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Analyzing the Mobile WiMAX System Resource Exploitation of the Downlink Direction

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Abstract Worldwide Interoperability for Microwave Access (WiMAX) constitutes a viable networking technology, providing wireless connectivity in an efficient manner. In this work, on the grounds of the defined frame structure, a novel performance analysis of the mobile WiMAX standard in terms of the downlink resource utilization is presented. The analysis is validated via simulation, the results of which are also provided along with numerical results, studying the effectiveness of the OFDMA-based downlink mapping. The results reveal the relationship among the frame dimensions, the mapped request size, and the number of unexploited OFDMA slots.

Keywords IEEE 802.16e, mapping, OFDMA, performance analysis, resource utilization, simulation, WiMAX

1. Introduction

Interoperability for Microwave Access (WiMAX) constitutes one of the most promising candidate networking technology for the realization of the 4G vision, supporting high-speed, sustainable, and long-distance wireless connections [1]. Either built in a mesh architecture or as an access network, WiMAX is based on the Orthogonal Frequency Division Multiple Access (OFDMA), which is responsible for arranging bandwidth requests from the Medium Access Control (MAC) to PHYsical (PHY) layer and vice versa in both time and frequency dimensions. Considering a WiMAX-based access network, the downlink scheduler, located in the Base Station (BS), receives the requests originating from all Mobile Stations (MSs) and forwards MAC requests to the mapper. The mapper decides on the final bandwidth distribution, taking into account the available resources in terms of slots, which define the minimum allocation unit in OFDMA systems.

One can find various analytical efforts of OFDMA systems in the literature. For example, in [2] the authors model an OFDMA system as a queuing network. They employ a quasi-birth-death

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process to obtain the queue occupancy distribution. The work in [3] provides findings of the signaling impact on the spectral efficiency in dynamic OFDMA systems. The authors infer that the throughput of dynamic OFDMA systems is significantly influenced by the signaling overhead. In [4] a subcarrier and power allocation method that minimizes the total transmitted power is proposed. The allocation strategy requires only the knowledge of the channel statistics and the rate requirements for all users. Furthermore, the authors provide an extension and a performance analysis of the suggested scheme in a multi-user environment. The work presented in [5] focuses on VoIP traffic in OFDMA systems. Both simulation and analytical models were employed to study the performance of scheduling algorithms for real-time data in the IEEE 802.16e uplink direction. Another approach in [6] involves a Markov chain to derive bounds for delay and queue backlog for various scheduling strategies. The results obtained indicate upper bounds on the buffer overflow probability, giving insights for practical buffer dimensioning problems in OFDMA systems. Authors in [7] use simulation to validate their proposed algorithm for QoS supportive resource allocation in OFDMA systems. They demonstrated the efficiency, feasibility, and fairness of their scheme, which is based on the awareness of the packet buffer state. QoS provision is also the main feature of the OFDMA rate allocation algorithm proposed in [8] with the name MARA (Maximum Achievement Rate Allocation). In the specific work, simulations were performed to exhibit the throughput improvement achieved by the algorithm in the downlink sub-frame.

However, to the best of our knowledge, there is no prior work focusing on deriving the intra-frame utilization of the OFDMA systems. Bearing in mind that the downlink mapping operation dramatically influences the efficiency of the OFDMA-based systems [9], in this study, we endeavor to cover this gap by providing a rigorous analytical approach studying the performance of the IEEE 802.16e systems with respect to downlink sub-frame dimensions and the number of connected subscribers. Simple Packing Algorithm (SPA) [10], a very common downlink mapping algorithm is adopted as the main allocation policy that simply creates allocation rectangles to provide full support of the various downlink requests. The remainder of this paper is organized as follows. Section 2 introduces SPA along with its variations and presents the analytical solution modeling its performance. Section 3 validates the analysis and Section 4 concludes the paper.

2. Analysis of the SPA Unexploited Resources

2.1. Slot Utilization in SPA

SPA is a well-known and simple to implement mapping scheme, arranging MS requests in the grid of OFDMA slots provided by the WiMAX frame. There are two variants of the specific algorithm; the vertical and the horizontal. The difference between these two variants lies in the way consecutive slots are allocated to a MS. In the vertical case, a request occupies slots in a column-by-column manner, where each column corresponds to an OFDMA timeslot. On the other hand, horizontal SPA assigns OFDMA slots in a row-by-row basis; starting with the slots of the first sub-channel, the algorithm then proceeds with the allocation of the second sub-channel's slots and so on. Both SPA variants have to align with a basic condition derived by the 802.16 physical layer specifications for the downlink; each MS request must form a rectangular area of OFDMA slots called data region. On this ground, vertical SPA always allocates a MS request either to a part of a single column or to multiple whole columns and horizontal SPA always allocates a MS request either to a part of a single row or to multiple whole rows. Each request forwarded by the scheduler is examined to decide whether it is small enough to fit into a single column/row (vertical/horizontal SPA) or it has to be mapped to multiple columns/rows. It is noted that one column/row is allowed to include multiple MS requests as long as the rectangular shape constraint is satisfied. This process carries on until all requests are examined or the available slots are exhausted and there is no adequate frame space to handle the remaining requests.

In general, the procedure of mapping MS requests to OFDMA slots involves idle slots. This may be attributed to the rectangular restriction of the data region. Specifically, a request that is allocated slots in new columns/rows (vertical/horizontal case) may leave idle slots in the previously assigned column/row or in its last assigned column/row. Moreover, idle slots may appear towards the end of the downlink sub-frame, either due to the lack of requests or the lack of adequate frame space. A graphic representation example of the above described slot distribution technique is illustrated in Fig. 1, where four MS requests are mapped according to vertical SPA. The unexploited slots are depicted in this resource grid as idle slots and the formed data regions are highlighted with double line borders. In the downlink sub-frame, idle slots are inactive blocks that correspond to no data reception. Hence, they lead to lower bandwidth utilization and consequently to performance degradation. The symmetric operation of the vertical and horizontal variants of SPA allows us to analyze both schemes together, since the number of occupied slots is identical in both cases, depending exclusively on the requests and the respective resource space dimensions.

2.2 Analytical Solution

In this subsection, the unexploited part of the downlink sub-frame resource space when SPA is employed is mathematically estimated. The available OFDMA slots are allocated to MS requests on a per frame basis. That is for each frame the requests provided by the scheduler are examined and mapped to data regions according to the SPA technique. It is noted that the specific analysis refers to the vertical SPA case; however, it is straightly applicable to the horizontal case, because of the symmetric operation of the two variants regarding the number of idle slots. Actually, interchanging in the following formulae sub-frame height with width causes the transition from the vertical case to the horizontal one and vice versa.

When estimating system's behavior in such type of analytical approaches, it is a common practice to assume Poisson arrivals. Thus, it is hereafter considered that the scheduler feeds the mapping entity with MS requests of a random number of OFDMA slots that follows Poisson distribution with parameter λ. In fact, the specific random variable could never be nullified nor be higher than the total number of available slots in the whole downlink sub-frame, since a MS could never request zero slots or more than the resource space. Hence, the request's size actually follows a restricted Poisson distribution, so parameter λ is not exactly equal to its expected value.

The first step in this analysis is to estimate the number of idle slots, when a MS is assigned downlink slots in a new column (since we examine the vertical SPA variant). In more specific, a MS request that occupies s slots starting in a new column leaves idle slots, when it does not fill an integer number of columns. Considering that each column is of height H OFDMA slots, a request leaves no idle slots only in the case that its size is an integer multiple of H. In the general case, d idle slots are left when s is a multiple of H, minus d. For instance, a request of 2H – 1 slots occupies one full column and H – 1 slots in the second column, where it leaves the last slot idle. Based on this fact and taking into account that the maximum feasible number of columns occupied by a request is W, which is defined as the sub-frame width, the probability a MS that is assigned slots in a new column leaves d idle slots is given by:

$$q[d] = \frac{\sum_{i=1}^{W} e^{-\lambda} \frac{\lambda^{(iH-d)}}{(iH-d)!}}{\sum_{i=1}^{W\times H} e^{-\lambda} \frac{\lambda^{i}}{i!}}, \quad \lambda > 0,\ 0 \le d \le H-1,\ d, W, H \in \mathbb{Z}^{+}. \quad (1)$$

Note that in Equation (1), the numerator is directly derived by the assumption that the request size follows the Poisson distribution [11], as it is already explained. However, the resulted value is normalized, so that the sum of the probabilities of all possible outcomes is one, on the ground that the request size is actually bounded in the range of 1 to WH, since a practical system does not allow mapping in a downlink sub-frame of zero slots nor requests exceeding the maximum capacity of the sub-frame. Thus, the aggregated Poisson probability in Equation (1) is divided by the probability the number of mapped slots lies in the abovementioned limits, which quantity is actually the normalizing constant [12]. It is noted that the specific normalization practice, which ensures real system conditions, is followed throughout the presented analysis. Next, the estimated number of idle slots a MS, which is assigned slots in a new column, leaves is denoted by D and calculated as follows:

$$D = \sum_{d=1}^{H-1} q[d] \times d, \quad W \in \mathbb{Z}^{+}. \quad (2)$$

The subsequent step is to estimate the number of MSs per frame that are served in new downlink columns. For that reason, we calculate the mean number of columns a MS that is assigned slots in a new column occupies. The probability that it occupies k columns derives by:

$$p[k] = \frac{\sum_{i=(k-1)\times H+1}^{k\times H} e^{-\lambda} \frac{\lambda^{i}}{i!}}{\sum_{i=1}^{W\times H} e^{-\lambda} \frac{\lambda^{i}}{i!}}, \quad \lambda > 0,\ 1 \le k \le W,\ k, W, H \in \mathbb{Z}^{+}. \quad (3)$$

Consequently, the mean number of columns a MS that is assigned slots in a new column occupies is equal to:

$$M = \sum_{k=1}^{W} p[k] \times k, \quad W \in \mathbb{Z}^{+}. \quad (4)$$

As a result, the number of MSs per frame that are served with new downlink columns is estimated to be:

$$R = \frac{W}{M}, \quad M \le W, \quad W \in \mathbb{Z}^{+}. \quad (5)$$

We proceed with the estimation of the number of MSs that are served with columns already occupied by another MS. The probability that l requests are mapped to a single column of height H is given by:

$$Q[l] = \sum_{i=0}^{H-l} P_l[H-i] \times P_1[>i], \quad 1 \le l \le H, \quad l, H \in \mathbb{Z}^{+}. \ (6)$$

where $P_y[x]$ is the probability that the total number of slots requested by y MSs is equal to x and $P_y[>x]$ gives the probability that the aggregate number of requested slots is higher than x. Regarding the computation of $P_y[x]$, it would not be precise to employ the additive property of the Poisson distribution, according to which the sum of Poisson variables is also Poisson distributed, because in this analysis the sum of the random variables does not follow the exact Poisson distribution, as already explained. For this reason, we provide here the detailed analysis outcome:

$$P_y[x] = \sum_{i=1}^{x-(y-1)} P_{y-1}[x-i] \times P_1[i], \quad x \ge y, \quad x, y \in \mathbb{Z}^{+}. \ (7)$$

Please note that $P_y[x]=0$, when $x < y$, since there are no empty downlink requests. Furthermore, $P_y[x]=0$, when $x > y \times W \times H$, as there can be no request exceeding the downlink sub-frame size. The respective probability for a single MS request is:

$$P_1[x] = \frac{e^{-\lambda} \frac{\lambda^x}{x!}}{\sum_{i=1}^{W \times H} e^{-\lambda} \frac{\lambda^i}{i!}}, \quad \lambda > 0, \quad x \in \mathbb{Z}^{+}. \ (8)$$

Referring to (6), we can easily calculate $P_y[>x]$ using the following formula:

$$P_y[>x] = \sum_{i=x+1}^{\infty} P_y[i] = 1 - \sum_{i=y}^{x} P_y[i], \quad x \ge y, \quad x, y \in \mathbb{Z}^{+}. \ (9)$$

Based on (6), the estimated number of requests mapped to a single column already occupied by another MS is:

$$A = \sum_{l=2}^{H} Q[l] \times (l-1), \quad H \in \mathbb{Z}^{+}. \ (10)$$

Our goal to estimate the mean number of MSs served with columns already occupied by another MS is now achieved via:

$$E = p[1] \times A \times \frac{W}{M} \xrightarrow{(5)} E = p[1] \times A \times R, \quad W \in \mathbb{Z}^{+}. \ (11)$$

Next, the analysis proceeds with the calculation of the mean number of slots allocated to MS requests mapped to columns already occupied by another MS. The probability that m slots are allocated to an already occupied column is given by:

$$u[m] = \sum_{i=1}^{H-m} \left(P_1[i] \times P_1[> H - m - i] \times \sum_{j=1}^{m} P_j[m] \right), \quad (12)$$
$$1 \leq m \leq H, \quad l, H \in \mathbb{Z}^+.$$

The mean number of slots allocated to an already occupied column is then equal to:

$$B = \sum_{m=1}^{H-1} u[m] \times m, \quad H \in \mathbb{Z}^+. \ (13)$$

The mean number of idle slots per downlink sub-frame (I) finally results by computing the number of idle slots related with the MSs that are served with new columns and then subtracting the slots allocated in the already occupied columns. Specifically, it holds:

$$I = R \times D - E \times \frac{B}{A}. \ (14)$$

Equation (14) provides an estimation of the number of unexploited OFDMA slots per downlink sub-frame, given that the number of requests scheduled for mapping is unlimited. Thus, we come up with a theoretical value corresponding to the saturation case. In what follows, the mean number of idle downlink slots per frame is estimated for a limited number of MS requests. Considering that there are N MSs requesting for resources, the following formula provides the mean number of idle OFDMA slots per downlink sub-frame:

$$I_N = \begin{cases} I & , N \geq R + E \\ \frac{NRD}{R+E} + \left(W - \frac{NRM}{R+E} \right) H - \frac{NEB}{(R+E)A}, & N < R + E \end{cases}, \quad N, W, H \in \mathbb{Z}^+. \ (15)$$

3 Numerical Results and Discussion

A simulation program[1] was developed in MATLAB to validate the mathematical approach presented above. The simulator implements a model of the vertical SPA technique, mapping requests that are generated according to the described restricted Poisson distribution to the two-dimension slot space.

The frame characteristics considered by this work are as follows. The Time Division

[1] The simulator source code is readily available and provided upon request.

Duplexing (TDD) technique is adopted. The frequency diversity mode is PUCH (Partially Used sub-Channelization), according to which H equals to 30 sub-channels. The downlink sub-frame width is determined by the downlink-to-uplink sub-frame ratio and the frame duration. Specifically, taking into account the control information fields, for the default 5ms frame, W is equal to 9, 12, and 15 timeslots corresponding to downlink-to-uplink sub-frame ratios of 1:1, 2:1, and 3:1, respectively, whereas for 10ms frame, W is equal to 21, 27, and 33 timeslots corresponding to downlink-to-uplink sub-frame ratios of 1:1, 2:1, and 3:1, respectively.

In the presented graphs, it is obvious that the analytical and the simulation results coincide to a satisfactory degree. Fig. 2 and Fig. 3 assume unlimited requests and depict the mean number of idle slots with respect to the Poisson parameter λ, for different downlink-to-uplink sub-frame ratios and considering frame duration 5ms and 10ms, respectively. Generally, it is evident that the larger the downlink sub-frame is the more idle slots are left. Initially, the number of idle slots increases as λ increases, since more “blank” spaces take place. This is due to the increasing request size that disallows mapping multiple requests in single columns. When the mean request size reaches the frame height, an upwards concavity is noticed, since a request tends to “fit” in a whole column. This local minimum value, that is approximately equal to 26% of the total number of slots, occurs for λ close to 25, which corresponds to a mean request size approximately equal to 30 on the basis of the considered restricted Poisson distribution. Then and up to the maximum considered λ value of 50, there is a decreasing trend justified by the large regions occupied by the requests leaving limited idle slots, since they expand to multiple columns and cause less "blank" spaces. The two highest numbers of idle slots, corresponding approximately to 34% and 31% of the total slots, are observed for λ=35 and λ=19, respectively. In Fig. 4 and Fig. 5, the estimated number of idle slots is plotted versus the number of MS requests considering λ=20, for different downlink-to-uplink sub-frame ratios for frame duration 5ms and 10ms, respectively. Apparently, an increased number of requests leads to reduced idle slots, until a saturation point is reached, the exact value of which depends on the frame duration and the downlink-to-uplink sub-frame ratio. This value converges close to 30% of the total number of slots in the resource grid.

4 Conclusions

A mathematical analysis on the unexploited downlink resources of the 802.16e frame was conducted for varying frame duration, number of MSs, and request size. The considered request mapping technique is the commonly employed SPA scheme and the analysis outcome concerns both vertical and horizontal variants, because of their symmetrical operation. The numerical results, which were validated via MATLAB simulation, have shown that the number of idle OFDMA slots depends on the available resource space size as well as the length of the MS requests. The minimum slot utilization amounts approximately to 66% and is observed for request size about 20% higher than the time/frequency grid height, in case of vertical SPA, or width, in case of horizontal SPA.

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Time Axis

Frequency Axis

RE1	RE1	RE2	RE4	i
RE1	RE1	RE2	RE4	i
RE1	RE1	RE2	RE4	i
RE1	RE1	RE2	RE4	i
RE1	RE1	RE3	RE4	i
RE1	i	RE3	RE4	i
RE1	i	RE3	i	i
RE1	i	RE3	i	I
RE1	i	I	i	I
RE1	i	I	i	I

Fig. 1 Example of mapping four MS requests in an OFDMA resource space on the basis of vertical SPA (RE: request, i: idle slot).

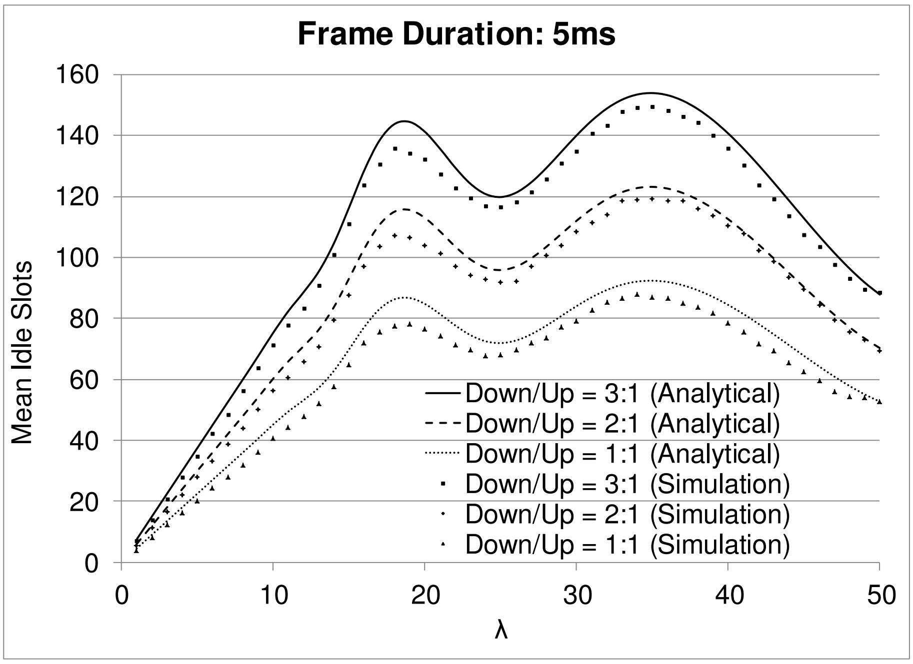

Fig. 2 Mean number of idle slots per DL sub-frame versus the Poisson λ parameter related to the requested slots in a 5ms long frame.

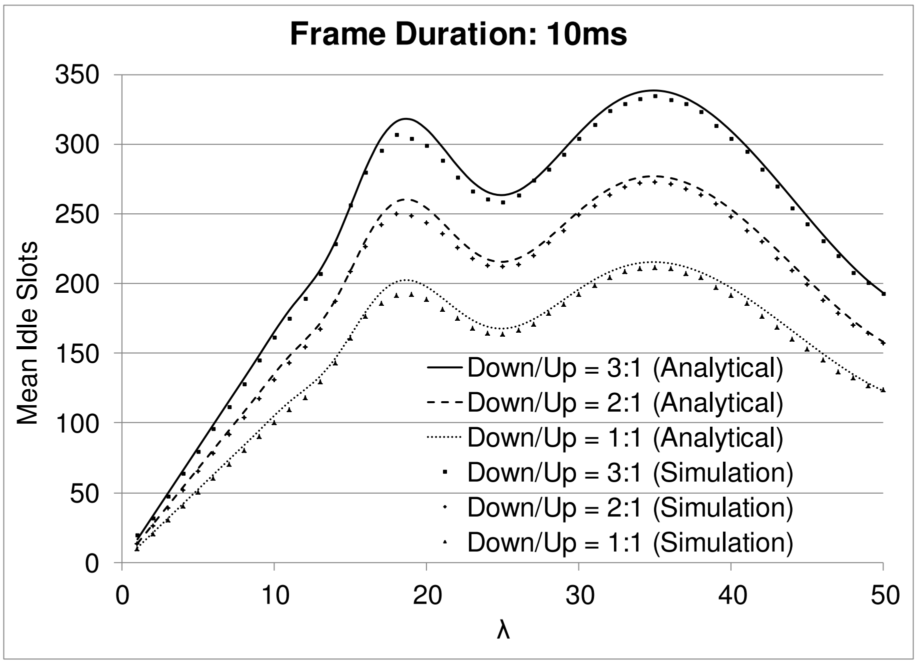

Fig. 3 Mean number of idle slots per DL sub-frame versus the Poisson λ parameter related to the requested slots in 10ms long frame.

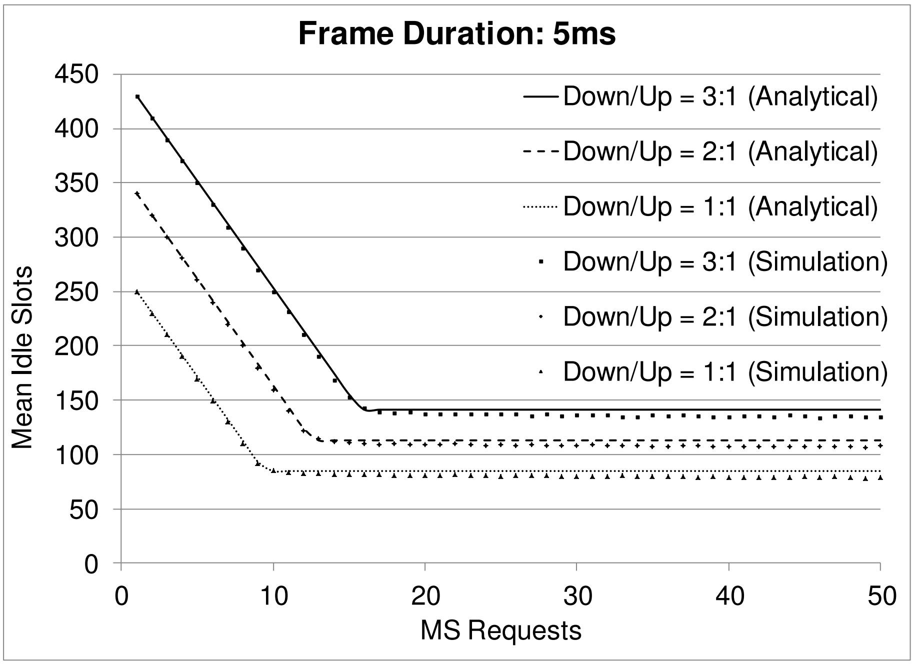

Fig. 4 Mean number of idle slots per DL sub-frame versus number of MS requests in 5ms long frame.

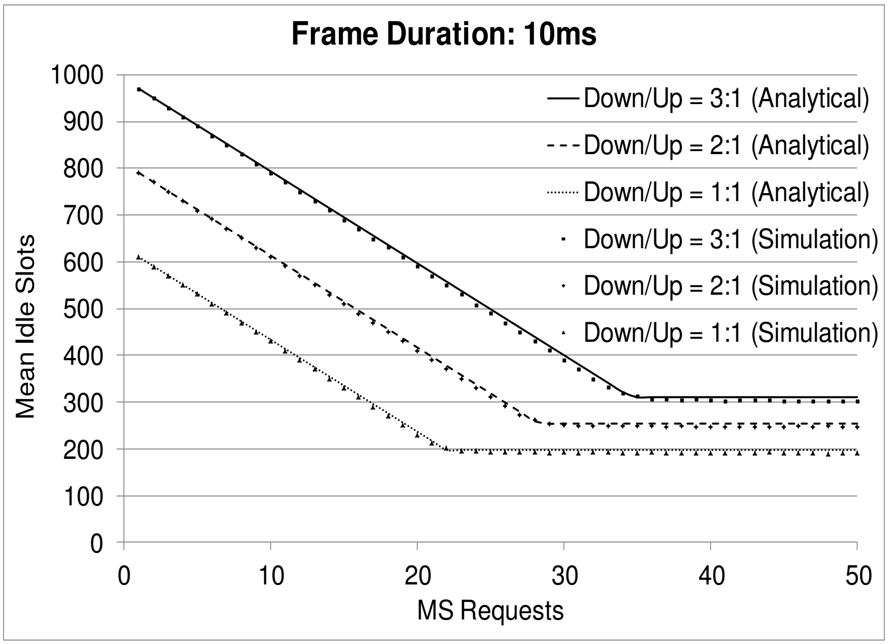

Fig. 5 Mean number of idle slots per DL sub-frame versus number of MS requests in 10ms long frame.