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Parental and child time investments and the cognitive development of adolescents

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Abstract

While a large literature has focused on the impact of parental investments on child cognitive development, very little is known about the role of child's own investments alongside that of the parents. By using the Child Development Supplement of the Panel Study of Income Dynamics, we model the cognitive production function for adolescents using an augmented value-added model and adopt an estimation method that takes account of unobserved child characteristics. We find that a child's own investments made during adolescence matter more than the mother's. Our empirical results appear to be robust to several sensitivity checks.

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1 Introduction

This study analyzes the impact of parental time investments on children's cognitive outcomes during adolescence in relation to the impact of time investments made by children themselves. Previous studies have focused either on the effect of parental investments (see Cunha et al. 2006; Cunha and Heckman 2007) or on the effect of children's own investments as measured by study effort, such as time spent on homework or studying (see Stinebrickner and Stinebrickner 2004 and 2008; Cooper et al. 2006; Eren and Henderson 2011; Kalenkoski and Pabilonia 2014), but not on both. This is the first analysis comparing the impact of parental and child time investments on cognitive outcomes in adolescence.

Empirical evidence suggests that the effect of parental investments on child development declines during adolescence, (see Carneiro et al. 2003; Cunha and Heckman 2008; Del Boca et al. 2014), whereas the effect of time spent on homework increases (see Cooper et al. 2006). This implies that there is scope for policy interventions targeting adolescents rather than their families. It is during adolescence, in fact, that teenagers start taking responsibility for their own actions, and that their cognitive investments begin to depend on their own decisions, such as how much time and effort to spend doing homework or reading instead of watching television.

Our paper differs from previous studies in that (i) we use data from time-use diaries to distinguish the effect of time investments made by parents from that made by children by considering the amount of time spent by the child in the mother's supervision versus the time spent alone doing formative activities, (ii) we extend the definition of a child's own time investment to include time spent on homework as well as time spent on other activities such as playing an instrument or going to the theater, (iii) we use a new identification strategy that exploits both the within-family between-siblings variation in investments and the within-child between-ability variation in cognitive test scores.

To investigate whether parental investments or child self-investments matter more during adolescence, we estimate a cognitive production function using the Child Development Supplement (CDS) of the Panel Study of Income Dynamics (PSID). We measure cognitive abilities using a revised version of a set of intelligence tests developed by Woodcock and Johnson in 1977. More specifically, we use three test scores measuring symbolic learning and reading, comprehension and vocabulary, and mathematical abilities.

The main econometric challenge in evaluating the effect of investments on child cognitive outcomes is accounting for unobserved child characteristics. Three of the most common strategies adopted are: controlling for the lagged test score in an attempt to reduce the bias caused by omitted past inputs (value-added model), considering child fixed effects estimation using the within-child across time variation in a panel data approach, and using an instrumental variable estimation (see Todd and Wolpin 2003 for a review). While papers focusing on the effect of parental investments on child's cognitive abilities have adopted one of these three strategies, most studies looking at the time spent by children on their homework have generally neglected the issue of unobserved child characteristics (see Cooper 2006 for a review of these papers). Among the few exceptions are Aksoy and Link (2000), who use a child fixed (as well as random) effect estimation that exploits repeated observations over time, Stinebrickner and Stinebrickner (2008), who instrument the time a child spends studying with her roommate's characteristics, and Dolton et al. (2003), who consider both a valued added model and an instrumental variable estimation. Another approach to control for unobserved child characteristics that has been adopted when inputs and test scores are subject-specific is the within-pupil between-subject estimation (e.g. Dee 2005 and 2007), which uses repeated observations of inputs and school test scores across subjects to control for child fixed effects. Eren and Henderson (2011) adopt this approach to control for unobserved child characteristics when evaluating the effect of homework on school test scores in mathematics, science, English and history.

Our empirical strategy is an improvement over valued added models because we relax two strong assumptions (discussed thoroughly in Todd and Wolpin, 2003): first, that past inputs are irrelevant after controlling for the lagged test, and second, that unobserved child-specific characteristics are independent of lagged test scores. While we relax the first assumption by controlling for the lagged test, current inputs and lagged inputs, i.e., by adopting what Todd and Wolpin (2003) call an augmented value-added model, we relax the second assumption by resorting to the within-pupil between-subject estimation. We implement the within-pupil between-subject estimation using three cognitive test scores rather than school test scores in different subjects. More specifically, we use test scores for symbolic learning and reading, comprehension and vocabulary, and mathematical abilities to estimate the effect of the lagged cognitive ability on the contemporaneous ability with a child fixed effects approach to control for unobserved child characteristics. We then use this estimated effect of the lagged cognitive ability in a second-step estimation which, by exploiting within-family between sibling variation to control for family fixed effects, allows us to evaluate the effect of investments. Therefore, the novelty of our procedure is to introduce a two-step estimation to evaluate the effect of the lagged cognitive ability as well as of the mother's and child's time investments on the contemporaneous cognitive ability.[1]

Our estimation results show that adolescent cognitive development seems to be affected much more by the time invested by the child during adolescence than by the time invested during childhood. In contrast, maternal time investments during childhood matter more than during adolescence. When comparing the time children spend on their own versus the time they spend with their mother doing formative activities during adolescence, we find that the child's own time investment affects their test scores much more than the time investment of their mother. This finding highlights the importance of self-investments during adolescence and suggests potential channels through which cognitive development can be influenced at later ages, such as policies using financial transfers to encourage student effort and educational activities.

[1] See Nicoletti and Rabe (2012) for an application of this method to evaluate the effect of expenditure per pupil using school test scores in different subjects.

2 Background

Several surveys have shown that parental time investments on children have important impacts on child cognitive and non-cognitive outcomes (see Carneiro and Heckman 2003, Ermisch and Francesconi 2005, Haveman and Wolfe 1995). Since the majority of socioeconomic surveys lack appropriate measures of parental time, most studies have been forced to use proxy measures, such as mothers' employment (Bernal 2008, Todd and Wolpin 2007, Liu et al. 2010, Bernal and Keane 2011). A more accurate measure of the time investments in children is provided by the time diary surveys,[2] which usually contain detailed information on the time children spend in different activities together with their mother, their father and other adults. Nevertheless, only a few papers have actually used time diaries to measure investments in children. Among these few exceptions are Hsin (2007, 2009) Carneiro and Rodriguez (2009) and Del Boca et al. (2014), who have used the Child Development Supplement (CDS) of the Panel Study of Income Dynamics (PSID) for the US. These papers estimate the effect on children's skills of different measures of parental time investments. Carneiro and Rodriguez (2009) consider the total time spent with the mother; Hsin (2007) defines measures of maternal total time, engaged time and quality time; Del Boca et al. (2014) distinguish between the time the children spend with their mother and with their father and between the time when the parents are actively engaged and when they are simply around.

As in these previous papers, we use time diary surveys to measure parental time investments, but the novelty of our paper is that we also consider the time children spend on their own. How children spend time on their own becomes important as children grow into teenagers (Kooreman 2007). This is because adolescents begin to take independent decisions on how to spend their time, and these decisions can affect their cognitive development. There are only a few examples of economic models that consider both children and parents as decision makers. Dauphin et al. (2011) estimate a collective model and provide evidence that children aged 16 and over and living with their parents influence the household consumption and labour supply decisions. Lundberg et al. (2009) adopt a non-cooperative model to distinguish between children's decisions taken on their own and those shared with their parents, finding that the probability of taking independent decisions increases sharply between age 10 and 14.

Given that during adolescence children begin to take decisions on their own on how to use their time, cognitive production models for adolescents should include the time children spend on their own engaged in formative activities. The question is then how to define formative activities and time investments made by children.

In the economic literature there are a few papers that have defined time investments by parents (see, beside the papers cited at the beginning of this section, Price, 2008 and Guryan et al. 2008). The common approach is to consider the time parents spend with their children in formative activities such as reading, doing homework, playing sports, and exclude activities which are usually considered detrimental or not beneficial to the

[2] Time investments measured using time diaries are not completely free of measurement errors (see Stinebrickner and Stinebrickner 2004) and we address this issue in our empirical analysis in Section 6.3.

child's development, such as, for example, watching television. A natural extension of this definition to time investments by the children themselves would consider the time the child spends on her own doing formal and informal educational activities, as well as socializing and sports activities which can contribute to the child's development. This is actually the definition which we will adopt in our empirical application (see Section 3 for more details).

Different definitions of children's time investments have been used in other papers, but none of them distinguishes between the time the child spends on her own and the time the child spends actively supervised by a parent. Fiorini and Keane (2014) consider time-use diaries from the Longitudinal Study of Australian Children to estimate the effect of the time children spend on doing a set of different activities (e.g. school-day care, educational activities with parents and social activities). There are also several studies that have looked at the time invested by children on doing homework or studying, but again they do not distinguish between the time spent by children on their own and supervised by their parents. Dolton et al. (2003) consider the time spent by children on educational activities done on their own, but they only analyze adult students using data from one university in Spain. Similarly, Stinebrickner and Stinebrickner (2004 and 2008) consider the time students invest on studying by using data from a liberal arts college in Kentucky (Berea College). Eren and Henderson (2011) and Aksoy and Link (2000) use the National Education Longitudinal Study of 88 and analyze the effect of homework for children in high school in grade 8 and between grades 8 and 12 respectively. All these studies find that there is a positive effect of time spent studying or doing homework (especially in mathematics) on cognitive achievements. However, this positive effect of spending more time doing homework does not seem to extend to primary school students (see Farrow et al. 1999; Cooper et al. 2006).

The above-mentioned research suggests that children's time investments are important inputs in their cognitive development process. If we split the children's investments into the time invested on their own and the time invested under the active supervision of an adult, the former will presumably be increasingly important as they get older. On the contrary, the effect of parental investments on cognitive skills has been shown to decrease rapidly with age (see Cunha and Heckman 2007, 2008). In particular, looking at mothers' and fathers' time investments, Del Boca et al. (2014) find that the time parents spend actively engaged with their child has an effect on cognitive skills that decreases with the child's age.

Policies aimed at parents are still relevant to child development. In fact, when children become adolescents, parents may still have some influence on the way their child uses her time when she is on her own. For instance, parents may set strict rules on what their child can and cannot do or they may be able to transmit to their child time-use habits (some evidence on the transmission of time-use habits is provided in Mancini et al. 2015). Nevertheless, children during adolescence have more freedom in deciding how to use their own time and they can potentially disobey parental advice; therefore, the time they spend on their own studying or doing other formative activities can be considered the result of their own choice and a measure of self-investments.

The importance of adolescents' self-investments has raised interest in policies that encourage study effort and educational activities. For example, Angrist and Lavy (2009) have analyzed a randomized trial where cash awards were given to students in low-achieving schools conditional on passing their matriculation exam at the end of high school, which is a prerequisite for enrolling at university in Israel. They find that these cash incentives increase students' effort, measured by the number of exams taken, and ultimately the matriculation success rate. Over the last decade, similar conditional cash transfer (CCT) programs have been used as a tool to reduce poverty and improve human capital development in several developing countries (see Aber and Rawlings 2011). Some CCT programs aiming at improving child development have also been adopted in the US. An example is provided by the Opportunity New York City Family Rewards, which introduced different types of cash incentives including cash transfers conditional on educational outcomes (such as school attendance and requirement levels on standardized test scores). Evaluation of this program indicates that these CCT have led to changes in the time-use of teenagers, in particular in encouraging more engagement in educational activities (see Morris et al. 2012).

3 Data and preliminary evidence

Our analysis relies on the Child Development Supplement (CDS), funded by the National Institute of Child Health and Human Development (NICHD). The CDS covers a maximum of two children for a subsample of households interviewed in the Panel Study of Income Dynamics.[3] About 3500 children aged 0-12 (from about 2400 households) were first interviewed in 1997, and then followed in two subsequent waves, 2002/03 and 2007. The number of successful re-interviews was quite high: 91% in the second wave, 90% in the third one. The CDS collects information on cognitive and non-cognitive development of the sampled children, as well as their time-use diaries and other individual and family characteristics. All the household and parental variables included in the PSID survey are also available for the CDS children. In our analysis we include teenagers aged between 11 and 15 and living with both biological parents. To avoid small sample size issues, we pool two cohorts of children, born respectively in 1982-1986 (adolescents in 2002) and in 1987-1992 (adolescents in 2007) and obtain a sample of 726 children. This makes available two repeated observations for each adolescent: one during adolescence, when she is between 11 and 15 years old (either in 2002 or in 2007), and the other during childhood, when she is between 6 and 10 years old (either in 1997 or in 2002). This is the main sample used in the descriptive statistics in this section. For the estimation of our production models we will use the subsample of siblings, *sibling sample*, which allows us to consider the family fixed effects estimation. We have 202 pairs of siblings (404 children out of the 726 included

[3] The Panel Study of Income Dynamics is a USA longitudinal survey of a nationally representative sample of individuals and families, started in 1968 with a sample of 4800 families. It collects yearly individual information on economic, demographic, sociological, psychological and well-being variables.

in the main sample). The main summary statistics for the main and sibling samples are reported in the Appendix in Tables A1 and A2 respectively.

3.1 Time investments

Crucial to our research question is the availability of detailed information on child's time-use allocation for one randomly selected weekday and one randomly selected weekend-day. Time diaries for each day contain recording of activities performed in the 24 hours on a continuous basis.[4] Each spell of a given activity comes with information on its duration, location and on whether the activity was done by the child on her own, in the presence of somebody not actively participating or in the presence of somebody actively engaged.

This allows us to define a measure of weekly parental time investment as well as a measure of weekly child own time investments.[5] These time investments are measured in a specific week when parents and children are interviewed, but we assume that these represent the usual or average time inputs during the last 5 years.

We measure the parental time investment as the time the parent spends actively engaged with the child reading, doing homework, doing arts and crafts, doing sport, playing, attending performances and museums, engaging in religious activity, having meals and talking with the child, or providing personal care for the child. This aggregate measure of parental investment corresponds to the parent's quality time defined by Price (2008).[6] It is meant to include all the activities in which either the child is the primary focus or there is a sufficient interaction between the parent and the child. The positive relationship between the frequency of activities such as reading, playing or eating with children and their outcomes is well-documented in the literature (see Price, 2008, Section II for a concise review). The positive productivity of both mother's and father's active time has also been very recently documented by Del Boca et al. (2014) who have estimated a structural model of household choice on a sample of children in the age group 3-16 from the PSID CDS data set.

In order to take the novel perspective of the child's own investments in her development process, we select from the above-listed activities those that improve the child's human capital when performed independently by the child (i.e., either on his own or without anyone actively engaged). The resulting aggregate measure of the child's own investment includes - beside the time spent doing homework - all active leisure components such as reading, doing arts and crafts, doing sport, playing, attending performances and museums, and engaging in religious activity. Both intuition and scientific evidence highlight that human capital includes components other than formal knowledge, such as personal interaction skills that can be enhanced by time spent with friends or engaging in physical

[4] Activities are coded and registered from midnight of one day (00:00) to midnight of the following day (24:00), using a 24 hour clock. The ending time of an activity coincides with the starting time of the following activity, so that there are no gaps in time.

[5] The weekly measure is obtained multiplying by five the week-day time, and summing the result with the weekend-day time multiplied by two.

[6] Price (2008) derives parental time inputs from the parents time diaries, which are available in the American Time Use Survey.

activities. Cardoso et al. (2010) consider socializing, together with reading and studying, as activities related to the acquisition of human capital, as opposed to passive leisure such as television watching, often portrayed as detrimental and crowding out other useful activities. Felfe et al. (2011) report that a positive link between participation in active leisure sport activities and educational attainment is well established for adolescence, and show that sport club participation during kindergarten and primary school has a positive effect on school performance.

The upper part of Table 1 contains the composition of the child's own time investments in childhood age (6-10) and adolescence (11-15). The total active time spent by children on their own increases by about one hour a week (25%), on average, across the two stages of their life. The reading and homework activities bring the largest contribution to this increase (respectively about 16 and 48 minutes per week on average), followed by the playing category (with an average increase of about 13 minutes per week). On the contrary, sport and arts activities appear less frequently performed on average during adolescence compared to childhood. The bottom panel of the same table shows a sharp decrease in the mother's time investments from childhood to adolescence. Mothers spend on average about 9 hours and a half per week actively engaged with their children aged 6 to 10 years, but only 5 hours and a half when their children become adolescents. All categories of mother's time investment except for religious activity diminish across the two life stages. In the Appendix, Table A3, we report the father's composition of time investments. The total time fathers spend with children declines with the child's age: on average they spend 6 hours a week with their children aged 6 to 10 years, but only 4 when the children are 11 to 15. However, time spent on helping with homework, talking and attending performances increases slightly.

Table 1
Mother's and child's time investment - Main sample

	Child's age range 6-10				Child's age range 11-15			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Own time investment								
Total time investment	4.08	5.15	0.00	30.92	5.12	6.86	0.00	41.25
Reading	0.69	1.79	0.00	24	0.96	2.5	0.00	21.83
Homework	0.46	1.72	0.00	17.50	1.25	3.52	0.00	29.00
Playing	2.27	3.81	0.00	24.75	2.48	5.10	0.00	41.25
Arts and craft	0.27	1.14	0.00	11.25	0.20	1.24	0.00	19.75
Sport	0.28	1.30	0.00	22.10	0.16	0.95	0.00	15.00
Attending performances	0.00	0.00	0.00	0.00	0.01	0.20	0.00	5.33
Attending museums	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Religious activity	0.11	0.70	0.00	6.33	0.08	0.56	0.00	7.17
Mother time investment								
Total time investment	9.47	7.08	0.00	40.42	5.46	5.20	0.00	35.42
Reading	0.50	1.21	0.00	11.25	0.11	0.84	0.00	12.33
Homework	0.24	1.12	0.00	10.83	0.11	0.84	0.00	11.17
Playing	1.11	2.56	0.00	25.17	0.31	1.49	0.00	21.25
Talking	0.37	0.98	0.00	8.33	0.57	1.48	0.00	12.42
Arts and craft	0.11	0.78	0.00	14.92	0.04	0.35	0.00	4.97
Sport	0.41	1.47	0.00	15.00	0.09	0.68	0.00	10.67
Attending performances	0.14	1.01	0.00	13.33	0.10	0.90	0.00	13.33
Attending museums	0.05	0.56	0.00	9.50	0.00	0.00	0.00	0.00
Religious activity	0.78	2.07	0.00	14.32	0.78	2.21	0.00	20.00
Meals	4.57	3.18	0.00	22.17	3.11	2.91	0.00	21.75
Personal care	1.20	2.50	0.00	24.17	0.24	1.21	0.00	16.17

NOTE.- Weekly time investments in hours. No. observations: 726.

3.2 Cognitive outcomes

The cognitive tests we use come from the Woodcock-Johnson Revised Tests of Achievement, "a well-established and respected measure that provides researchers with information on several dimensions of intellectual ability" (CDS User Guide). The CDS provides three cognitive test scores measuring symbolic learning and reading, comprehension and vocabulary, and mathematical abilities: the Letter-Word Identification, Passage-Comprehension, and Applied-Problems test scores. These tests were administered to respondents aged 6 years and older by the interviewer, following a standardized administrative protocol and adjusting the test by difficulty according to the respondent's age (see CDS User Guide for details). Each of these three tests provides a score which is a measure of the child's cognitive ability. The Letter-Word Identification Score (LWS) measures symbolic learning (matching pictures with words) and reading identification skills (identifying letters and words). It starts from the easiest items (identification of letters and pronunciation of simple words), progressing to the more difficult items. The Passage Comprehension Score (PCS) assesses comprehension and vocabulary skills through multiple-choice and fill-in-the-blank formats. The Applied Problems Score (APS) measures mathematical skills in analyzing and solving practical problems. The test scores are available in both raw and standardized formats. The former essentially counts the number of items correctly answered, while the latter are obtained by standardizing the raw scores according to the respondent's age.[7] We use the standardized measures throughout our analysis.

3.3 Time investments and cognitive ability: preliminary evidence

In Tables 2 and 3 we provide descriptive evidence on the link between time investments and child cognitive outcomes. In Table 2 we look at the differences between average test scores for adolescents, dividing them into two groups: those receiving a high level of investments from their mother (higher than average) and those receiving a low level of investments (lower than average). It can be noticed that children receiving low time investments from their mothers during adolescence have essentially the same outcomes in adolescence as children receiving high time investments, while the time spent with the mother actively engaged in childhood displays some association with adolescents' cognitive outcomes (the difference is statistically different at 1% level for PCS and marginally significant, at 15% level, for APS).

Table 2

Average test scores during adolescence by mother's time investment

	Obs	LWS Average	PCS Average	APS Average
Main Sample	726	105.842	104.055	107.135
	Mother's time investment in adolescence			
Subsample with mother's time investment				
higher than average	288	106.028	104.653	106.833
lower than average	438	105.719	103.662	107.333
Difference between the two subsamples		0.308	0.990	-0.500
Standard Error		(1.275)	(1.135)	(1.150)
	Mother's time investment in childhood			
Subsample with mother's time investment				
higher than average	320	106.700	105.872	108.028
lower than average	406	105.165	102.623	106.431
Difference between the two subsamples		1.534	3.249***	1.597
Standard Error		(1.254)	(1.112)	(1.131)

NOTE.- LWS, PCS, APS = Letter-Word Identification, Passage Comprehension, Applied Problems Scores. *, **, *** statistically significant at 10%, 5%, 1% level respectively (two-sided t-test for H_0: Difference of means=0).

[7] The age standardization process allows for comparison of children of different ages, eliminating the discrepancy in the results due to age differences.

Turning to child's own investments in Table 3, the pattern is reversed, and investments during adolescence display a much stronger relationship with adolescents' outcomes than investments during childhood. The highly significant differences in the test scores between children with high own time investments and those with low own time investments strongly support our investigation about the relevance of autonomous decisions taken by children at this stage of life.

Table 3
Average test scores during adolescence by child's own time investments

	Obs	LWS Average	PCS Average	APS Average
Main Sample	726	105.842	104.055	107.135
		Own time investment in adolescence		
Subsample with mother's time investment				
higher than average	249	108.566	107.365	110.438
lower than average	477	104.419	102.327	105.411
Difference between the two subsamples		4.147***	5.038***	5.026***
Standard Error		(1.305)	(1.155)	(1.170)
		Own time investment in childhood		
Subsample with mother's time investment				
higher than average	268	108.160	105.944	108.585
lower than average	458	104.484	102.950	106.286
Difference between the two subsamples		3.675***	2.994***	2.300**
Standard Error		(1.285)	(1.145)	(1.162)

NOTE.- LWS, PCS, APS = Letter-Word Identification, Passage Comprehension, Applied Problems Scores. *, **, *** statistically significant at 10%, 5%, 1% level respectively (two-sided t-test for H_0: Difference of means=0).

4 Modeling cognitive achievement production function during adolescence

We model the cognitive achievement production function considering inputs that reflect decisions by schools and families as well as by the children themselves. We also take into account the fact that cognitive development is a cumulative process, by allowing the production function to depend on both contemporaneous and past investments.

By assuming that the production function be additive, separable and linear in its inputs, we specify the achievement production model during the adolescent stage, i.e., between age 11 and 15, as follows:

$$Y_{ijt} = \beta_0 + \beta_1 \mathbf{X}_{ijt} + \beta_2 \mathbf{X}_{ijt-5} + \beta_3 \mathbf{X}_{ijt-10} + \mu_{ij} + \eta_{ijt}. \quad (1)$$

where the outcome Y_{ijt} is a general measure of cognitive ability for adolescent i in household j at t years old, $t = 11, ..., 15$ and the arguments are given by:

- the vector of contemporaneous cognitive investments during adolescence by the child herself, X^C_{ijt}, her family, X^F_{ijt}, and her school, X^S_{ijt}, $\mathbf{X}'_{ijt} = [X^{C\prime}_{ijt}, X^{F\prime}_{ijt}, X^{S\prime}_{ijt}]$;
- the corresponding vector of inputs during childhood (5 years earlier), $\mathbf{X}'_{ijt-5} = [X^{C\prime}_{ijt-5}, X^{F\prime}_{ijt-5}, X^{S\prime}_{ijt-5}]$;
- the corresponding vector of inputs during early childhood (10 years earlier), $\mathbf{X}'_{ijt-10} = [X^{C\prime}_{ijt-10}, X^{F\prime}_{ijt-10}, X^{S\prime}_{ijt-10}]$;
- her unobserved cognitive endowment μ_{ij};
- a random (idyonsincratic) shock in period t, η_{ijt}.

Notice that we assume that the parameters of the above model are invariant during the stage of adolescence, i.e., for children aged between 11 and 15 ($t = 11, ..., 15$), but we do not impose that this model is invariant across different child life stages. The following specification for children during the childhood stage (ages 6-10 years old) is useful in some cases:

$$Y_{ijt-5} = \gamma_0 + \gamma_1 \mathbf{X}_{ijt-5} + \gamma_2 \mathbf{X}_{ijt-10} + \mu^c_{ij} + \eta_{ijt-5}. \tag{2}$$

where the outcome(s) and inputs are observed 5 years earlier than in equation (1), μ^c_{ij} captures the unobserved cognitive endowment during childhood, which can differ from the corresponding endowment during adolescence, and the parameters γ_0, γ_1 and γ_2 are not imposed to be equal to β_0, β_1 and β_2.

Our production function is similar to the one considered by previous works on child cognitive development, with the main difference being that it considers the investments made by the children themselves alongside the inputs by families and schools (see Todd and Wolpin 2003 and 2007).

In our sample, we do not observe a general measure of cognitive ability Y_{ijt}, but we observe three different specific skills measured by the Letter-Word Identification, Passage-Comprehension, and Applied-Problems test scores. We indicate these three observed skills with Y_{kijt}, where the subscript k denotes each of the three cognitive test scores, and we impose the following assumptions, which we call *maintained* assumptions because they are imposed throughout the rest of paper:[8]

M1 The specific measure of ability k in adolescence follows the model:

$$Y_{kijt} = Y_{ijt} + \epsilon_{kijt}, \tag{3}$$

where $t = 11, ..15$, ϵ_{kijt} measures the deviation of skill k, Y_{kijt}, from the general latent ability, Y_{ijt}, and it is assumed to be identically and independently distributed across skills, individuals and households, with mean 0 variance σ^2_ϵ, uncorrelated with the production function inputs, the latent general ability and the unobserved endowment, but it is allowed to be correlated across time;

[8] We abstract from any measurement error in skills in our main analysis, but we explore the issue in section 6.4.

M2 The specific measure of ability k in the childhood period follows the model:

$$Y_{kijt-5} = Y_{ijt-5} + \epsilon_{kijt-5}, \tag{4}$$

where $t-5 = 6, ..10$, ϵ_{kijt-5} measures the deviation of skill k, Y_{kijt-5}, from the general latent ability, Y_{ijt-5}, and it is assumed to be identically and independently distributed across skills, individuals and households, with mean 0 variance σ^2_ϵ, uncorrelated with the production function inputs, the latent general ability and the unobserved endowment, but it is allowed to be correlated across time.

Under assumption M1, the production function during adolescence (1) can be rewritten as:

$$Y_{kijt} = \beta_0 + \beta_1 \mathbf{X}_{ijt} + \beta_2 \mathbf{X}_{ijt-5} + \beta_3 \mathbf{X}_{ijt-10} + \mu_{ij} + \epsilon_{kijt} + \eta_{ijt}. \tag{5}$$

Model (5) is similar to what Todd and Wolpin (2003) call the cumulative model, where the child's outcome during adolescence at age t depends on current and past inputs. Since we only observe inputs every five years, the outcome during adolescence at age t ($t = 11, ..., 15$) depends only on inputs observed at age t, $t-5$ and $t-10$, i.e., we assume that inputs during adolescence, childhood and early childhood can be approximated by inputs observed at three points in time, t, $t-5$ and $t-10$.

In the following sections, we list the assumptions needed to obtain consistent estimators of the cumulative model during adolescence (Section 4.1), and of an extended model that includes the lagged cognitive score Y_{kijt-5} as an additional input, called the augmented value-added model (Section 4.2). For the cumulative model, we discuss consistency of the Ordinary Least Squares (OLS), the family fixed effects (FE) estimator and the estimator obtained taking differences across time, which we call the time difference (TD) estimator. For the augmented value-added model, we do not consider the TD estimator, which is not applicable in our context,[9] and, in addition to OLS and family FE, we discuss a two-step estimator that we propose below.

4.1 Cumulative model

Breaking down investments by children, families and schools, the cumulative model during adolescence (5) can be written as:

$$\begin{aligned} Y_{kijt} \;=\; & \beta_0 + \beta_1^C X_{ijt}^C + \beta_1^F X_{ijt}^F + \beta_1^S X_{ijt}^S + \beta_2^C X_{ijt-5}^C + \beta_2^F X_{ijt-5}^F \\ & + \beta_2^S X_{ijt-5}^S + \beta_3^C X_{ijt-10}^C + \beta_3^F X_{ijt-10}^F + \beta_3^S X_{ijt-10}^S + \mu_{ij} + \epsilon_{kijt} + \eta_{ijt}, \end{aligned} \tag{6}$$

where β_0 is the intercept, $\beta_1 = [\beta_1^C, \beta_1^F, \beta_1^S]$, $\beta_2 = [\beta_2^C, \beta_2^F, \beta_2^S]$ and $\beta_3 = [\beta_3^C, \beta_3^F, \beta_3^S]$ are vectors of coefficients corresponding to contemporaneous, 5-year and 10-year lagged inputs.

[9] This would require observing the test scores Y_{kijt} in more than two periods.

Estimation of the above model is quite demanding in terms of data on current and past investments. In our empirical application we are able to measure parental investments by looking at the time the mother spends actively engaged with her child, whereas we measure child investments by the time children spend on formative activities on their own without the supervision of an adult (see Section 3 for details on these definitions). We are able to observe these parental and child investments during late childhood and adolescence, whereas we are unable to observe school inputs, $[X^S_{ijt}, X^S_{ijt-5}, X^S_{ijt-10}]$, and early childhood inputs, X_{ijt-10}. For this reason we have to collapse these investments into the idyosincratic error of the model, which becomes $\widetilde{\eta}_{ijt} = \eta_{ijt} + \beta^S_1 X^S_{ijt} + \beta^S_2 X^S_{ijt-5} + \beta^S_3 X^S_{ijt-10} + \beta^C_3 X^C_{ijt-10} + \beta^F_3 X^F_{ijt-10}$:

$$Y_{kijt} = \beta_0 + \beta^C_1 X^C_{ijt} + \beta^F_1 X^F_{ijt} + \beta^C_2 X^C_{ijt-5} + \beta^F_2 X^F_{ijt-5} + \mu_{ij} + \epsilon_{kijt} + \widetilde{\eta}_{ijt}. \qquad (7)$$

To consistently estimate the cumulative model (7) using OLS, beside M1, the following condition must hold:

A1 the observed inputs are uncorrelated with the unobserved endowment μ_{ij} and with the unobserved inputs, i.e., with the idiosyncratic shock $\widetilde{\eta}_{ijt}$.

Clearly assumption A1 is quite restrictive because it is hardly credible that parental and child investments are uncorrelated with the child's unobserved endowment, the school inputs and the early childhood investments. Omitting the time investment by the child herself during early childhood, X^C_{ijt-10}, is not a major concern, because children aged 0-5 spend zero or very little time on their own (i.e., without the supervision of an adult). On the contrary, the omission of early parental investments and of school inputs can be relevant.

Next, let us consider the family FE estimation. In our empirical application we observe up to two siblings for each household, and we can therefore compute the family FE estimator by regressing sibling differences in test scores on the sibling differences in their inputs and endowments:[10]

$$\begin{aligned} Y_{kijt} - Y_{ki'jt} \;=\; & \beta^C_1 (X^C_{ijt} - X^C_{i'jt}) + \beta^F_1 (X^F_{ijt} - X^F_{i'jt}) + \beta^C_2 (X^C_{ijt-5} - X^C_{i'jt-5}) \\ & + \beta^F_2 (X^F_{ijt-5} - X^F_{i'jt-5}) + (\mu_{ij} - \mu_{i'j}) + (\epsilon_{kijt} - \epsilon_{ki'jt}) + (\widetilde{\eta}_{ijt} - \widetilde{\eta}_{i'jt}), \end{aligned} \qquad (8)$$

where the subscripts i and i' denote the two siblings in household j. The consistency of the family FE estimation requires the following assumption:

B1 sibling differences in observed inputs are uncorrelated with sibling differences in their unobserved endowment, $(\mu_{ij} - \mu_{i'j})$ and sibling differences in unobserved inputs, i.e., sibling differences in the idiosyncratic shock, $(\widetilde{\eta}_{ijt} - \widetilde{\eta}_{i'jt})$.

[10] The difference in the variables between two siblings is taken either at the same calendar period (year) or in the two available periods.

Assumption B1 is likely to be less restrictive than assumption A1, because inputs are allowed to depend on the unobserved family-specific endowment and on unobserved inputs which do not vary between siblings. In fact, in model (8), we actually control for all unobserved family-specific characteristics using sibling differences. The consistency of the family FE estimation still requires that the inputs do not respond to the unobserved child-specific endowment. Rather than requiring a zero response of parental and child investments to changes in omitted school inputs (as assumption A1 in model 7 does), it only requires that sibling differences in parental and child's investments do not react to sibling differences in omitted school inputs. In our empirical section, we will test the validity of such an assumption in the augmented value-added model, which will be our preferred specification.

An alternative estimation strategy for model (7) is analogous to first difference estimation in the context of panel data, which exploits the test scores and time investments available at different points in time for the same child. In our framework the time difference for a variable is between the variable observed in t and in $(t-5)$ and the corresponding estimation, we call time difference (TD) estimation, is implemented by differencing model (7):

$$\begin{aligned} Y_{kijt} - Y_{kijt-5} &= \beta_1^C (X_{ijt}^C - X_{ijt-5}^C) + \beta_1^F (X_{ijt}^F - X_{ijt-5}^F) + \beta_2^C (X_{ijt-5}^C - X_{ijt-10}^C) \\ &\quad + \beta_2^F (X_{ijt-5}^F - X_{ijt-10}^F) + (\epsilon_{kijt} - \epsilon_{kijt-5}) + (\widetilde{\eta}_{ijt} - \widetilde{\eta}_{ijt-5}). \end{aligned} \tag{9}$$

The consistency of the TD estimation requires, beside M1-M2, the following assumptions:

C1 the production models in adolescence and in childhood are identical, i.e., $\beta_0 = \gamma_0, \beta_1 = \gamma_1; \beta_2 = \gamma_2$, and $\mu_{ij}^c = \mu_{ij}$.

C2 time differences in observed inputs are uncorrelated with time differences in unobserved inputs, i.e., time difference in the idiosyncratic shock, $(\widetilde{\eta}_{ijt} - \widetilde{\eta}_{ijt-5})$;

C3 time differences in observed inputs are uncorrelated with $(\epsilon_{kijt} - \epsilon_{kijt-5})$.

While condition C3 is satisfied because of the assumptions M1-M2, conditions C1 and C2 are quite strong. Condition C1 is hardly credible, especially in light of recent literature which emphasizes that the child development process is a multistage process and that some inputs can be more productive in some stages and less in others (Cunha et al. 2006, Cunha et al. 2010). Condition C2 is also not credible because investments in t, X_{ijt}^C and X_{ijt}^F, are taken by the child and her parents after observing Y_{kijt-5}, or some other correlated measure of cognitive ability, and are likely to respond to Y_{kijt-5}. Because Y_{kijt-5} depends on $\widetilde{\eta}_{ijt-5}$, we cannot exclude that X_{ijt}^C and X_{ijt}^F are correlated with $\widetilde{\eta}_{ijt-5}$, which implies that the assumption C2 cannot be satisfied. In other words, every time that investments in t react to cognitive ability in $(t-5)$, the TD estimation is biased by a reverse causality issue. In the case of the family FE, this reverse causality issue does not occur because decision on investments in t, $(X^C_{ijt} - X^C_{i'jt})$ and $(X^F_{ijt} - X^F_{i'jt})$, are taken before observing sibling differences in test score in t, $(Y_{kijt} - Y_{ki'jt})$. Nevertheless, if investments respond to past cognitive abilities and past cognitive abilities are relevant in the production of current cognitive abilities, then both the TD and family FE estimation are inconsistent. For this reason, in next section we extend the production model to include past cognitive abilities as inputs.

4.2 Augmented value-added model

The family FE estimator of the cumulative model allows the inputs to depend on the unobserved endowment and characteristics that are identical between siblings, but, as with the OLS estimator, it is unable to take account of the possible dependence of inputs on the unobserved child-specific endowment or on past cognitive achievements. Parents may respond to the child's past cognitive abilities or to differences in the past cognitive abilities between their children with reinforcing or compensating behaviors, and these are sources of inconsistency for the OLS and the family FE estimators.

To control for this dependence between lagged cognitive ability and inputs, we add the lagged true cognitive ability Y_{ijt-5} as an explanatory variable in the cumulative production model during adolescence (1), which yields the *augmented value-added model* (as defined by Todd and Wolpin 2007):

$$Y_{ijt} = \delta_0 + \delta_1 \mathbf{X}_{ijt} + \delta_2 \mathbf{X}_{ijt-5} + \rho Y_{ijt-5} + \mu^a_{ij} + \eta^a_{ijt}, \tag{10}$$

where μ^a_{ij} is the new unobserved child-specific endowment and η^a_{ijt} is an idiosyncratic shock.

For this augmented value-added model we consider, beside M1 and M2, the following new maintained assumption:

M3 the persistence[11] in each of the three k-specific abilities, Y_{kijt} $(k = 1, 2, 3)$, ρ_k, is identical to the persistence in latent general ability, Y_{ijt}, and equal to ρ.

Assumption M3 states that each of the three different abilities (Letter-Word Identification, Passage-Comprehension, and Applied-Problems) depreciates at the same rate from $(t-5)$ to t. This seems a reasonable pattern which is supported by the empirical evidence we provide in Section 6.2.[12]

By replacing the unobserved latent general ability with the observed k-specific ablity we can rewrite model (10) as:

$$Y_{kijt} = \delta_0 + \delta_1 \mathbf{X}_{ijt} + \delta_2 \mathbf{X}_{ijt-5} + \rho Y_{kijt-5} + \mu^a_{ij} + u_{kijt}, \tag{11}$$

[11] By persistence we mean the net autocorrelation, i.e. the correlation between a variable in t and the corresponding variable in $(t-5)$ net of the explanatory variables in the production model.

[12] M2 also implies that the correlation between ϵ_{kijt} and ϵ_{kijt-5} is equal to ρ for each k. Since ϵ_{kijt} and ϵ_{kijt-5} are not errors but measures of extra ability of child i in subject k with respect to her general ability, Y_{ijt}, an equal persistence in this extra ability and in the general latent ability seems reasonable.

where $u_{kijt} = \epsilon_{kijt} - \rho\epsilon_{kijt-5} + \eta^a_{ijt}$.

Consistency of the OLS estimator of model (11) requires, beside M1-M3, the additional assumptions:

D1 conditionally on the past cognitive ability Y_{kijt-5}, the observed inputs are uncorrelated with the unobserved endowment μ^a_{ij} and with the unobserved inputs, i.e., with the idiosyncratic shock, η^a_{ijt} (but they are allowed to be correlated with the past cognitive ability Y_{kijt-5});

D2 the past cognitive ability, Y_{kijt-5}, is uncorrelated with both the unobserved endowment μ^a_{ij} and the idiosyncratic shock η^a_{ijt}.

Notice that in model (11) Y_{kijt-5} and the error term u_{kijt} are correlated, since both Y_{kijt-5} and u_{kijt} depend on ϵ_{kijt-5} This correlation would generally bias the estimation, but, under the above assumptions, we can prove that the asymptotic bias cancels out, and the OLS estimator of ρ converges asymptotically to:

$$\begin{aligned} plim\widehat{\rho}_{OLS} &= \frac{Cov(M_X Y_{kijt}, M_X Y_{kijt-5})}{Var(M_X Y_{kijt-5})} \\ &= \rho + \frac{Cov(\mu^a_{ij}, M_X Y_{kijt-5})}{Var(M_X Y_{kijt-5})} + \frac{Cov(\epsilon_{kijt}, \epsilon_{kijt-5})}{Var(M_X Y_{kijt-5})} - \rho\frac{Var(\epsilon_{kijt-5})}{Var(M_X Y_{kijt-5})} \\ &= \rho \end{aligned} \tag{12}$$

where M_X is the projection matrix on the space orthogonal to the one generated by the variables $\mathbf{X}' = (\mathbf{X}'_{ijt}, \mathbf{X}'_{ijt-5})$. The consistency is guaranteed by the fact that:

- assumption D2 implies that the asymptotic bias caused by the omission of the unobserved individual endowment cancels out, i.e., $\frac{Cov(\mu^a_{ij}, M_X Y_{kijt-5})}{Var(M_X Y_{kijt-5})} = 0$;
- $\left[\frac{Cov(\epsilon_{kijt},\epsilon_{kijt-5})}{Var(M_X Y_{kijt-5})} - \rho\frac{Var(\epsilon_{kijt-5})}{Var(M_X Y_{kijt-5})}\right]$, which is the asymptotic bias caused by the correlation between $(\epsilon_{kijt} - \rho\epsilon_{kijt-5})$ and the lagged test Y_{kijt-5}, is also zero because of assumptions M1-M3:

$$\frac{Cov(\epsilon_{kijt}, \epsilon_{kijt-5})}{\sqrt{Var(\epsilon_{kijt-5})Var(\epsilon_{kijt})}} = \frac{Cov(\epsilon_{kijt}, \epsilon_{kijt-5})}{Var(\epsilon_{kijt-5})} = \rho. \tag{13}$$

Assumptions D1-D2 can be quite restrictive, so we also consider family FE estimation, i.e., express model (11) as differences between siblings:

$$\begin{aligned} Y_{kijt} - Y_{ki'jt} &= \delta^C_1(X^C_{ijt} - X^C_{i'jt}) + \delta^F_1(X^F_{ijt} - X^F_{i'jt}) + \delta^C_2(X^C_{ijt-5} - X^C_{i'jt-5}) \\ &\quad + \delta^F_2(X^F_{ijt-5} - X^F_{i'jt-5}) + \rho(Y_{kijt-5} - Y_{ki'jt-5}) + (\mu^a_{ij} - \mu^a_{i'j}) + (u_{kijt} - u_{ki'jt}). \end{aligned} \tag{14}$$

Family FE estimation is consistent under the following assumptions (beside the maintained assumptions M1-M3):

E1 conditionally on the sibling difference in the past cognitive ability $(Y_{kijt-5} - Y_{ki'jt-5})$, sibling differences in observed inputs are uncorrelated with sibling differences in their unobserved endowment, $(\mu^a_{ij} - \mu^a_{i'j})$, and with sibling differences in unobserved inputs, i.e., the sibling difference in the idiosyncratic shock, $(\eta^a_{ijt} - \eta^a_{i'jt})$; but are allowed to be correlated with the sibling difference in past cognitive attainment, $(Y_{kijt-5} - Y_{ki'jt-5})$;

E2 the sibling difference in the past cognitive test score, $(Y_{kijt-5} - Y_{ki'jt-5})$, is uncorrelated with the sibling difference in unobserved endowment, $(\mu^a_{ij} - \mu^a_{i'j})$, and in the idiosyncratic shock, $(\eta^a_{ijt} - \eta^a_{i'jt})$.

Assumptions E1-E2 are likely to be less restrictive than assumptions D1-D2 because using differences between siblings eliminates the unobserved family-specific characteristics and endowment that do not vary between siblings. Assumption E1 is in line with the view that there exists exogenous sibling variation in time investments which can explain sibling differences in cognitive abilities after controlling for the lagged abilities and other variables. Borrowing from the seminal paper of Ashenfelter and Rouse (1998), this variation can be seen as originated by random deviations from optimal investment choices caused, for example, by unexpected influence of school peers and friends (e.g. Stinebrickner and Stinebrickner 2008) or by experiencing events that change the child's preferences about time-use but do not directly impact test scores.

The assumption that the unobserved child-specific endowment is uncorrelated with the lagged test (assumption E2) is likely to be less restrictive than assumption D2, but it is still likely to be invalid. If assumption E2 does not hold, then being unable to control for sibling differences in the unobserved endowment will lead to an overestimation of the effect of the lagged test score, the persistence ρ, which can contaminate the input coefficients as well (Andrabi et al., 2011).

We solve this further issue of endogeneity by adopting a two-step estimation procedure.

In the first step we use the observed scores for the three different skills available for each child in t and $(t-5)$ to compute an individual fixed effects (individual FE) estimation, which controls for the child-specific endowment, μ^a_{ij}. This individual FE estimation can be implemented by considering model (11) expressed in deviations from the mean:

$$Y_{kijt} - \bar{Y}_{.ijt} = \rho(Y_{kijt-5} - \bar{Y}_{.ijt-5}) + (u_{kijt} - \bar{u}_{.ijt}), \quad (15)$$

where the bar indicates the mean over the three skills. This individual FE estimation method is identical to the within-pupil between-subject estimation used by Dee (2005 and 2007) to estimate the effect of teacher characteristics on test scores. Because none of the right hand side variables in model (11) changes across the three skills except the test score, the individual fixed effects estimation provides an estimate only for the persistence parameter, $\widehat{\rho}_{IndFE}$.[13]

[13]Notice that, if the inputs changed across different skills, then we could include them in equation (15) and we would be able to estimate their effect directly in the first stage estimation, with no need for a second stage.

In the second step, we replace ρ in model (14) with its estimate from the first step:

$$\begin{aligned} Y_{kijt} - Y_{ki'jt} - \widehat{\rho}_{IndFE}(Y_{kijt-5} - Y_{ki'jt-5}) &= \delta_1^C(X_{ijt}^C - X_{i'jt}^C) + \delta_1^F(X_{ijt}^F - X_{i'jt}^F) \\ &+ \delta_2^C(X_{ijt-5}^C - X_{i'jt-5}^C) + \delta_2^F(X_{ijt-5}^F - X_{i'jt-5}^F) \\ &+ (\mu_{ij}^a - \mu_{i'j}^a) + (u_{kijt} - u_{ki'jt}), \end{aligned} \tag{16}$$

and we use family FE estimation to produce estimates for the coefficients $\delta_1^C, \delta_1^F, \delta_2^C$ and δ_2^F. Thanks to this novel *two-step estimation,* we obtain results that are purged of the bias induced by the lagged test regressor. We are actually not the first to assume that different cognitive test scores are related to a same latent cognitive ability and to use the multiplicity of measures to solve the issue of endogeneity of the lagged test. For example, Cunha and Heckman (2008) use multiple measures of tests and inputs to derive three latent measures corresponding to cognitive and non-cognitive abilities and investment. Furthermore, they use multiple measures of tests and inputs to instrument the lagged tests and inputs in their cognitive development model (see Pudney 1982 for more details on this other type of estimation). Our procedure imposes some different restrictions, but it is simpler and has the advantage of distinguishing between parents' and child's inputs and therefore allows us to evaluate the contribution of children's decisions to their cognitive development process.

Under assumptions M1-M3 it can be proven that the individual fixed effects estimation of the persistence, ρ_{IndFE}, is consistent because

$$plim \widehat{\rho}_{IndFE} = \frac{Cov(\epsilon_{kijt}, \epsilon_{kijt-5})}{Var(\epsilon_{kijt-5})} = \rho. \tag{17}$$

The two-step estimation, which uses the child individual estimation in the first step and the family fixed effects estimation in the second step is consistent under assumption E1 beside the maintained assumptions M1-M3. These assumptions are identical to those required for the consistency of the family FE estimation of the augmented value-added model, except for assumption E2, which is now relaxed.

Notice that, as for the family fixed effects estimation of the augmented value-added model, the two-step estimation does not require that parental and school investments be identical between siblings or that they be uncorrelated with lagged test scores. Since the seminal paper of Behrman et al. (1982), several studies have tried to explain why parental investments differ between siblings and have examined whether these investments compensate or reinforce children's differences in abilities. Bernal (2008), for example, finds that compensating behavior seems to dominate when looking at time investments of mothers. We take into account that the mother's investment may compensate for or reinforce differences between her children's abilities by controlling for lagged test score realizations. However, we assume that any other unobserved ability or input is either identical between siblings or that, if a difference exists, it is uncorrelated with the sibling differences in observed inputs once controlling for their gaps in the lagged test and other variables. Clearly, sibling differences in unobserved characteristics that cause a response in investments would make E1 invalid and lead to an overestimation (or underestimation) of the investment effect if the investments reinforce (or compensate for) the sibling gap in cognitive abilities. In Section 6.1, we test empirically whether omitted inputs are a cause of concern by considering three sets of potential omitted variables: (i) school inputs, (ii) early childhood inputs, (iii) child's health shocks.

5 Estimation results of the cognitive production model

In Tables 4 and 5, we report our main estimation results for the cumulative model (5) and the augmented value-added model (11). For the cumulative model, we report the results of the ordinary least squares (OLS), family fixed effects (family FE) and time difference (TD) estimations (columns 1 to 3 of Table 4); whereas for the augmented valued added model we report the estimates of the OLS, family FE and two-step estimation methods (columns 1 to 3 of Table 5). Both the cumulative and the augmented value-added models include the same explanatory variables except for the lagged test, which is included only in the augmented model. The outcome variable is measured by the three standardized test scores described in Section 3: the Letter-Word Identification Score (LWS), the Passage Comprehension Score (PCS) and the Applied Problems Score (APS). We treat the three tests as repeated measures of the child's ability, so that our number of observations increases from 404 (the number of siblings) to 1,212 (the number of siblings multiplied by the number of tests available for each child). We estimate the production models using the sibling sample for all estimations except for the TD estimation, which also requires information on twice lagged time investments, and is therefore based on the subsample that excludes missing cases for these investments.[14]

Our main coefficients of interest are the effects of time investments, which we measure by the weekly number of hours the child and his/her mother invest in formative activities during adolescence (child's and mother's time investments) and during childhood (child's and mother's lagged time investments). We focus our discussion mainly on these four coefficients and on the coefficient of the lagged test, which captures the correlation between the contemporaneous and lagged test net of the explanatory variables and allows us to assess whether a bad test result today may create a trap into low cognitive achievements for the child's future.

There are differences across different specifications and estimations, but two findings clearly emerge from all but the TD estimation: (i) the mother's investment during childhood matters, while the mother's investment during adolescence does not (see rows 1 and 3 in Table 4 and rows 2 and 4 in Table 5); (ii) the child's own investment during childhood matters less than the child's investment during adolescence (see rows 2 and 4 in Table 4 and rows 3 and 5 in Table 5).

The TD estimation is the only model for which the above finding (i) is not confirmed, but we think that this might be caused by the failure of assumptions C1 and C2. Because

[14] In Table A2 in the Appendix we report some summary statistics of the variables used.

mothers and children take decisions on time investments in t before observing the test results in t but after observing the test results (or some correlated measures of cognitive abilities) in $t-5$, the TD estimation is biased by a reverse causality issue that invalidates assumption C2. We also think that assumption C1, which imposes an identical production model for children aged 6-10 and aged 11-15, is hard to believe. For these reasons, we judge the TD estimation to be inadequate for the estimation of our cumulative model. Stinebrickner and Stinebrickner (2008) reach a similar conclusion for their first difference estimation applied to evaluate the effect of students' study effort on test scores during college.

The finding that the mother's investment during childhood matters more than the mother's investment during adolescence in explaining adolescents' cognitive ability suggests a decreasing importance of the mother's time investment as the child grows. Empirical evidence on the decreasing importance of the mother's investment is also provided by our descriptive statistics in Table 1, where we can see that the mother's time investment decreases from about 9 hours per week to 5 hours per week when children move from childhood to adolescence. The fact that only child self investments during adolescence and not during childhood affect their cognitive outcomes in adolescence suggests that the importance of child own investments increases with age, as children become more independent.

A third clear finding emerges within the results of the augmented value-added model (see Table 5): (iii) the lagged test coefficient is always highly significant, suggesting a very high persistence in the test score results. Nevertheless, this coefficient decreases from 0.528 to 0.352 when we control for the family fixed effects (see columns 1 and 2 in Table 5) and to 0.279 when we also control for the individual fixed effects (see column 3), revealing that part of the test persistence is explained by the unobserved ability endowment.

Table 4
Cumulative model estimation results

	OLS	Family FE	TD
Mother's time investment	-0.004	-0.007	0.001
	(0.006)	(0.007)	(0.004)
Child's time investment	0.022***	0.010*	0.014***
	(0.004)	(0.005)	(0.004)
Lag(Mother's time investment)	0.009**	0.010*	0.002
	(0.004)	(0.005)	(0.002)
Lag(Child's time investment)	0.013**	0.007	0.005
	(0.005)	(0.006)	(0.003)
Child's age	-0.185	0.045	
	(0.427)	(0.411)	
Child's age sq.	0.004	-0.001	
	(0.016)	(0.016)	
Male	-0.107*	-0.099	
	(0.055)	(0.063)	
Mother's age	0.302***	-0.144	
	(0.070)	(0.233)	
Mother age sq.	-0.003***	-0.002	
	(0.001)	(0.002)	
Birth order	-0.225***	0.011	
	(0.037)	(0.085)	
Born 1982-1987	-0.051	1.587	
	(0.058)	(1.026)	
Constant	-5.081*	8.498	0.009
	(3.079)	(8.426)	(0.037)
R-squared	0.126		
No. observations	1212	1212	1029
No. sibl. Groups		202	
Sibl. correlation		0.918	

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling sample (cols 1-2) and subsample with details on twice lagged test (col 3).
Standard errors are in brackets. FE = fixed effects, TD = time difference.
*, **, *** statistically significant at 10%, 5%, 1% level.

Table 5
Augmented value-added model estimation results

	OLS	Family FE	Two-step
Lag(test)	0.528***	0.352***	0.279***
	(0.023)	(0.028)	(0.044)
Mother's time investment	0.003	0.000	-0.001
	(0.005)	(0.007)	(0.006)
Child's time investment	0.022***	0.014***	0.013**
	(0.004)	(0.005)	(0.005)
Lag(Mother's time investment)	0.010***	0.009*	0.010*
	(0.003)	(0.005)	(0.005)
Lag(Child's time investment)	0.005	0.005	0.005
	(0.004)	(0.005)	(0.006)
Child age	-0.631*	-0.476	-0.368
	(0.355)	(0.384)	(0.414)
Child age sq.	0.022	0.018	0.014
	(0.014)	(0.015)	(0.016)
Male	-0.020	-0.087	-0.092
	(0.046)	(0.058)	(0.262)
Mother's age	0.139**	-0.079	-0.002
	(0.058)	(0.216)	(0.002)
Mother's age sq.	-0.001*	-0.002	-0.089
	(0.001)	(0.002)	(0.065)
Birth order	-0.106***	-0.021	-0.014
	(0.031)	(0.079)	(0.088)
Born 1982-1987	-0.045	1.024	1.139
	(0.048)	(0.953)	(1.219)
Constant	1.025	8.385	8.409
	(2.573)	(7.815)	(9.471)
R-squared	0.396		
No. observations	1212	1212	1212
No. sibl. Groups		202	202
Sibl. correlation		0.860	

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS). Sibling sample. Standard errors are in brackets (bootstrapped for the two-step estimation). FE = fixed effects. *, **, *** statistically significant at 10%, 5%, 1% level.

By estimating a regression of the difference in mother's time investment between her two siblings on the sibling differences in the lagged test scores, we found that mothers' investments compensate for sibling differences in cognitive abilities. This evidence corroborates our choice of including the lagged test score in the production function, obtaining the augmented value-added specification model. Therefore, hereafter we discuss the differences across the estimation results we obtain for this model (see Table 5). We are concerned with the potential omission of family characteristics and endowment, and for this reason we compare the OLS and the family fixed effects estimations. The results seem to change when moving from the OLS to the family fixed effects estimation (compare columns 1 and 2 in Table 5) and this suggests that the OLS estimation suffers from a variable omission problem.

The next question is whether considering the lagged test and family fixed effects is enough to control for all unobserved characteristics that are associated with the explanatory variables and relevant in explaining the cognitive tests. It is certain that family fixed effects estimation fails to control for unobserved individual abilities that differ between siblings. As explained in section 4.2 we have an issue of endogeneity of the lagged cognitive test, that we can address by means of a two-step estimation. The results of this two-step estimation are reported in the last column of Table 5, where standard errors have been bootstrapped using 1,000 replications. These are our preferred results because the two-step estimation takes account of all our main econometric concerns. The main difference in the results between columns (2) and (3) in Table 5 is an attenuation of the coefficient of the lagged test, and this confirms that the family fixed effects estimation presented in column (2) is inadequate to control for unobserved individual characteristics that differ between siblings.[15] Nevertheless, we find that the coefficients of the time investments as well as the effects of all variables remain almost unaltered in size and statistical significance.

Considering our preferred estimates (see column 3 in Table 5), an increase of 10 hours per week in the mother's time investment during childhood seems to have an effect similar to an increase of 10 hours per week in the child's own time investment during adolescence: both changes lead to an increase of about 10-13% of a standard deviation of the cognitive test. The effect of decreasing child's time investments during adolescence by 10 hours per week is identical to the effect of having a mother working full-time and using child care during one year on children's cognitive tests measured in the preschool period, as found by Bernal (2008) using National Longitudinal Survey of Youth 79 (NLSY79) in the US. A similar effect is found also in Bernal and Keane (2011) when evaluating the effect of an increase of one year in full time child care using again the NLSY79, but considering exogenous changes in the work/child care decisions caused by the introduction of new welfare policy rules for single mothers in the US.

In conclusion, the main results of our empirical analysis may be summarized in the following three main points. First, the time children spend on their own during adolescence explains their test scores much more than the time the mothers spend with them during adolescence. Second, time investments during childhood by the mother are relevant to explain adolescents' test scores (even after controlling for lagged test scores), while children's own time investments during childhood are not as important as the quality time they spend with their mother. Third, the test scores are highly persistent, which implies

[15] We check formally whether the lagged test is independent of the unobserved individual characteristics in the augmented value added model by testing whether there is no difference between the lagged test coefficient obtained considering the family fixed effect estimation (Table 5 column 2) and the one obtained using the individual fixed effect estimation, i.e., the first step estimation of our two step procedure (Table 5 column 3).

that if a child obtains a bad result on a test during childhood, there is a strong probability that she will get a bad result again during adolescence.

6 Testing the model's assumptions

In this section we present a set of robustness checks providing evidence on the validity of the main assumptions required for the consistency of the proposed Two-Step Augmented value-added Estimator (see column 3 in Table 5). Each robustness analysis is performed either on the sibling sample (1212 observations) or on a subsample whose size is dictated by the availability of the additional information needed.

6.1 Omission of variables

One of our maintained assumptions is that we do not neglect differences between siblings in unobserved inputs or characteristics that have a direct effect on test results and are correlated with differences in time investments by children and parents (assumption E1). To convince ourselves that this is not a main concern we considered a set of potential omitted variables that have been found to be relevant to explain investments as well as child's abilities by previous papers, which are: (i) school inputs, (ii) early childhood inputs, and (iii) children's health shocks (see Datar and Mason 2008; Currie and Almond 2011; Almond and Mazumder 2013; Yi et al. 2014).

In Table 6 we begin by considering the subsample of children for whom we can observe the class size and the main teacher's experience (number of years of total teaching experience) during primary school, and we evaluate the two step estimator of an extended augmented value-added model that includes these school inputs.[16] The magnitude of the time investment coefficients is very similar to that observed in our main estimation (compare the first column of Table 6 and the last column of Table 5), despite the increased standard errors caused by the smaller sample size. In the second column of Table 6 we test for omission of early childhood inputs, exploiting information about whether the child was breast-fed and whether the mother was working in the year after childbirth. Again, we do not observe any change in our coefficients of interest. Finally, we check for a potential bias caused by the omission of child's health shocks by including a dummy variable capturing whether the child experienced any hospitalization in the last 5 years. Column 3 in Table 6 shows that our main results on time investments are robust to inclusion of this measure of health shocks.[17] The last column of the table reports the estimation results obtained by including all potential omitted variables and again we do not observe any relevant changes in the estimated coefficients of interest. We conclude from our sensitivity analysis that our results are relatively invariant with respect to these changes in the model specification.

[16]These two school measures have been extensively used in previous papers to control for school inputs and reflect to some extent the quality and quantity of teachers (e.g. Hanushek 2006, Jepsen and Rivkin 2009, Altinok and Kingdon 2012, Mueller 2013).

[17]We also used as alternative measure of health shocks a dummy indicating more than one doctor visit in the last 12 months, which leads to the same conclusion.

Table 6
Augmented valued added model including school, early inputs and health shocks

	School inputs	Early inputs	Health shocks	All factors
Lag(test)	0.279***	0.279***	0.279***	0.279***
	(0.059)	(0.043)	(0.043)	(0.058)
Mother's time investment	-0.003	-0.001	-0.001	-0.002
	(0.014)	(0.006)	(0.006)	(0.015)
Child's time investment	0.015*	0.013**	0.013**	0.016*
	(0.009)	(0.005)	(0.005)	(0.009)
Lag(Mother's time investment)	0.007	0.010*	0.010*	0.007
	(0.009)	(0.005)	(0.005)	(0.009)
Lag(Child's time investment)	0.003	0.005	0.005	0.002
	(0.008)	(0.005)	(0.006)	(0.008)
Primary School Class size	0.004			-0.002
	(0.008)			(0.013)
Primary School Teacher Experience	-0.002			0.004
	(0.012)			(0.008)
Breast-fed		0.019		0.097
		(0.128)		(0.242)
Mother's employed during first year of life		-0.002		-0.157
		(0.081)		(0.181)
Any hospital admission in last 12 month			0.010	-0.081
			(0.129)	(0.185)
Constant	18.420	8.468	8.446	21.107
	(14.801)	(9.131)	(9.047)	(15.329)
No. observations	726	1212	1212	726

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling sample (columns 2-3) and subsample with details on school inputs (colums 1 and 4).
Two-step estimation. Bootstrapped standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.
Controls include: Child's age, Child's age sq., Male, Mother's ag e, Mother's age sq.,
Birth order, Born 1982-1987.

6.2 Equal persistence in the three test scores

Our augmented value-added model imposes the assumption of equal persistence in the three test scores (see assumption M3). To show that this assumption is not too restrictive, we compute our estimation results again by allowing each of the three lagged test scores to have a different effect on the corresponding current test score in the first step of our two-step estimation.[18] In Table 7 we report the results of the first step estimation (the individual fixed effects estimation). The coefficients corresponding to the three test scores are very similar and we do not reject the equality of the three coefficients when looking at the Wald test whose p-value is 0.51.

Table 7
Augmented value-added model allowing for different ρ across tests

	Child fixed effects
Lag(LWS)	0.283***
	(0.043)
Lag(PCS)	0.248***
	(0.044)
Lag(APS)	0.301***
	(0.043)
dummy (LWS)	-0.009
	(0.044)
dummy (PCS)	-0.028
	(0.044)
Constant	0.093***
	(0.031)
Test for equality of Lag(test score) coefficients. F(2, 803)	0.67
p-value	0.51

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS). Sibling sample. No. observations 1212. Child fixed effects estimation - First step of the two-step estimation. Standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.

We also carried out a factor analysis for the three test scores, finding that the first component explains more than 70% of the total variance and that its factor loadings are very similar for the three tests (varying between 0.813 and 0.882). This supports the representation in equations (3) and (4) and suggests that the three test scores measure the same latent cognitive ability (see assumptions M1 and M2).

6.3 Measurement errors in time investments

Our analysis builds on the assumption that the time investments we observe represent a reasonable proxy of the long-run time investments. It is acknowledged that the short reference period and the collection methodology (exhaustive recording of all activities performed) make time diary data much more accurate than retrospective survey questions that are affected by recall bias, but this comes at the cost of other measurement errors.

[18] See Lavy et al. (2013) for a similar approach in relaxing the equality of the effect of the lagged test scores.

These arise from different sources such as the day-to-day variation in time-use patterns of individuals, or from the possible low frequency of the analyzed activity, with high proportion of false zeros typically observed for infrequent activities (see Frazis and Stewart, 2012 and Foster and Kalenkoski, 2013). Aggregation over different activities and/or multiple days is a way to mitigate measurement error problems (see Stinebrickner and Stinebrickner, 2004). We resort to both types of aggregation in our analysis. Firstly, we adopt a broad definition of time investments, which includes a whole set of formative activities. Secondly, we are able to define an aggregate measure of weekly time investment, thanks to the availability of time diary information for two days, one weekday and one weekend-day, for each child in the CDS.[19] The first and second columns of Table 8 display the results of separate regressions where we consider time investments during weekdays and during weekend-days. It can be observed that adopting this different definition of time investments based on a single day period makes the time investment effects very imprecisely estimated (compare - for example - the standard errors of the child's time coefficient in the first column of Table 8 and in the last column of Table 5). In the presence of measurement errors, it could also be argued that our result of a diminished importance of parental time and an increased importance of child's own time during adolescence might be caused by the variability over time of the measurement errors. In particular, our pattern of estimates could be explained by time investments being more variable for parents of older children (than for parents of younger children) and for younger children (than for older children). However, we were not able to find any evidence in the literature on variability of measurement errors of time investments - as defined in this paper - over child life periods, and therefore have no reason to expect this to be a pattern leading to our main findings.

[19] This is a considerable advantage offered by the CDS design survey, since most time use data are single-diary or include two consecutive days, not necessarily a week-day and a weekend-day (Frazis and Stewart, 2010).

Table 8.
Augmented value-added model considering daily investments and typical days

	Daily investment		Weekly investment
	weekday	Weekend-day	typical days
Lag(test)	0.279***	0.279***	0.284***
	(0.043)	(0.043)	(0.057)
Mother's time investment	0.019	-0.023	-0.001
	(0.045)	(0.025)	(0.011)
Child's time investment	0.050	0.063***	0.019**
	(0.035)	(0.022)	(0.009)
Lag(Mother's time investment)	0.047	0.022	0.009
	(0.031)	(0.023)	(0.008)
Lag(Child's time investment)	0.019	0.022	0.001
	(0.036)	(0.027)	(0.008)
Constant	9.110	6.934	11.623
	(9.139)	(8.930)	(11.440)
No. observations	1212	1212	798

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling sample (columns 1-2) and subsample for typical days (col 3). Two-step estimation.
Bootstrapped standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.
Controls include: Child's age, Child's age sq., Male, Mother's age, Mother's age sq.,
Birth order, Born 1982-1987.

In order to test for possible measurement error bias caused by day-to-day variation we also performed the following sensitivity analysis (the full set of results is available upon request). We purged the daily time investments from the potential effect of the type of the day (and of the year) by evaluating the residuals of the regression of daily time investments on dummy variables for the different days of the week and for the different years.[20] We then estimated two separate augmented value-added models (one for weekday, one for weekend-day) using these daily investments net of the effect of the type of day and year and compared the coefficients with the corresponding coefficients of the daily time investments in columns 1 and 2 of Table 8. Since we found very similar figures, we argue that the day-to-day variation does not represent an important source of bias. This evidence is corroborated by the regression results we obtain using the sub-sample of children filling in the diary on typical or very typical days (about 66% of our sibling sample), for which the observed time investments should be much less affected by day-to-day variation (column 3 of Table 8). The coefficients of the weekly time investments are of similar size with respect to our benchmark model, but are less precisely estimated because of the decreased sample size. The slightly higher value observed for the effect of the child

[20] We run separate regressions for week-days and weekend-days.

investment seems to suggest the presence of a classical measurement error causing some attenuation bias in our main sample.

6.4 Measurement errors in test scores

So far we have assumed that each of the three test scores is an accurate measure of the corresponding skill (Letter-Word Identification, Passage-Comprehension and Applied-Problems). In this section we allow for the presence of measurement errors in the test scores and let the observed measure of skill k in t follow the model:

$$Y_{kijt} = Y_{ijt} + \epsilon_{kijt} + v_{kijt}, \tag{18}$$

where $t = 6, ..., 15$, $(Y_{ijt} + \epsilon_{kijt})$ is the true measure of ability k in t and v_{kijt} is a classical measurement error identically and independently distributed across skills, individuals, households and time, with mean zero and variance σ_v^2, uncorrelated with ϵ_{kijs}, Y_{ijs}, the inputs in the production function during childhood and adolescence and the unobserved endowment. When we regress Y_{kijt} on Y_{kijt-5} in our first step estimation, the classical measurement error in Y_{kijt-5} can lead to an attenuation bias of the persistence, ρ. To correct for this attenuation bias we adopt an analytic correction formula (see for details Appendix B), i.e., we multiply the ρ coefficient by a correction factor given by $Var(\epsilon_{kijt-5} + v_{kijt-5})/Var(\epsilon_{kijt-5})$. The estimation results by assuming that the correction factor be equal to 1.613 and 2.327 (see Appendix B for a justification of these two choices) are reported in columns 2 and 3 in Table 9. These results seem to suggest that measurement errors in the test scores do not cause any bias in the effect of the mother's and child's time investments. In Table B1 of Appendix B we also show that our main conclusions hold for a wide range of ρ from 0.1 to 0.9.

Table 9
Augmented value-added model with correction for errors in test scores

	Correction factor		
	1 (Benchmark)	1.613	2.327
Lag(test)	0.279***	0.450***	0.648***
	(0.044)	(0.071)	(0.103)
Mother's time investment	-0.001	0.002	0.006
	(0.006)	(0.006)	(0.007)
Child's time investment	0.013**	0.015**	0.018**
	(0.005)	(0.005)	(0.005)
Lag(Mother's time investment)	0.010*	0.009*	0.009*
	(0.005)	(0.005)	(0.005)
Lag(Child's time investment)	0.005	0.004	0.003
	(0.006)	(0.006)	(0.006)
Constant	8.409	8.354	8.290
	(9.471)	(9.536)	(9.983)

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS). Sibling Sample.
No. observations 1212.
Corrected two-step estimation using the correction factor $Var(\epsilon_{kijt-5} + \upsilon_{kijt-5})/Var(\epsilon_{kijt-5})$
Bootstrapped standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.
Controls include: Child's age, Child's age sq., Male, Mother's age, Mother's age sq.,
Birth order, Born 1982-1987.

7 Sensitivity analysis

In this section we present our sensitivity analysis, which allows us to check the robustness of our empirical results to (i) alternative definitions of mother and child time investments, (ii) the inclusion of father time investments, (iii) the extension of the sample to non-intact families, and (iv) the adoption of specifications which allow for a non-linear effect of the time investments on the child's cognitive skill.

7.1 Alternative definitions of time investments

We begin by considering the robustness of our results to finer definitions of time investments, with the idea of capturing the measure of investment which is more relevant for child development. For the mother's investment we consider a new definition that excludes the time the mother spends with her child playing and having meals, to take account of the fact that these two activities might be less relevant for the child's development, especially during adolescence. The results are reported in the first column of Table 10 and show that the effect of the time a mother spends with her child during adolescence is still not statistically significant, while the effect of the time a mother spends with her child during childhood, which was statistically significant in our benchmark estimation, becomes insignificant. This might indicate that playing and having meals are important activities during childhood that have a long term effect even during adolescence, but may also be in part the consequence of a finer definition of mother's investments, leading to larger measurement errors and less precise estimates.

We also run two new regressions where the child's investment is specified, including a) two separate variables: the time a child spends doing homework or reading and the time she spends doing other formative activities, b) only the child's homework and reading time. In theory, we would expect a larger effect on cognitive abilities of the time a child spends doing homework and reading; but, because we are using time dairies, the use of a narrower definition of time investment can come at the cost of larger measurement errors. The results in column 2 and 3 of Table 10 seem to confirm this and to support the adoption of a broader concept of investment, as used in our benchmark estimation.

7.2 Investments by fathers

We consider here two new model specifications which include the father's time investments. In the first column of Table 11 we report, for comparison, the estimates obtained by considering the mother's time investments (which were already reported in the last column of Table 5), while in the second column we show the estimates obtained by replacing the mother's time investments with the father's. Finally, in the last column of Table 11, we report the results computed by using both the mother's and the father's time investments. The effect of the child's time investments remains the same across specifications. Similarly, the coefficients of the lagged test and the lagged mother's time are almost unaffected. We find that the effect of the father's time investments is not significantly different from zero. The difference between the effect of the mother's and father's time investment during childhood might be explained in part by the fact that the father's time investment during childhood is on average much lower than the mother's (about 40% lower). We also checked whether the effect of the father's investment might depend on gender and be more relevant for boys than for girls (see Bertrand and Pan 2013), but again we find a non-significant effect (results available upon request).

7.3 Non-intact families

In our main analysis, we have focused on families where the children live with their biological parents. In many countries, the proportion of children growing up with both biological parents has declined dramatically over time. Using an extended sample, which includes children living in households where the biological father is absent (16.5% of the sample), leads to results that are similar to those obtained considering just families with both biological parents (results available upon request).

Table 10
Augmented value-added model with alternative definitions of investments

	Mother's time investment with no meals and playing	Child's time investment with separate homework and reading	Child's time investment including only homework and reading
Lag(test)	0.279***	0.279***	0.279***
	(0.043)	(0.043)	(0.043)
Mother's time investment	-0.009	-0.001	-0.003
	(0.011)	(0.006)	(0.006)
Child's time investment	0.012**	0.014**	
	(0.005)	(0.006)	
Child's homework and reading		0.012	0.010
		(0.008)	(0.008)
Lag(Mother's time investment)	0.014	0.010*	0.010*
	(0.009)	(0.005)	(0.005)
Lag(Child's time investment)	0.005	0.003	
	(0.006)	(0.006)	
Lag(Child's homework and reading)		0.014	0.016
		(0.015)	(0.015)
Constant	7.803	8.534	7.586
	(9.017)	(9.017)	(8.933)

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling sample. No. observations 1212. Two-step estimation.
Bootstrapped standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.
Controls include: Child's age, Child's age sq., Male, Mother's age, Mother's age sq., Birth order, Born 1982-1987.

Table 11

Augmented value-added model with child's, mother's and father's investments

	Mother	Father	Both parents
Lag(test)	0.279***	0.279***	0.279***
	(0.044)	(0.044)	(0.045)
Mother's time investment	-0.001		-0.004
	(0.006)		(0.008)
Father's time investment		0.002	0.003
		(0.007)	(0.009)
Child's time investment	0.013**	0.014***	0.013**
	(0.005)	(0.005)	(0.005)
Lag(Mother's time investment)	0.010*		0.012**
	(0.005)		(0.006)
Lag(Father's time investment)		0.000	-0.006
		(0.006)	(0.007)
Lag(Child's time investment)	0.005	0.003	0.005
	(0.006)	(0.005)	(0.006)
Constant	8.409	9.723	7.331
	(9.471)	(8.449)	(8.731)

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling Sample. No. observations 1212. Two-step estimation.
Bootstrapped standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.
Controls include: Child's age, Child's age sq., Male, Mother's age, Mother's age sq.,
Birth order, Born 1982-1987.

7.4 Non-linearities in time investments

Finally, in Table A4 in the Appendix, we introduce some non-linearities in the effect of the mother's and child's time investments. We estimate three different specifications: (i) a model where the coefficient of each type of time investment is allowed to differ for levels of investment below and above the corresponding median (switching coefficients); (ii) a model with an additional dummy variable for each time investment, which takes value one when the time investment is zero and zero otherwise; (iii) a model where all time investments are expressed in logarithms (see respectively first, second and third column in Table A4). The first specification allows the effect of each time investment to be different for values that are below and over the median. The results suggest that each of the time investments has a coefficient that does not vary significantly below and over the median, so that our linear specification is not rejected. The second model allows for a discontinuity at zero, so that when a time investment is zero its effect is not imposed to be null. The dummy variables indicating zero time investments have coefficients which are not significantly different from zero, suggesting again that our linear specification is not rejected. Lastly, the third model allows for a further form of non-linearity of the partial effects, by resorting to the log transformation of the various time investment variables. In this specification the estimated coefficients are interpretable as semi-elasticities, and this explains the observed change in magnitude, which is, however, coherent with our benchmark model results.

8 Conclusions

While a large literature has focused on the effects of parental time on child's outcomes, there are no studies that evaluate and compare the time investments made by parents and children. In our paper, we model the cognitive production function for adolescents using an augmented value-added specification and considering parental time investments alongside child time self investments. We account for different sources of endogeneity that typically undermine the identification of the inputs' coefficients. First, we are able to control for the endogeneity of parents' and children's time investments arising from unobserved household-specific inputs by way of family fixed effects estimation. Second, we take account of the endogeneity of the lagged test, which is caused by its dependence on the unobserved child-specific characteristics, by applying an individual fixed effects estimation which makes use of the multiplicity of cognitive tests available in our data.

We show that the time investments made by children during adolescence affect their test scores much more than the time investments made by their mothers. Our results suggest that one way to improve the cognitive abilities of adolescents is to influence their time allocation decisions and their investments in formative activities. The fact that adolescents become important actors in their development process has important policy implications, suggesting that educational policies should target adolescents directly rather than their parents. Recent educational policies such as conditional cash transfers are in line with our findings, since they target not only parental time investments, but also children's time investments in themselves.

References

Aber, Lawrence, and Laura B. Rawlings. 2011. North-South knowledge sharing on incentive-based conditional cash transfer programs. SP Discussion Paper no. 1101, The World Bank, Washington, DC.

Aksoy, Tevfik, and Charles R. Link. 2000. A panel analysis of student mathematics achievement in the US in the 1990s: does increasing the amount of time in learning activities affect math achievement? *Economics of Education Review* 19, no. 3:261-277.

Almond, Douglas, and Bhashkar Mazumder. 2013. Fetal origins and parental responses. *Annual Review of Economics*, 5, no. 1:37-56.

Altinok, Nadir, and Geeta Kingdon. 2012. New evidence on class size effects: A pupil fixed effects approach. *Oxford Bulletin of Economics and Statistics*, 74, no. 20: 203-234.

Andrabi, Tahir, Jishnu Das, Asim Ijaz Khwaja, and Tristan Zajonc. 2011. Do value-added estimates add value? Accounting for learning dynamics. *American Economic Journal: Applied Economics*, 3, no. 3:29-54.

Angrist, Joshua, and Victor Lavy. 2009. The effects of high stakes high school achievement awards: evidence from a randomized trial. *American Economic Review*, 99, no. 4:1384-1414.

Ashenfelter, Orley, and Cecilia Rouse. 1998. Income, schooling and ability: evidence from a new sample of identical twins. *The Quarterly Journal of Economics*, 113, no. 1:253-284.

Behrman, Jere R., Pollak, Robert A., and Paul Taubman. 1982. Parental preferences and provision for progeny. *Journal of Political Economy*, 90, no. 1:52-73.

Bernal, Raquel. 2008. The effect of maternal employment and child care on children's cognitive development. *International Economic Review*, 49, no. 4:1173-1209.

Bernal, Raquel, and Michael P. Keane. 2011. Child care choices and children's cognitive achievement: the case of single mothers. *Journal of Labor Economics*, 29, no. 3:459-512.

Bertrand, Marianne, and Jessica Pan. 2013. The trouble with boys: social influences and the gender gap in disruptive behavior. *American Economic Journal: Applied Economics*, 5, no. 1:32–64.

Cardoso, Ana R., Fontaihna, Elsa, and Chiara Monfardini. 2008. Children's and parents' time use: empirical evidence on investment in human capital in France, Germany and Italy. *Review of Economics of the Household*, 8, no. 4:479-504.

Carneiro, Pedro, and James J.Heckman. 2003. Human capital policy. In *Inequality in America: What Role for Human Capital Policy?* ed. J. Heckman and A. Krueger. MIT Press.

Carneiro, Pedro, and Margarida Rodrigues. 2009. Evaluating the effect of maternal time on child development using the generalized propensity score. Unpublished manuscript, Institute for the Study of Labor, 12th IZA European Summer School in Labor Economics.

Cooper, Harris, Robinson Jorgianne C., and Erika A. Patall. 2006. Does homework improve academic achievement? A synthesis of research, 1987–2003. *Review of Educational Research*, 76, no. 1:1–62.

Cunha, Flavio, Heckman, James J., and Lance Lochner. 2006. Interpreting the evidence on life cycle skill formation. In *Handbook of the Economics of Education*, Vol. 1, ed. Eric A. Hanushek, Stephen Machin and Ludger Woessmann. Amsterdam: Elsevier.

Cunha, Flavio, and James J. Heckman. 2007. The technology of skill formation. *American Economic Review: Papers and Proceedings*, 97, no. 2:31-47.

Cuhna, Flavio, and James J. Heckman. 2008. Formulating, identifying and estimating the technology of cognitive and noncognitive skill formation. *Journal of Human Resources*, 43, no. 4:738-782.

Cunha, Flavio, Heckman, James J., and Susanne M. Schennach. 2010. Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, 78, no. 3:883-931.

Currie, Janet, and Douglas Almond. 2011. Human capital development before age five. In *Handbook of Labor Economics*, Vol. 4B, ed. D. Card and O. Ashenfelter. Amsterdam: Elsevier.

Datar, Ashlesha, and Bryce Mason. 2008. Do reductions in class size crowd out parental investment in education? *Economics of Education Review*, 27, no. 6:712-723.

Del Boca, Daniela, Flinn, Christopher, and Matthew Wiswall. 2014. Household choices and child developement. *Review of Economic Studies*, 81:137-185.

Dauphin, Anyck, El Lahga, Abdel-Rahmen, Fortin, Bernard, and Guy Lacroix. 2011. Are children decision-makers within the household? *Economic Journal*, 121:871-903.

Dee, Thomas S. 2005. A teacher like me: does race, ethnicity, or gender matter? *American Economic Review: Papers and Proceedings*, 95, no. 2:158-165.

Dee, Thomas S. 2007. Teachers and the gender gaps in student achievement. *Journal of Human Resources*, 42, no. 3:528-554.

Dolton, Peter, Marcenaro, Oscar D., and Lucia Navarro. 2003. The effective use of student time: a stochastic frontier production function case study. *Economics of Education Review* 22, no. 6:547-560.

Ermisch, John, and Marco Francesconi. 2005. Parental employment and children's welfare. In *Women at work: an Economic Perspective*, ed. Tito Boeri, Christopher Pissarides, Daniela Del Boca. New York: Oxford University Press.

Eren, Ozkan, and Daniel J. Henderson. 2011. Are we wasting our children's time by giving them more homework? *Economics of Education Review*, 30:950-961.

Farrow, Steve, Tymms, Peter, and Brian Henderson. 1999. Homework and attainment in primary school. *British Educational Research Journal*, 25, no. 3:323-341.

Felfe, Christina, Lechner, Michael, and Andreas Steinmayr. 2011. Sports and child development. CESifo Working Paper Series no. 3629.

Fiorini, Mario, and Michael P. Keane. 2014. How the allocation of children's time affects cognitive and noncognitive development. *Journal of Labor Economics*, 32, no. 4:787-836.

Foster, Gigi, and Charlene M. Kalenkoski. 2013. Tobit or OLS? An empirical evaluation under different diary window lenghth. *Applied Economics*, 45, no. 20:2994-3010.

Frazis, Harley, and Jay Stewart. 2012. How to think about time-use data: What inferences can we make about long- and short-run time use from time diaries? *Annales d'Economie et Statistique*, 105/106: 231-246.

Guryan, Jonathan, Hurst, Erik, and Melissa Kearney. 2008. Parental education and parental time with children. *Journal of Economic Perspectives*, 22, no. 3:23-46.

Hanushek, Eric A. 2006. School resources. In *Handbook of the Economics of Education*, Vol. 1, ed. E. A. Hanushek and F. Welch. Amsterdam: Elsevier.

Haveman, Robert, and Barbara Wolfe. 1995. The determinants of children's attainments: A review of methods and findings. *Journal of Economic Literature*, 33:1829-1878.

Hsin, Amy. 2007. Mothers' time with children and the social reproduction of cognitive skills. Working paper, California Center for Population Research Working Paper Series.

Hsin, Amy. 2009. Parent's time with children: does time matter for children cognitive achievement? *Social Indicators Research*, 93:123-126.

Jepsen, Chistopher, and Steven Rivkin. 2009. Class size reduction and student achievement: the potential tradeoff between teacher quality and class size. *Journal of Human Resources*, 44, no. 1: 223-250.

Lavy, Victor, Silva, Olmo, and Felix Weinhardt. 2012 The good, the bad, and the average: evidence on ability peer effects in schools. *Journal of Labor Economics*, 30, no. 2:367-414.

Kalenkoski, Charlene M., and Sabrina Wulff Pabilonia. 2014. Does high school homework increase academic achievement? IZA Discussion Paper no. 8142.

Kooreman, Peter. 2007. Time, money, peers, and parents; some data and theories on teenage behavior. *Journal of Population Economics*, 20, no. 1:9-33.

Liu, Haiyong, Mroz, Thomas, and Wilbert Van der Klaauw. 2010. Maternal employment, migration, and child development. *Journal of Econometrics*, 156, no. 1:212-228.

Lundberg, Shelly, Romich, Jennifer, and Kwock Tsang. 2009. Decision-making by children. *Review of Economics of the Household*, 7, no. 1:1-30.

Mancini, Anna L., Monfardini, Chiara, and Silvia Pasqua. Forthcoming. Is a good example the best sermon? Children's imitation of parental reading. *Review of Economics of the Household.* DOI 10.1007/s11150-015-9287-8.

Morris, Pamela, Lawrence Aber J., Wolf, Sharon, and Juliette Berg. 2012. Using incentives to change how teenagers spend their time. The effects of New York City's conditional cash transfer program. MDRC Report, September 2012.

Mueller, Steffen. 2013. Teacher experience and the class size effect. Experimental evidence. *Journal of Public Economics*, 98:44-52.

Nicoletti, Cheti, and Birgitta Rabe. 2012. The effect of school resources on test scores in England. ISER Working Paper 2012-13, Colchester, University of Essex.

Price, Joseph. 2008. Parent-child quality time: does birth order matter? *Journal of Human Resources*, 43, no. 1:240–265.

Pudney, Steven E. 1982. Estimating latent variable systems when specification is uncertain: generalized component analysis and the eliminant method. *Journal of the American Statistical Association*, 77, no. 380:883-89.

Stinebrickner, Ralph, and Todd R. Stinebrickner. 2004. Time use and college outcomes. *Journal of Econometrics*, 121:243-269.

Stinebrickner, Ralph, and Todd R. Stinebrickner, 2008. The causal effect of studying on academic performance. *The B.E. Journal of Economic Analysis & Policy*, 8, no. 1:1-55.

Todd, Petra, and Ken Wolpin. 2003. On the specification and estimation of the production function for cognitive achievement. *Economic Journal*, 113:F3-F33.

Todd, Petra, and Ken Wolpin. 2007. The production of cognitive achievement in children: home, school and racial test score gaps. *Journal of Human Capital*, 1:91-136.

Yi, Junjian, Heckman, James J., Zhang, Junsen, and Gabriella Conti. 2014. Forthcoming. Early health shocks, intrahousehold resource allocation, and child outcomes. *Economic Journal*.

Appendix A: Additional Tables

Table A1.
Summary statistics. Main Sample.

Variable	Mean	Std. Dev.
Tests		
LWS	105.842	16.792
PCS	104.055	14.956
APS	107.135	15.149
Lag(LWS)	109.649	16.530
Lag(PCS)	110.299	14.261
Lag(APS)	110.745	16.940
Time investments		
Mother's time investment	5.463	5.197
Lag(Mother's time investment)	9.472	7.082
Father's time investment	4.078	5.045
Lag(Father's time investment)	5.996	5.943
Child's time investment	5.123	6.859
Lag(Child's time investment)	4.076	5.149
Control variables		
Age	13.025	1.410
Mother's age	41.397	5.276
Male	0.479	0.500
Birth order	1.886	0.847
Born 1982-1987	0.528	0.500

NOTE.- No obsevations: 726.

LWS, PCS, APS= Letter-Word Identification, Passage Comprehension, Applied Problem Scores.

Table A2
Summary statistics for the Sibling sample and differences with the Main sample

Variable	Sibling sample Mean	Sibling sample Std. Dev.	Mean differences between Sibling and Main samples
Tests			
LWS	107.606	16.266	1.765*
PCS	105.255	14.686	1.200
APS	108.973	14.914	1.838**
Lag(LWS)	110.906	16.966	1.257
Lag(PCS)	111.196	14.318	0.897
Lag(APS)	112.347	16.806	1.601
Time investments			
Mother's time investment	5.253	4.918	-0.210
Lag(Mother's time investment)	9.711	6.951	0.239
Father's time investment	4.096	4.812	0.017
Lag(Father's time investment)	6.067	5.875	0.069
Child's time investment	5.148	6.458	0.025
Lag(Child's time investment)	4.201	5.265	0.125
Control variables			
Age	12.998	1.403	-0.270
Mother's age	41.354	4.912	-0.043
Male	0.475	0.500	-0.004
Birth order	1.839	0.785	-0.047
Born 1982-1987	0.525	0.500	-0.003

NOTE.- No. observations 404.

LWS, PCS, APS= Letter-Word Identification, Passage Comprehension, Applied Problem Scores.

*, **, *** statistically significant at 10%, 5%, 1% level (two-sided t-test for H_0: Difference of means=0).

Table A3.
Father's time investment - Main sample.

	Child's age range 6-10				Child's age range 11-15			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Father time investment								
Total time investment	6.00	5.94	0.00	45.92	4.08	5.04	0.00	36.25
Reading	0.15	0.58	0.00	6.67	0.06	0.58	0.00	12.33
Homework	0.05	0.46	0.00	7.50	0.09	0.78	0.00	11.17
Playing	0.99	2.49	0.00	23.33	0.35	1.65	0.00	25.67
Talking	0.23	0.83	0.00	7.73	0.33	1.10	0.00	13.25
Arts and kraft	0.13	1.38	0.00	33.75	0.05	0.60	0.00	11.00
Sport	0.44	1.60	0.00	15.00	0.17	1.13	0.00	16.5
Attending performances	0.04	0.48	0.00	7.50	0.08	0.73	0.00	13.33
Attending museums	0.02	0.39	0.00	9.50	0.00	0.00	0.00	0.00
Religious activity	0.60	1.84	0.00	15.27	0.55	1.92	0.00	20.00
Meals	3.04	2.8	0.00	20.50	2.34	2.74	0.00	21.75
Personal care	0.31	1.16	0.00	15.25	0.07	0.47	0.00	6.00

NOTE.- Weekly time investment in hours. No observations: 726.

Table A4
Augmented value-added model with non linearities in time investments

	Switching coeff for time < median	Including dummy for 0 time	Time in logs
Lag(test)	0.279***	0.279***	0.279***
	(0.043)	(0.045)	(0.043)
Mother's time investment	0.000	-0.000	0.005
	(0.007)	(0.007)	(0.041)
Mother's time if below the median	0.021		
	(0.033)		
Dummy for zero mother's time		0.025	
		(0.102)	
Child's time investment	0.014**	0.011*	0.075**
	(0.005)	(0.006)	(0.033)
Child's time if below the median	-0.009		
	(0.071)		
Dummy for zero child's time		-0.051	
		(0.085)	
Lag(Mother's time investment)	0.010*	0.010*	0.084
	(0.006)	(0.005)	(0.053)
Lag(Mother's time) if below the med	0.002		
	(0.014)		
Dummy for zero Lag(Mother's time)		-0.058	
		(0.190)	
Lag(Child's time investment)	0.007	0.003	0.024
	(0.006)	(0.006)	(0.033)
Lag(Child's time) if below the med	0.042		
	(0.050)		
Dummy for zero Lag(Child's time)		-0.044	
		(0.095)	
Constant	9.144	8.951	9.310
	(9.433)	(9.243)	(9.144)

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling sample. No observations 1212. Two-step estimation.
Bootstrapped standard errors in brackets.
*, **, *** statistically significant at 10%, 5%, 1% level.
Controls include: Child's age, Child's age sq., Male, Mother's age, Mother's age sq.,
Birth order, Born 1982-1987 dummy.

Appendix B: Measurement error correction

In Section 6.4 we reported the estimation results of the production model during adolescence in the presence of measurement error on the test scores and using an analytic formula for correcting for the consequent attenuation bias for the persistence ρ. In this section, we provide details on how to compute this correction formula.

Our first step estimation of the persistence ρ is given by the individual fixed effects estimation of the regression of the test score observed during adolescence $Y_{kijt} = Y_{ijt} + \epsilon_{kijt} + v_{kijt}$ on the lagged test score observed 5 years earlier, during childhood, $Y_{kijt-5} = Y_{ijt-5} + \epsilon_{kijt-5} + v_{kijt-5}$. When relaxing the assumption of no measurement errors, i.e., the assumption that v_{kijt} and v_{kijt-5} have degenerate distribution with zero mean and zero variance, the individual fixed effects estimator of the persistence ρ converges to:

$$\begin{aligned} plim\widehat{\rho}_{IndFE} &= \frac{Cov(\epsilon_{kijt} + v_{kijt}, \epsilon_{kijt-5} + v_{kijt-5})}{Var(\epsilon_{kijt-5} + v_{kijt-5})} = \frac{Cov(\epsilon_{kijt}, \epsilon_{kijt-5})}{Var(\epsilon_{kijt-5} + v_{kijt-5})} \\ &= \rho \frac{Var(\epsilon_{kijt-5})}{Var(\epsilon_{kijt-5} + v_{kijt-5})}. \end{aligned} \tag{19}$$

In other words, the error term in the lagged test scores, v_{kijt-5}, is an example of classical measurement error and causes an attenuation bias of the ρ coefficient estimated in the first step. Instead, the error term in the current test score, v_{kijt}, simply causes a decrease in the precision of the estimation of ρ. To correct for the attenuation bias we simply need to multiply the ρ coefficient estimated in the first step by an estimate of the following correction factor:

$$\frac{Var(\epsilon_{kijt-5} + v_{kijt-5})}{Var(\epsilon_{kijt-5})}. \tag{20}$$

We do not observe the above correction factor, but we can compute it using information on the reliability ratio $Var(Y_{ijt-5} + \varepsilon_{kijt-5})/Var(Y_{ijt-5} + \varepsilon_{kijt-5} + v_{kijt-5})$, and on the share of the variance of the observed test score in ability k explained by the latent ability Y_{ijt-5}, i.e., $Var(Y_{ijt-5})/Var(Y_{ijt-5} + \varepsilon_{kijt-5} + v_{kijt-5})$. This is because, under our maintained assumptions there is no correlation between Y_{ijt-5}, ε_{kijt-5} and v_{kijt-5}, and $Var(Y_{kijt-5}) = Var(Y_{ijt-5}) + Var(\varepsilon_{kijt-5}) + Var(v_{kijt-5}) = 1$.[21]

Previous studies on the reliability of the Woodcock-Johnson Revised tests we use in this paper suggest that the reliability ratio is always above 0.8 and often above 0.9. By implementing a factor analysis for the three observed lagged test scores, we find that 77.0% (79.2% and 65.0%) of the variance of the Letter-Word Identification (Passage-Comprehension and Applied-Problems) test score is explained by the main common factor. By considering this common factor as a measure of the latent ability Y_{ijt-5}, we can impute to $Var(Y_{ijt-5})/Var(Y_{ijt-5} + \varepsilon_{kijt-5} + v_{kijt-5})$ a value of 0.737, which is the average of the share of variance explained by the common factor across the three observed test scores.

By imposing a reliability ratio of 0.9 and 0.85 and $Var(Y_{ijt-5})/Var(Y_{ijt-5} + \varepsilon_{kijt-5} + v_{kijt-5})$ =0.737, we can assume that $Var(\epsilon_{kijt-5} + v_{kijt-5})/Var(\epsilon_{kijt-5})$ takes values 1.613

[21] $Var(Y_{kijt-5}) = 1$ because our test scores are standardized by skill.

and 2.327 and we can apply the analytic error correction for the ρ estimation suggested by equation (19). These two corrections factors are the ones used in Table 9 where we reported the estimation results corrected for measurement error in the test scores.

In Table B1, we also report results for our second step estimation when imposing different values for the persistence that range from a 0.1 to 0.9. The aim of this exercise is to show how much the effects of mother's and child's time investments can be biased by measurement errors or by any other issue that might affect the estimation of ρ.

Table B1
Augmented value-added model with imposed values for ρ

	Imposed values for ρ				
	0.1	0.2	0.3	0.4	0.5
Mother's time investment	-0.005	-0.003	-0.001	0.001	0.003
	(0.007)	(0.007)	(0.006)	(0.006)	(0.007)
Child's time investment	0.011**	0.012**	0.013***	0.015***	0.016***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Lag(Mother's time investment)	0.010**	0.010**	0.010**	0.009*	0.009*
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Lag(Child's time investment)	0.006	0.006	0.005	0.004	0.004
	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)
Constant	8.466	8.434	8.402	8.370	8.338
	(8.131)	(7.929)	(7.824)	(7.823)	(7.924)
	Imposed values for ρ				
	0.6	0.7	0.8	0.9	
Mother's time investment	0.005	0.007	0.009	0.011	
	(0.007)	(0.007)	(0.007)	(0.008)	
Child's time investment	0.017***	0.018***	0.019***	0.021***	
	(0.005)	(0.005)	(0.005)	(0.006)	
Lag(Mother's time investment)	0.009*	0.009*	0.009	0.008	
	(0.005)	(0.005)	(0.005)	(0.006)	
Lag(Child's time investment)	0.003	0.003	0.002	0.002	
	(0.006)	(0.006)	(0.006)	(0.006)	
Constant	8.306	8.273	8.241	8.209	
	(8.124)	(8.415)	(8.790)	(9.236)	

NOTE.- Dependent variable: standardized test scores (LWS, PCS, APS).
Sibling sample. No. observations: 1212.
Two-step estimation (First step estimates results replaced with given values).
*, **, *** statistically significant at 10%, 5%, 1% level.
Standard errors in partenthesis. Controls include: See Table 9.