Article:

Pitenis, AA, Dowson, D orcid.org/0000-0001-5043-5684 and Gregory Sawyer, W (2014) Leonardo da Vinci’s Friction Experiments: An Old Story Acknowledged and Repeated. Tribology Letters, 56 (3). pp. 509-515. ISSN 1023-8883

https://doi.org/10.1007/s11249-014-0428-7

Leonardo da Vinci’s Friction Experiments: An Old Story Acknowledged and Repeated

Angela A. Pitenis[1], Duncan Dowson[2], W. Gregory Sawyer[1,3]

[1]Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611, USA

[2]School of Mechanical Engineering, University of Leeds, Leeds, West Yorkshire LS2 9JT, UK

[3]Department of Materials Science and Engineering, University of Florida, Gainesville, FL 32611, USA

Abstract

Leonardo da Vinci (1452–1519) is universally regarded as a brilliant polymath, designer, astronomer, artist, philosopher, and a visionary engineer of the Renaissance era. Interestingly, due to the delayed discovery of several caches of his notebook pages (as late as the 1960s), his immense contribution to the field of tribology has only recently surfaced. From these salvaged documents, da Vinci’s three notable observations that preceded the development of the laws of friction were uncovered: (1) Friction is independent of apparent contact area, (2) the resistance of friction is directly proportional to applied load, and (3) friction has a consistent value of $\mu = 0.25$. In this work, we have attempted to construct a nearly faithful recreation of Leonardo da Vinci’s apparatus for measuring friction based on his notebook illustrations and investigate the conditions under which Leonardo da Vinci’s experiments produced his findings. Our experiments, performed roughly 500 years later, reproduced Leonardo da Vinci’s findings of friction coefficients with wood of $\mu = 0.25$, but only under conditions of roughly cut and brusquely squared samples of dry wood that were handled and sullied by hand in a fashion typical of wood working but inconsistent with the modern laboratory practice. Thus, our interpretation of Leonardo da Vinci’s findings is that these first tribological studies were actually performed on roughly cut and unpolished samples that had been handled extensively prior to and during testing; Such a procedure of sample preparation is entirely reasonable for the time period and suggests an active, dusty, and dynamic laboratory environment.

Keywords Friction; Friction coefficient; Static friction; Friction laws; Leonardo da Vinci

1 Historical Introduction

The advantages and disadvantages of friction and wear have been recognized for thousands of years, and records from early civilizations clearly indicate an awareness of their importance in everyday life. There is evidence that Stone Age man developed skis and sharp tools for hunting and that over 10,000 years ago, fire was started by rubbing together wooden sticks or the percussion of flint stones.

Remarkably, some 5,000 years ago, wheeled vehicles first emerged in the Middle East and China [1]. In Assyria roughly 4,500 years ago, wear-resistant stone socket bearings were used to support temple doors [2–4]. As early as 3500 BC, the importance of friction and wear was recognized in the manufacture and use of tools (e.g., thong and bow drills), simple machines (e.g., Potter's wheels), and sharp weapons (e.g., spears and knives) [5, 6]. During the time period of 3500 BC–30 BC, large blocks of stone, and even complete stone statues, were transported over sand and soft ground in Mesopotamia and Egypt [7–9].

The feasibility and marvel of transporting heavy objects in antiquity continue to inspire the analysis of friction for conditions of simple contacts and natural materials (e.g., wood, water, ice, and sand) [10–12]. Recently, Li, Chen, and Stone performed an analysis of the ice-assisted movement associated with the transportation of enormous stone slabs that formed the foundation of buildings in the Forbidden City in China during the time period 1407–1420 AD; the transport of these slabs, which were estimated to be more than 300 tons, occurred over a distance of at least 70 km and was completed within a single month. The authors make a compelling case, based on both the historical record and modern analysis, that the transport of these foundations was achieved through the use of water, ice, and wooden sledges. They went on to suggest that such a method of transport is both more reliable and efficient than any other methods available at the time.

In some cases, the historical record of transporting massive objects has been better preserved, such as in the much-quoted paintings found in the tombs of Ti at Saqqara (c 2400 BC) and of Tehuti-Hetep at El-Bersheh (c 1880 BC). These paintings indicate the enormous physical effort required to transport heavy stone carvings [13, 14]. Both paintings illustrate that some reduction in friction could be achieved by applying a liquid lubricant from large jars onto wooden planks over which the statues were pulled. It has been estimated that the coefficient of friction was probably about $\mu = 0.23$ [12], a figure that supports the range for wood-on-wood reported in 1950 by Bowden and Tabor for wet and dry conditions, $\mu = 0.2$ and $\mu = 0.25$–0.5, respectively [15]. These findings have significance in relation to Leonardo da Vinci's experiments [16] and those reported in the present paper.

The notebooks of Leonardo da Vinci offer a remarkable view of the first recorded qualitative application of a scientific methodology to tribological problems. Although Leonardo da Vinci is widely credited with the introduction of the friction coefficient, his experiments took place over 200 years before the concepts of force were worked out and presented by Sir Isaac Newton. In Leonardo da Vinci's notebooks he concluded that objects had a resistance of friction equal to one-quarter of the object's weight—and while this is now known to be an incorrect conclusion, it is interesting to explore the events that may have led to this finding. In the interim between da Vinci's experiments and their eventual revelation, researchers independently arrived at similar means of measuring dry friction of solids [17–22] though the focus of this work is on da Vinci's experiments.

In this study, effort was put forth to faithfully recreate a tribological apparatus and perform a series of friction experiments described within a page from Leonardo da Vinci's Codex Arundel (dating between 1480 and 1518) [12]. We have chosen to reproduce the experiments that involved the application of a force to a sample block transmitted through a pulley and resisted by friction. All sliding surfaces were prepared in a dry condition and allowed to equilibrate for over 1 year before experiments were performed. The most notable modern convenience employed in this recreation is a hidden pair of precision rolling element bearings and a non-rotating steel shaft that was discreetly concealed within the pulley assembly. A digital tracing of an illuminated photograph from the Codex Arundel and a digital sketch of the recreated apparatus both are shown in Fig. 1.

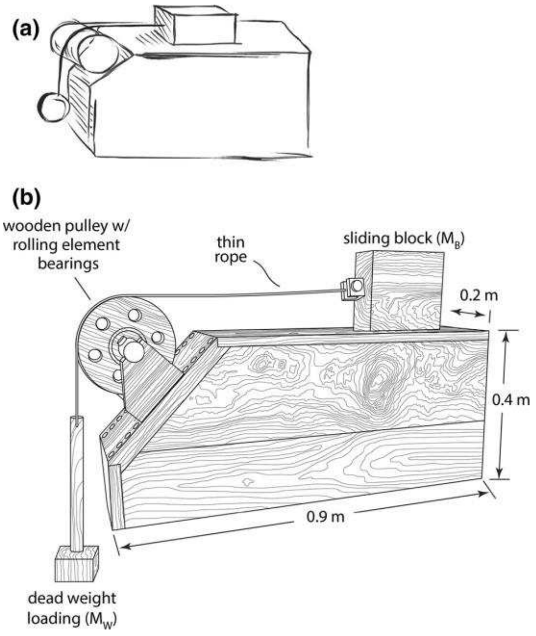


Fig. 1 **a** A digital tracing of Leonardo da Vinci's experimental schematic taken from an illuminated digital photograph by the British Library of Arundel 263 from Leonardo da Vinci's Codex Arundel (dating between 1480 and 1518), f. 40 V as displayed in bifolium with f. 41, but reproduced here without accompanying notes and diagrams [12]. **b** A detailed illustration of the wooden recreation of the experiment described in the Codex Arundel, including notes on the hardware, and the associated dimensions. The entire assembly was made from wood (Maple for all planar surfaces), with the exception of a pair of rolling element bearings and a precision steel shaft that were inserted and hidden within the wooden pulley assembly.

In the following experiments, we explore Leonardo da Vinci's notable observations that led to the development of his laws of friction. First, his finding that a friction coefficient has a consistent value of ¼ ($\mu = 0.25$); this (friction coefficient) being defined as a ratio of the hanging applied masses or weights (M_W) to the mass of the sliding block (M_B), and $\mu_s = M_W/M_B$. Second, for blocks sliding against horizontal surfaces, we explore his finding that the friction coefficient is independent of the apparent area of contact by preparing blocks with planar surfaces that differed in area by over a factor of 2. To add a modern touch, further experiments were performed on patterned surfaces as well as surfaces with the intentional addition of third-body debris in the form of fine Olive wood dust. Finally, we explore his finding that the force of friction is directly proportional to the applied load by doubling the weight of the sliding block (M_B).

2 Experimental Methods and Apparatus

2.1 Recreating da Vinci's Experimental Apparatus

The reproduced sketch of da Vinci's original experimental setup (Fig. 1a) shows the simplistic projection of a rectangular table built to accommodate a cylindrical pulley at one end. On top of this table is a rectangular block connected to a hanging mass by a thin rope over the pulley.

The modern recreation (Fig. 1b) comprised a wooden testing table, a pulley system, a thin smooth rope, a threaded wooden tackle for attachment, a hanging wooden assembly for the addition of weights, and a series of wooden sliding blocks. See Online Resource 1 for more details.

The top surface of the flat wooden table was fabricated from a solid piece of machined and sanded curly Maple (Acer saccharum) with only a very slight orientation in grain in the direction of sliding. The remainder of the apparatus was entirely constructed from wooden threaded connections, tapers, hand-pegged joints, or an occasional biscuit-joined and glued structural interface (non-tribological surface). The wooden pulley system contained a pair of precision rolling element bearings embedded into a carefully machined wooden pulley made from West African Bubinga (Guibourtia pellegriniana). These bearings were mounted onto a precision hardened steel shaft concealed within a false non-rotating wooden axle made from Olive wood (Olea europaea).

The sliding blocks were manufactured from African Padauk (Pterocarpus soyauxii), Olive wood, and West African Bubinga. Fastened to the sliding block was a threaded Olive wood tackle connected to a long, thin, smooth nylon rope. The sliding blocks and Olive wood tackle had a combined mass of 3.347 kg for the African Padauk block, 3.477 kg for the Olive wood block, and 2.075 and 4.157 kg for the Bubinga wood blocks. The other end of the nylon rope was connected to a hanging wooden assembly machined from Olive wood (0.302 kg).

Applied frictional weights were stacked onto this hanging wooden assembly and were machined from a block of African Ebony (Diospyros mespiliformis) into precise masses ranging from 5 g to over 500 g. The uncertainty in each applied frictional mass was <100 mg, and the average uncertainty for mass in these experiments was <1 g. Mass assemblies and wooden blocks were recorded and weighed after each experiment.

For these experiments, the top of the pulley system and the centerline of the rope pass through the center of mass of each of the sliding blocks to within a measured angle of better than 1°. The flat Maple surfaces were leveled using a machinist's spirit level to approximately 1° over the entire surface. Scanning white-light interferometry revealed the average surface roughness of the extensively hand-polished sliding blocks of the African Padauk, and the curly Maple deck reached values less than Ra = 200 nm, while the machined surfaces had Ra's > 1,000 nm. The roughly cut, brusquely squared, and sullied surfaces of the Bubinga had an average roughness Ra > 3,000 nm. The measured moisture content of all of the wooden testing surfaces was ~7 %, and the temperature was a steady 20 ℃ with 45 % relative humidity for the duration of the friction experiments.

2.2 Sample Preparation

2.2.1 Cleaned and Polished Samples

Friction experiments were performed using a series of wooden sliding blocks that were machined and prepared dry without the addition of any oils, varnishes, or waxes; samples were prepared and equilibrated in a climate-controlled environment for over a year.

The sliding surfaces of the first and most extensive series of experiments (the Maple deck and two sliding faces of the African Padauk block) were gently sanded with 600-grit silicon carbide abrasive paper approximately 50 times in a random orbital pattern. The surfaces were thoroughly cleaned and wiped with a new low-lint laboratory cloth. The sliding block was then placed in either a vertical (apparent area of contact ~150 cm2) or a horizontal (apparent area of contact ~420 cm2) position.

2.2.2 Third-Body Debris

A second series of ten experiments was conducted using the same previously sanded surfaces (African Padauk, apparent area of contact ~420 cm^2, and Maple table) were gently dusted with very fine, clean, and dry Olive wood sawdust prior to each experiment (particle diameter range: 60–200 μm). The sawdust was obtained by dry cutting a solid block of Olive wood without oils or greases with a new saw blade. The wood dust produced from this cutting operation was gently collected and applied to the sliding surfaces using an Olive wood spatula cut from the same wooden block of Olive wood to reduce contamination.

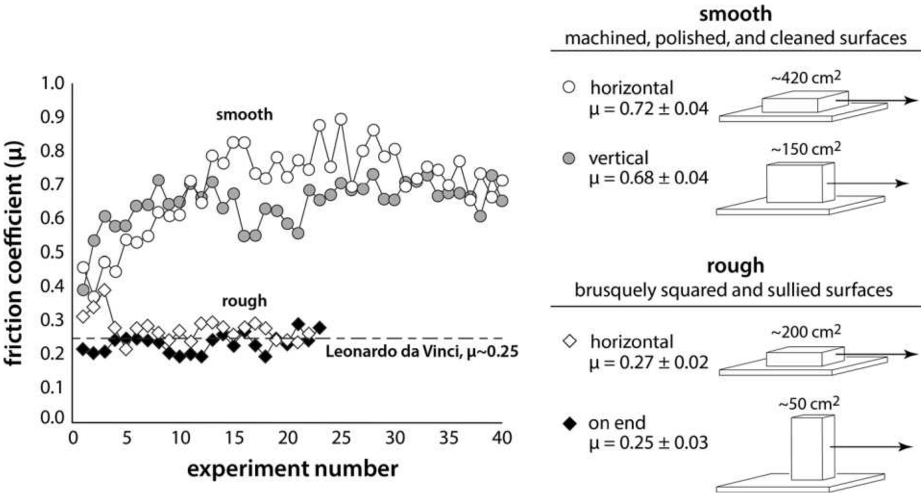


Fig. 2 Experiments performed on the block of African Padauk in both a horizontal (open circle) and vertical (gray circle) configuration with the sliding direction aligned with the grain. Additional experiments were performed in the horizontal (open diamond) and on end (black diamond) configuration using a block of roughly squared

Bubinga wood (M_B ~ 2.1 kg) with sullied surfaces in the sense that the sliding surfaces were not polished, sanded, or cleaned with anything other than a casual wipe of the hand.

2.2.3 Sullied Surfaces

A third series of experiments was performed on roughly sawn and brusquely squared Bubinga wood. The sliding surfaces of the block (apparent area of contact in the horizontal position: ~200 cm^2 and apparent area of contact in the vertical position: ~50 cm^2) and the curly Maple deck were intentionally exposed to fingerprint oils and airborne dust, resulting in a ‘‘sullied’’ environment. The same block of Bubinga wood was also used for a set of experiments in which additional weight was placed on top of the sliding block to double the mass; this enabled the use of the same sliding surface for both experiments.

2.3 Experimental Procedure

The initial positions of the wooden blocks were located using mechanical gauging with a repeatability of significantly better than 1 mm. As soon as the block’s position was defined, a digital timer was started and frictional masses were gradually added to the wooden hanger until the block experienced a slip event and began to slide across the Maple tabletop toward the pulley system. The mass at which limiting static friction was detected and the block rapidly accelerated was recorded as the total applied frictional mass, M_W, which was divided by the sliding block’s mass, M_B, to arrive at the friction coefficient.

The time of the slip event and the total mass, M_W, that resulted in slip were recorded by hand into a laboratory notebook and documented in Online Resources 2–5.

3 Results

3.1 Apparent Contact Area

In accordance with da Vinci’s findings, both sliding configurations of the African Padauk wooden block that were sanded between experiments yielded similar static friction coefficients (averaged over the last ten experiments): $\mu_s = 0.72 \pm 0.04$ (horizontal) and $\mu_s = 0.68 \pm 0.04$ (vertical), as shown in Fig. 2 and delineated in Online Resource 2. It is evident that approximately 25 experiments were required to ‘‘condition’’ the wooden blocks. 3.2 Surface Preparation and Texture Frictional dependence on surface texture is shown and described in Fig. 3. For sliding directions aligned with the grain, the average of the last ten friction coefficients of the meticulously sanded and cleaned African Padauk wooden block and Maple deck was $\mu_s = 0.72 \pm 0.04$, while the clean, not sanded ‘‘as-machined’’ face of the African Padauk sliding block resulted in $\mu_s = 0.43 \pm 0.03$. This is consistent with the initial friction coefficients shown in Fig. 2 that correspond to the tribological system still transitioning from machined to polished and cleaned surfaces.

The inclusion of Olive wood dust into the wood-on-wood sliding interface decreased the average friction coefficient ($\mu_s = 0.35 \pm 0.03$) to nearly half of the steady state friction coefficient produced by sanding and thoroughly cleaning the wooden block prior to each experiment. See Online Resource 3 for additional information.

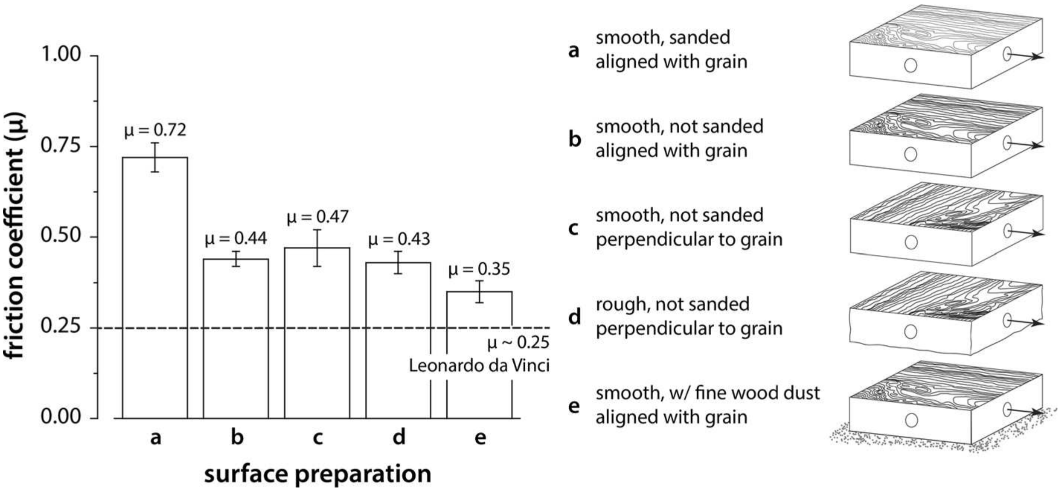


Fig. 3 Experiments performed on African Padauk in a horizontal configuration but varying grain orientation and the degree of wood particulate dust on the surfaces. All surfaces were unsullied and machined surfaces.

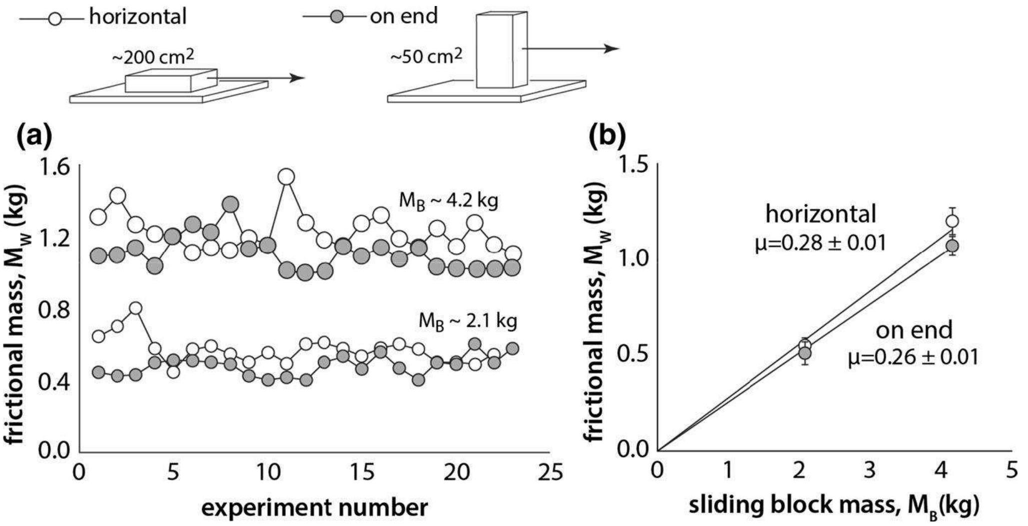


Fig. 4 Additional experiments using a block of brusquely squared Bubinga wood whose surfaces were sullied in the sense that they were not polished, sanded, or cleaned with anything other than a casual wipe of the hand were tested in the horizontal (open circle) and on end (gray circle) configurations. **a** Larger circles correspond to experiments conducted with the Bubinga block's weight doubled, M_B ~ 4.2 kg. **b** Friction coefficient is the slope of the frictional mass, M_W, over the sliding block mass, M_B, through zero. Circles represent the average frictional mass for the last ten experiments for the horizontal and on end configurations. Error bars represent the standard deviation.

The sullied and brusquely squared Bubinga block (M_B ~ 2.1 kg) yielded an average friction coefficient of $\mu_s = 0.25 \pm 0.03$ when slid on end, as shown in Fig. 2.

This value is within one standard deviation of the value that Leonardo da Vinci reported over 500 years ago.

The effect of grain alignment on the coefficient of friction was not very significant, as shown in Figs. 3 and 4 and discussed further in Online Resource 4.

3.3 Load

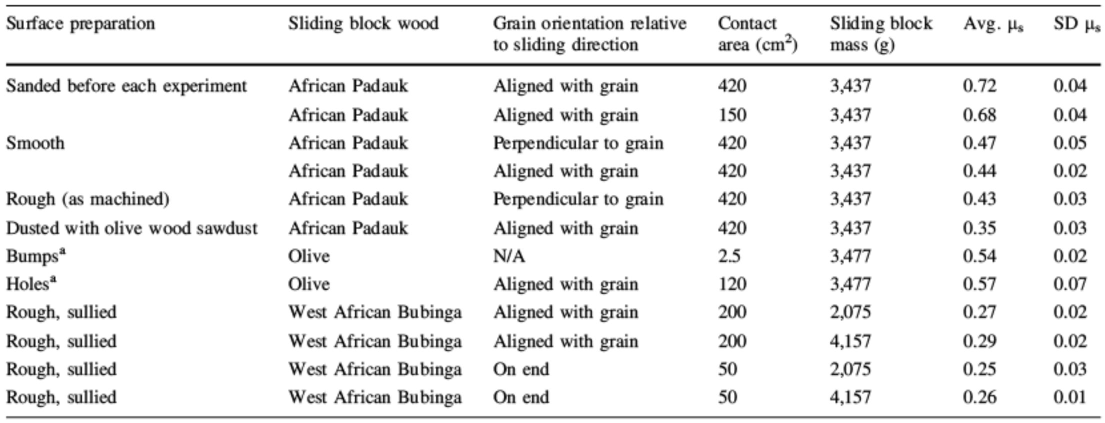
In keeping with Leonardo da Vinci's findings, when the load was doubled on the same sliding block, the frictional mass required to overcome breakloose friction also doubled as shown in Fig. 4a. The friction coefficients for both sliding configurations were calculated from the slope of these mass measurements as plotted in Fig. 4b. When slid in the horizontal configuration, the friction coefficient was $\mu_s = 0.28 \pm 0.01$, and when slid on one end, the friction coefficient was $\mu_s = 0.26 \pm 0.01$. For these experiments, the same wooden block made from Bubinga was used in both horizontal and vertical configurations (Online Resource 5), and simply stacking a nearly identical block on top of the sample doubled the mass. These findings too are consistent with Leonardo da Vinci's ratio of ¼.

4 Discussion

4.1 Leonardo da Vinci's Laws of Friction

It is charming to imagine Leonardo da Vinci postulating his thoughts on friction after seeing similar data (see Table 1), and selective passages from the translations of his writings from the Codex Atlanticus and the Codex Arundel provide a remarkably modern interpretation on the findings reported in this manuscript.

Table 1 Overview of experimental results



Surface preparation	Sliding block wood	Grain orientation relative to sliding direction	Contact area (cm^2)	Sliding block mass (g)	Avg. μ_s	SD μ_s
Sanded before each experiment	African Padauk	Aligned with grain	420	3,437	0.72	0.04
	African Padauk	Aligned with grain	150	3,437	0.68	0.04
Smooth	African Padauk	Perpendicular to grain	420	3,437	0.47	0.05
	African Padauk	Aligned with grain	420	3,437	0.44	0.02
Rough (as machined)	African Padauk	Perpendicular to grain	420	3,437	0.43	0.03
Dusted with olive wood sawdust	African Padauk	Aligned with grain	420	3,437	0.35	0.03
Bumps[a]	Olive	N/A	2.5	3,477	0.54	0.02
Holes[a]	Olive	Aligned with grain	120	3,477	0.57	0.07
Rough, sullied	West African Bubinga	Aligned with grain	200	2,075	0.27	0.02
Rough, sullied	West African Bubinga	Aligned with grain	200	4,157	0.29	0.02
Rough, sullied	West African Bubinga	On end	50	2,075	0.25	0.03
Rough, sullied	West African Bubinga	On end	50	4,157	0.26	0.01

Regarding the effects of the orientation of the blocks and the mass of the sliding blocks on friction, da Vinci wrote [16]: ‘‘The friction made by the same weight will be of equal resistance at the beginning of its movement although the contact may be of different breadths and lengths’’[1]

‘‘Friction produces double the amount of effort if the weight be doubled’’[2]

The above observations, while more restrictive, are wholly in agreement with Amontons’ two laws of friction published about two centuries later [17]:

1. The force of friction is independent of the apparent area of contact

2. The force of friction is directly proportional to the applied load

Intriguingly, Leonardo da Vinci also observed that frictional bodies with ‘‘polished and smooth’’ surfaces have lower friction [12] although the details of how his surfaces were polished or oiled is unknown. Coulomb [21] extended the work of Leonardo da Vinci and Amontons by considering a wider range of materials (including wood), the time of repose, and the role of interlocking asperities upon static friction. He found that kinetic (sliding) friction was generally lower than static friction and independent of sliding velocity. This became known as the third law of friction.

Coulomb was convinced that interlocking asperities played a major role in static friction. He found that the friction of metal-on-metal interfaces was essentially proportional to load, whereas for softer materials, such as wood, there was a marked increase in static friction with time of repose: a process that could take several days. For softer woods, such as pine-on-pine, the static friction was $\mu_s = 0.56$, while for oak-on-pine, it was $\mu_s = 0.66$. For harder woods, such as elm-on-elm and oak-on-oak, lower values of $\mu_s = 0.46$ and $\mu_s = 0.43$, respectively, were recorded [22].

---

[1] Forster Bequest ms. II, 133r and 132 v: see E. MacCurdy (1938), Vol. 1, pp. 615, 616.
[2] Forster Bequest ms. III, 72r: see ibid, Vol. 1, p. 621.

While the age of contact was recorded for many experiments in this investigation, no significant trends with friction coefficient were observed, perhaps due to much shorter time scales. Experiments to examine the breakloose dynamics were conducted and are included in Online Resource 6.

4.2 Evaluating da Vinci’s Conclusions

Leonardo da Vinci concluded that an object’s resistance of friction was equal to one-quarter of its weight, $\mu = 0.25$.

There is a clear distinction between the reported value of da Vinci’s measurements and the values in this investigation measured under dry, clean, smooth, and sanded conditions $\mu_s = 0.72 \pm 0.04$ (Figs. 2, 3; Table 1). Eliminating sanding from the experimental procedure lowers the static friction coefficient ($\mu_s = 0.44 \pm 0.04$), and introducing fine Olive wood sawdust on the sliding surfaces achieves an average static friction coefficient of $\mu_s = 0.35 \pm 0.03$.

It is only when sliding blocks are used with roughly cut surfaces sullied by the natural oils in fingertips and hands and by dust in the air that static friction coefficients approach those of da Vinci’s, $\mu_s = 0.25 \pm 0.03$ (Figs. 2, 4; Table 1). This, coupled with the modern benefit of closely examining precursor slip events (see Online Resource 6), suggests that the values obtained in da Vinci’s laboratory are consistent with an active and dusty environment, likely with little attention paid to low vibration levels or surface cleanliness. We also noted a consistent trend over a variety of surface patterns; the same empirical findings of da Vinci’s ‘‘friction laws’’ may have originated from his use of roughly cut wooden blocks and subjecting all sliding surfaces in his experiments to extensive and repeated handling. Interestingly, this experimental result from da Vinci’s work agrees quite nicely with the modern perspective that highlights the importance of adsorbed layers on the origin of static friction [23].

It is truly remarkable that Leonardo da Vinci initiated his masterly experimental studies of friction about half a millennium ago. His findings were confirmed by Amontons 200 years later and again by Coulomb in 1785. The simple laws that the limiting coefficient of static friction is independent of the apparent area of contact and directly proportional to the applied load for a wide range of materials were to form the basis of major advances in civil and mechanical engineering, and they still resonate in the minds of students and engineers alike.

5 Closing Remarks

We humbly honor Leonardo da Vinci and recognize his experimental investigations of friction some 500 years ago as one of the outstanding studies in tribology. His studies of friction provide an exemplary demonstration of the profound impact of the scientific revolution upon the Renaissance era. Although his contemporaries could not benefit from his findings and were forced to independently discover and develop the laws of friction, da Vinci’s work has remained formative to the scientific community. From this investigation, we suggest that rough preparation and surface contamination (i.e., brusquely squared and sullied surfaces) are likely the best explanations of the consistent and repeatedly low value of friction found in his experiments. In agreement with da Vinci's conclusions, we observed area-independent friction and increasing frictional mass in proportion to normal load in all of our experiments, regardless of sample preparation and surface conditions.

Acknowledgments

Special thanks to Michael Braddock and Luther Lloyd for their assistance in crafting the wooden apparatus, and the former and current members of the University of Florida Tribology Laboratory for their support, encouragement, and thoughtful discussions.

References

1. Kranzberg, M., Pursell, C.W.: Technology in Western Civilization, vol. 2. Technology in the Twentieth Century (1967)
2. Derry, T.K.: A Short History of Technology From the Earliest Times to AD 1900, vol. 231. Courier Dover Publications, Mineola (1960)
3. Brown, K.S., Marean, C.W., Herries, A.I., Jacobs, Z., Tribolo, C., Braun, D., Roberts, D.L., Meyer, M.C., Bernatchez, J.: Fire as an engineering tool of early modern humans. Science **325**(5942), 859–862 (2009)
4. Childe, V.G.: Archaeological ages as technological stages. J. Anthropol. Inst. G. B. Irel. **74**, 7–24 (1944)
5. Harrison, H.: Fire-Making, Fuel and Lighting. A History of Technology, pp. 216–237. Oxford, Clarendon (1954)
6. Newberry, P.E.: The Life of Rekhmara: Vezîr of Upper Egypt Under Thothmes III and Amenhetep II (circa B.C. 1471–1448). A. Constable and Co., Westminster (1900)
7. Layard, A.H.: Nineveh and its Remains, vol. 2. Murray, London (1849)
8. Layard, A.H.: Discoveries in the Ruins of Nineveh and Babylon: With Travels in Armenia, Kurdistan and the Desert. John Murray, London (1853)
9. Woolley, C.L.: Digging Up The Past. Ernest Benn Limited, London (1930)
10. Li, J., Chen, H., Stone, H.A.: Ice lubrication for moving heavy stones to the Forbidden City in 15th- and 16th-century China. Proc. Natl. Acad. Sci. **110**(50), 20023–20027 (2013)
11. Fall, A., Weber, B., Pakpour, M., Lenoir, N., Shahidzadeh, N., Fiscina, J., Wagner, C., Bonn, D.: Sliding friction on wet and dry sand. Phys. Rev. Lett. **112**(17), 175502 (2014)
12. Dowson, D.: History of Tribology. Longman Group Ltd., London (1979)
13. Steindorff, G.: Das Grab Des Ti. J.C. Hinrichs, Leipzig (1913)
14. Newberry, P.E., Griffith, F.L., Fraser, G.W.: Beni Hasan. K. Paul, Trench, Trübner and Co., London (1893)

15. Bowden, F.P., Tabor, D.: The Friction and Lubrication of Solids. Oxford University Press, London (1950)
16. McCurdy, E.: The Notebooks of Leonardo da Vinci, vol. 1. Reynal & Hitchcock, New York (1938)
17. Amontons, G.: De la résistance causée dans les machines. Mém. Acad. Roy. Sci. 206–226 (1699)
18. Rennie, G.: Experiments on the friction and abrasion of the surfaces of solids. Philos. Trans. R. Soc. Lond. 119 (ArticleType: research-article/Full publication date: 1829), 143–170 (1829). doi:10.2307/107775
19. Vince, S., Shepherd, A.: On the Motion of Bodies Affected by Friction. By the Rev. Samuel Vince, A. M. of Cambridge; Communicated by Anthony Shepherd, D. D. F. R. S. Plumian Professor of Astronomy and Experimental Philosophy at Cambridge. Philos. Trans. R. Soc. Lond. 75 (ArticleType: researcharticle/Full publication date: 1785/), 165–189 (1785). doi:10.2307/106753
20. Thurston, R.H.: Friction and Lubrication: Determinations of the Laws and Coefficients of Friction by New Methods and with New Apparatus. Railroad gazette (1879)
21. Coulomb, C.: Théorie des machines simples, en ayant égard au frottement de leurs parties, et a la roideur dews cordages. Memoires de Math. Phys. Acad. Sci. 10, 161–331 (1785)
22. Coulomb, C.A.: Théorie Des Machines Simples. Bachelier, Paris (1821)
23. He, G., Müser, M.H., Robbins, M.O.: Adsorbed layers and the origin of static friction. Science 284(5420), 1650–1652 (1999)