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Did Poverty Reduction Reach the Poorest of the Poor? Complementary Measures of Poverty and Inequality in the Counting Approach*

Suman Seth† and Sabina Alkire‡

Abstract

A number of multidimensional poverty measures have recently been proposed, within counting approach framework, respecting the ordinal nature of dimensions. Besides ensuring a reduction in poverty, however, it is important to monitor distributional changes to ensure that poverty reduction has been inclusive in reaching the poorest. Distributional issues are typically captured by adjusting a poverty measure to be sensitive to inequality among the poor. This approach however has certain practical and conceptual limitations. It conflicts, for example, with some policy-relevant measurement features, such as the ability to decompose a measure into dimensions post-identification, and does not create an appropriate framework for assessing disparity in poverty across population subgroups. In this paper, we propose and justify the use of a separate decomposable inequality measure – a positive multiple of 'variance' – to capture the distribution of deprivations among the poor and to assess disparity in poverty across population subgroups. We demonstrate the applicability of our approach through two contrasting inter-temporal illustrations using Demographic Health Survey (DHS) datasets for Haiti and India.

Keywords: Inequality decomposition, Inequality among the poor, Multidimensional poverty, Counting approach, Variance decomposition, Haiti, India.

JEL classification: D63, O1, I32

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† Economics Division, Leeds University Business School, Maurice Keyworth Building, Leeds, LS2 9JT, UK, and Oxford Poverty and Human Development Initiative (OPHI), University of Oxford, Oxford OX1 3TB, UK, S.Seth@leeds.ac.uk.

‡ Oxford Poverty & Human Development Initiative (OPHI), Queen Elizabeth House (QEH), Department of International Development, University of Oxford, sabina.alkire@qeh.ox.ac.uk.

1 Introduction

The progress of a society remains incomplete without improving the conditions of those stricken with poverty. It is commonly agreed that there are three important aspects in poverty measurement – incidence, intensity and inequality – that should receive consideration because each may differently influence policy incentives.[1] Measures that only capture incidence, such as the World Bank's $1.90/day poverty headcount ratio, create incentives for a policy maker, who is keen on showing a large reduction in overall poverty, to improve the lives of the least poor, as doing so will have the same poverty impact at a lower cost than addressing those experiencing the severest poverty. On the other hand, measures that capture both incidence and intensity, such as the Multidimensional Poverty Index (MPI) published in the United Nation Development Programme's (UNDP) *Human Development Reports*, create incentives for a policy maker to address the poorest as well as the least poor, but may not provide over-riding incentives to prioritize the poorest. Such a priority can, however, be provided by monitoring the distribution of deprivations among the poor, which would ensure that the fruits of poverty alleviation are shared by all and that the poorest individuals or groups are not left behind.

The classical approach to incorporate distributional sensitivity into poverty measurement, following the seminal work of Sen (1976), has been to adjust a poverty measure to make it sensitive to the distribution across degrees of deprivations among the poor. We will refer to this approach as the *assimilated approach* to poverty measurement. A number of poverty measures using the assimilated approach have been developed in the context of unidimensional poverty measurement as well as multidimensional poverty measurement following a growing consensus that poverty is not just a reflection of deprivation in any single dimension.[2] We classify the multidimensional approaches into two types. One constructs measures under the assumption that one or more cadinally measurable indicators are available for all underlying dimensions, whose wider applicability is hindered by the fact that indicators available for most dimensions in practice are ordered categorical or binary. The other takes into consideration this practical nature of the indicators while constructing poverty measures. Following the extensive literature (Atkinson 2003), we refer to this approach as the counting approach, where the poor are identified by counting their number (or weighted sum) of deprivations in different dimensions. In our paper, we focus on the multidimensional counting approach owing to its practicality and a number of recent applications.[3]

In the counting approach framework, the indicators of all dimensions are binary or dichotomized between deprived and non-deprived. Because the indicators are dichotomised, the only way to capture inequality or distributional changes is by observing the distribution of the simultaneous

[1] Jenkins and Lambert (1997) refer to these three aspects as 'three I's of poverty'.

[2] Single dimensional measures include Thon (1979), Clark, Hemming and Ulph (1981), Chakravarty (1983), Foster, Greer and Thorbecke (1984), and Shorrocks (1995). Multidimensional measures include Chakravarty, Mukherjee and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Massoumi and Lugo (2008), Alkire and Foster (2011, 2016), Bossert, Chakravarty and D'Ambrosio (2013), Jayaraj and Subramanian (2009), and Aaberge and Peluso (2012).

[3] A particular counting measure, the adjusted headcount ratio developed by Alkire and Foster (2011), has been applied by international organizations and country governments. The UNDP used it to introduce the Global MPI (Alkire and Santos 2010), the Colombian and Mexican governments used it to create official poverty measures (Foster 2007; CONEVAL 2011; Angulo, Diaz and Pardo 2011); the Bhutanese government adapted it to create the Gross National Happiness Index (Alkire, Ura, Wangdi and Zangmo 2012).

deprivations that poor people suffer. More specifically, each person's deprivation profile is summarized in a cardinally meaningful deprivation score by obtaining a weighted sum of their deprivations. The distribution of such deprivation scores across the poor can be used to capture inequality among the poor. In order to incorporate distribution sensitivity, the assimilated approach has been adopted in the counting approach framework by Bossert, Chakravarty and D'Ambrosio (2013), Jayaraj and Subramanian (2009), Aaberge and Peluso (2012) and Alkire and Foster (2016).

The assimilated approach, however, suffers from certain practical and conceptual limitations. First, measures based on the assimilated approach are useful for poverty comparisons across space and time, but the overall measure may become rather intricate to interpret and the underlying policy message may become obscure. Second, some assimilated measures are broken down into different partial indices – each separately capturing the incidence, intensity and inequality across the poor – in order to study their contribution to overall poverty. However, their relative contribution to the overall poverty assessment – whether cross-sectional or inter-temporal – is seldom made transparent. Third, assimilated measures do not provide the appropriate framework for capturing disparity in poverty across population subgroups. An overall improvement in poverty may come with an improvement in the distribution among the poor or with a more uniform reduction in intensities across the poor, but, simultaneously, accompany a non-uniform improvement in poverty across different population subgroups. Monitoring uneven progress is important in order to avoid aggravating horizontal inequality (Stewart 2008). Finally, and most importantly, assimilated measures in a multidimensional framework compromise a crucial policy-relevant property – *dimensional breakdown*. This property allows overall poverty to be expressed as a weighted sum of dimensional deprivations, and it is required to construct many of the existing public policy responses to multidimensional poverty, such as policies addressing the composition of poverty, allocating resources across sectors, and designing multi-sectoral policies (Alkire and Foster 2016). The final limitation leads to the impossibility result established by Alkire and Foster (2016) – because one must either choose a measure that respects the dimensional breakdown property or a measure that is sensitive to the distribution among the poor.

In this paper, we explore the possibility of using a separate inequality measure that may capture inequality among the poor alongside the widely adopted 'Adjusted Headcount Ratio', which is an intuitive measure of poverty in the counting framework but is not sestitive to distribution among the poor.[4] A separate inequality measure captures the inequality across the degree of deprivations adding valuable information besides the intuitive poverty measure. If a poverty alleviation program has reduced poverty by leaving the poorest behind, then even though the poverty measure would show a reduction in poverty, the inequality measure will reflect the deterioration in the distribution of deprivations among the poor. If the inequality measure is decomposable then its between-group component can provide valuable information by assessing whether changes in poverty have been uniform across population subgroups. Finally, our approach breaks the impasse by allowing one to capture the distribution across the degrees of deprivations through the inequality measure and at the same time allowing the choice of a poverty measure that respects dimensional breakdown.

[4] Separate pie diagrams, in addition to MPIs, have been used to capture the distribution across deprivation scores among the poor (see Chapter 10.1, Alkire *et al.* 2015). The pie diagram provides a good visual depiction of a distribution, but its applicability is limited when the number of countries or sub-national regions under consideration is large.

We motivate our proposed measure of inequality across the poor in the counting framework through certain normative value judgments and some desirable properties. We propose to ensure that our inequality measure (i) is additively decomposable (allowing the overall inequality to be broken down into two components: inequality within population subgroups and inequality between population subgroups) in order to facilitate the study of disparity across subgroups; (ii) allows the total within-group measure of inequality to be independent of mere changes in subgroup averages, and (iii) reflects the same level of inequality whether the magnitudes of deprivation are computed by counting deprivations or by counting attainments. We show that the only inequality measure that ensures these requirements is the positive multiple of 'variance'.

We support our methodological development with two inter-temporal illustrations using Demographic Health Survey (DHS) datasets of Haiti and India. In order to assess poverty, we use the global MPI, which is a counting measure respecting the dimensional breakdown property (Alkire and Santos 2014). In Haiti, we find that between 2006 and 2012 the MPI fell by 0.014 points per annum, which was accompanied by large and statistically significant reductions in incidence and intensity as well as inequality among the poor. Improvement was visible in the distribution across the degrees of deprivations among the poor within every sub-national region and disparity between sub-national MPIs also went down. A contrasting scene was visible in the case of India, where, although the pace of MPI reduction between 1999 and 2006 was half the pace of the MPI reduction in Haiti, reductions in both incidence and intensity were statistically significant. However, the reduction in inequality among the poor was modest and barely significant. Inequality among the poor within certain subgroups did not show any sign of improvement and disparity between sub-national MPIs went up.

The rest of the paper is structured as follows. Section 2 presents the counting approach framework for measuring poverty. Section 3 reviews and evaluates the assimilated approach to poverty measurement and discusses how using a separate inequality measure provides valuable information. In Section 4, we propose and justify the inequality measure that is suitable for the purpose of the paper and present some policy-relevant decompositions. Supporting empirical illustrations are given in Seciton 5. Section 6 concludes.

2 The Counting Approach to Poverty Measurement

This section presents the counting approach framework, which is the mainstay of our paper. We begin by assuming that there is a hypothetical society containing $n \geq 2$ persons and their well-being is assessed by a fixed set of $d \geq 2$ dimensions.[5] These d dimensions may not contribute equally to the overall well-being and so a relative weight w_j is assigned to each dimension j based on its value relative to other dimensions, such that $w_j > 0$ and $\sum_j w_j = 1$. These d weights are summarized by vector w. The achievement of each person i in dimension j is denoted by $x_{ij} \in \mathbb{R}_+$ and the achievements of all n persons in d dimensions are summarized by an $n \times d$. achievement matrix $X \in \mathbb{R}_+^{n \times d}$. The set of all achievement matrices of population size n is denoted by $\mathcal{X}_n$ and the set of all possible $n \times d$ matrices of any population size is denoted by $\mathcal{X} = \bigcup_n \mathcal{X}_n$.

[5] In many studies, the terms 'dimensions' and 'indicators' are used differently, where dimensions are assumed to be the pillars of well-being and each dimension is measured using one or more indicators.

2.1 Identification of Deprivations and of Poverty

In the counting approach framework, first the deprivations are identified and the information on deprivations is used to identify the poor. In order to identify deprivations, each dimension j is assigned a deprivation cutoff $z_j \in \mathbb{R}_{++}$. If $x_{ij} < z_j$, the person is considered *deprived* in dimension j or equivalently considered to have *failed to attain* the threshold in that dimension. On the other hand, if $x_{ij} \geq z_j$, then person i is considered *non-deprived* in dimension j or equivalently considered to have *attained* the threshold. The d deprivation cutoffs are summarized by vector z and the set of all possible deprivation cutoff vectors is denoted by $\boldsymbol{z}$. Note that the poor are identified either by counting deprivations or equivalently by counting attainments. The identification by counting deprivations is a dual to the identification by counting attainments. We present notation for both alternatives as they are required in the subsequent analysis.

2.1.1 Counting Deprivations

For any $X \in \mathcal{X}$, each person i is assigned a *deprivation status value* $g_{ij} = 1$ in dimension j if $x_{ij} < z_j$ and $g_{ij} = 0$ otherwise. The *deprivation score* of person i, π_i, is obtained by the weighted sum of the deprivation status values, i.e., $\pi_i = \sum_j w_j g_{ij}$. By definition, $\pi_i \in [0,1]\ \forall i$ and $\pi_i > \pi_{i'}$ implies that person i' suffers a higher sum of deprivation(s) than person i. The deprivation scores of all n persons in the society are summarized by vector $\pi = (\pi_1, \dots, \pi_n)$.[6]

2.1.2 Identification of Poverty

After the deprivation scores are obtained, any person i is identified as poor if $\pi_i \geq k$ for any poverty cutoff $k \in (0,1]$.[7] We define the identification function as $\rho_i(k) = 1$ if $\pi_i \geq k$ and $\rho_i(k) = 0$ otherwise. The post-identification *censored deprivation score* of person i is denoted by $c_i = \pi_i \rho_i(k)$ and the corresponding vector by $c = (c_1, \dots, c_n)$. Thus, $c_i = \pi_i$ if $\pi_i \geq k$ and $c_i = 0$, otherwise.[8] We denote the number of poor after identification by q and the set of poor by Z. The share of poor population or the *incidence* is denoted by $H(\pi; k) = H(c) = q/n$. Without loss of generality, we assume that people are ordered by deprivation score from high to low such that $c_1 \geq \cdots \geq c_n$. Thus, if $q > 0$, then the first q persons are identified as poor. We summarize the deprivation scores of the poor by a containing q elements such that $a_i = c_i$ for all $i = 1, \dots, q$. The average of all elements in a, is the average deprivation score among the poor or *intensity*, which is denoted by $A(\pi; k) = \left[\sum_{i=1}^{q} a_i\right]/q$.

2.1.3 Counting Attainments

An alternative but equivalent approach that can be used to assess the extent of multiple deprivations is counting attainments, where each person i is assigned an *attainment status value* $\tilde{g}_{ij} = 1$ in dimension j if $x_{ij} \geq z_j$ and $\tilde{g}_{ij} = 0$ otherwise (see Alkire and Foster 2016). The *attainment*

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[6] We use a slightly different notation than Alkire and Foster (2011) for denoting deprivation score vectors to simplify presentations. Alkire and Foster denote the deprivation score vector by c and the corresponding censored vector by $c(k)$. We instead use π to denote the deprivation score vector and c to denote the corresponding censored vector.

[7] If $k = 1$, then it is the intersection approach. If $k \in (0, \min_j\{w_j\}]$, it is the union approach. If $\min_j\{w_j\} < k < 1$, it is the intermediate approach (Alkire and Foster 2011).

[8] Note that $c = \pi$ when a union approach is used for identifying the poor.

score of person i can be obtained by $\tilde{\pi}_i = \sum_j w_j \tilde{g}_{ij}$ and the attainment scores of all persons are summarized by vector $\tilde{\pi}$. In this case, the lower is the attainment score, the higher is the extent of deprivation. Note that by construction, for the same achievement matrix $X \in \mathcal{X}$, the same deprivation cutoff vector $z \in \boldsymbol{z}$ and the same weight vector w, we have $\tilde{\pi}_i = 1 - \pi_i \; \forall i$. Moreover, if the same poverty cutoff $k \in (0,1]$ is used such that any person i is identified as poor whenever $\tilde{\pi}_i \leq (1-k)$ and non-poor otherwise, then the same set of people Z are identified as poor. Thus, $\tilde{\rho}_i(k) = 1$ if $\tilde{\pi}_i \leq (1-k)$ and $\tilde{\rho}_i(k) = 0$ otherwise. The share of poor population, as earlier, is denoted by $H(\tilde{\pi}; k) = q/n$. The post-identification *censored attainment score* $\tilde{c}_i$ can be obtained as $\tilde{c}_i = \tilde{\pi}_i \tilde{\rho}_i(k)$ and the corresponding vector is denoted by $\tilde{c}$. As in case of deprivations, it can be assumed that $\tilde{c}_1 \leq \cdots \leq \tilde{c}_n$. The attainment score vector among the poor is denoted by $\tilde{a}$ containing q elements such that $\tilde{a}_i = \tilde{c}_i \; \forall i = 1, \ldots, q$. By definition, $\tilde{a}_i = 1 - a_i \;\; \forall i$ and $\tilde{A}(\tilde{\pi}; k) = \left[\sum_{i=1}^{q} \tilde{a}_i\right]/q = 1 - \left[\sum_{i=1}^{q} a_i\right]/q = 1 - A(\pi; k)$.

2.2 Aggregation

After identification in the counting approach framework, the information on the censored achievements is used for measuring the level of poverty in the society using a poverty index $P(X; z, k, w)$, where $P(X; z, k, w) = 0$ represents the lowest level of poverty. For any $X, X' \in \mathcal{X}$, $P(X'; z, k, w) > P(X; z, k, w)$ implies that X' has the higher level of poverty than X, irrespective of whether the identification is based on counting deprivations or counting attainments. In Table 1, we present different poverty measures that have been proposed in the counting approach framework. The measures proposed by Alkire and Foster (2011, 2016) apply an intermediate approach for identification; whereas, the other three measures use a union criterion for identification. Aaberge and Peluso's (2012) measure requires dimensions to be equally weighted.

Table 1: Poverty Measures based on Counting Approaches

Literature	Poverty Measure	Identification criterion/weights
Alkire and Foster (2011)[9]	$P_{AF1} = M_0 = \frac{1}{n}\sum_{i=1}^{n} c_i = H \times A$	Intermediate
Chakravarty and D'Ambrosio (2006)	$P_{CD} = \frac{1}{n}\sum_{i=1}^{n} \pi_i^{\beta}$; with $\beta \geq 1$	Union
Bossert, Chakravarty and D'Ambrosio (2013)	$P_{BCD} = \left(\frac{1}{n}\sum_{i=1}^{n} \pi_i^{\beta}\right)^{1/\beta}$; with $\beta \geq 1$	Union
Aaberge and Peluso (2012)[10]	$P_{AP} = d - \sum_{j'=0}^{d-1} \Gamma\left(\sum_{j=0}^{j'} \bar{p}_j\right)$; Γ is increasing in its argument with $\Gamma(0) = 0$ and $\Gamma(1) = 1$, and $\bar{p}_j$ is the share of people simultaneously deprived in j dimensions	Union and equal weight
Alkire and Foster (2016)	$P_{AF2} = \frac{1}{n}\sum_{i=1}^{n} c_i^{\gamma}$; with $\gamma \geq 0$	Intermediate

2.3 Population Subgroups

We introduce the following subgroup notation in order to facilitate the decomposition analysis. We assume that there are $m \geq 2$ mutually exclusive and collectively exhaustive population subgroups within the hypothetical society. Population subgroups may be geographic regions or

[9] Alkire and Foster (2011) propose an entire class of indices. Here by P_{AF1} we refer to a particular member in their class – the adjusted headcount ratio or M_0.

[10] For an extension of this approach, see Silber and Yalonetzky (2014).

social groups. The number of all persons and the number of poor persons in subgroup ℓ are denoted by n^ℓ and q^ℓ, respectively, $\forall \ell = 1, \dots, m$ such that $\sum_{\ell=1}^{m} n^\ell = n$ and $\sum_{\ell=1}^{m} q^\ell = q$. Vectors $\underline{n} = (n^1, \dots, n^m)$ and $\underline{q} = (q^1, \dots, q^m)$ summarize the subgroup population and subgroup poor population, respectively. The censored deprivation score vector and the deprivation score vector for the poor for subgroup ℓ are denoted by c^ℓ and a^ℓ, respectively. As earlier, without loss of generality, we assume that within each subgroup ℓ, $c_i^\ell \geq k$ for all $i \leq q^\ell$ if there is at least one poor person in the subgroup and $c_i^\ell = 0$ for all $i > q^\ell$.

2.4 Additional Operators

Notation on the following mathematical relations and operators that we will be using subsequently is crucial. The *mean* of all elements in $u \in \mathbb{R}^d$ is denoted by $\mu(u)$. For any $u, v \in \mathbb{R}^d$, operator $\vee$ is the *join* of u and v such that $u' = (u \vee v)$ implying $u'_j = \max[u_j, v_j]\ \forall j$ and operator $\wedge$ is the *meet* of u and v such that $v' = (u \wedge v)$, implying $v'_j = \min[u_j, v_j]\ \forall j$.

2.5 Two Useful Properties

Poverty measures in the counting approach framework are required to satisfy certain desirable properties such as a set of invariance properties, dominance properties, subgroup properties and technical properties (see Chapter 2 of Alkire *et al.* 2015). In this section, we present two crucial properties that are central to our discussion in the next section. The first requires that the overall poverty measure can be expressed as a weighted sum of post-identification dimensional deprivations. This property allows one to see how different dimensions have contributed to overall poverty. The second property is related to distribution sensitivity among the poor, which requires the overall poverty measure to reflect any change in the distribution of deprivations among the poor.

In order to state these properties formally, we need to introduce two concepts. One is the post-identification dimensional deprivation of each dimension j, which we denote by $P_j(x_{\cdot j}; z, k, w)$, where $x_{\cdot j}$ is the jth column of matrix X summarizing the achievements of all persons in dimension j. Note that P_j not only depends on its own deprivation cutoff z_j but also on w, z and k whenever a non-union identification criterion is used. The other concept is related to rearrangement of dimensional deprivations among the poor, which reflects inequality in joint deprivations among the poor. Suppose for any $X \in \mathcal{X}_n$, the censored *deprivation status values* of all n persons in d dimensions are summarized by the $n \times d$-dimensional matrix $g(k)$. Thus, the ij^{th} element of $g(k)$ is $g_{ij}(k) = g_{ij} \times \rho_i(k)$. We denote the ith row of matrix $g(k)$ by $g_{i\cdot}$. Suppose there exists another matrix $g'(k)$ corresponding to any $X' \in \mathcal{X}_n$ with the same set of poor persons, yet $g'(k) \neq g(k)$, $g'(k)$ is not a permutation of $g(k)$, and for any i_1 and i_2, $g'_{i_1\cdot} = (g_{i_1\cdot} \vee g_{i_2\cdot})$, $g'_{i_2\cdot} = (g_{i_1\cdot} \wedge g_{i_2\cdot})$, and $g'_{i\cdot} = g_{i\cdot}\ \forall i \neq i_1, i_2$. A *regressive dimensional rearrangement among the poor* is stated to have taken place whenever X' is obtained from X; whereas a *progressive dimensional rearrangement among the poor* is stated to have taken place whenever X is obtained from X'.[11]

[11] For other versions of rearrangement properties in poverty measurement, but defined across achievements, see Tsui (2002), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011). All of them have been motivated by Boland and Proschan (1988).

We now state the two properties, *dimensional breakdown* and *dimensional transfer*, using the same terminology as in Alkire and Foster (2016). The dimensional breakdown property is the same as the *factor decomposability* property of Chakravarty, Mukherjee and Ranade (1998) for a union approach to identification.

Dimensional breakdown: For any $X \in \mathcal{X}$, $P(X; z, k, w) = \sum_{j=1}^{d} w_j \, P_j(x_{\cdot j}; z, k, w)$.

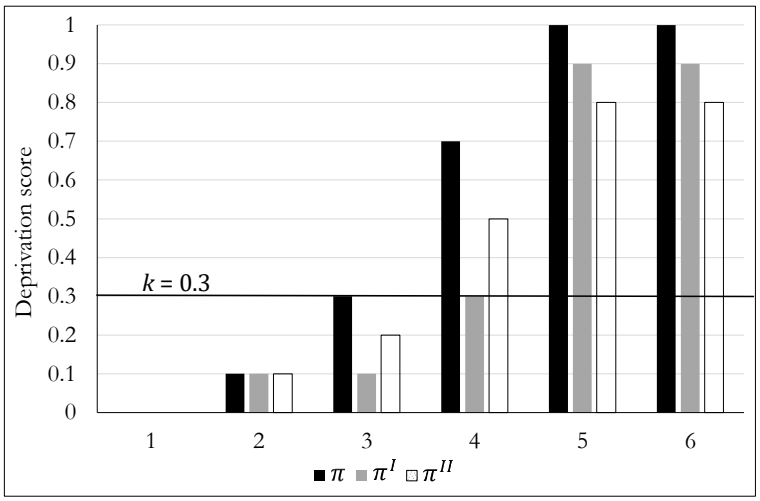
Dimensional transfer: For any $X, X' \in \mathcal{X}_n$, if X' is obtained from X by a progressive dimensional rearrangement among the poor, then $P(X'; z, k, w) < P(X; z, k, w)$.

Having introduced the counting approach framework, we now move on to discuss how distributional considerations are incorporated into counting poverty measurement.

3 Capturing Distribution of Deprivations among the Poor

The primary objective of any poverty alleviation program is to eradicate poverty. Poverty eradication may take years or even decades. Often a reduction in poverty is assessed by merely looking at the reductions in incidence and intensity, which ignores the distribution of deprivations among the poor (Sen 1976). For example, consider a society with the initial deprivation score vector $\pi = (0, 0.1, 0.3, 0.7, 1, 1)$ as depicted in Figure 1 with black bars. The height of each bar represents a deprivation score. If $k = 0.3$, which is represented by the solid black horizontal line in Figure 1, then the poor people are found above the poverty cutoff (because their deprivations meet or exceed k), so $H(\pi; k) = 4/6$ and $A(\pi; k) = 0.75$.

Figure 1: Changes in the distribution of deprivation scores due to two alternative policies



Now suppose two alternative policies – Policy I and Policy II – lead to two different distributions of deprivation scores: $\pi^I = (0, 0.1, 0.1, 0.3, 0.9, 0.9)$, represented by gray bars, and $\pi^{II} = (0, 0.1, 0.2, 0.5, 0.8, 0.8)$, represented by spotted bars in Figure 1. Then, for $k = 0.3$, $H(\pi^I; k) = H(\pi^{II}; k) = 1/2$ and $A(\pi^I; k) = A(\pi^{II}; k) = 0.7$. Both policies have resulted in similar improvements in terms of H and A, but only by overlooking an important difference. Policy I has resulted in marked improvements in the conditions of the two least poor persons with initial deprivation scores 0.3 and 0.7 (the first became non-poor), but only slight improvements in the conditions of the two poorest persons. Policy II, in contrast, has resulted in modest improvements in the conditions of all poor persons. Policy I has not been as 'pro-poorest' as Policy II, but this

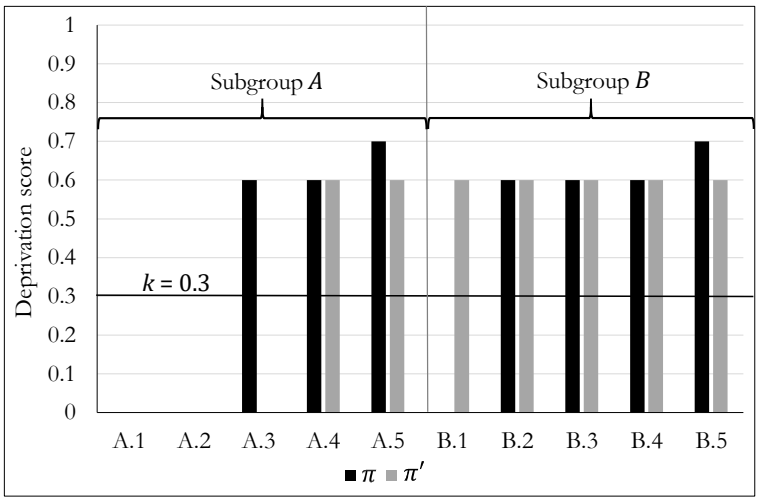
difference is not provided either by H or by A.

We should point out at this stage that the concern for inequality in the context of welfare measurement is slightly different from the distributional concerns in the context of poverty measurement. In welfare measurement, one is concerned with the entire distribution of achievements, where a transfer of achievements from a richer person to a poorer person is attributed to a reduction of inequality within the distribution. One may question the justification of extending this concept to the distribution of achievements among the poor on the grounds that if poor persons already suffer lower levels of achievements, why is it crucial to consider a transfer of achievements among two poor persons rather than focusing on a transfer of achievements between a poor and a non-poor person, and the complete eradication of poverty? This is a valid question, but we should clarify that our goal is not to merely capture inequality among the poor irrespective of the level of poverty. Our objective is to capture and evaluate situations where overall poverty reduction has not been inclusive in the sense that it has left the poorest behind in order to inform corrective and efficient action.

Another illustration with $k = 0.3$ will clarify our point. Suppose the initial deprivation score vector *among* the poor is $a = (0.5, 0.5, 0.9, 0.9)$, which becomes $a' = (0.9, 0.9, 0.9, 0.9)$ over time. Then, inequality among the poor has definitely decreased, but this reduction is accompanied by a large increase in poverty. This type of case is not of interest to us because the poverty measure would have already reflected it. Given that overall poverty has worsened, the fact that inequality among the poor has decreased does not seem beneficial. Now, consider another situation where a becomes $a'' = (0.4, 0.4, 0.9, 0.9)$ over time. In this case, a'' has been obtained from a by a reduction in overall poverty (captured by any measure that is sensitive to intensity of poverty), but this leaves the poorest behind, which is certainly reflected by increasing inequality among the poor. Our goal is to capture the distributional changes of this second type.

We would also like to point out that a reduction in inequality among the poor does not guarantee that poverty is reduced uniformly across all population subgroups. Alkire and Seth (2015), for example, found in the Indian context that multidimensional poverty went down between 1999 and 2006, but the reductions were slowest among the poorest population subgroups (poorest state, poorest caste and the poorest religion). We clarify this point with an example using a ten-person hypothetical society containing two subgroups – Subgroup A and Subgroup B – consisting of five persons each. For simplicity, we suppose that every dimension is equally weighted and $= 0.3$. The initial deprivation score vector of the society is $\pi = (0, 0, 0, 0.6, 0.6, 0.6, 0.6, 0.6, 0.7, 0.7)$, of Subgroup A is $\pi_A = (0, 0, 0.6, 0.6, 0.7)$ and of Subgroup B is $\pi_B = (0, 0.6, 0.6, 0.6, 0.7)$. In Figure 2, the heights of the black bars represent the deprivation scores. Clearly, there is inequality in deprivation scores among the poor within each subgroup. Besides, any poverty measure satisfying standard properties would conclude that Subgroup B has more poverty than Subgroup A. Now, suppose that over time the society's deprivation score vector becomes $\pi' = (0, 0, 0, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6)$. This signifies a reduction in overall poverty as well as a reduction in inequality among the poor. As represented by the grey bars in Figure 2, the deprivation score vectors of Subgroups A and B now become $\pi'_A = (0, 0, 0, 0.6, 0.6)$ and $\pi'_B = (0.6, 0.6, 0.6, 0.6, 0.6)$, respectively. Clearly, disparity in poverty between these two subgroups has not gone down despite the overall improvement in the distribution among the poor.

Figure 2: Changes in the distributions of deprivation scores for two subgroups across two periods



How can inequality among the poor and disparity in poverty across population subgroups be captured in the counting approach framework? The classical approach, which we refer to as the *assimilated approach*, has been, since the seminal article of Sen (1976), to fine-tune a poverty measure so that it is sensitive to inequality among the poor, both in the unidimensional and in the multidimensional contexts. Similar paths have been undertaken in the counting approach framework, where inequality can be captured across multiple deprivations or across deprivation scores among the poor. Among the measures presented in Table 1, the ones proposed by Chakravarty and D'Ambrosio (2006), Bossert, Chakravarty and D'Ambrosio (2013), Aaberge and Peluso (2012) and Alkire and Foster (2016) fall in the category of assimilated measures, which satisfies the *dimensional transfer* property presented in the previous section.

Measures pursuing the assimilated approach are primarily used for ordering purposes. For example, Jayaraj and Subramanian (2009) found that the ranking of Indian states altered when inequality-sensitive poverty indices were used instead of a poverty index insensitive to inequality. The ranking altered owing to the different levels of inequality in deprivation scores among the poor within states. If the assimilated measures are, in addition, additively decomposable, then the overall poverty can be expressed as a population-weighted average of subgroup poverty, which allows an understanding of how subgroups contribute to overall poverty.

The assimilated approach, however, suffers from four practical as well as conceptual limitations. First, the final index obtained from an assimilated approach often lacks intuitive and policy appeal. For some measures, the final figures are broken down into various partial indices of incidence, intensity and inequality. However, the relative weights that the measure places on each of these aspects are not transparent, which is important. For example, consider the following two breakdowns of the poverty measure proposed by Chakravarty and D'Ambrosio (2006) presented in Table 1 assuming $\beta = 2$. One is, following Aristondo et al. (2010),

$$P_{CD}(\pi) = H \times [\mu(a)]^2 \times [1 + 2GE(a; 2)], \tag{3.1}$$

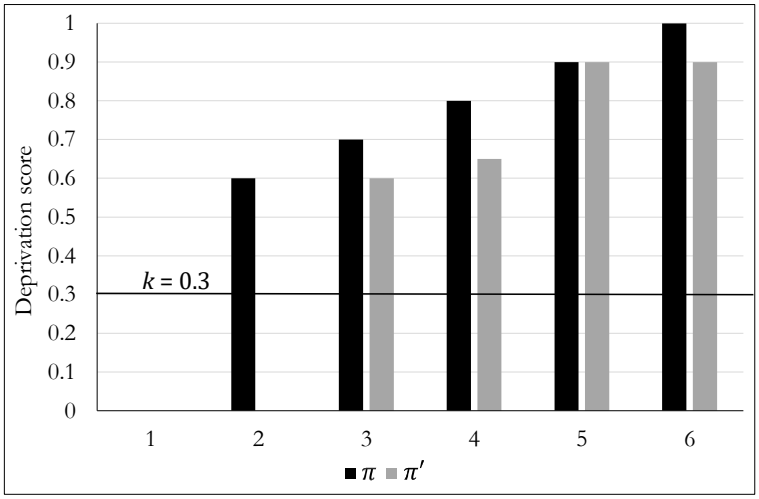
where $\mu(a)$ is the intensity among the poor identified by the union approach and $GE(a; 2)$ is the Generalized Entropy measure of order 2 capturing inequality in deprivation scores among the

poor. The other breakdown is

$$P_{CD}(\pi) = [\sigma(\pi)]^2 + [\mu(\pi)]^2 = H \times [[\sigma(a)]^2 + [\mu(a)]^2], \tag{3.2}$$

where $\sigma(a)$ is the standard deviation capturing inequality in deprivation scores among the poor. These two breakdowns would attach quite different weights to incidence, intensity and inequality across the poor depending on whether the value judgment of inequality is absolute or relative.[12]

Figure 3: Two hypothetical distributions of deprivation scores over time



Let us consider the two distributions of deprivation scores depicted in Figure 3, where $\pi = (0, 0.6, 0.7, 0.8, 0.9, 1)$, represented by black bars, becomes $\pi' = (0, 0, 0.6, 0.65, 0.9, 0.9)$, represented by grey bars, over time. The corresponding distribution of deprivation scores among the poor are: $a = (0.6, 0.7, 0.8, 0.9, 1)$ and $a' = (0.6, 0.65, 0.9, 0.9)$. Thus, $P_{CD}(\pi) = 0.55$, $H(\pi) = 83.2\%$, $\mu(a) = 0.80$, $\sigma(a) = 0.141$, and $GE(a; 2) = 0.016$; whereas, $P_{CD}(\pi') = 0.40$, $H(\pi') = 66.7\%$, $\mu(a') = 0.76$, $\sigma(a') = 0.139$, and $GE(a'; 2) = 0.017$. Clearly, $P_{CD}(\pi') < P_{CD}(\pi)$ and thus poverty has certainly gone down over time according to the P_{CD} measure. Which of the three components of poverty has contributed to this reduction? If one uses the decomposition formulation in Equation (3.2), then the reduction in P_{CD} has been due a reduction in incidence since $H(\pi') < H(\pi)$, a reduction in intensity since $\mu(a') < \mu(a)$ as well as a reduction in inequality among the poor since $\sigma(a') < \sigma(a)$. However, if one uses the decomposition formulation in Equation (3.1), then the reduction in P_{CD} is indeed due to reductions in both incidence and intensity, but not due to a reduction in inequality among the poor since $GE(a'; 2) > GE(a; 2)$. One decomposition suggests a favourable effect of inequality among the poor in poverty reduction; whereas the other decomposition suggests otherwise for the same poverty measure across two identical distributions. Which of these two decompositions should thus provide the reasonable interpretation of the final result? The proponents of the assimilated approaches, in general, do not take a position on this question, but when inequality is measured as inequality among the poor, additional considerations are relevant.

[12] We discuss various implications of this value judgment in the next section. In fact, Zheng (1994) shows in the unidimensional context that the only poverty index that is both absolute and relative is related to the headcount ratio. Also, there can be no meaningful index of inequality that can be both relative and absolute.

In particular, the fundamental aim of poverty reduction is not to merely reduce inequality among the poor, nor the intensity of poverty. Rather, it is to eradicate poverty, bringing the incidence to zero. While it is certainly better to have lower inequality among the poor than higher inequality among the poor, even with low inequality across the poor it is far better to have this situation with a low than a high intensity. The example above is not intended to criticize a particular poverty measure, but to highlight the important point that these different components play very different roles in interpreting the final results and their relative roles are often not clarified. The M_γ class of measures proposed by Alkire and Foster (2016), for example, go step-by-step from the multidimensional headcount ratio for $\gamma = 0$, the adjusted headcount ratio for $\gamma = 1$, to a squared count measure for $\gamma = 2$, making the incremental contribution of inequality among the poor clear.

Second, the assimilated measures often involve an inequality aversion parameter, whose value depends on how averse an evaluator is to inequality among the poor. The parameter discounts for larger inequality by increasing an assimilated index. For the same distribution across the poor, a more inequality-averse evaluator would conclude that poverty in the distribution was higher than a less inequality-averse evaluator would. Depending on the particular value of the parameter chosen, one may have different ranking of regions. Although, ideally an agreement across different parameters is expected to allow one to make robust conclusions, yet in practice this additional parametric decision-making can be a subject of significant debate.

Third, the assimilated approaches are not generally accompanied by appropriate frameworks for measuring disparity in poverty levels across different population subgroups. As we have shown previously in the example involving Figure 2, a reduction in poverty and even a reduction in inequality among the poor may not necessarily be accompanied by a reduction in disparity across subgroup poverty levels. Disparity across subgroup poverty levels should not be misconstrued as between-group inequality among the poor, which represents disparity across subgroup intensities. For a reasonably small number of subgroups, a visual examination may be sufficient for understanding the direction of change in disparity in poverty levels, but for a larger number of subgroups, a proper framework for assessing subgroup disparity in poverty levels is required. The consideration of disparity in poverty levels between subgroups is no less important than inequality in deprivation counts among the poor because a large disparity may reflect large horizontal inequalities that may create an environment for potential conflict across groups, which may have further adverse consequences on poverty (Stewart 2008).

Finally, in an assimilated approach, two properties outlined in the previous section – dimensional breakdown and dimensional transfer – conflict with each other. In fact, no counting poverty measure exists that simultaneously respects dimensional breakdown and dimensional transfer (Alkire and Foster 2016). The measure proposed by Alkire and Foster (2011) in Table 1 satisfies the dimensional breakdown property but not the dimensional transfer property; whereas the other three measures presented in the table satisfy the dimensional transfer property but not the dimensional breakdown property. In the M_γ class, dimensional breakdown (but not transfer) is satisfied when $\gamma = 1$ and dimensional transfer (but not breakdown) is satisfied when $\gamma = 2$.

In sum, assimilated poverty measures are certainly useful for ranking, yet they suffer from a number of practical limitations that may hinder their applicability in practice. The conflict between dimensional breakdown and dimensional transfer creates an impasse where one is forced to choose a poverty measure that satisfies only one of these two properties. Is there a way to come out of

this impasse? In this paper, we propose an alternative: using a poverty measure that satisfies dimensional breakdown alongside a separate, linked inequality measure that depicts inequality across the poor and disparity across population subgroups.[13] There are certain advantages to this approach. The additional measure provides complements information on incidence and intensity with information on inequality. Furthermore, the inequality measure may be reported along with a poverty measure that satisfies dimensional breakdown such as the Adjusted Headcount Ratio. If the inequality measure is decomposable, then its between-group component may be used to assess disparity in poverty across population subgroups.

4 Which Inequality Measure?

The inequality measure that we should use depends on crucial normative value judgments, which we present in the form of properties. The most important normative value judgment is whether the concept of inequality across deprivation scores should be judged in relative or absolute sense. If the normative assessment of inequality depends on absolute distance, then a change in every deprivation score by the same *amount* leaves the level of inequality unchanged. If, on the other hand, the assessment of inequality is relative, then a change in every deprivation score by the same *proportion* leaves the level of inequality unchanged.[14]

To further the discussion, let us provide an example. For simplicity, we assume there are ten dimensions that are equally weighted and a union approach is used for identification. Suppose the deprivation score vector $x' = (0.1, 0.1, 0.3, 0.3)$ is obtained from vector $x = (0.4, 0.4, 0.9, 0.9)$ over time. Looking at these two distributions, several questions may arise. Has poverty gone down? How has poverty gone down? Has the share of poor been reduced? Has the average deprivation score improved? Have the poorest been left behind?

Indeed, overall poverty has gone down because every person's deprivation score has gone down. The incidence has not changed but the intensity has improved; the poorest are not left behind because the two poorest persons have had much larger reductions in their number of deprivations. If we use any relative inequality measure to capture this distributional improvement, then any relative inequality measure would reflect an increase in inequality. Normatively, this appears counter-intuitive because the poorest persons in distribution x' have two additional deprivations whereas the poorest in distribution x have five additional deprivations. It might seem that measured inequality in the second distribution should be higher, but by every relative measure it will be lower.

The reason behind this counter-intuitive result may be that it is not appropriate to understand relative inequality across deprivation scores as higher values representing worse outcomes. Traditionally, while measuring inequality from a welfare point of view where higher values represent better outcome, assessment of relative inequality assigns larger weights to lower values. What happens then if we transform the deprivation scores into attainment scores and then assess

[13] Some existing empirical studies use a separate inequality measure for capturing inequality among the poor. See the study on child poverty by Delamonica and Minujin (2007) which was followed by Roche (2013).

[14] Relative measures have frequently been used when assessing income inequality. Atkinson (1970) proposed considering inequality in a relative sense in order to make the measure of inequality independent of mean. The other appealing reason is that the property of unit consistency (Zheng 2007) is satisfied. Kolm (1976), on the other hand, discussed the social disadvantages of considering inequality in a relative rather than an absolute sense.

relative inequality? Does it provide the expected result? Suppose the attainment score vectors corresponding to x and x' are $\tilde{x} = (0.6, 0.6, 0.1, 0.1)$ and $\tilde{x}' = (0.9, 0.9, 0.7, 0.7)$, respectively. Clearly, any relative inequality measure in this case would show improvement in the distribution among the poor when $\tilde{x}'$ has been obtained from $\tilde{x}$.

Does this approach always produce the desired result? Let us look at another example, where the attainment score vector $\tilde{x}' = (0.9, 0.9, 0.2, 0.2)$ has been obtained from $\tilde{x} = (0.5, 0.5, 0.1, 0.1)$ over time. Clearly, the poorest have been left behind (had an increase of only one attainment) while improving the situations of the least poor (improvements in four attainments). However, any relative inequality measure would conclude that there has been a distributional improvement among the poor. Again, we obtain a counter-intuitive result, which questions the efficacy of a relative inequality measure in a counting approach.

4.1 Properties

To formally present the properties, we introduce some additional notation in this section. We present the properties in terms of a general t-dimensional vector $x \in \mathbb{R}^t$ and use these properties to characterise the inequality measure. We then show in the next subsection how the measure is applicable in the counting approach framework. We define an inequality measure as a *continuous function* $I: \mathbb{R}^t \mapsto \mathbb{R}$. If x is divided into m mutually exclusive and collectively exhaustive subgroups, then x^ℓ and t^ℓ represent the subgroup vector and the size of subgroup ℓ, respectively, for all $\ell = 1, \dots, m$. Following Section 2.3, $\underline{t} = (t^1, \dots, t^m)$ and $\underline{\mu}_x = (\mu(x^1), \dots, \mu(x^m))$ summarize the subgroup population sizes and the averages of vector elements of subgroups, respectively.

First, in order to avoid the counter-intuitive conclusions presented at the beginning of this section, we impose the value judgment that if all deprivation scores improve by the same amount (proportion of weighted deprivations), then this leaves the level of inequality unchanged. In the inequality measurement literature, this property is known as *translation invariance*. In this case, if the amount of improvement among the poorer is slower, inequality should rise, and if the amount of improvement among the poorer is faster, inequality should fall. An added advantage of this value judgment is that the same level of inequality is reflected no matter whether inequality is assessed across deprivation scores or equivalently across attainment scores.[15]

Translation Invariance: For any t-dimensional vector x and $\delta > 0$,

$$I(x) = I(x + \delta \mathbf{1}^t),^{16}$$

where, $\mathbf{1}^t$ is a t-dimensional vector of ones.

The next three are standard properties that any inequality measure should satisfy. The second property, *anonymity*, requires that an inequality measure should not change by a permutation of elements in x.

[15] While measuring the inequality of bounded variables, Lambert and Zheng (2011) have proposed using absolute inequality measures to reflect consistent inequality comparisons whether inequality is assessed across attainments or across attainment-shortfalls. Seth and Yalonetzky (2016) have also used absolute inequality measures and absolute Lorenz curve to study cross country convergence for bounded variables that can me expressed either in terms of attainments or attainment-shortfalls.

[16] If the elements of x are bounded, we may additionally require that $\max\{x_1, \dots, x_t\} \leq 1 - \varepsilon$ and $\varepsilon \geq \delta > 0$.

Anonymity: For any two t-dimensional vectors x and x', if x' is a permutation of x, then

$$I(x') = I(x).$$

The third property, *replication invariance*, requires that the inequality measure should enable comparison across societies with different population sizes. Technically, if a society is obtained from another society by a merely duplicating or replicating the entire population, then the level of inequality should not alter.

Replication Invariance: For any t-dimensional vector x, if x' is obtained from x by replicating x twice or more, then

$$I(x') = I(x).$$

The fourth property, *transfer,* is fundamental in inequality measurement and requires that an inequality measure should increase due to a *regressive transfer*. What is a regressive transfer? Suppose, x' is obtained from x, such that $x'_{i_1} = x_{i_1} - \delta \geq 0$, $x'_{i_2} = x_{i_2} + \delta \leq 1$, $x_{i_1} < x_{i_2}$, $\delta > 0$ and $x'_i = x_i \; \forall i \neq i_1, i_2$.

Transfer: For any two t-dimensional vectors x and x', if x' is obtained from x by a regressive transfer, then

$$I(x') > I(x).$$

The next set of two properties link subgroup inequalities to the overall inequality. Given that we are interested in within-group and between-group inequalities, it is meaningful for the inequality measure to be *additively decomposable* so that overall inequality can be decomposed into a within-group term (I_W) and a between-group term (I_B).

Additive Decomposability: For any t-dimensional vector x,

$$I(x) = I_W(x) + I_B(x) = \sum_{\ell=1}^{m} \omega^{\ell}\left(\underline{t}, \underline{\mu}_x\right) I\left(x^{\ell}\right) + I\left(\underline{\mu}_x; \underline{t}\right);^{17}$$

The overall within-group term is a weighted average of within-group inequalities of the population subgroups, i.e., $I_W(x) = \sum_{\ell=1}^{m} \omega^{\ell}\left(\underline{t}, \underline{\mu}_x\right) I(x^{\ell})$, where $\omega^{\ell}(\underline{t}, \underline{\mu}_x)$ is the weight attached to inequality within subgroup ℓ which depends on both the subgroups' populations and the subgroups' means.[18] The between-group term is $I_B(x) = I(\underline{\mu}_x; \underline{t})$, where $I\left(\underline{\mu}_x; \underline{t}\right) = I(\mu(x^1)\mathbf{1}^{t^1}, \dots, \mu(x^m)\mathbf{1}^{t^m})$ and $\mathbf{1}^{t^{\ell}}$ is a t^{ℓ}-dimensional vector of ones. While computing between-group inequality, elements within each group x^{ℓ} receive the average of the group $\mu\left(x^{\ell}\right)$, which is incorporated by using the t^{ℓ}-dimensional vector of ones.

What does it imply when weights attached to within-group terms depend on subgroup means? It implies that if the means of the subgroups change, but the level of inequality and the population shares within these subgroups do not change, the overall within-group inequality *may change* without

[17] This is the usual definition of additive decomposability also used by Shorrocks (1980), Foster and Shneyerov (1999) and Chakravarty (2001).

[18] Note that the weight $\omega^{\ell}(\bar{t}, \bar{\mu}_x)$ for subgroup ℓ is different from weight or value w_j assigned to dimension j.

any justifiable reason. In order to avoid such circumstances, we impose a restriction such that the overall within-group inequality should not change when the inequality level and population size of each group remains unchanged but subgroup means change.

Within-group Mean Independence: For any two t-dimensional vectors x and x' and for any additively decomposable inequality measure I, if $t^{\ell} = t'^{\ell}$ and $I(x^{\ell}) = I(x'^{\ell})\ \forall \ell = 1, \dots, m$, then $I_W(x) = I_W(x')$.[19]

There are various inequality measures that are either translation invariant or additively decomposable, but the following proposition provides the only class of inequality measures that satisfy the above-mentioned properties.

Proposition: An inequality measure $I: \mathbb{R}^t \mapsto \mathbb{R}$ satisfies translation invariance, anonymity, replication invariance, transfer, additive decomposability and within-group mean independence if and only if there is an $\alpha > 0$ such that, for any $x \in: \mathbb{R}^t$:

$$I(x) = \frac{\alpha}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2. \tag{4.1}$$

Proof: See Appendix.[20]

Thus, the only class of inequality measures that satisfies the required properties is a positive multiple (α) of *variance*.[21] By construction, the minimum possible value that $I(x)$ takes is zero, which is attained when all elements in x take equal value. This is the situation of perfect equality. The maximum possible value that variance takes is one-fourth of the range of x, which is attained when half of the population have the lowest possible performance and the other half have the highest possible performance. For convenience and ease of interpretation, the value of α can be chosen in such a way that the value of the inequality measure is bounded between zero and one, as it is true for many well-known inequality measures. For example, if the elements of x range between 0.2 to 1, the maximum possible variance is 0.16 and so we suggest setting $\alpha = 1/0.16 = 6.25$.

4.2 Application to the counting approach framework

Depending on the situation, vector x may represent π, c, or a (and thus $\tilde{x}$ may represent $\tilde{\pi}$, $\tilde{c}$, or $\tilde{a}$) and t may represent n or q as required. Whenever $x = \pi$ or c and $t = n$, then $x_i \in [0,1]$ for all $i = 1, \dots, t$. Whenever $x = a$ and $t = q$, then $x_i \in [k, 1]$ for all $i = 1, \dots, t$. Table 2 presents the different values that x and t may take. It should be noted that the deprivation scores in vectors π, c, or a for any achievement matrices $X \in \mathcal{X}$ are obtained by applying dimensional weights,

[19] Note that the property is analogous to the *path independence* property of Foster and Shneyerov (2000) for relative inequality measures. The within-group mean independence property does not require an index to be absolute or relative *a priori*. The additive decomposability property along with the within-group mean independence implies path independence.

[20] The proposition is analogous to Theorem 1 of Chakravarty (2001). However, we do not assume differentiability and population share weighted decomposability as Chakravarty did.

[21] Note that the unbiased sample estimate for variance is $\sum_{i=1}^{t}[x_i - \mu(x)]^2 / (t-1)$, but this formulation does not satisfy population replication invariance. Lambert and Zheng (2011) also showed that the positive multiple of variance is the only decomposable inequality measure that assesses inequality *consistently*.

deprivation cutoffs, and poverty cutoff on X. Also, note that the domain of x, depending on the situation is bounded in this case and x may contain non-continuous values. The class of inequality measures presented in Equation (**4.1**), however, also applies to these situations.

Table 2: Values of x and t Under Different Circumstances

	Deprivation Score Vector (x)	Number of Elements (t)	Range of Each Element (x_i)
All Deprivation Scores	π	n	[0,1]
Censored Deprivation Scores	c	n	[0,1]
Deprivation Scores of the poor	a	q	$[k, 1]$

We should point out at this stage that in the counting approach framework, the *regressive transfer* property presented in the previous subsection is conceptually equivalent to the regressive version of the *dimensional transfer* property introduced in Section 2. This relationship can be verified easily. Suppose X' is obtained from X by a regressive dimensional rearrangement among the poor. Then, by definition, for any i_1 and i_2, $g'_{i_1\cdot} = (g_{i_1\cdot} \vee g_{i_2\cdot})$, $g'_{i_2\cdot} = (g_{i_1\cdot} \wedge g_{i_2\cdot})$, and $g'_{i\cdot} = g_{i\cdot}\ \forall i \neq i_1, i_2$. If we look in terms of deprivation scores, then clearly $\pi'_{i_1} > \pi_{i_1}, \pi_{i_2} > \pi'_{i_2}$. Given that dimensional weights are unchanged, then without loss of generality, $\pi'_{i_1} = \pi_{i_1} + \delta$ and $\pi'_{i_2} = \pi_{i_2} - \delta$, where δ is the sum of dimensional weights that are involved in dimensional rearrangement. It can be also be easily verified that the relationship holds in terms of attainment scores.

Furthermore, all measures in Equation (**4.1**) conclude the same level of inequality, whether the identification involves counting deprivations or counting attainments. The following corollary summarizes the result, which can be verified very easily by plugging $\tilde{x}_i$ in Equation (4.1).

Corollary: If $\tilde{x}$ is obtained from any x such that $\tilde{x}_i = 1 - x_i\ \forall i = 1, \dots, t$, then $I(\tilde{x}) = I(x)$.

We now show how the inequality measure presented in Equation (4.1) can be decomposed in order to be useful for policy-relevant applications.

$$I(x) = \left[\sum_{\ell=1}^{m} \frac{t^\ell}{t} I\left(x^\ell\right)\right] + \alpha \sum_{\ell=1}^{m} \frac{t^\ell}{t} [\mu\left(x^\ell\right) - \mu(x)]^2\,; \tag{4.2}$$

where t^ℓ/t is the population share of subgroup ℓ and $I\left(x^\ell\right)$ is the level of inequality in subgroup ℓ. The first term in Equation (4.2) captures the total within-group inequality and the second term captures the between-group inequality in x across population subgroups.

We now present two interesting cases that we also apply to empirical illustrations in the next section. In the first case, the focus remains only among the poor and thus $x = a$ and $t = q$. Note that the average of all elements in a is the average deprivation score among the poor, i.e., $A = \mu(a)$. The following expression computes inequality across deprivation scores of the poor:

$$I^q(a) = \frac{\alpha}{q} \sum_{i=1}^{q} [a_i - A]^2. \tag{4.3}$$

Equation (4.3) can be decomposed as:

$$I^q(a) = I_W^q(a) + I_B^q(a) = \left[\sum_{\ell=1}^{m} \frac{q^\ell}{q} I^q(a^\ell)\right] + \alpha \sum_{\ell=1}^{m} \frac{q^\ell}{q} [A^\ell - A]^2 \,; \tag{4.4}$$

where $I^q(a^\ell)$ is inequality among the poor in subgroup ℓ and A^ℓ is the intensity of poverty in subgroup ℓ. The first term in the right-hand side of Equation (4.4), $I_W^q(a)$, captures the total **within**-group inequality and the second term, $I_B^q(a)$, captures disparity **between**-subgroup intensities.

In the second case, the focus remains on the entire censored deprivation score vector and thus $x = c$ and $t = n$. Notice that the average of all elements in c is the measure M_0 proposed by Alkire and Foster (2011) presented in Table 1, i.e., $M_0(c) = \mu(c)$. Thus,

$$I^n(c) = I_W^n(c) + I_B^n(c) = \left[\sum_{\ell=1}^{m} \frac{n^\ell}{n} I^n(c^\ell)\right] + \alpha \sum_{\ell=1}^{m} \frac{n^\ell}{n} [M_0(c^\ell) - M_0(c)]^2. \tag{4.5}$$

The between-group term $I_B^n(c)$ in Equation (4.5) assesses disparity between subgroup poverty, or, more specifically, disparity between the subgroups' M_0. The term $I_B^n(c)$ thus adds valuable information by capturing disparity across the subgroups' M_0.[22]

5 Empirical Illustration

We now apply the method developed in the previous section to illustrate how it can be applied in practice and how it can add valuable information besides a meaningful poverty measure. For our purpose, we choose two developing countries with high poverty levels: Haiti and India. Haiti is the poorest country in Latin America and the Caribbean; whereas India is the most populous country in South Asia and not surprisingly houses the largest number of poor people. These two countries yield contrasting results even though poverty went down statistically significantly in both countries during the studied period. Haiti provides a story of success, where poverty reduction between 2006 and 2012 was pro-poorest; i.e., Haiti alleviated multidimensional poverty through a relatively larger reduction in poverty among the poorest, thus reducing inequality among the poor as well as reducing the disparity in poverty across sub-national regions. India, on the other hand, did not enjoy similar success between 1999 and 2006. Multidimensional poverty reduction in India was accompanied by only a modest reduction in the inequality among the poor and an increase in disparity in poverty across sub-national regions.

We have already discussed previously that our approach to using a separate inequality measure resolves the impasse created by two conflicting properties: dimensional breakdown and dimensional transfer. To show the practical efficacy of our approach, we use a poverty measure that respects dimensional breakdown. The measure proposed by Alkire and Foster (2011), P_{AF}, is the only counting measure in Table 1 that satisfies the dimensional breakdown property. We use an empirical adaptation of their approach: the global MPI developed by Alkire and Santos (2010,

[22] We have discussed further possible theoretical decompositions in our previous working paper version. See Seth and Alkire (2014).

2014). The MPI is composed of ten indicators grouped in three dimensions: education, health and standard of living. All three dimensions are equally weighted and indicators within each dimension are also equally weighted. The identification of poor takes place at the household level, where a household is identified as poor if the household's deprivation score is one-third or higher ($k = 1/3$). Every person living in a poor household is identified as poor; whereas, every person living in a non-poor household is identified as no-poor. For detailed information on the MPI methodology, see Alkire, Conconi and Seth (2014).[23]

For both countries, we use the Demographic Health Survey (DHS) datasets. For Haiti, we use the DHS datasets for the years 2006 and 2012, and, for India, we use the DHS datasets for the years 1999 and 2006.[24] DHS datasets are nationally representative as well as representative at the sub-national level, allowing us to conduct analysis across population subgroups. Samples are collected through multi-stage stratification. Appendix B presents the number of clusters and sample sizes (number of households) for India and Haiti in both periods, both nationally and across sub-national regions.

Table 3: Change in MPI, Incidence, Intensity, and Inequality among the Poor in Haiti and India

Haiti (2006–2012)											
MPI			**Incidence (H)**			**Intensity (A)**			**Inequality (I^q)**		
2006	**2012**	**Change**	**2006**	**2012**	**Change**	**2006**	**2012**	**Change**	**2006**	**2012**	**Change**
0.335	0.248	-0.087***	0.606	0.494	-0.112***	0.553	0.503	-0.050***	0.253	0.190	-0.062**
India (1999–2006)											
MPI			**Incidence (H)**			**Intensity (A)**			**Inequality (I^q)**		
1999	**2006**	**Change**	**1999**	**2006**	**Change**	**1999**	**2006**	**Change**	**1999**	**2006**	**Change**
0.300	0.251	-0.050***	0.568	0.485	-0.083***	0.529	0.517	-0.012***	0.224	0.219	-0.005*

The statistical tests of differences are one-tailed tests. ***Statistically significant at 1%, **Statistically significant at 5%, and *Statistically significant at 10%.

Source: Alkire and Seth (2015), Alkire, et al (2017) and authors' own computation.

In this paper, we use a standard bootstrap procedure for statistical inference considering this stratified sampling process. Two primary reasons for using the bootstrap technique are: (i) it automatically takes into account the natural bounds which are [0,1] in this case, and (ii) it mostly achieves the same accuracy as the delta-method (Biewen 2002, Davidson and Flachaire 2007). Given that some of the between group inequality values are very low, we may not rule out the possibility of negative lower bound of confidence intervals.[25] For applications of bootstrap techniques in inequality and poverty measurement, see Mills and Zandvakili (1997) and Biewen (2002). Our bootstrap resampling process involved one thousand replications occuring at the observation level.[26] The resampling was conducted taking into account the stratified DHS survey

[23] Minor adjustments in the deprivation cutoffs were made to preserve strict inter-temporal comparability. Details may be found for India in Alkire and Seth (2015) and for Haiti in Alkire, Roche and Vaz (2017).

[24] The years for the Indian datasets are 1998/99 and 2005/06, respectively. Given that samples covering 80.5% of the population in the 1998/99 DHS were collected in 1999, and, in the 2005/06 DHS, samples covering 92.6% of the population were collected in 2006, we consider 1999 and 2006 as the reference years for the surveys.

[25] We have not developed asymptotic properties of the estimators in this paper and instead relied on the bootstrap resampling process for statistical inference. Asymptotic properties may be developed following Cowell (1989), expressing the inequality measure in terms of moments under certain assumptions, including normality and bounded support. The corresponding standard error can be shown to be based on the Chi-squared distribution.

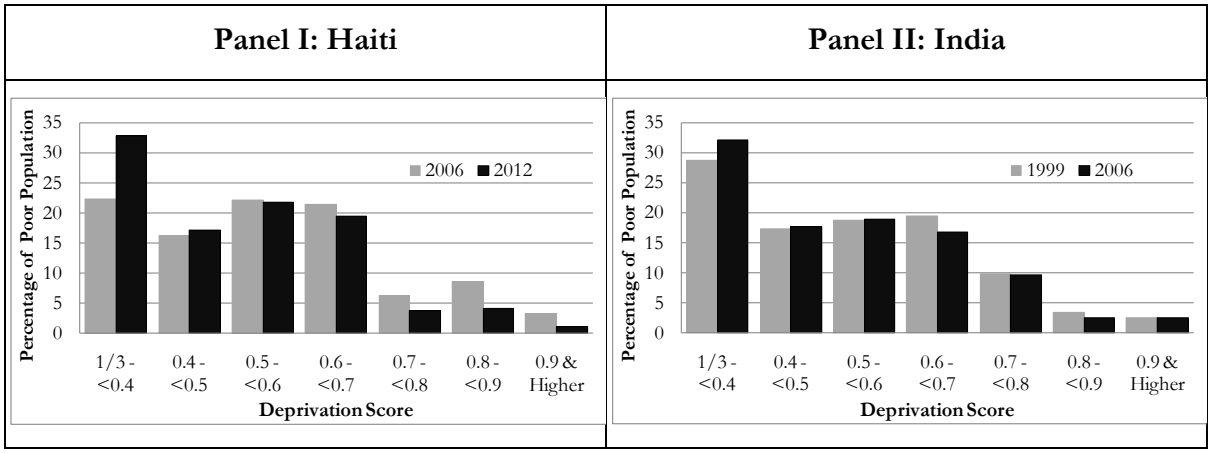
[26] The one-thousand replications provide us fifty replications to conclude 5% level of statistical significance and a

design, i.e., the resampling represented the strata and clusters (primary sampling units).

5.1 Change in Inequality among the Poor Nationally

It is evident from Table 3 that Haiti's MPI estimate has gone down from 0.335 to 0.248 between 2006 and 2012. The estimated incidence has dropped from 0.606 to 0.494 and the estimated intensity from 0.553 to 0.503. What has happened to inequality among the poor? Has the poverty reduction been pro-poorest? We use Equation (4.3) to compute the level of inequality among the poor I^q. We normalize by setting $\alpha = 9$ because deprivation scores among the poor range between $1/3$ and 1 and thus the maximum possible value that variance may take is $1/9$. It ensures that I^q is bounded between zero and one. We find that the inequality estimate among the poor in Haiti has gone down statistically significantly between 2006 and 2012.[27] For India, the national MPI estimate has gone down from 0.300 in 1999 to 0.251 in 2006 as well as the estimated incidence from 0.568 to 0.485 and intensity from 0.529 to 0.517.[28] All estimated reductions have been statistically significant, but they have not been as pro-poorest as in Haiti, which is reflected by the merely modest reduction in inequality estimate among the poor.

Figure 4: Distribution of Deprivation Scores among the Poor in Haiti and India



Source: Authors' computation.

Figure 4 presents the distributions of deprivation scores among the poor for both countries. Panel I presents the distribution of deprivation scores among the poor in Haiti in 2006 and in 2012. Panel II presents the distribution of deprivation scores among the poor in India in 1999 and in 2006. In both panels, the horizontal axes present ranges of deprivation scores and the vertical axes present the percentage of poor population suffering each range of deprivation scores. In both panels, the grey bars represent the distribution of deprivation scores in the first period and the black bars represent the distribution of deprivation scores in the second period. For Haiti, we clearly find that the reduction among the poorest with a deprivation score of 0.7 and higher has been much larger. When we look at the case of India, a stark difference is visible. The situation of the poorest, as a share of all poor persons, with a deprivation score of 0.7 and higher has not

hundred replications to conclude 10% level of statistical significance.

[27] Note that we are not using panel datasets and so are unable to track changes in the deprivation score of particular poor persons.

[28] Our result confirms the findings of Jayaraj and Subramanian (2009) and Mishra and Ray (2013), who use the measure proposed by Chakravarty and D'Ambrosio (2006) in Table 1. Both studies found that the national reduction in poverty was not accompanied by uniform reductions across different population subgroups.

changed much, slowing down the pace of reduction in inequality among the poor.

Table 4: Changes in MPI and Inequality among the Poor across Sub-national Regions in Haiti and India

	Ten Departments of Haiti					
	MPI			Inequality among the Poor		
	2006	2012	Change	2006	2012	Change
Aire Métropolitaine	0.195	0.162	-0.033*	0.189	0.182	-0.007*
Artibonite	0.418	0.316	-0.102***	0.229	0.196	-0.032**
Centre	0.545	0.391	-0.154***	0.313	0.213	-0.100**
Grand-Anse	0.455	0.378	-0.078*	0.242	0.201	-0.041**
Nippes	0.381	0.257	-0.124***	0.207	0.139	-0.067**
North	0.399	0.244	-0.155***	0.319	0.198	-0.121**
North-East	0.358	0.323	-0.035	0.238	0.217	-0.021*
North-West	0.395	0.311	-0.084**	0.240	0.147	-0.092**
South	0.336	0.249	-0.087**	0.218	0.192	-0.026**
South-East	0.398	0.307	-0.091**	0.223	0.147	-0.075**
	Seventeen Large States of India					
	MPI			Inequality among the Poor		
	1999	2006	Change	1999	2006	Change
Andhra Pradesh	0.299	0.194	-0.105***	0.223	0.153	-0.070***
Bihar#	0.442	0.416	-0.026**	0.252	0.268	0.016*
Goa	0.112	0.057	-0.055***	0.127	0.099	-0.027*
Gujarat	0.248	0.175	-0.073***	0.207	0.182	-0.025*
Haryana	0.190	0.154	-0.036**	0.166	0.158	-0.008
Himachal Pradesh	0.154	0.100	-0.054***	0.073	0.066	-0.007
Jammu & Kashmir	0.226	0.146	-0.080***	0.177	0.141	-0.037**
Karnataka	0.255	0.173	-0.082***	0.202	0.152	-0.049**
Kerala	0.136	0.038	-0.098***	0.080	0.059	-0.021**
Madhya Pradesh#	0.368	0.329	-0.040***	0.238	0.221	-0.018**
Maharashtra	0.226	0.155	-0.071***	0.182	0.151	-0.031**
Orissa	0.381	0.309	-0.072***	0.222	0.225	0.003
Punjab	0.117	0.088	-0.029***	0.172	0.126	-0.046**
Rajasthan	0.341	0.310	-0.031**	0.234	0.243	0.008
Tamil Nadu	0.195	0.110	-0.085***	0.132	0.083	-0.048**
Uttar Pradesh#	0.348	0.314	-0.034***	0.211	0.205	-0.007
West Bengal	0.339	0.283	-0.055***	0.231	0.211	-0.021**

The statistical tests of differences are one-tailed tests. ***Statistically significant at 1%, **Statistically significant at 5%, and *Statistically significant at 10%.

We have combined Bihar and Jharkhand, Madhya Pradesh and Chhattisgarh, and Uttar Pradesh and Uttarakhand as these states were not partitioned in 1999.

Source: Alkire and Seth (2015) and authors' own computation.

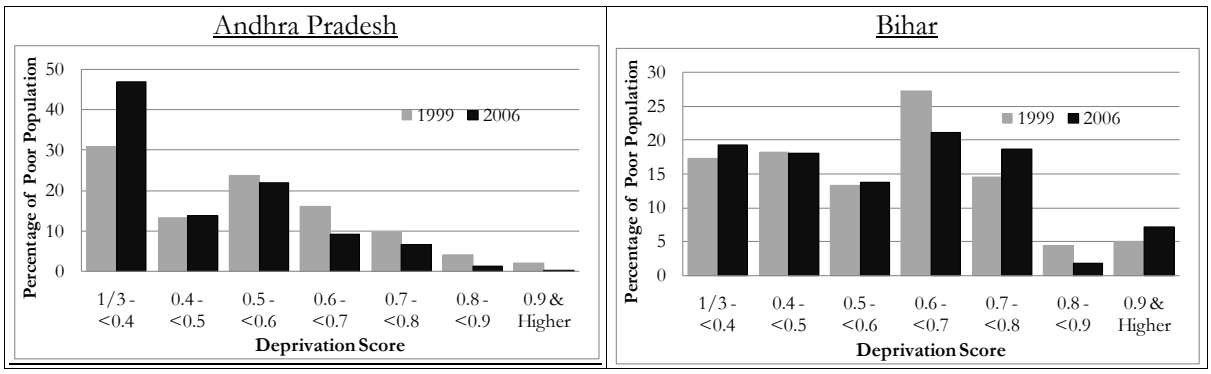
5.2 Change in Inequality among the Poor within Sub-national Regions

Has the national pattern of reduction in poverty and inequality among the poor been replicated within sub-national regions? We answer this question by computing the inequality measure $I^q(a^\ell)$ using Equation (4.3). We set $\alpha = 9$ because the deprivations scores among the poor in this case still range between 1/3 and one. Note that the weighted average of the within-group inequalities among the poor provides the total within-group inequality I_W^q in Equation (4.4). In Table 3, we present the changes in all ten sub-national MPIs as well as the changes in sub-national inequality among the poor for the ten departments of Haiti and seventeen Large States of India.

In Haiti, the national pattern of estimated poverty reduction has been replicated within almost all sub-national regions. The MPI estimates have gone down statistically significantly in all regions. The inequality estimate among the poor within each region has gone down statistically significantly but indeed with variation across regions. In five regions, the pace of reduction in estimated inequality among the poor has been faster than the national average. The pace of reduction in both MPI and inequality among the poor was slowest for Aire Métropolitaine.

Again, a contrasting picture is obtained when we look at the seventeen large states of India. The MPI estimate in each of the seventeen sub-national regions has gone down statistically significantly, with the largest reduction in MPI estimate being in Andhra Pradesh. Unlike Haiti, however, every sub-national region has not seen a reduction in inequality among the poor. Large reductions in inequality are visible in states like Andhra Pradesh, Karnataka, Punjab and Tamil Nadu. Inequality has risen statistically significantly in Bihar. In order to understand which part of the distribution is responsible for an increase or reduction in inequality among the poor, we present the distribution of deprivation scores across the poor for two states at the extremes: Andhra Pradesh and Bihar. Clearly, in Andhra Pradesh, a reduction in poverty estimate has taken place by improving the situation of those with deprivation scores of 0.6 and higher. In the case of Bihar, however, the reduction in poverty estimate has not been inclusive in the sense that the overall reduction has not helped those who are more severely deprived, in comparison to those who are less poor. In fact, the shares of the poor with deprivation scores of 0.7 and above have increased in 2006.

Figure 5: Distribution of Deprivation Scores in Two States of India: Andhra Pradesh and Bihar



Source: Authors' own computation.

5.3 Disparity across Population Subgroups

We have looked at inequality estimates among the poor nationally and by sub-national regions. Now, after looking at the reduction in national poverty estimate, an obvious question comes to mind: Has the fruit of national reduction been shared by all population subgroups? Before closing this section, we explore the answer to this question by computing disparity across subgroups using the term $I_B^n(c)$ in Equation (4.5). Given that MPI estimates may vary between zero and one, in this case we choose $\alpha = 4$. Thus,

$$I_B^n(c) = 4\sum_{\ell=1}^{m}\frac{n^\ell}{n}[MPI_\ell - MPI]^2.$$

Changes in disparities across population subgroups are reported in Table 4. For Haiti, we compute disparity in poverty only across sub-national regions. The sub-national disparity estimate has gone down statistically significantly from 0.054 to 0.025. This means that poorer sub-national regions had faster estimated poverty reduction and thus there has been a convergence in estimated poverty across sub-national regions. India, however, has a different story to tell. Sub-national disparity estimates in India increased statistically significantly from 0.031 to 0.041. When we look at disparity across castes and religious groups, no changes are visible.[29] Thus, unlike Haiti, we did not find any evidence of a pro-poorest convergence in poverty estimates across population subgroups.

Table 5: Disparity across Different Population Subgroups Haiti and India

Subgroups	First Period	Second Period	Change
Haiti: Sub-national Regions (2006–2012)	0.054	0.025	−0.029**
India: Sub-national Regions (1999–2006)	0.031	0.041	0.009**
India: Castes (1999–2006)	0.021	0.021	0.000
India: Religions (1999–2006)	0.004	0.004	0.000

Source: Authors' own computation.

6 Concluding Remarks

There have been recent developments in both theory and practice in the measurement of multidimensional poverty within the counting approach framework. The categorical or binary nature of many indicators and the fact that the counting measures of poverty are based on direct deprivations make the use the counting approaches more practicable. Even in counting approaches, however, it is important that all three 'I's of poverty – incidence, intensity and inequality among the poor – can be incorporated. If the object of a policy maker is to reduce only the incidence of poverty, then only marginally poor people would be lifted out of poverty, ignoring the poorest of the poor completely. If the objective is to reduce both the incidence and intensity of poverty, then while the policy maker has no reason to focus on the marginally poor instead of the poorest of the poor, the policy maker has no strong incentive to assist the poorest of the poor either. It is only when the consideration of inequality is brought to the table that a policy maker has greater incentives to assist the poorest.

The most common approach to incorporating inequality into poverty measurement, what we refer to as the *assimilated approach*, has been to adjust a poverty measure so that the measure is sensitive to the distribution of poverty among the poor. This approach has been used by a number of authors, including Alkire and Foster 2016, who propose a new M_γ class of measures that include their previous Adjusted Headcount Ratio and a new assimilated measure known as squared count.

Complementing the discussion of assimilated measures, this paper explores a different analytical approach because of certain limitations in assimilated approaches that may be relevant in some policy contexts. First, assimilated poverty measures may lack intuitive interpretations. Even when they combine incidence, intensity and inequality, the relative weight that the measure places on each of these aspects is not made transparent. Second, assimilated measures often involve selecting

[29] We divide the population in India into five religious subgoups: Hindu, Muslim, Christian, Sikh, and others. Hindus comprise nearly 80% of the overall population; Muslims comprise nearly 14%; Sikhs and Christians comprise around 2% each; and other religious subgroups combined comprise the rest. We also divided the population into four castes: Scheduled Caste, Scheduled Tribe, Other Backward Castes (OBC), and General (consisting of none of the three). The distribution of population across these four categories did not remain unchanged between 1999 and 2006.

a particular value for an inequality-aversion parameter, which may become a subject of debate. Third, these measures may not explicitly assess the disparity in poverty between population subgroups. Finally, they do not allow the possibility of breaking down a measure by dimensions in order to understand dimensional contributions to overall poverty (Alkire and Foster 2016).

In this paper, we propose the use of a separate inequality measure to capture inequality among the poor and disparity across population subgroups. Our choice of inequality measure is determined by certain desirable properties, in addition to the standard properties. First, we require that the inequality measure is additively decomposable so that it can be expressed as a sum of total within-group inequality and between-group inequality. Moreover, the total within-group inequality should not change as long as the population share and inequality within each population subgroup does not change. Second, we require that inequality across deprivation scores to remain unchanged when all deprivation scores increase by the same amount. In other words, we require that inequality should be perceived through absolute distances between deprivation scores. The only inequality measure that satisfies our requirements is a positive multiple of variance.

We provide an illustration comparing the changes in the situation of the poor in two countries: Haiti and India. We use the MPI to assess poverty, which is an implementation of the adjusted headcount ratio poverty measure proposed by Alkire and Foster (2011), satisfying dimensional breakdown. We find that in Haiti the overall poverty reduction was pro-poorest. Nationally, inequality among the poor went down as did inequality among the poor within every sub-national region. Furthermore, poorer sub-national regions reduced MPI estimates more, resulting in a reduction in sub-national disparity in poverty. The Indian experience, however, was not so positive. There was a strong reduction in poverty nationally as well as within some sub-national regions such as Andhra Pradesh and Tamil Nadu. However, inequality among the poor did not go down in some states like Bihar. Also, subgroup disparities between sub-national regions and religious and caste groups did not go down, so no convergence in poverty estimates across groups was evident.

So what is the value added of using the proposed inequality measure alongside a poverty measure? First, the inequality measure adds valuable information to any poverty measure that respects the dimensional breakdown property – such as the adjusted headcount ratio proposed by Alkire and Foster (2011), which has been adopted by international organizations and country governments. Second, the inequality measure does not require an inequality-aversion parameter, whose selection may be contested. Also, the additive decomposability property allows overall poverty to be decomposed into within-group and between-group components. Although the contribution of within-group and between-group components to overall poverty is subject to debate (Kanbur 2006), we show with empirical illustrations how understanding their changes over time may provide valuable information. Finally, the inequality measure reflects the same level of inequality whether the poor are identified by counting attainments or by counting deprivations.

At the same time, this research agenda raises a number of interesting questions regarding the dynamics of inequality among the poor. For example, in situations in which the intensity of poverty is exceedingly high – approaching 100% – then progress in reducing the intensity of poverty is likely to involve a temporary increase in inequality among the poor as the intensity of deprivations for some are reduced. Using the proposed inequality measure 'variance' alongside an intuitive measure of poverty, such as the adjusted headcount ratio, may enable researchers to identify

various patterns of progression of inequality among the poor and to link these to other patterns such as conflict, migration, and local or regional activities. It will also be interesting to compare multidimensional 'variance' with income inequality among the income poor in order to assess whether diverse kinds of inequality among the poor converge or diverge.

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Appendix A: Proof of the proposition

The proof has two parts: sufficiency and necessity. For the first, it is straightforward to show that the inequality measure $I(x) = \frac{\alpha}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2$ satisfies anonymity, transfer, replication invariance, subgroup decomposability and translation invariance.

Exploiting the subgroup notation, we may decompose $I(x)$ as:

$$I(x) = \left[\sum_{\ell=1}^{m} \frac{t^\ell}{t} I(x^\ell)\right] + \alpha \sum_{\ell=1}^{m} \frac{t^\ell}{t}[\mu(x^\ell) - \mu(x)]^2.$$

Clearly, each $I(x^\ell)$ is weighted by the corresponding population share t^ℓ/t, which does not depend on the mean and so the total within-group term remains unaltered as long as each $I(x^\ell)$ and each t^ℓ/t remains unchanged. Thus, $I(x)$ further satisfies within-group mean independence.

Let us show that this is the only inequality measure that satisfies the six properties. An inequality measure that satisfies the *additive decomposability* property also satisfies the decomposability property in Bosmans and Cowell (2010), which requires that $I(x) = F(I(x^1), \dots, I(x^m), \underline{\mu}_x, \underline{t})$ for some function F.

Now, we know following Bosmans and Cowell (2010) that the class of inequality measures that satisfies anonymity, transfer, replication invariance, decomposability and translation invariance, satisfies:

$$f(I(x)) = \begin{cases} \frac{1}{t}\sum_{i=1}^{t}\{\exp(\gamma[x_i - \mu(x)]) - 1\} & \text{if } \gamma \neq 0 \\ \frac{1}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2 & \text{if } \gamma = 0 \end{cases};$$

where γ a real number and $f : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function, with $f(0) = 0$.

The additive decomposability property along with $f(0) = 0$ and the functional restriction on f requires $f(y) = y/\alpha$ for any $\alpha > 0$. Thus,

$$I(x) = \begin{cases} \frac{\alpha}{t}\sum_{i=1}^{t}\{\exp(\gamma[x_i - \mu(x)]) - 1\} & \text{if } \gamma \neq 0 \\ \frac{\alpha}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2 & \text{if } \gamma = 0 \end{cases}.$$

Next, we show which of these measures satisfies the *within-group mean independence*. Consider a partition into $m \geq 2$ mutually exclusive and collectively exhaustive subgroups, where the vector and the population size of any subgroup ℓ are denoted by x^ℓ and t^ℓ.

Consider $\gamma \neq 0$. The corresponding measures can be decomposed into within-group inequalities and between-group inequality components as:

$$I(x) = \sum_{\ell=1}^{m} \frac{t^{\ell} \exp[\gamma\mu(x^{\ell})]}{t \exp[\gamma\mu(x)]} I(x^{\ell}) + I(\underline{\mu}_x; \underline{t}).$$

The measures with $\gamma \neq 0$ do not satisfy the property of within-group mean independence, which can be shown as follows. Consider two vectors x and u with population size t and τ, such that $t^{\ell} = \tau^{\ell}$ and $I(x^{\ell}) = I(u^{\ell})\ \forall \ell = 1, \dots m$, but $\mu(x^{\ell'}) \neq \mu(u^{\ell'})$ and $I(x^{\ell'}) > 0$ for some ℓ' and $\mu(x^{\ell}) = \mu(u^{\ell})\ \forall \ell \neq \ell'$. The difference between the overall within-group inequality terms of x and u, with some manipulation, turns out to be:

$$I_W(x) - I_W(u) = \frac{t^{\ell'}}{t} I(x^{\ell'}) \left(\exp\left[\gamma\left(\mu(x^{\ell}) - \mu(x)\right)\right] - \exp\left[\gamma\left(\mu(u^{\ell}) - \mu(u)\right)\right]\right).$$

In the above expression, $\mu(x^{\ell}) - \mu(x) \neq \mu(u^{\ell}) - \mu(u)$ by construction. If $t^{\ell} > 0$ and $I(x^{\ell'}) > 0$, as assumed, clearly $I_W(x) - I_W(u) \neq 0$ whenever $\gamma \neq 0$.

Thus, the class of inequality measures satisfying all the required properties is $I(x) = \frac{\alpha}{t} \sum_{i=1}^{t} [x_i - \mu(x)]^2$.

Appendix B: Distribution of sample clusters and households in Haiti and India

Haiti				
	Clusters		Households	
Region	**2006**	**2012**	**2006**	**2012**
Aire Métropolitaine	84	142	2,239	4,083
Artibonite	31	38	880	1,103
Centre	27	34	813	982
Grand-Anse	28	31	834	898
Nippes	27	30	822	871
North	29	36	844	1,063
North-East	29	31	846	912
North-West	28	33	830	958
South	28	35	844	1,021
South-East	28	35	843	1,010
Overall	**339**	**445**	**9,795**	**12,901**

India				
	Clusters		Households	
Region	**1999**	**2006**	**1999**	**2006**
Andhra Pradesh	133	195	3,818	6,364
Bihar	233	197	6,110	5,339
Eastern States	450	753	11,894	18,639
Goa	50	126	1,552	3,005
Gujarat	133	113	3,834	3,117
Haryana	100	91	2,790	2,245
Himachal Pradesh	100	106	3,348	2,716
Jammu	117	97	2,646	2,312
Karnataka	133	176	4,129	5,049
Kerala	100	125	2,723	2,962
Madhya Pradesh	233	280	6,598	8,457
Maharashtra	218	289	5,603	7,684
New Delhi	100	112	2,545	3,039
Orissa	133	115	4,636	3,795
Punjab	100	99	2,887	2,870
Rajasthan	233	106	5,968	3,249
Tamil Nadu	158	214	5,223	6,248
Uttar Pradesh	333	451	7,388	11,937
West Bengal	158	205	4,556	5,883
Overall	**3,215**	**3,850**	**88,248**	**104,910**