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Flexible Bearing-Only Rendezvous Control of Mobile Robots

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***Abstract*— In this paper we study rendezvous control of multiple mobile robots. We propose a control law that merely requires each robot to measure the relative bearings of their neighbors in their local coordinate frames. Distance measurement or relative position estimation is not required. In theory, the proposed control law verifies that distance information is redundant in rendezvous control tasks though the objective of rendezvous is to decrease the inter-robot distances. In practice, the control law provides a simple solution to vision-based rendezvous tasks where bearings can be measured by visual sensing. Moreover, we generalize the proposed control law by introducing an additional heading vector into the control law. This heading vector may preserve the system convergence and, in the meantime, provides great flexibility to adapt the control law for nonholonomic robot models or obstacle avoidance.**

I. INTRODUCTION

Multi-robot rendezvous is one of the basic tasks for multi-robot coordination. Its objective is to steer each robot so that all the robots converge to the same location. The rendezvous problem would become the same as the well-known consensus problem [1] when the kinematics of each robot can be modeled as a single integrator and each robot can measure the relative positions of their neighbors. However, the practical kinematic model of a ground or aerial robot is usually more complicated than a single integrator. This motivates many researchers to study multi-robot rendezvous with nonholonomic models [2], [3].

Sensing capability is an important practical problem that should be considered in multi-robot rendezvous. Most of existing rendezvous control laws assumed that each robot is able to measure the relative positions of their neighbors. This assumption can be realized by two methods in practice. The first is based on external navigation systems such as GPS. In particular, each robot must localize themselves with GPS and then share their locations with their neighbors via wireless communication. This method is, however, not applicable in GPS-denied or communication-denied environments such as indoor or hostile environments. The second method is based on onboard sensors carried by each robot. Visual sensing is one of the most promising and popular sensing approaches. It can be realized easily with low-cost cameras and, more importantly, it is a passive sensing approach that does not require wireless communication.

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Though powerful, visual sensing may not be able to give reliable relative position measurements. For example, once a target has been recognized in an image, its bearing can be immediately calculated based on the pixel coordinate of the target and intrinsic parameters of the camera. It would be, however, much more complicated to obtain the distance of the target since additional geometric information of the target is required to estimate the distance. Distance information can be alternatively estimated by stereo vision systems where the target position is triangulated by using the bearings captured by multiple cameras. However, due to the small baseline of a stereo vision system, the accuracy of distance estimation drops fast when the target is far from the camera. In summary, a fundamental feature of visual sensing is that it is easy to get bearing but difficult to get distance. This feature motivates us to study multi-robot rendezvous with bearing-only measurements.

Despite the practical importance of bearing-only rendezvous, this topic has merely received limited research attention up to now. The study in [4] proposed a quantized control scheme to achieve multi-robot rendezvous based on quantized bearing angle measurements. The proposed control scheme is merely applicable to the case where each robot has one single neighbor. The work in [5] proposed continuous control laws for unicycle robots which are applicable to more general sensing topologies. The works in [6], [7] also studied rendezvous with bearing measurements, but the bearing measurements are used to estimate position information. In our work, bearing measurements are directly applied to control and no estimation is required.

The main contributions of this paper are summarized below. We first propose and analyze a nonlinear rendezvous control law for single-integrator kinematic models. The control law merely relies on bearing measurements expressed in each robot's local coordinate frame. The control law is proven to be globally stable. We then generalize this control law by introducing a flexible heading vector for each robot. The heading vector may preserve the convergence of the entire system and, in the meantime, provides great flexibility for each robot to adjust their velocity heading. By selecting proper heading vectors, the rendezvous control law proposed for the single-integrator model can be adapted for nonholonomic models in two- and three-dimensional spaces. By choosing appropriate heading vectors, we may also achieve obstacle avoidance. This obstacle avoidance approach mainly relies on the bearing information of obstacles and is different from the potential-based approaches that generate repulsive forces to push a robot away from obstacles and require the robot-obstacle distance information [8].

II. Bearing-Only Rendezvous Control Law

In this section we propose and analyze a distributed control law to solve the multi-robot rendezvous problem with local bearing measurements.

Consider n robots in $\mathbb{R}^d$ $(d = 2, 3)$. Let $\mathcal{V} := \{1, \dots, n\}$. The location of robot $i \in \mathcal{V}$ is denoted as $p_i \in \mathbb{R}^d$. The sensing topology of the robots is described by a fixed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. If $(i, j) \in \mathcal{E}$, robot i can measure the relative bearing of robot j or, in other words, robot i can "see" robot j. If $(i, j) \in \mathcal{E}$, we say robot j is adjacent to robot i or a neighbor of robot i. Let $\mathcal{N}_i$ denote the set of robot i's neighbors.

The proposed bearing-only rendezvous control law is

$$\dot{p}_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij} g_{ij}(t), \tag{1}$$

where a_{ij} is positive constant and

$$g_{ij}(t) = \frac{p_j(t) - p_i(t)}{\|p_j(t) - p_i(t)\|}.$$

The unit vector g_{ij} represents the relative bearing of p_j with respect to p_i. Control law (1) is nonlinear. Its interpretation is that each robot should move towards its neighbor robots.

The quantities in control law (1) are all expressed in a global coordinate frame. This control law, however, can be implemented with locally measured bearings. To see that, let v_i be the velocity of robot i in the global coordinate frame. Let the superscript (i) indicate a quantity expressed in the local coordinate frame of robot i and $R_i \in \mathbb{R}^{d \times d}$ the rotational transformation from the local frame to the global frame. Then, we have $v_i = R_i v_i^{(i)}$ and $g_{ij} = R_i g_{ij}^{(i)}$. Substituting into control law (1) gives

$$v_i^{(i)}(t) = \sum_{j \in \mathcal{N}_i} a_{ij} g_{ij}^{(i)}(t),$$

which indicates that the control law can be implemented with locally measured bearings.

We next analyze the convergence of the proposed control law. In order to do that, we need some assumptions.

Assumption 1 (Undirected Graph)**.** *The sensing graph is undirected which means $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$ and $a_{ij} = a_{ji}$.*

The assumption on undirected graphs enable us to analyze the nonlinear system via a Lyapunov approach. The directed case will be studied in the future.

Assumption 2 (Robot Merge)**.** *For any $(i, j) \in \mathcal{E}$, if robots i and j are sufficiently close so that $\|p_i - p_j\| < r$ where r is a positive threshold, the two robots will merge into a new robot i'. The new robot i' is adjacent with robot $k \in \mathcal{V}$ if either i or j is adjacent with k.*

The assumption on robot merge guarantees that the bearing between any pair of neighbors is well defined. This assumption has also been adopted in previous works on bearing-only rendezvous [4], [5]. The threshold r could be thought of as a measure of the physical size of each robot.

Under Assumption 2, rendezvous is achieved when all the robots merge into one single robot. The convergence result for control law (1) is given below.

Theorem 1. *Given a fixed undirected graph $\mathcal{G}$, the bearing-only control law* (1) *solves the rendezvous problem if and only if the graph is connected.*

Proof. The necessity is obvious. That is if the graph is not connected rendezvous cannot be achieved. We next prove the sufficiency. If the graph is connected, consider the Lyapunov function

$$V = \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{N}_i} a_{ij} \|p_i - p_j\|.$$

It is clear that $V = 0$ if and only all the agents merge into one single robot whose $\mathcal{N}_i$ is empty. The time derivative of V is

$$\begin{aligned}
\dot{V} &= \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{N}_i} a_{ij} \frac{(p_i - p_j)^T}{\|p_i - p_j\|} (\dot{p}_i - \dot{p}_j) \\
&= 2 \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{N}_i} a_{ij} \frac{(p_i - p_j)^T}{\|p_i - p_j\|} \dot{p}_i \\
&= -2 \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{N}_i} a_{ij} g_{ij}^T \dot{p}_i.
\end{aligned}$$

Substituting control law (1) into the above equation gives

$$\dot{V} = -2 \sum_{i \in \mathcal{V}} \left(\sum_{j \in \mathcal{N}_i} a_{ij} g_{ij} \right)^T \left(\sum_{j \in \mathcal{N}_i} a_{ij} g_{ij} \right) \leq 0.$$

According to the invariance principle [9], the trajectory of the system converges to the invariance set where $\dot{V} = 0$. We next show that $\dot{V} = 0$ if and only if all robots merge to one single robot. First, it is clear that

$$\dot{V} = 0 \Longleftrightarrow \sum_{j \in \mathcal{N}_i} a_{ij} g_{ij} = 0, \quad \forall i \in \mathcal{V}. \tag{2}$$

Equation (2) indicates that each robot is located inside the convex hull spanned by its neighbors. To see that, rewrite (2) as

$$\begin{aligned}
&\sum_{j \in \mathcal{N}_i} a_{ij} \frac{p_j - p_i}{\|p_j - p_i\|} = \sum_{j \in \mathcal{N}_i} \bar{a}_{ij} (p_j - p_i) = 0, \\
\Longleftrightarrow &\left(\sum_{j \in \mathcal{N}_i} \bar{a}_{ij} \right) p_i = \sum_{j \in \mathcal{N}_i} \bar{a}_{ij} p_j, \\
\Longleftrightarrow &p_i = \sum_{j \in \mathcal{N}_i} \tilde{a}_{ij} p_j, \quad \forall i \in \mathcal{V},
\end{aligned} \tag{3}$$

where $\bar{a}_{ij} = a_{ij} / \|p_j - p_i\| > 0$ and $\tilde{a}_{ij} = \bar{a}_{ij} / \sum_{j \in \mathcal{N}_i} \bar{a}_{ij}$. Equation (3) clearly indicates that p_i is in the convex hull spanned by $\{p_j\}_{j \in \mathcal{N}_i}$ since $\tilde{a}_{ij} > 0$ and $\sum_{j \in \mathcal{N}_i} \tilde{a}_{ij} = 1$. If there are more than one robot, it is impossible for all the robots to satisfy (3). For example, if we consider the convex hull spanned by all the robots, then at least one vertex of the convex hull is not in the convex hull spanned by its neighbors. As a result, $\dot{V} = 0$ if and only if there is merely one single robot; in other words, V would keep decreasing if there are more than one robots.

In the above derivation, robot merge is not considered. When multiple robots merge into one single robot, the value of V would have a discontinuous decrease. Since there are merely a finite number of discontinuous decreases, the time horizon $[0,\infty)$ may be divided into a finite number of time intervals and the above argument can be applied to each of them. □

III. A Flexible Bearing-Only Rendezvous Control Law

In this section we generalize control law (1) and propose a more flexible control law by introducing a heading vector. In particular, the flexible control law is

$$\dot{p}_i(t) = h_i(t)h_i^T(t)\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}(t), \tag{4}$$

where $h_i(t)\in\mathbb{R}^d$ is a nonzero heading vector which may be time-varying. The purpose of $h_i(t)$ is to deflect the velocity direction of robot i. Since $h_i(t)h_i^T(t)$ is a projection matrix (it becomes an orthogonal projection when $h_i(t)$ is a unit vector), the velocity of robot i is the projection of $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$ onto h_i. Specifically, the velocity direction is parallel to $h_i/\|h_i\|$ and the velocity magnitude is $\|h_i\|(h_i^T\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij})$.

The heading vector $h_i(t)$ introduces great flexibility into the control law. By selecting appropriate $h_i(t)$, we may obtain control laws for unicycle robots. We will show in the next section how to choose appropriate $h_i(t)$ according to different tasks. In practice, $h_i(t)$ can be chosen by robot i based on its local information and hence control law (4) remains distributed. We next show that $h_i(t)$ preserves rendezvous convergence under a mild condition.

Theorem 2. *Given a fixed, undirected, and connected graph $\mathcal{G}$, control law* (4) *solves the rendezvous problem if the nonzero heading vector $h_i(t)$ is uniformly continuous and not orthogonal to $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}(t)$ for all $t\geq 0$ and $i\in\mathcal{V}$.*

Proof. Note that system (4) is nonautonomous and we derive the stability by using Barbalat's Lemma [9, Lemma 8.2]. Consider the Lyapunov function $V=\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}\|p_i-p_j\|$, which is the same as the one in Theorem 1. Then, we have

$$\begin{aligned}\dot{V} &= -2\sum_{i\in\mathcal{V}}\left(\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}\right)^T\dot{p}_i\\ &= -2\sum_{i\in\mathcal{V}}\underbrace{\left(\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}\right)^T}_{f_i^T} h_ih_i^T \underbrace{\left(\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}\right)}_{f_i}\leq 0.\end{aligned}$$

Since V is nonincreasing and bounded from below, V converges as $t\to\infty$. It can be proved that f_i is uniformly continuous in t and consequently $\dot{V}$ is uniformly continuous in t. It then follows from Barbalat's Lemma that $\dot{V}$ converges to zero as $t\to 0$. It is clear that $\dot{V}=0\Leftrightarrow h_i^Tf_i=0$ for all i. Since h_i is not orthogonal to f_i for all t as assumed, then $h_i^Tf_i=0\Leftrightarrow f_i=0$. As a result, $\dot{V}=0\Leftrightarrow f_i=0$ for all i and hence the invariant set is exactly the same as that of control law (1). The rest of the proof is same as Theorem 1. □

The convergence condition on h_i in Theorem 2 is mild since it merely requires h_i to be uniformly continuous and not to be orthogonal to $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$. This mild condition provides great flexibility to design appropriate h_i for various tasks as we show in the following sections. This mild condition can be even further relaxed. For example, if h_i is orthogonal to $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$ only for a finite time period, the convergence is still guaranteed. We will demonstrate this point when we study the application of the proposed control law in unicycle robots (see Theorem 3).

IV. Applications of the Flexible Control Law

A. Application to Two-Dimensional Nonholonomic Robots

Although control law (4) is designed for the single-integrator model, we now show that this control law can be adapted for nonholonomic models.

Consider a group of unicycle robots in $\mathbb{R}^2$. Let $p_i=[x_i,y_i]^T\in\mathbb{R}^2$ and $\theta_i\in\mathbb{R}$ denote the position coordinate and heading angle of robot i, respectively. The motion of robot i is governed by the unicycle model

$$\begin{aligned}\dot{x}_i &= v_i\cos\theta_i,\\ \dot{y}_i &= v_i\sin\theta_i,\\ \dot{\theta}_i &= w_i,\end{aligned}\tag{5}$$

where $v_i\in\mathbb{R}$ and $w_i\in\mathbb{R}$ are the linear and angular velocities to be designed.

We next use the flexible control law (4) to design a rendezvous control law for the unicycle model (5). Since the velocity of a unicycle must be aligned with its heading angle, we choose the heading vector to be

$$h_i=\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}.$$

Substituting h_i into (4) yields

$$\begin{aligned}\dot{p}_i &= \begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix}\\ &= h_ih_i^T\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}\\ &= \left([\cos\theta_i,\sin\theta_i]\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}\right)\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}.\end{aligned}\tag{6}$$

By comparing (6) with the unicycle model, we design the linear velocity v_i to be

$$v_i=[\cos\theta_i,\sin\theta_i]\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}\tag{7}$$

such that the nonholonomic constraint is satisfied.

The design of angular velocity w_i can be very flexible. The rule of thumb is that w_i may be designed arbitrarily as long as the heading of the unicycle robot is not always

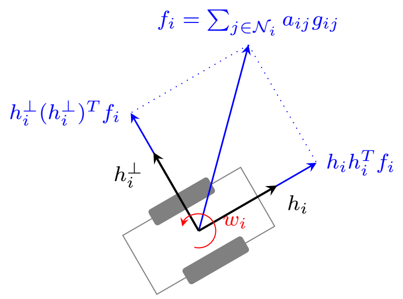

Fig. 1: An illustration of the unicycle control law in (7) and (8).

orthogonal to $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$. For example, we can simply choose $w_i = \cos t$ as in [2]. We next present a new angular velocity control law

$$w_i = [-\sin\theta_i, \cos\theta_i] \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}. \tag{8}$$

Let $h_i^\perp = [-\sin\theta_i, \cos\theta_i]^T$. Then, equation (8) can be rewritten as $w_i = (h_i^\perp)^T \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$. Note $h_i^\perp \perp h_i$. The control law in (7) and (8) has a clear geometric interpretation: The linear and angular velocities are equal to the magnitudes of the orthogonal projection of $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$ onto h_i and $h_i^\perp$, respectively. See Figure 1.

The convergence result under the control law in (7) and (8) is given below.

Theorem 3. *For the unicycle model* (5)*, given a connected graph $\mathcal{G}$, the linear and angular velocity control laws in* (7) *and* (8) *solves the bearing-only rendezvous problem.*

Proof. Let $f_i = \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$. The convergence of (7) and (8) does not follows from Theorem 2. That is because h_i may be orthogonal to f_i for certain time t. The time derivative of V satisfies $\dot{V} = -\sum_{i\in\mathcal{V}} f_i^T \dot{p}_i = -\sum_{i\in\mathcal{V}} (f_i^T h_i)^2 \leq 0$. Assume $\dot{V} = 0$ but $f_i \neq 0$ for certain i. Then, we know $h_i \perp f_i$. In this case, $w_i = h_i^\perp f_i \neq 0$ and hence the corresponding state is not in the invariant set. As a result, the system will converge to the invariant set where f_i must be zero. The rest of the proof is similar to Theorem 1. □

The control law in (7) and (8) can be implemented with locally measured bearings. By rewriting the control law in terms of local bearing measurements, the control law becomes the bearing-only rendezvous control law proposed in [5]. As a result, some existing control laws may be viewed as special expressions of the flexible control law (4).

B. Application to Three-Dimensional Nonholonomic Robots

Consider a group of nonholonomic robots in $\mathbb{R}^3$. Let $p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3$ be the position coordinate robot i. The direction of the velocity of robot i is characterized by the yaw and pitch angles α_i and β_i, respectively. The motion of robot i is governed by the three-dimensional nonholonomic model

$$\begin{aligned}
\dot{x}_i &= v_i \cos\beta_i \cos\alpha_i, \\
\dot{y}_i &= v_i \cos\beta_i \sin\alpha_i, \\
\dot{z}_i &= v_i \sin\beta_i, \\
\dot{\alpha}_i &= w_{\alpha_i}, \\
\dot{\beta}_i &= w_{\beta_i},
\end{aligned} \tag{9}$$

where $v_i, w_{\alpha_i}, w_{\beta_i} \in \mathbb{R}$ are the linear and angular velocities to be designed. The three-dimensional nonholonomic model can be used to characterize unmanned aerial vehicles.

We now use the flexible control law (4) to design a rendezvous control law for the three-dimensional nonholonomic model (9). Since the direction of the velocity is constrained by the azimuth and altitude angles, we design the heading vector h_i to be

$$h_i = \begin{bmatrix} \cos\beta_i \cos\alpha_i \\ \cos\beta_i \sin\alpha_i \\ \sin\beta_i \end{bmatrix}. \tag{10}$$

Substituting h_i into (4) yields

$$\begin{aligned}
\dot{p}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} &= h_i h_i^T \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} \\
&= \Big(h_i^T \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} \Big) \begin{bmatrix} \cos\beta_i \cos\alpha_i \\ \cos\beta_i \sin\alpha_i \\ \sin\beta_i \end{bmatrix}.
\end{aligned} \tag{11}$$

By comparing (11) with the nonholonomic model (9), we design the linear velocity v_i to be

$$v_i = h_i^T \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} \tag{12}$$

such that the nonholonomic constraint is satisfied.

Again, the angular velocities w_{α_i} and w_{β_i} are flexible to design. Here we also present a specific one that is intuitively easy to understand. Let $w_i \in \mathbb{R}^3$ be the angular velocity vector satisfying

$$\dot{h}_i = w_i \times h_i = [w_i]_\times h_i = -[h_i]_\times w_i, \tag{13}$$

where $[\cdot]_\times$ denotes the associated skew-symmetric matrix for a vector [10, Chapter 2]. Specifically, if $x = [x_1, x_2, x_3]^T \in \mathbb{R}^3$, then

$$[x]_\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

The vector w_i is orthogonal to h_i and characterizes how the heading vector h_i changes. We design the angular velocity as

$$w_i = h_i \times \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} = [h_i]_\times \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}. \tag{14}$$

The control input w_i attempts to rotate h_i so that h_i aligns with $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}$. Substituting (14) into (13) gives

$$\dot{h}_i = -[h_i]_\times^2 \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} = P_{h_i} \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}, \tag{15}$$

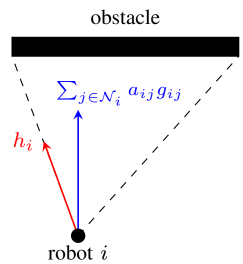

Fig. 2: A simple and effective way to design h_i for obstacle avoidance. The heading vector is always pointing to the leftmost (or rightmost) point on the obstacle.

where $P_{h_i} = I_3 - h_i h_i^T$ is an orthogonal projection matrix that projects any vector onto the orthogonal complement of h_i. Note $P_{h_i} x = 0$ for any $x \in \mathbb{R}^3$ if and only if x is parallel to h_i. The last equability in the above equation is due to the fact $[x]_\times^2 = xx^T - \|x\|^2 I_3$ for any $x \in \mathbb{R}^3$ [10, Thm 2.11].

On the other hand, take the time derivative of (10) gives

$$\dot{h}_i = \underbrace{\begin{bmatrix} -\cos\beta_i \sin\alpha_i & -\sin\beta_i \cos\alpha_i \\ \cos\beta_i \cos\alpha_i & -\sin\beta_i \sin\alpha_i \\ 0 & \cos\beta_i \end{bmatrix}}_{A_i} \begin{bmatrix} \dot{\alpha}_i \\ \dot{\beta}_i \end{bmatrix}. \quad (16)$$

By comparing (16) and (15), we have

$$\begin{bmatrix} \dot{\alpha}_i \\ \dot{\beta}_i \end{bmatrix} = (A_i^T A_i)^{-1} A_i^T P_{h_i} \sum_{j\in\mathcal{N}_i} a_{ij} g_{ij}. \quad (17)$$

The convergence result is given below.

Theorem 4. *For the three-dimensional nonholonomic model* (9)*, given a connected graph* $\mathcal{G}$*, the linear and angular velocity control laws in* (12) *and* (17) *solves the bearing-only rendezvous problem.*

Proof. Let $f_i = \sum_{j\in\mathcal{N}_i} a_{ij} g_{ij}$. The time derivative of V satisfies $\dot{V} = -\sum_{i\in\mathcal{V}} f_i^T \dot{p}_i = -\sum_{i\in\mathcal{V}} (f_i^T h_i)^2 \leq 0$. Assume $\dot{V} = 0$ but $f_i \neq 0$ for certain i. Then, we know $h_i \perp f_i$. In this case, $w_i = P_{h_i} f_i \neq 0$ and hence the corresponding state is not in the invariant set. As a result, the system will converge to the invariant set where f_i must be zero. The rest of the proof is similar to Theorem 1. □

C. Application to Bearing-Only Obstacle Avoidance

We next demonstrate how to apply the flexible control law (4) to obstacle avoidance. One simple and effective way is to design h_i such that h_i points to the leftmost or rightmost point on an obstacle (see Figure 2). Loosely speaking, if h_i is not always orthogonal to $\sum_{j\in\mathcal{N}_i} a_{ij} g_{ij}$, convergence of rendezvous can be guaranteed. When there are multiple obstacles and h_i must be orthogonal to $\sum_{j\in\mathcal{N}_i} a_{ij} g_{ij}$ in order to avoid obstacles, then the convergence of rendezvous may not be ensured.

Since we merely need to choose appropriate h_i to avoid obstacles, the bearing information of the obstacle is sufficient to achieve obstacle avoidance by the proposed approach. In practice, distance information of the obstacle may be required to trigger the obstacle avoidance mechanism, but it does not have to be accurate because it is not used in the control law. Our approach is different from the conventional artificial potential approaches where distance information is used to calculate repelling forces generated by obstacles. In the case of using visual sensing, we assume the obstacle does not block the line of sight for the neighbors for robot i. If the obstacle blocks some of the neighbors of robot i, the sensing graph in this case is time-varying, which will be studied in the future.

To demonstrate, simulation results are given in Figures 3 and 4, where there are three robots and the underlying graph is complete. The weight a_{ij} is set as 1 for all edges and the threshold for robot merge is set as 0.2 meter. Figure 3 demonstrates the case without obstacle avoidance. It can be seen that rendezvous can be successfully achieved under the proposed bearing-only control law (1). Figure 4 shows that obstacles can be successfully avoided and, in the meantime, rendezvous can also be achieved under the flexible control law (4). In this example, each agent is assumed to have limited distance sensing ability. When a robot senses that the minimum distance between the robot and any point on an obstacle is less than 2 meters, the obstacle avoidance mechanism is triggered and the heading vector is designed to point at the leftmost or rightmost point on the obstacle. It can be seen that the Lyapunov function always decreases and there are some discontinuous decreases caused by robot merging.

V. Conclusions

The first contribution of this paper is to propose a multi-robot rendezvous control law that merely requires local bearing measurements. The control law provides a simple solution to vision-based multi-robot rendezvous problems and verifies that inter-robot distance information is not necessarily required in order to achieve rendezvous. The second contribution of this paper is to generalize the proposed bearing-only rendezvous control law by introducing an extra heading vector. It has been proved that the convergence of rendezvous is still guaranteed under some mild conditions. The heading vector provides great flexibility to adapt the control law to adapt for nonholonomic models and obstacle avoidance.

Many problems regarding bearing-only rendezvous are still unsolved. For example, this paper merely considered undirected and fixed sensing graphs. Directed and switching sensing graphs should be addressed in the future. In addition, the simulation has shown that the control law achieves rendezvous within finite time. How to prove finite-time convergence and estimate the convergence time need to be studied.

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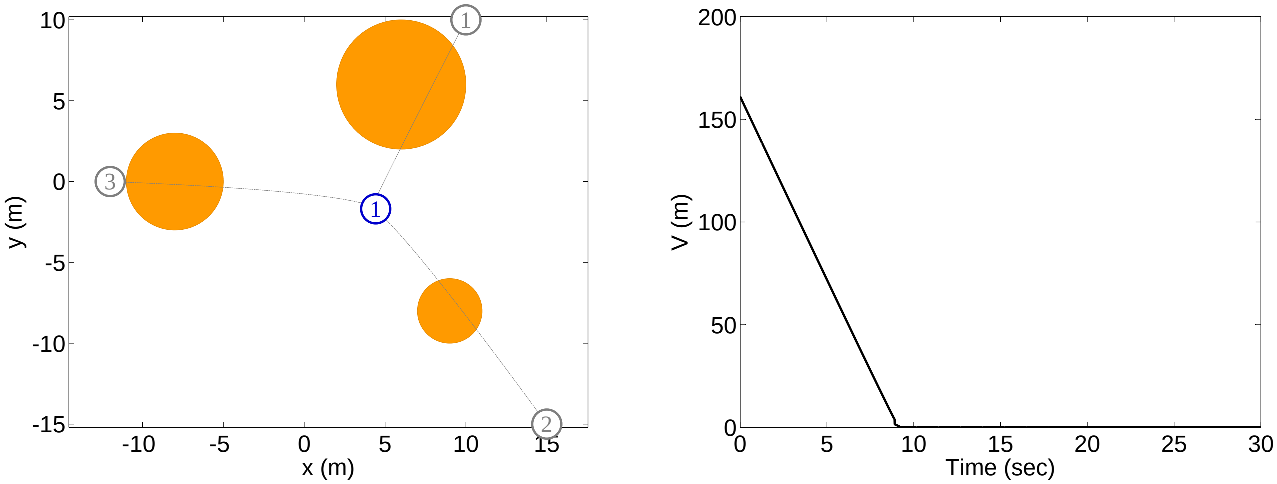
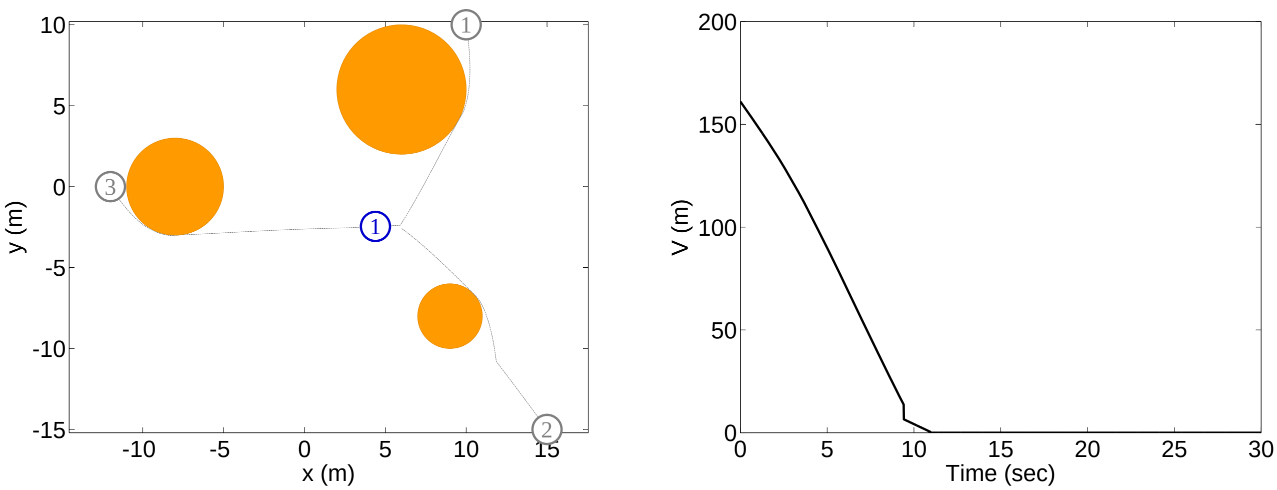


(a) Trajectory (orange areas represent obstacles)

(b) Lyapunov function ($V = \sum_{(i,j)\in\mathcal{E}} \|p_i - p_j\|$)

Fig. 3: Rendezvous without obstacle avoidance.



(a) Trajectory (orange areas represent obstacles)

(b) Lyapunov function ($V = \sum_{(i,j)\in\mathcal{E}} \|p_i - p_j\|$)

Fig. 4: Rendezvous with obstacle avoidance.

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