+2)/2(n+1)}. \quad (12)$$

Now  $\rho_{\text{sun}}/\rho_{\text{lab}} \approx 10^{14}$  and, with  $n = 0(1)$ , one finds

$$m_{\text{sun}} \sim 10^{-2} \text{ GeV} \gg 10^{-5} \text{ GeV} \quad (13)$$

implying no production of chameleons deep inside the sun. Chameleon particles can also be produced at the surface of the sun where the density  $\rho_{\text{out}}$  is much lower. Taking  $\rho_{\text{out}}/\rho_{\text{lab}} \approx 10^9$ , we find that the CAST results can be explained when  $n \leq 1$ . Hence, the CAST experiment is in agreement with the chameleon model due to the fact that the chameleon field is very massive in the sun.

The PVLAS experiment puts constraints on the mass of the scalar field inside the field zone and the coupling

constant  $M$ . Since the mass of the chameleon field depends on the ambient energy density, the theory predicts that the results of the PVLAS experiment would have been different if the density of matter inside the field zone and the magnetic field strength were different. Since the amplitude of the dichroism depends on  $m_\phi^{-4}$  [5, 9], we would expect that, from eq. (9), the amplitude of dichroism would decrease as  $\rho_{\text{gas}}$  increases (if all other parameters are fixed). In our theory, the amplitude of dichroism depends in a very non-trivial way on both the pressure and the strength  $B$  of the magnetic field. Observing such variations would be a test of the theory described here. If the magnetic field strength is kept fixed at  $B = 5.5$  T and decreasing  $\rho_{\text{lab,gas}}$ , according to our theory there would be a saturation soon, since the total energy density inside the field zone is dominated by  $\rho_{\text{lab,field}}$  and the chameleon mass becomes independent of  $\rho_{\text{lab,gas}}$ . If the density of gas is further increased, the mass of the chameleon field changes and therefore the amplitude of the observed effect. Fixing the gas density but changing  $B$  would result in a non-trivial dependence of the amplitude of the dichroism on  $B$ , since the mass of the chameleon depends on  $B$ . The detailed analysis of these effects is in progress [11].

Let us now analyse the gravitational tests on earth and in the solar system. As we will now argue, current experiments will not be able to detect a new force mediated by the chameleon field, even if the coupling  $\beta$  is large. The argument is as follows:

On earth, and for experiments performed in the atmosphere where the density is  $10^{-3} \text{ g/cm}^3$ , the mass of the chameleon field is  $m_{\text{atm}} \approx 10^{-5} \text{ GeV}$  leading to a very short-ranged interaction, hence not detectable in gravity experiments. Similarly in the solar system where  $\rho_{\text{solar}} = 10^{-24} \text{ g/cm}^3$ , the mass of the chameleon becomes  $m_{\text{solar}} = 10^{-22} \text{ GeV}$ , with a range of  $10^6 \text{ m}$ , too small to affect the motion of planets. Finally let us consider the satellite gravity experiments. As the range of the chameleon force in the galactic vacuum is much larger than the size of a satellite, one would expect large deviations from Newton's law for satellite experiments. This is only the case if the thin shell mechanism is not at play. A test mass of a gedanken experiment aboard a satellite has a thin shell provided  $\phi_{\text{solar}} \leq \beta \Phi_N$  where  $\Phi_N$  is Newton's potential at the surface of the test body. For a typical test body of mass 40 g and radius 1 cm, Newton's potential is  $\Phi_N \approx 10^{-27} m_{\text{Pl}}$  implying that a thin shell exists for  $\phi_{\text{solar}} \leq 100 \text{ GeV}$ . We find that

$$\phi_{\text{solar}} \approx 10^{-(1+12n)/(n+1)} \ll 10^2 \quad (14)$$

implying that satellite experiments would not detect any deviation from Newton's law due to a force mediated by the chameleon field.

A detailed analysis of theories with scalar fields strongly coupled to matter has been carried out in [10]. A range of different constraints have been investigated and, for the model at hand, current constraints coming from local experiments are fulfilled. The large coupling in the

theory implied by the PVLAS experiment ensures that the chameleon effect is very efficient, resulting in a large effective mass for the scalar field and, consequently, short interaction range of the force mediated by the chameleon.

For  $\beta \gg 1$ , Casimir force experiments provide tight bounds on the model parameter. For large  $\beta$ , it was found that the scale  $\Lambda$  cannot be much larger than that set by the cosmological constant  $\Lambda = 10^{-3} \text{ eV}$  [10]. However, current experiments are compatible with the parameter  $M$  and  $\Lambda$  in our model. More precisely, the ratio of the chameleon force over the Casimir force for a two plate geometry is given by [10]

$$\frac{F_\phi}{F_{\text{Cas}}} = \frac{240}{\pi^2} K_n(\Lambda d)^{\frac{2(n+4)}{n+2}} \quad (15)$$

where  $K_n$  is expressed in terms of Euler's beta function  $K_n = (\sqrt{2} \frac{B(1/2, 1/n+1/2)}{n})^{2n/(n+2)}$  and  $\frac{240}{\pi^2} K_n \approx 40$  for  $n \leq O(1)$ ;  $d$  is the interplate distance. Casimir forces have been measured up to  $d = 10 \text{ } \mu\text{m}$  implying that  $\Lambda d \leq 0.1$  and therefore  $F_\phi/F_{\text{Cas}} \leq 10^{-2}$ . Such an accuracy is below the present experimental levels. Detailed work on Casimir constraints is in progress [14].

Astrophysical constraints, such as those coming from neutron stars and white dwarfs also constrain the existence of strongly coupled scalar fields. The force mediated by  $\phi$  could alter the stability of such stars. However, the constraints coming from these considerations are not very strong and for the parameter at hand the theory is compatible with observations.

On scales relevant for cosmology, the chameleon mediates a force which could affect structure formation. The interaction range is given by  $\lambda = V_{,\phi\phi}^{-1/2}$ , below which the effective gravitational constant is  $G_{\text{eff}} = G_N [1 + 2\beta^2]$ . For the potential (5), the interaction range is (assuming  $\Lambda = 10^{-3} \text{ eV}$ )

$$\lambda_{\text{cham}} \approx 10^{-2} \left( \frac{\phi}{\Lambda} \right)^{1+n/2} \text{ cm}. \quad (16)$$

As an example, with  $n = 1$ , eq. (7) gives  $\phi = 10^{-4} \text{ GeV}$  in the minimum today, so that, for  $\Lambda = 10^{-3} \text{ eV}$ ,  $\lambda_{\text{cham}} = 10^{10} \text{ cm}$ , which is of the order of the radius of the sun and hence cosmologically irrelevant. We remark that the interaction range is much larger if  $\beta$  is smaller:  $\lambda_{\text{cham}} \approx 100 \text{ pc}$  for  $\beta = \mathcal{O}(1)$  [4]. As is the case with local experiments, cosmologically the chameleon mechanism is more effective for large  $\beta$ .

As analysed in [4], the chameleon must be stuck at the bottom of the potential since before BBN. Thus the chameleon has the following evolution

$$\phi_{\text{cos}} \approx (\Lambda^n M)^{1/(n+1)} (1+z)^{-3/(n+1)} \quad (17)$$

which is a valid approximation as long as  $\phi \geq \Lambda$ . In particular, the fine-structure constant is such that

$$\frac{\alpha^{-1}(z) - \alpha^{-1}(0)}{\alpha^{-1}(0)} = \left( \frac{\Lambda}{M} \right)^{n/(1+n)} \left( (1+z)^{-3/(n+1)} - 1 \right) \quad (18)$$

The prefactor is very small ( $10^{-8}$  for  $n = 1$  and smaller for larger  $n$ ). This implies that the variation of the fine structure constant is negligible compared to the results presented in [12].

Hence the chameleon field is not observable either in current gravitational experiments or cosmologically. In addition to its role in the PVLAS experiment, the chameleon Lagrangian possesses terms like

$$\lambda \frac{H\phi}{M} \psi \bar{\psi} \quad (19)$$

coupling two fermions, one Higgs field and the chameleon field. After electroweak symmetry breaking when the Higgs field picks up a vev, the effective coupling becomes

$$\frac{m_\psi}{M} \phi \bar{\psi} \psi \quad (20)$$

so the chameleon couples like an almost massless Higgs boson to the standard model fermions. The only difference is that the Higgs coupling is suppressed by the electroweak vev  $v$ . The weakness of the chameleon coupling is measured by the ratio  $v/M \sim 10^{-4}$ . Such a small coupling makes the chameleon detection unrealistic even at LHC scales.

However, such a Yukawa coupling could lead to deviations from standard model results in high precision experiments. For example, the anomalous magnetic moment of the muon (or the electron) could be affected. New contributions occur due to the chameleon coupling to fermions, as in eqn. (20). These can be evaluated by replacing photon lines with chameleon lines in the relevant Feynman diagrams. This gives a contribution to  $(g - 2)$  of  $(m_{e,\mu}/M)^2$ , which is of order  $10^{-12}$  for the muon, instead of the usual  $\alpha_{QED}$ . It is thus suppressed. Another contribution comes from hadronic loops which is about two orders of magnitude larger than that of the chameleon (see e.g. [13] and references therein for a recent discussion). Hence the effect on the anomalous magnetic

moment is negligible. Similarly, the couplings in eqn. (20) could lead to corrections to the hyperfine structure of the hydrogen atom. As for  $(g - 2)$ , the effect spring from one loop contributions obtained by replacing photon propagators by those for chameleons. Hence, the effect on the energy levels is of order  $(m_e/M)^2$  compared to  $\alpha_{QED}$ , and is thus negligible.

In conclusion, a scalar field strongly coupled to matter, with coupling strength as suggested by the new PVLAS results, and non-linear self-interactions, is compatible with current fifth-force experiments and cosmology. Moreover a coupling strength with  $M \approx 10^6 \text{ GeV}$  is favoured when considering compatibility with dark energy. In such a range, chameleons provide a natural explanation for the discrepancy with the CAST results, since the (effective) mass of the scalar field depends on the matter density of the environment. Future laboratory experiments, such as Casimir force experiments, could be designed to detect the force mediated by the scalar field and would be an independent test from the PVLAS experiment. We have pointed out that, according to the model discussed in this paper, the amplitude of the effect in the PVLAS experiment depends on the mass density and the magnetic field inside the field zone. Therefore it would be interesting to investigate both experimentally and theoretically whether there is any dependence on both the density of ambient matter and the magnetic field. Work on this topic is in progress [11].

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