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Compressive Sensing Based Sparse Antenna Array Design for Directional Modulation

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Abstract. Directional modulation (DM) can be achieved based on uniform linear arrays (ULAs) where the maximum spacing between adjacent antennas is half wavelength of the frequency of interest in order to avoid spatial aliasing. To exploit the additional degrees of freedom (DOFs) provided in the spatial domain, sparse antenna arrays can be employed for more effective DM. In this work, the spare array design problem in the context of DM is formulated from the viewpoint of compressive sensing (CS), so that it can be solved using standard convex optimisation toolboxes in the CS area. In detail, we need to find a common set of active antennas for all modulation symbols generating a response close to the desired one. The key to the solution is to realise that we have to employ the group sparsity concept, as a common antenna set cannot be guaranteed if we optimise antenna locations for each modulation symbol individually. Moreover, we have also considered two practical scenarios for our proposed design: robust design with model errors, and design with practical non-zero-sized antennas, and corresponding solutions are found by modifying the proposed standard solution.

Keywords: Directional modulation, sparse array, compressive sensing, group sparsity, robust design, size constraint.

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1 Introduction

In conventional wireless communication systems, since the same constellation mappings are used in all directions of the transmit antennas, it is possible for the signals to be captured and demodulated by highly sensitive eavesdroppers even if they are located at sidelobe regions of the antennas. To avoid this, the directional modulation (DM) technique has been developed to improve security by keeping known constellation mappings in a desired direction or directions, while scrambling them for the remaining ones [1, 2].

In [3], a four-element reconfigurable array was designed by switching elements for each symbol to change its amplitude and phase of the element radiation pattern to make their constellation points not scrambled in desired directions, but distorted in other directions. A method named dual beam DM was introduced in [4]. Unlike the methods where I and Q data are transmitted by the same antennas, in this technique they are transmitted by different antennas. In [5], phased arrays are employed to show that DM can be implemented by phase shifting the transmitted antenna signals properly. The bit error rate (BER) performance of a system based on a two-antenna array was studied using the DM technique for eight phase shift keying (PSK) modulation in [6]. The particle swarm optimization technique was employed for DM transmitter synthesis by linking the BER performance to the settings of phase shifters in [7], and a more systematic pattern synthesis approach was presented in [8], followed by an energy-constrained design in [9]. Recently in [10], the time modulation technique was introduced to DM to form a four-dimensional (4-D) antenna array, where radiation pattern changes with time.

However, most existing research in DM is based on uniform linear arrays (ULAs) with a maximum half wavelength spacing to avoid grating lobes. To have a larger aperture and a higher spatial resolution given a fixed number of antennas, sparse arrays are normally employed in traditional array signal processing [11, 12]. The increased degrees of freedom (DOFs) in the spatial domain allow the system to incorporate more constraints into the design of various beamformers. Many methods have been proposed to design such a sparse array, including the genetic algorithm (GA) [13–17], simulated annealing (SA) [18], and compressive sensing (CS) [19–24].

In this work, we extend the CS-based sparse array design to the area of DM and try to optimise the antenna locations for a given set of modulation symbols and desired transmission directions by matching designed beam responses to desired ones. To our best knowledge, it is the first time to address this important problem for directional modulation. The key to the solution is to realise that we can not perform this optimisation individually for each symbol; otherwise we would end up with different antenna locations for different transmission symbols. Rather we need to find a common set of optimised antenna locations for all required transmission symbols with the desired directions. As a result, the traditional CS-based narrowband sparse array design methods will not work and a group sparsity based approach is proposed to tackle the problem. The new CS-based formulation for sparse array design in the context of DM can then be solved using standard convex optimisation toolboxes in the CS area.

One common issue in practical design of antenna arrays is the robustness of the resultant system against various model perturbations, such as errors in antenna locations, mutual coupling and discrepancies in individual antenna responses. Many methods have been proposed to design robust adaptive arrays, such as diagonal loading, worst case optimisation and robust Capon beamformers [25–28], where it is usually assumed that there is a norm-bounded steering vector error. In this paper this idea is used to place an extra constraint on the CS-based design process. As a result, the difference between the designed and achieved modulation responses can be kept below an acceptable level.

Another problem is the size of the antenna. In the design of antenna arrays, the antennas are often considered to be an ideal point without a physical size. As a result, it is possible that the resulting antenna locations will be too close for the antennas to physically fit in, especially for multiband or wideband arrays, where the antenna size may be much larger than $\lambda/2$ [29] . Following the approach in [24], we also consider the design of sparse arrays with physical size constraint in the context of directional modulation.

The remaining part of this paper is structured as follows. A review of the DM technique based on phased arrays is given in Sec. 2. A class of CS-based design methods is presented in Sec. 3, including l_1 norm minimisation and reweighted l_1 norm minimisation. Two practical scenarios are considered in Sec. 4, including a robust design in the presence of steering vector errors, and a design considering the nonzero size of antennas. In Sec. 5, design examples are provided, with conclusions drawn in Sec. 6.

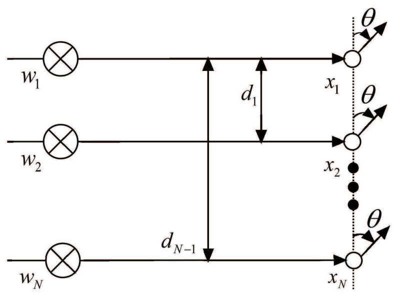


Figure 1: A narrowband transmit beamforming structure.

2 Review of Directional Modulation

2.1 Narrowband beamforming based on ULAs

A narrowband linear array for transmit beamforming is shown in Fig. 1, consisting of N equally spaced omnidirectional antennas with the spacing from the first antenna to its subsequent antennas represented by d_n for $n = 1, \ldots, N-1$, where the transmission angle $\theta \in [0^\circ, 180^\circ]$. The output signal and weight coefficient for each antenna are respectively denoted by x_n and w_n for $n = 1, \ldots, N$. The steering vector of the array is a function of angular frequency ω and transmission angle θ, given by

$$\mathbf{s}(\omega,\theta) = [1, e^{j\omega d_1 \cos\theta/c}, \ldots, e^{j\omega d_{N-1}\cos\theta/c}]^T, \tag{1}$$

where $\{\cdot\}^T$ is the transpose operation, and c is the speed of propagation. For a ULA with a half-wavelength spacing ($d_n - d_{n-1} = \lambda/2$), the steering vector is simplified to

$$\mathbf{s}(\omega,\theta) = [1, e^{j\pi\cos\theta}, \ldots, e^{j\pi(N-1)\cos\theta}]^T. \tag{2}$$

Then, the beam response of the array is given by

$$\mathrm{p}(\theta) = \mathbf{w}^H \mathbf{s}(\omega,\theta), \tag{3}$$

where $\{\cdot\}^H$ represents the Hermitian transpose, and $\mathbf{w}$ is the weight vector including all corresponding coefficients

$$\mathbf{w} = [w_1, w_2, \ldots, w_N]^T. \tag{4}$$

2.2 DM design for a given array geometry

The objective of DM design for a given array geometry is to find the set of weight coefficients giving the desired constellation values in the directions of interest while scrambling the values and simultaneously maintaining a magnitude response as low as possible in other directions. For M-ary signaling, such as multiple phase shift keying (MPSK), there are M sets of desired array responses $p_m(\theta)$, with a corresponding weight vector $\mathbf{w}_m = [w_{m,1}, \ldots, w_{m,N}]^T$, $m = 1, , \ldots, M$. Each desired response $p_m(\theta)$ as a function of θ is split into two regions: the mainlobe and the sidelobe. We sample each region and put the sampled desired responses into two vectors $\mathbf{p}_{m,ML}$ and $\mathbf{p}_{m,SL}$, respectively. Without loss of generality, we consider only one point θ_{ML} in the mainlobe and $R-1$ points $\theta_1, \theta_2, \ldots, \theta_{R-1}$ in the sidelobe region. Therefore, we have

$$\begin{aligned} \mathbf{p}_{m,SL} &= [p_m(\theta_1), p_m(\theta_2), \ldots, p_m(\theta_{R-1})] \\ \mathbf{p}_{m,ML} &= p_m(\theta_{ML}) \ . \end{aligned} \tag{5}$$

All constellation points for a fixed θ share the same steering vector and we put all the $R-1$ steering vectors at the sidelobe region into an $N \times (R-1)$ matrix $\mathbf{S}_{SL}$, and the steering vector at the mainlobe direction θ_{ML} is denoted by $\mathbf{s}(\theta_{ML})$. For the m-th constellation point, its corresponding weight coefficients can be found by

$$\begin{aligned} \min \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H \mathbf{S}_{SL}||_2 \\ \text{subject to} \quad & \mathbf{w}_m^H \mathbf{s}(\theta_{ML}) = p_{m,ML}, \end{aligned} \tag{6}$$

where $||\cdot||_2$ denotes the l_2 norm. The objective function and constraint in (6) ensure a minimum difference between desired and designed responses in the sidelobe, and a desired constellation value to the mainlobe or the direction of interest. To guarantee scrambled constellations in the sidelobe, the phase of the desired response $\mathbf{w}_m^H \mathbf{S}_{SL}$ at different sidelobe directions can be randomly generated.

3 Proposed design method

3.1 Group sparsity based design

For a standard sparse array design method, a given aperture is densely sampled with a large number of potential antennas. First, consider Fig. 1 as a grid of potential active antenna locations. Then d_{N-1} is the aperture of the array and the values of d_n, for $n = 1, 2, \ldots, N-1$, are selected to give a uniform grid, with N being a very large number. Through selecting the minimum number of non-zero valued weight coefficients to generate a response close to the desired one, sparseness is introduced. In other words, if a weight coefficient is zero-valued, the corresponding antenna will be inactive and therefore can be removed, leading to a sparse result. Assume $\mathbf{p}$ is the vector holding the desired responses at the R sampled angles, and $\mathbf{S}$ is the $N \times R$ matrix composed of the R steering vectors. Then the design can be formulated as follows

$$\min \quad ||\mathbf{w}||_1 \quad \text{subject to} \quad ||\mathbf{p} - \mathbf{w}^H\mathbf{S}||_2 \leq \alpha, \tag{7}$$

where the l_1 norm $|| \cdot ||_1$ is used as an approximation to the l_0 norm $|| \cdot ||_0$, and α is the allowed difference between the desired and designed responses.

Now, in the context of sparse array design for DM, we could modify (6) and find the sparse set of weight coefficients $\mathbf{w}_m$ through the following formulation

$$\begin{aligned} \min \quad ||\mathbf{w}_m||_1 \;\; \text{subject to} \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H\mathbf{S}_{SL}||_2 \leq \alpha \\ & \mathbf{w}_m^H\mathbf{s}(\theta_{ML}) = p_{m,ML}. \end{aligned} \tag{8}$$

However, the solution to (8) cannot guarantee the same set of active antenna positions for all constellation points. If a weight coefficient is zero in an antenna position for one constellation point, but non-zero for others, the corresponding antenna still cannot be removed. To solve the problem, similar to [30], group sparsity is introduced here, which imposes zero-valued coefficients at the same antenna locations for all constellation points simultaneously. To achieve this, we first construct the following matrices

$$\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_M] \tag{9}$$

$$\mathbf{P}_{SL} = [\mathbf{p}_{1,SL}, \mathbf{p}_{2,SL}, \ldots, \mathbf{p}_{M,SL}]^T \;, \tag{10}$$

and the vector

$$\mathbf{p}_{ML} = [p_{1,ML}, p_{2,ML}, \ldots, p_{M,ML}]^T \;. \tag{11}$$

Each row of the $N \times M$ weight matrix $\mathbf{W}$ holds the weight coefficients at the same antenna location for different constellation points and it is denoted by $\tilde{\mathbf{w}}_n = [w_{n,1}, \ldots, w_{n,M}]$ for $n = 1, \ldots, N$. Now define $\hat{\mathbf{w}}$ as a vector of l_2 norm of $\tilde{\mathbf{w}}_n$, given by

$$\hat{\mathbf{w}} = [||\tilde{\mathbf{w}}_1||_2, ||\tilde{\mathbf{w}}_2||_2, \ldots, ||\tilde{\mathbf{w}}_N||_2]^T. \tag{12}$$

Then the group sparsity based sparse array design for DM can be formulated as

$$\begin{aligned} \min \;\; ||\hat{\mathbf{w}}||_1 \quad \text{subject to} \quad & ||\mathbf{P}_{SL} - \mathbf{W}^H\mathbf{S}_{SL}||_2 \leq \alpha \\ & \mathbf{W}^H\mathbf{s}_{ML} = \mathbf{p}_{ML} \;. \end{aligned} \tag{13}$$

The problem in (13) can be solved using cvx, a package for specifying and solving convex programs [31, 32].

3.2 Reweighted l_1 norm minimisation

Different from l_0 norm which uniformly penalises all non-zero valued coefficients, the l_1 norm penalises larger weight coefficients more heavily than smaller ones. To make the l_1 norm a closer approximation to the l_0 norm, a reweighted l_1 norm minimisation method can be adopted here [33–35], where a larger weighting term is introduced to those coefficients with smaller non-zero values and a smaller weighting term to those coefficients with larger non-zero values. This weighting term will change according to the resultant coefficients at each iteration. Applying this idea to the group sparsity problem in (13), for the i-th iteration, it is formulated as follows

$$\begin{aligned} \min \quad & \sum_{n=1}^{N} \delta_n^i ||\tilde{\mathbf{w}}_n^i||_2 \\ \text{subject to} \quad & ||\mathbf{P}_{SL} - (\mathbf{W}^i)^H \mathbf{S}_{SL}||_2 \leq \alpha \\ & (\mathbf{W}^i)^H \mathbf{s}_{ML} = \mathbf{p}_{ML} \ , \end{aligned} \tag{14}$$

where the superscript i indicates the value of the corresponding parameters at the i-th iteration, and δ_n is the reweighting term for the n-th row of coefficients, given by $\delta_n^i = (||\tilde{\mathbf{w}}_n^{i-1}||_2 + \gamma)^{-1}$. The iteration processes are described as follows:

1. For the first iteration ($i = 1$), calculate the initial value $||\tilde{\mathbf{w}}_n||_2$ by solving (13).
2. Set i = $i + 1$. Use the value of the last $||\tilde{\mathbf{w}}_n^{i-1}||_2$ to calculate δ_n^i, and then find $\mathbf{W}^i$ and $||\tilde{\mathbf{w}}_n^i||_2$ by solving the problem in (14).
3. Repeat step 2 until the positions of non-zero values of the weight coefficients do not change any more for some number of iterations (three in our design examples).

Here $\gamma > 0$ is required to provide numerical stability to prevent δ_n^i becoming infinity at the current iteration if the value of a weight coefficient is zero at the previous iteration, and it is chosen to be slightly less than the minimum weight coefficient that will be implemented in the final design (i.e. the value below which the associated antenna will be considered inactive and therefore removed from the obtained design result), where $\delta_n^i ||\tilde{\mathbf{w}}_n^i||_2 = \frac{||\tilde{\mathbf{w}}_n^i||_2}{||\tilde{\mathbf{w}}_n^i||_2 + \gamma}$.

3.3 Discussion with multiple-point constraints in the mainlobe

The proposed design can work irrespective of the number of points chosen at the mainlobe area. However, one potential problem is, if we choose multiple points at the mainlobe and still want to make sure the transmission is in the desired modulation pattern over those chosen direction points, we would have to sacrifice the performance of the whole system on other aspects such as sidelobe level and main beamwidth. The reason is, each additional modulation constraint on the mainlobe area will take up one degree of freedom (DOF) away from the system and therefore leave less number of DOFs to meet other requirements of the design.

For r sample points in the mainlobe and $R-r$ points in the sidelobe, the reweighted l_1 norm minimisation formulation for sparse array design in context of DM becomes

$$\begin{aligned} \min \quad & \sum_{n=1}^{N} \delta_n^i ||\tilde{\mathbf{w}}_n^i||_2 \\ \text{subject to} \quad & ||\mathbf{P}_{SL} - (\mathbf{W}^i)^H \mathbf{S}_{SL}||_2 \leq \alpha \\ & (\mathbf{W}^i)^H \mathbf{S}_{ML} = \mathbf{P}_{ML} \ , \end{aligned} \tag{15}$$

where $\mathbf{W}$ and $\mathbf{P}_{SL}$ are unchanged, $\mathbf{S}_{ML}$ is the $N \times r$ matrix composed of the r steering vectors at the mainlobe directions and the $M \times r$ matrix $\mathbf{P}_{ML}$ holds the M desired modulation responses at the r mainlobe directions, given by

$$\mathbf{P}_{ML} = [\mathbf{p}_{1,ML}, \mathbf{p}_{2,ML}, \ldots, \mathbf{p}_{M,ML}]^T \ , \tag{16}$$

$$\mathbf{p}_{m,ML} = [p_{m,1}, p_{m,2}, \ldots, p_{m,r}] \ . \tag{17}$$

Note that for a fixed m (one of the M constellation points), $p_{m,1}$, $p_{m,2}$, ..., and $p_{m,r}$ should have the same value to make sure the same information is transmitted for all the r chosen direction points in the mainlobe.

4 Two Practical Scenarios

4.1 Steering vector error

The above design methods are based on an ideal situation where the designed steering vectors are the same as the actual ones. To have the resultant sparse array robust against various steering vector errors, we first introduce an error vector $\mathbf{e}$, and the actual steering vector is described by $\hat{\mathbf{s}} = \mathbf{s} + \mathbf{e}$, where $\mathbf{s}$ indicates the assumed steering vector. The difference between actual and designed array responses satisfies

$$|\mathbf{w}^H \hat{\mathbf{s}} - \mathbf{w}^H \mathbf{s}| = |\mathbf{w}^H \mathbf{e}| \leq \varepsilon ||\mathbf{w}||_2, \tag{18}$$

where ε is the upper norm-bound of $\mathbf{e}$. Then we can add a constraint to the previous formulations to make sure the difference between the actual and designed array responses does not exceed a predetermined threshold value β, and the new optimisation problem is formulated as

$$\begin{aligned} \min \quad & \sum_{n=1}^{N} \delta_n^i ||\tilde{\mathbf{w}}_n^i||_2 \\ \text{subject to} \quad & ||\mathbf{P}_{SL} - (\mathbf{W}^i)^H \mathbf{S}_{SL}||_2 \leq \alpha \\ & (\mathbf{W}^i)^H \mathbf{S}_{ML} = \mathbf{p}_{ML} \\ & \varepsilon ||\mathbf{w}_m^i||_2 \leq \beta \qquad \forall m = 1, 2, \ldots, M. \end{aligned} \tag{19}$$

4.2 Size constraint

In practice, the antennas may not fit into the optimised locations obtained by the above design methods since so far we have assumed that the antennas have no physical size, which is obviously not true. The most straightforward method is to merge closely located optimised antenna positions into a new one to meet the minimum spacing requirement, although clearly this may lead to a solution far away from the optimum one. To deal with this problem, two methods for enforcing a minimum spacing d_{min} between adjacent antennas in the design result are proposed.

4.2.1 Iterative sampling method

This method iteratively samples a remaining range to obtain its following optimised antenna location until the remaining range is less than d_{min}, where in each iteration the starting point of the sampling aperture is at least d_{min} away from the previous optimised locations. The details are as follows.

Step 1 At the first iteration, the first antenna is fixed at the starting point of the original aperture, i.e. $\hat{d}_{op(1)}$. We set a range from $\hat{d}_{op(1)} + d_{min}$ to the end of the original aperture as the sampling aperture, and by solving (14) we have all initial optimised locations $d_{op(i)}$ for $i = 1, 2, \ldots$, (i.e. $\hat{d}_{op(1)} = d_{op(1)}$). To make sure the first active location $\hat{d}_{op(1)}$ is included in the final result, the reweighting term for this location is set to be a very small value. Now the second active location $\hat{d}_{op(2)}$ is the average of the first cluster of optimised locations whose range is d_{min} away from $d_{op(2)}$.

Step 2 With the previously fixed active locations meeting the minimum spacing requirements, at the n-th iteration, $n = 2, 3, \ldots$, we set a range from $\hat{d}_{op(n)} + d_{min}$ to the end of the original aperture as the sampling aperture, and by solving (14) and taking an average of the new cluster which is within the range from $d_{op(n+1)}$ to $d_{op(n+1)} + d_{min}$ to find $\hat{d}_{op(n+1)}$. The process is repeated until the remaining range is less than d_{min}.

4.2.2 Modified reweighted l_1 norm minimisation method

It is based on (14) and the idea is to modify the reweighting term δ_n^i to make sure when the resultant active antenna locations are too close to each other, we will increase the value of the reweighting term significantly so that it will be penalised more heavily in the optimisation process. To achieve this, δ_n^i in (14) is modified as

$$\delta_n^i = \begin{cases} (||\tilde{\mathbf{w}}_n^{i-1}||_2 + \gamma)^{-1}, & n = 1 \\ (||\tilde{\mathbf{w}}_n^{i-1}||_2 + \gamma)^{-1}, & n > 1 \text{ \& constraint met} \\ \gamma^{-1}, & \text{otherwise} \end{cases} \quad (20)$$

The process is repeated until all spacings between adjacent active antennas are larger than d_{min}.

5 Design examples

In this section, we provide several design examples to show the performance of the proposed sparse designs in comparison with a standard ULA. The mainlobe direction is $\theta_{ML} = 90°$ and the sidelobe region is $\theta_{SL} \in [0°, 85°] \cup [95°, 180°]$, sampled every $1°$. The desired response is a value of one (magnitude) with $90°$ phase shift at the mainlobe (QPSK) and a value of 0.1 (magnitude) with random phase shifts over the sidelobe regions.

To have a fair comparison, we first obtain the DM result using the method in (6) based on a 24-element ULA with half-wavelength spacing. Based on the design result, we then calculate the error norm between the designed and the desired responses of this ULA and this value is used as α in the sparse array design formulations in (13) and (14).

To assess the performance of each design, we also calculate the bit error rate (BER) by setting the signal to noise ratio (SNR) at 12 dB in the main lobe direction. As we assume the additive white Gaussian noise (AWGN) level is the same for all directions, the SNR value will be much smaller at the sidelobe directions.

5.1 ULA design example

For the 24-element ULA with half-wavelength spacing, the resultant beam pattern for each constellation point is shown in Fig. 2(a), where all main beams are exactly pointed to $90°$ with a reasonable sidelobe level. Moreover, the phase at the main beam direction is $90°$ spaced and random in the sidelobe directions, as shown in Fig. 2(b).

5.2 Usual l_1 norm based design example

With the above ULA design, we obtain $\alpha = 2.5521$. Since the resultant sparse array may have a larger aperture than the ULA, we have set the maximum aperture to be 16.5λ, consisting of 500 equally spaced potential antennas.

By the standard group-sparsity based formulation in (13), 26 active antennas are obtained, with an average spacing of 0.655λ. To obtain the result, antennas with a coefficient value smaller than 0.001 are considered inactive and removed from the final result. In theory, we should only discard those antennas with a zero coefficient value, but in reality, it is almost impossible to have such an antenna. So a very small value is normally chosen. If this threshold value is too high, more antennas will be discarded in the final design, leading to a result with less number of antennas. This may seem desirable, but discarding antennas with a large coefficient value will also lead to a design result with a quite different beam pattern from the desired one. The change of beam pattern due to different threshold values for inactive antenna removal has been analysed in [36].

The resultant beam pattern for each constellation point is shown in Fig. 3(a), where all main beams are exactly pointed to 90° with a reasonable sidelobe level. The phase at the main beam direction is 90° spaced and random in the sidelobe directions, as shown in Fig. 3(b). As shown in Table 1, although its resultant value for $||\mathbf{p} - \mathbf{w}^H\mathbf{S}||_2$ is a little better than the ULA, the number of antennas is larger than the ULA, which is not desirable.

5.3 Reweighted l_1 norm based sparse design example

In this design, there is an additional parameter γ, which should be small enough, and in our simulations $\gamma = 0.001$ is chosen, which means that antennas associated with a weight coefficient value smaller than 0.001 will be considered inactive. With the other parameters same as in previous examples, it results in 19 active antennas with an average spacing of 0.660λ. So as expected, a sparser solution has been obtained compared to the design in (13). The array response for each constellation point is shown in Fig. 4(a) and the phase pattern in Fig. 4(b), all indicating a satisfactory design result. The array response is closer to the desired ones than the ULA according to the value of $||\mathbf{p} - \mathbf{w}^H\mathbf{S}||_2$, as shown in Table 1.

Table 1: Summary of performances of sparse arrays and ULAs.

	ULA	Usual l_1	Reweighted	Robust
Antenna number	24	26	19	20
Aperture/λ	11.5	16.37	11.87	11.87
Average spacing/λ	0.5	0.655	0.660	0.625
$\|\|\mathbf{p}-\mathbf{w}^H\mathbf{S}\|\|_2$	2.5521	2.3742	2.5478	2.6754

5.4 Robust design example

For the robust design, $\varepsilon = 1$ is set as the upper bound on the norm of the steering vector error and given the design result, this accounts for 22% of the real steering vector norm. $\beta = 0.23$ is chosen to allow maximum 23% change in the magnitude response at the main direction given the maximum allowable steering vector error. The result is a 20-antenna array with an average spacing of 0.625λ. The mean beam patterns obtained by averaging $L = 1000$ different responses resultant from randomly generated steering error vector **e** satisfying the norm-constraint are shown in Figs. 5(a), and the phase patterns are similar to the results in the earlier two designs. To show the robustness of the design, we also calculated the normalised variance of the beam pattern as follows,

$$var(\theta_r) = \frac{1}{L}\sum_{l=1}^{L}\frac{|p_l(\theta_r)-\bar{p}(\theta_r)|^2}{|\bar{p}(\theta_r)|^2}, \tag{21}$$

where $\bar{p}(\theta_r) = \frac{1}{L}\sum_{l=1}^{L} p_l(\theta_r)$ is the average achieved array response at θ_r for $r = 1, 2, \ldots, R$, and the results are shown in Fig. 5(b), with a value of almost zero in the designed main direction, less than 1 in other directions, indicating a robust geometrical layout of the antennas. The $||\mathbf{p}-\mathbf{w}^H\mathbf{S}||_2$ value is also shown in Table 1 as a comparison and we can see a comparable result has been obtained.

5.5 BER comparisions between ULA and sparse arrays

As shown in Fig. 6(a), the BERs of the ULA and sparse arrays obtained by the usual l_1 norm algorithm and the reweighted l_1 norm minimisation are all down to 10^{-5} in the mainlobe direction, while in other directions are around 0.5, further demonstrating the effectiveness of the designs. A very similar BER result is obtained for the robust design and the normalised variance of BER for the robust design is shown in Fig. 6(b), with a value of around 0.005 over sidelobe regions and 0.03 in mainlobe direction, indicating that BERs in the set are very close to the mean and each other.

Table 2: Optimised antenna locations based on the reweighted l_1 norm design (14)

n	d_n/λ	n	d_n/λ	n	d_n/λ
1	2.71	8	6.75	15	11.84
2	3.60	9	7.27	16	12.66
3	4.30	10	7.84	17	13.06
4	4.66	11	8.43	18	13.69
5	5.26	12	9.23	19	14.58
6	5.89	13	10.09		
7	6.38	14	10.94		

Table 3: Optimised antenna locations for the iterative sampling method

n	d_n/λ	n	d_n/λ	n	d_n/λ
1	0	7	6.61	13	11.95
2	2.70	8	7.60	14	12.88
3	3.49	9	8.41	15	13.71
4	4.29	10	9.30	16	14.75
5	5.04	11	10.19	17	15.41
6	5.83	12	11.07		

5.6 Reweighted l_1 norm based sparse array design with size constraints

The minimum spacing d_{min} between adjacent antennas is set to 0.55λ. For the design by (14), the spacing between the third and forth antennas, the spacing between the sixth and seventh, the spacing between the seventh and eighth, the spacing between the eighth and ninth, and the spacing between sixteenth and seventeenth are less than d_{min}, indicating an impractical design for an antenna with a physical size of 0.55λ, as shown in Table 2.

5.6.1 Iterative sampling method

By this method, all main beams in Fig. 7(a) are pointed to the mainlobe direction, and their phases are 90° spaced, as shown in Fig. 7(b). The locations listed in Table 3 show that the size constraint d_{min} has been met.

Table 4: Optimised antenna locations for the modified reweighted l_1 norm minimisation method

n	d_n/λ	n	d_n/λ	n	d_n/λ
1	0	8	5.13	15	10.94
2	0.73	9	5.89	16	11.84
3	1.49	10	6.55	17	12.66
4	2.08	11	7.37	18	13.69
5	2.68	12	8.20	19	14.58
6	3.47	13	9.06	20	15.38
7	4.30	14	10.09	21	15.97

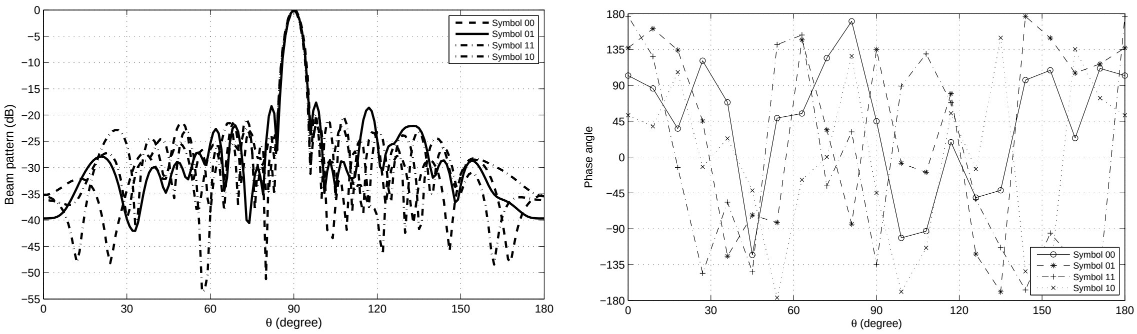
Table 5: Summary of performances of different designs with and without size constraint

	No size constraint	With size constraint	
	Reweighted	Iterative	Modified Reweighted
The number of antennas	19	17	21
Aperture/λ	11.87	15.41	15.97
Average spacing/λ	0.660	0.963	0.799
$\|\|\mathbf{p} - \mathbf{w}^H\mathbf{S}\|\|_2$	2.5478	2.6157	2.5336
Size constraint satisfied	No	Yes	Yes

5.6.2 Modified reweighted l_1 norm minimisation method

The array responses in Fig. 8(a), the phase patterns in Fig. 8(b), and the positions in Table 4 all indicate a satisfactory design result by this method. Moreover, according to the value of $||\mathbf{p} - \mathbf{w}^H\mathbf{S}||_2$, the array response is the closer to the desired one than the response resulted from the iterative sampling method, as shown in Table 5.

Note that, with the optimised non-symmetrical antenna locations and weights, the implementation of such a sparse antenna array system would be more complicated. However, it is still feasible as what we need for the proposed design is an individual tailor-made feed circuit (including phase shift and amplitude change) for each antenna.



(a) (b)

Figure 2: Design result for the uniform linear array using (6): (a) resultant beam responses, (b) resultant phase patterns.

6 Conclusions

The sparse antenna array design problem in the context of directional modulation has been studied for the first time. The main contribution is to formulate the problem from the viewpoint of CS so that it can be solved using standard convex optimisation toolboxes in the CS area. In detail, we need to find a common set of active antennas for all modulation symbols generating a response close to the desired one. The key to the solution is to realise the we have to employ the group sparsity concept, as a common antenna set cannot be guaranteed if we optimise antenna locations for each modulation symbol individually. Then, a class of compressive sensing based methods has been proposed, including the usual l_1 norm minimisation and the reweighted l_1 norm minimisation. Two practical scenarios are analysed where steering vector error happens and optimised locations are too close to each other. As shown in the provided design examples, in the context of DM, all sparse designs satisfy the mainlobe pointing to the desired direction with scrambled phases in other directions. In particular, the reweighted l_1 norm minimisation method can provide a sparer solution as expected, achieving a similar performance as the ULA but with less number of antennas.

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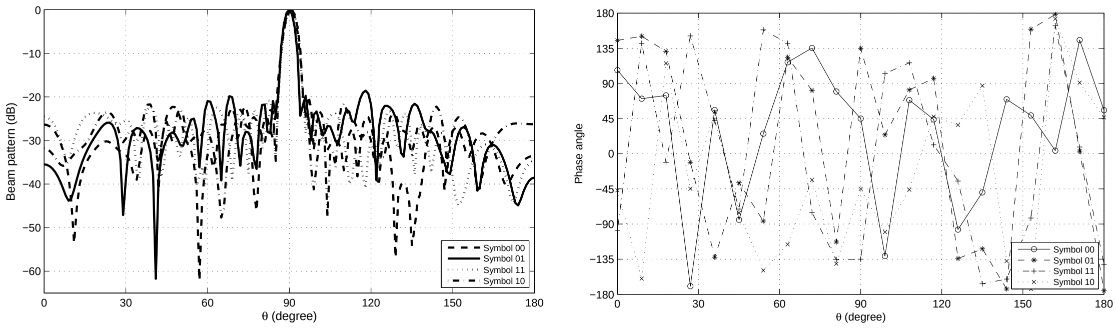
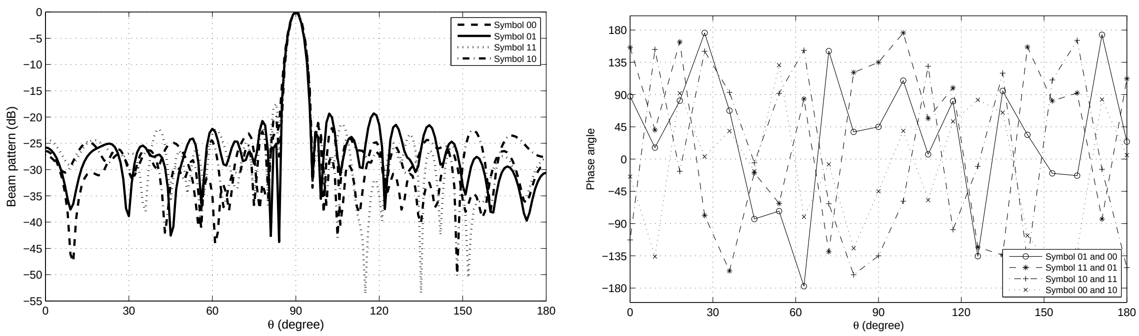


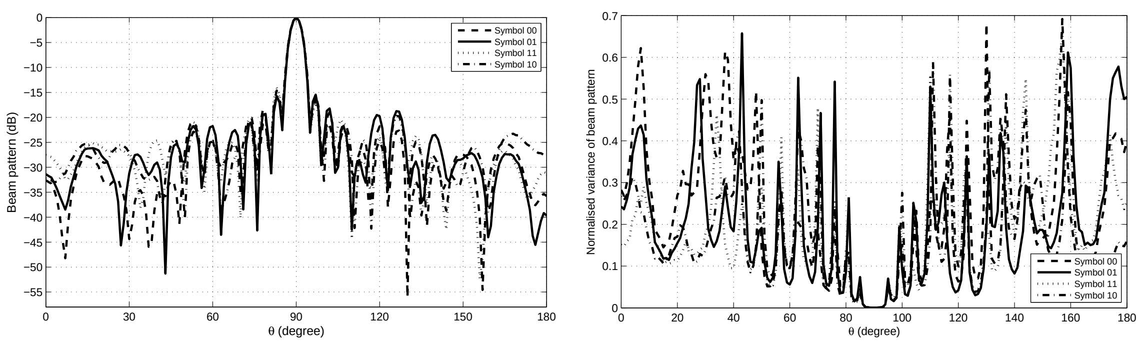
(a) (b)

Figure 3: l_1 norm based design (13): (a) resultant beam responses, (b) resultant phase patterns.



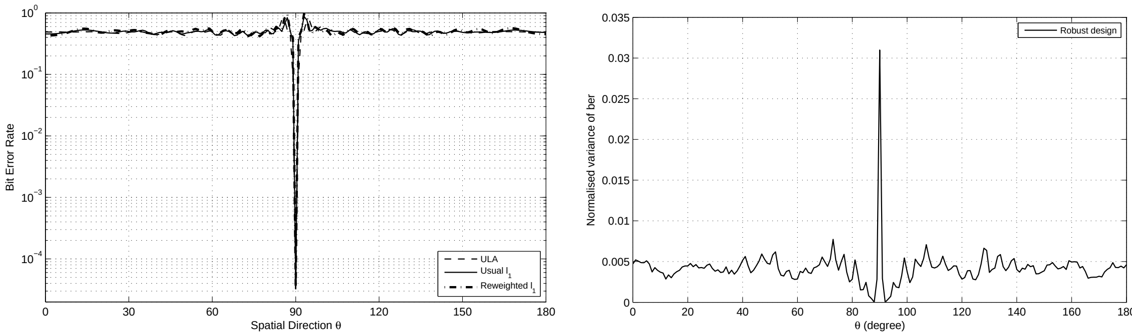
(a) (b)

Figure 4: Reweighted design using (14): (a) resultant beam responses, (b) resultant phase patterns.



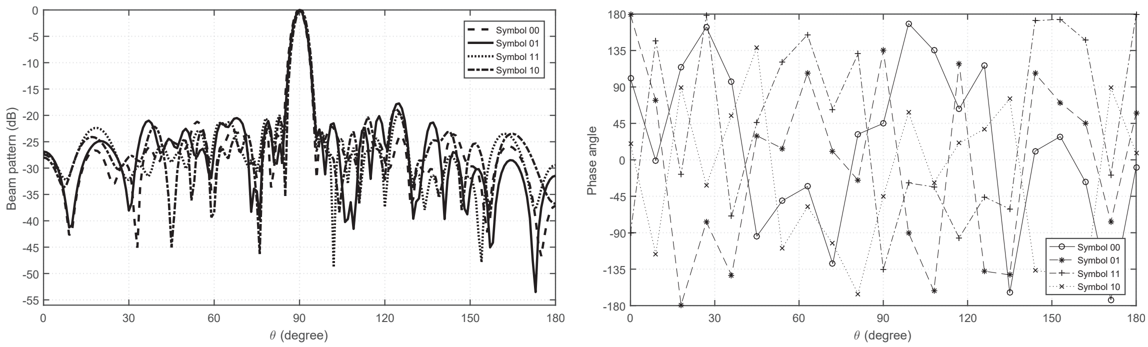
(a) (b)

Figure 5: Robust design using (19): (a) resultant beam responses, (b) normalised variance of beam pattern.



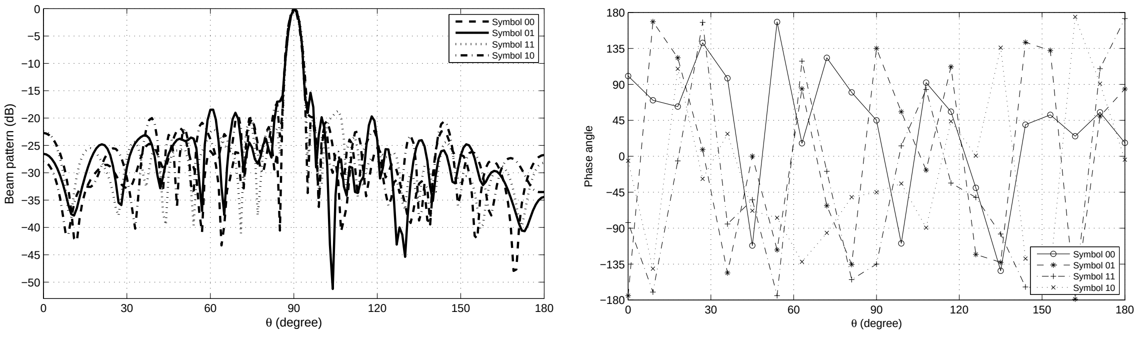
(a) (b)

Figure 6: BER performance for different design results: (a) BER spatial distributions, (b) normalised variance of BER.



(a) (b)

Figure 7: Design result for the iterative sampling method: (a) resultant beam responses, (b) resultant phase patterns.



(a) (b)

Figure 8: Design result for the modified reweighted l_1 norm minimisation method: (a) resultant beam responses, (b) resultant phase patterns.

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