



Deposited via The University of Sheffield.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/id/eprint/114240/>

Version: Published Version

Article:

Benone, C.L., de Oliveira, E.S., Dolan, S.R. et al. (2017) Addendum to "Absorption of a massive scalar field by a charged black hole". *Physical Review D*, 95 (4). ISSN: 2470-0010

<https://doi.org/10.1103/PhysRevD.95.044035>

Reuse

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.

Addendum to “Absorption of a massive scalar field by a charged black hole”

 Carolina L. Benone,^{1,*} Ednilton S. de Oliveira,^{1,†} Sam R. Dolan,^{2,‡} and Luís C. B. Crispino^{1,§}
¹*Faculdade de Física, Universidade Federal do Pará, 66075-110 Belém, Pará, Brazil*
²*Consortium for Fundamental Physics, School of Mathematics and Statistics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Kingdom*

(Received 25 January 2017; published 21 February 2017)

In [1] we studied the absorption cross section of a scalar field of mass m impinging on a static black hole of mass M and charge Q . We presented numerical results using the partial-wave method, and analytical results in the high- and low-frequency limit. Our low-frequency approximation was only valid if the (dimensionless) field velocity v exceeds $v_c = 2\pi Mm$. In this addendum we give the complementary result for $v \lesssim v_c$, and we consider the possible physical relevance of this regime.

DOI: 10.1103/PhysRevD.95.044035

In Ref. [1] we analyzed the scenario of a neutral scalar field of mass m and frequency ω absorbed by a Reissner-Nordström black hole of mass M and charge Q . Our stated aim was to provide a quantitative full-spectrum description of absorption, by bringing together numerical methods and analytical approximations. However, due to a tacit assumption, we gave an incomplete description of the low-frequency regime; with this addendum we fulfil our original objective.

The total absorption cross section σ may be written as a sum of partial absorption cross sections σ_l . In the low-frequency limit $M\omega \ll 1$, the dominant contribution arises from the monopole sector, $\sigma_{lf} \approx \sigma_{l=0}$. In Ref. [1] we obtained $\sigma_{lf} = \mathcal{A}/v$ [see Eq. (63)], where $v = \sqrt{1 - m^2/\omega^2}$ is the (dimensionless) velocity of the field, $\mathcal{A} = 4\pi r_+^2$ is the area of the black hole, and r_+ is the areal coordinate of the event horizon location.

However, our original result did not encompass the limit of small velocities. After taking this into account, the completed low-frequency approximation is

$$\sigma_{lf} = \begin{cases} \sigma_{lf}^{(1)} = \mathcal{A}/v, & v \gtrsim v_c, \\ \sigma_{lf}^{(2)} = \frac{4(\pi r_+)^2(2Mm)}{v^2}, & v \lesssim v_c, \end{cases} \quad (1)$$

where $v_c = 2\pi Mm$ is the velocity of the transition.

One may obtain the approximation valid for $v \lesssim v_c$ by starting from Eq. (62) of Ref. [1], namely

$$\sigma = \frac{4\pi r_+^2 \rho^2}{v}, \quad (2)$$

where we are considering only the first term in the denominator of Eq. (62). For this purpose, we write¹

$$\rho^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \quad (3)$$

as

$$\rho^2 = -\frac{2\pi Mm(1+v^2)}{v\sqrt{1-v^2}} \frac{1}{\exp\left(-\frac{2\pi Mm(1+v^2)}{v\sqrt{1-v^2}}\right) - 1}. \quad (4)$$

Substituting Eq. (4) in Eq. (2) and considering the limit for $v \rightarrow 0$ we obtain

$$\sigma_{lf}^{(2)} = \frac{4(\pi r_+)^2(2Mm)}{v^2}. \quad (5)$$

In the uncharged case $Q = 0$ we recover Unruh’s result for the case of a Schwarzschild black hole [cf. Eq. (97) of Ref. [2]].

In Fig. 1 we compare Eq. (1) with our numerical results. We can see that the numerical results present a transition between $\sigma_{lf}^{(1)}$ and $\sigma_{lf}^{(2)}$, which happens near $v = v_c$, with $v_c \approx 0.138$ in this case.

Recently a new dark matter candidate was proposed by Hui *et al.* [3], in the form of a scalar field with mass $m \approx 10^{-22}$ eV/ c^2 and de Broglie wavelength $\lambda_B \approx 1$ kpc. Its corresponding velocity is $v \approx 4 \times 10^{-4}$, found from $v = (1 + \lambda_B^2/\lambda_C^2)^{-1/2}$, where $\lambda_C = h/mc \approx 0.4$ pc is the Compton wavelength. For a black hole mass $M_1 = 3.6 \times 10^6 M_\odot$ (e.g. Sgr. A* [4]), we find $v_c \approx 1.7 \times 10^{-5}$ and thus $v > v_c$; whereas for a supermassive black hole of mass $M_2 = 2 \times 10^8 M_\odot$ (e.g. Andromeda’s supermassive black hole has mass $(1.1 - 2.3) \times 10^8 M_\odot$ [5]) we find $v_c = 9.4 \times 10^{-4}$ and thus $v < v_c$. This suggests that both regimes of Eq. (1) are potentially relevant in the scenario of

*lben.carol@gmail.com
 †esdeoliveira@gmail.com
 ‡s.dolan@sheffield.ac.uk
 §crispino@ufpa.br

¹There are 2π factors (in front of η) missing in Eq. (57) of Ref. [1].

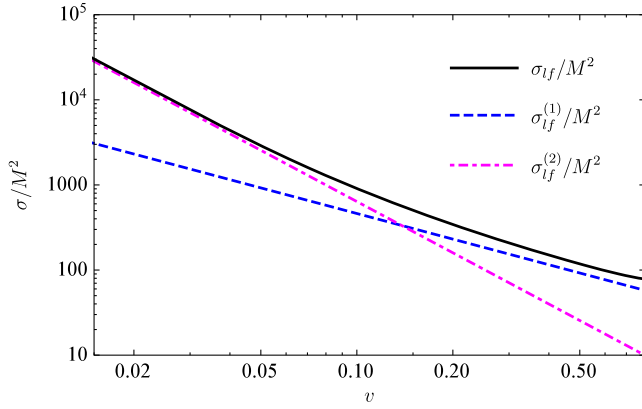


FIG. 1. Comparison between the partial absorption cross section $\sigma_{l=0}$ (solid line), and the approximate analytical results of Eq. (1) (broken lines), for the case $Q/M = 0.4$ and $Mm = 0.022$. A transition in behavior is visible near $v_c \approx 0.138$.

Hui *et al.*, and that disparate cross sections are possible. For example, $\sigma_{lf}^{(1)} \approx 3.6 \times 10^{24} \text{m}^2$ for M_1 (Sgr. A*) and $\sigma_{lf}^{(2)} \approx 2.6 \times 10^{28} \text{m}^2$ for M_2 (Andromeda).

ACKNOWLEDGMENTS

We thank E. Witten for raising a question by email that led to this addendum. We acknowledge Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Marie Curie action Grant No. NRHEP-295189-FP7-PEOPLE-2011-IRSES, Engineering and Physical Sciences Research Council Grant No. EP/M025802/1 and Science and Technology Facilities Council Grant No. ST/L000520/1 for partial financial support.

-
- [1] C. L. Benone, E. S. de Oliveira, S. R. Dolan, and L. C. B. Crispino, Absorption of a massive scalar field by a charged black hole, *Phys. Rev. D* **89**, 104053 (2014).
 - [2] W. Unruh, Absorption cross section of small black holes, *Phys. Rev. D* **14**, 3251 (1976).
 - [3] L. Hui, J. P. Ostriker, S. Tremaine, and E. Witten, On the hypothesis that cosmological dark matter is composed of ultralight bosons, [arXiv:1610.08297](https://arxiv.org/abs/1610.08297).
 - [4] R. Schödel, D. Merritt, and A. Eckart, The nuclear star cluster of the Milky Way: Proper motions and mass, *Astron. Astrophys.* **502**, 91 (2009).
 - [5] R. Bender *et al.*, HST STIS spectroscopy of the triple nucleus of M31: Two nested disks in Keplerian rotation around a supermassive black hole, *Astrophys. J.* **631**, 280 (2005).