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A Monte Carlo Simulation-Based Approach to Evaluate the Performance of Three Meteorological Drought Indices in Northwest of Iran

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Abstract

Although meteorological drought indices are considered as important tools for drought monitoring, they are embedded with different theoretical and experimental structures. Regarding the different geographic and climatic conditions around the world, the most meteorological drought indices have been commonly applied for drought monitoring in different parts of the world. Interestingly, it is observed that such indices in the published studies on drought monitoring have usually yielded inconsistent performance. On the other hand, most studies on drought monitoring as well as the performance of drought indices has been based on short-term historical data (less than 50 years). Therefore, this study aimed to analyze and compare the performance of three common indices of SPI, RAI and PNPI to predict long-term drought events using the Monte Carlo procedure and historical data. To do this end, the 50-year recorded or historical rainfall data across 11 synoptic stations in the Northwest of Iran were employed to generate 1000 synthetic data series so that the characteristics of long-term drought might be determined and the performance of those three indices might be analyzed and compared. The results indicated a very high comparative advantage of the SPI in terms of yielding a satisfactory and detailed analysis to determine the characteristics of long-term drought. Also, the RAI indicated significant deviations from normalized natural processes. However, these results could not reasonably and sufficiently predict long-term drought. Finally, the PNPI was determined as the most uncertain and spatial index (depending on average or coefficient of variation of rainfall data) in drought monitoring.

Key words: Data generation models, Drought, Drought indices, Monte Carlo simulation.

1 Introduction

Drought is a periodical natural phenomenon which occurs in a geographical area and is accompanied by continuous (e.g. few months or years) water shortage and human civilizations have been repeatedly caught in it throughout history. This environmental phenomenon is an inevitable part of climatic variation and it is repeated in different climatic zones of the world (Wilhite 2000). Natural habitats, ecosystems and a majority of economic and social sectors may be affected by droughts (Heim 2002). In recent years, droughts have repeatedly occurred in different parts of the world and as demand for water increases (largely due to population growth and climate change), their adverse damages are exacerbated. Therefore, the drought phenomenon has attracted much attention and many researchers have attempted to study the various aspects of this phenomenon (Mishra and Singh 2011).

Drought indices are commonly used for qualitative and quantitative evaluation of the drought phenomenon around the world (Barua et al. 2011). Specifically, drought indices are extracted by assimilating drought indicators into a single numerical value. A drought index provides a comprehensive picture of any given drought analysis and decision-making and this is more readily useable compared to raw data extracted from indices (Zargar et al. 2011).

Drought indices generally tend to use rainfall individually or simultaneously in combination with other climatic elements, including evapotranspiration, temperature or soil moisture. For example, a combination of meteorological variables, including rainfall and temperature, is used for Palmer Drought Severity Index (PDSI) (Palmer 1965) and Crop Moisture Index (Palmer 1968). Similarly, rainfall and soil moisture are used for Moisture Adequacy Index (McGuire and Palmer 1957), rainfall and evapotranspiration are used for Reconnaissance Drought Index (RDI; Tsakiris and Vangelis 2005; Tsakiris et al. 2007) and only the rainfall are used for the Standardized Precipitation Index (SPI) (McKee et al. 1993; Tsakiris and Vangelis 2004).

Single-parameter drought indices based on annual rainfall are among the most commonly used ones because it is easy to get access to annual rainfall data in different parts of the world (Moreira et al. 2008). Although annual time scale seems a long-time one, it can be used effectively to summarize the characteristics and regional behaviors of droughts (Mishra and Singh 2010). Meanwhile, monthly-time scale seems more appropriate for identifying the effects of drought on agriculture and water supply issues (Panu and Sharma 2002). The most commonly used drought indices that have been developed based on rainfall data include Rainfall Anomaly Index (RAI) (van Rooy 1965), Deciles Index (DI) (Gibbs and Maher 1967), Nitzche Index (Nitzche 1985), Standardized Precipitation Index (SPI) (McKee et al. 1993), Percent of Normal Precipitation Index (PNPI) (Willeke 1994), Z-Score, China Z Index (CZI) (Ju et al. 1997) its variants and the Modified China Z Index (MCZI) (Wu et al. 2001).

Generally, these indices have occupied a special place in drought monitoring and have been individually or collectively applied in a lot of drought events investigations. Usually, the main specific objectives of drought studies can be classified as follows: a) drought characteristics during data record length; b) comparison of drought indices performance; c) long-term or future drought events prediction. A number of such studies have been presented below.

Guttman (1998) compared historical time series of the PDSI with time series of the corresponding SPI through spectral analysis. The results showed that the spectral characteristics of the PDSI were spatially variable while those of SPI were spatially invariant. Besides, it was indicated that the PDSI had a complex structure with an exceptionally long memory but the SPI was spatially consistent and had a simple structure. For this reason, Guttman (1998) recommended the SPI as a better drought index for interregional comparison of drought events. Keyantash and Dracup (2002) compared 14 drought indices using historical data of annual rainfall in USA, and concluded that the SPI as one of the most valuable indices to estimate drought severity. Quiring and Papakryiakou (2003) used historical data and compared four drought indices; PDSI, Palmer's Z-Index, SPI and NOAA and found that the Palmer’s Z-index was the most suitable one to monitor agricultural drought in Canadian prairies. Loukas et al. (2003) examined the historical data of 40 years rainfall in Greece and compared three indices of SPI, RAI and Z-Score in 28 stations. They concluded that all three indices performances were similar in 12-month scale to determine wet and dry years and were in good agreement with Palmer’s drought severity. Morid et al. (2006) used the 32-years historical data record of annual rainfall in Tehran province and investigated the performance of seven drought indices namely DI, PNPI, SPI, CZI, MCZI, Z-Score and EDI in this province. They concluded that the SPI, CZI and Z-Score indices had similar behavior to detect drought, and they demonstrated slow reactions to the beginning of drought. Also, the DI index was shown to have an inconsistent behavior for temporal and spatial variations resulting from drought. Meanwhile, Morid et al. (2006) argued that SPI and EDI were able to better identify drought beginning and had the steady state of spatial and temporal variations. Finally, it became also clear that EDI had more capability for identifying drought and had better performance. Khalili et al. (2011) compared SPI and RDI in different climatic conditions in Iran using Markov chain model. They found that both SPI and RDI exhibit an overall similar behaviour. Also, they indicated that the RDI by utilizing the reference crop evapotranspiration can be very sensitive to climatic variability. Zargar et al. (2011) reviewed the general features and application of 74 drought indices and noted the superiority of popular drought indices such as SPI and NDVI. In another study, based on historical streamflow data from the Yarra River in Australia, Barua et al. (2011) compared the PNPI, SPI, SWSI and ADI drought indices and found the ADI as the best index in the area.

Sayari et al. (2013) evaluated the impact of climate change on drought conditions using SPI, PNPI and RAI indices and showed that the results of the PNPI were similar to the SPI and yielded similar behavior for both historical and future periods. Jain et al. (2015) evaluated six drought indices with historical data from 1901 to 2002 and different time scales in India and they found that the time scale had a significant impact on the performance of the drought indices. Besides, it was indicated that there was not a single best index for all locations and time scales.

Having reviewed the aforementioned studies, the following three basic points can be deduced:

1) Inconsistent performance of drought indices in different drought monitoring studies. Although the results of a study conducted by Loukas et al. (2003) indicated that the performance of three indices of SPI, RAI and Z-Score on drought monitoring was quite similar to each other, the results of a study conducted by Morid et al. (2006) on drought monitoring showed that the performance of seven compared indices was quite dissimilar to each other. Similarly, such inconsistency was evident between studies conducted by Sayari et al. (2013) and Barua et al. (2011) in terms of two indices of SPI and PNPI.

2) The application of historical rainfall data (often less than 50 years) to compare the performance of drought indices. Given the historical data as a observed small sample, they might not be considered as full and adequate representatives of rainfall population (Montaseri and Amirataee, 2016). Accordingly, using such rainfall data record to compare drought indices would lead to numerous uncertainties.

3) Prediction of future or long-term behavior of drought events using the historical data. Although this would happen if the historical rainfall data series will be repeated in the future or next years, while the historical data are considered as some discrete examples of thousands of probable samples that may occur in the future (Montaseri and Amirataee, 2016).

Therefore, this study aimed to analyze and compare the performance of three common indices of SPI, RAI and PNPI as representatives of three different classes of meteorological drought indices to predict long-term drought events using the Monte Carlo procedure and historical data across synoptic stations in the Northwest of Iran. As such, the following objectives were delineated for the present study: 1) Predicting long-term or future behavior of drought events. 2) Determining the inherent performance of aforementioned drought indices. 3) Determining the origin of the performance inconsistency across drought indices, and 4) Recognizing the limitations of historical data to predict long-term behavior of drought events and compare the performance of indices. To do this end, the 50-year rainfall data across 11 synoptic stations in the Northwest of Iran were used to generate 1000 synthetic rainfall data series.

1. 1 Drought Indices

The three drought indices under investigation included the Rainfall Anomaly Index (RAI), Percent of Normal Precipitation Index (PNPI) and Standardized Precipitation Index (SPI).

The RAI was presented by Van Rooy (1965). The index is aimed to calculate the deviation of rainfall from the normal amount of rainfall and it evaluates monthly or annual rainfall on a linear scale resulting from a data series (Oladipo 1985). The PNPI was presented in 1994 by Willeke. This index has been one of the easiest ways to assess the drought severity and it is calculable for various time intervals. The PNPI can be obtained by dividing the actual amount of rainfall by the average rainfall multiplied by 100. The SPI was developed by McKee et al. in 1993 in order to determine and monitor drought. The U.S. Colorado Climate Center, the U.S. Western Regional Climate Center, and the U.S. National Drought Mitigation Center, among others, use SPI to assess the present drought situation in the United States. The SPI is able to determine a wet and dry state for a specific time scale for each location having rainfall data (Mishra et al. 2009). To determine this index, first, the appropriate probability distribution is fitted to the long-term rainfall data, and then the cumulative distribution function is turned into the normal distribution via equal probabilities. Finally, transformed data to normal distribution are used to calculate SPI values (McKee et al. 1993; Morid et al. 2006; Cancelliere et al. 2007; Angelidis et al. 2012). The following equations calculate RAI, PNPI and SPI indices:

$$\mathrm{RAI_i} = \pm 3\left(\frac{\mathrm{P_i}-\overline{\mathrm{P}}}{\overline{\mathrm{E}}-\overline{\mathrm{P}}}\right) \tag{1}$$

$$\mathrm{PNPI_i} = \frac{\mathrm{P_i}}{\overline{\mathrm{P}}} \times 100 \tag{2}$$

$$\mathrm{SPI_i} = \frac{\mathrm{P_i}-\overline{\mathrm{P}}}{\delta} \tag{3}$$

where $\mathrm{P_i}$ is rainfall values in period i, $\overline{\mathrm{P}}$ is long-term average rainfall, $\overline{\mathrm{E}}$ is the mean of ten highest (for positive anomalies) and mean of ten lowest (for negative anomalies) values of P in the time series, δ is the standard deviation of rainfalls. The classifications of wet and dry states for RAI, PNPI and SPI indices are shown in Table 1.

In order to simplify and to realize the evaluation and comparison of the performance of mentioned drought indices, a common quantitative classification (CQC), including seven categories of extremely wet (+3), severely wet (+2), moderately wet (+1), near normal (0), moderate drought (-1), severe drought (-2) and Extreme drought (-3), may be was specified to classify the criteria for drought as shown in Table 1.

Table 1 Common quantitative classification of drought indices

CQC Values	Drought Class	SPI	PNPI	RAI
+3	Extremely wet	>2	>160	>3
+2	Severely wet	1.5 to 1.99	145 to 160	1.2 to 2.1
+1	Moderately wet	1 to 1.49	130 to 145	0.3 to 1.2
0	Near normal	-0.99 to 0.99	70 to 130	-0.3 to 0.3
-1	Moderate drought	-1 to -1.49	55 to 70	-1.2 to -0.3
-2	Severe drought	-1.5 to -1.99	40 to 55	-2.1 to -1.2
-3	Extreme drought	<-2	<40	<-3

1.2 Assessing Drought Indices Performance within a Monte Carlo Framework

Most studies have been conducted on the evaluation of performance of different drought indices and the analysis of long-term drought characteristics have been based on historical data record. It was assumed that a- historical time series or historical data representes the full features of rainfall population and b- the expected rainfall which may occur in the future will duplicate the behavior of historical data. Whereas, the historical data is an example of thousands probable samples that may occur in future and it is not representatives of future events or population of rainfall values. Thus, although it is not possible to develop complete long-term drought characteristics and compare the inherent performance of drought indices based on historical rainfall data, the application of stochastic procedures to generate large samples of synthetic rainfall time series will becomes a necessity and is useful for drought event analysis (Salas 1993; Montaseri and Adeloye 1999; Douglas 2000). The generated stochastic rainfall data describe more realistic features of rainfall that may occur in the future (Salas 1993; Montaseri and Adeloye 2004).

In this study, annual and monthly rainfall data are required to investigate drought indices performance at both annual and monthly levels. However, efforts must be made to ensure that annual and within-year statistical characteristics are equally preserved in the generated rainfall sequences; hence rather than generating monthly rainfalls directly, annual rainfalls were firstly generated and later disaggregated to monthly rainfalls using the Valencia–Schaake (Valencia–Schaake 1973) approach. The advantage of disaggregation schemes over models with monthly changing parameters like ARMA (p,q) model and Thomas-Fiering model is that the former can preserve both the statistics of annual and monthly rainfalls, which are significant for drought events analysis at both monthly and annual levels. It is noted that the ARMA (p,q) model and Thomas-Fiering model can only preserve the monthly rainfall statistics (Santos and Salas 1992; Salas 1993; McMahon and Adeloye 2005).

2 Methodology

2.1 Study Area and Data Characteristics

In this study, the annual rainfall time series of 11 synoptic stations geographically located in the Northwest of Iran (Fig. 1) were used thereof. The rainfall data spanned a 50-year period (1961-2010) and the general characteristics of the data records were presented in Table 2. The average annual rainfall in the study area varied between a high rate of 1755 mm (Site-3) to a low rate of 235 mm (Site-5) which covered humid to semi-arid climates. However, all stations (except Site-3) had average annual rainfall between 283mm (Site-4) and 503mm (Site-6) and were therefore placed in semi-arid climate.

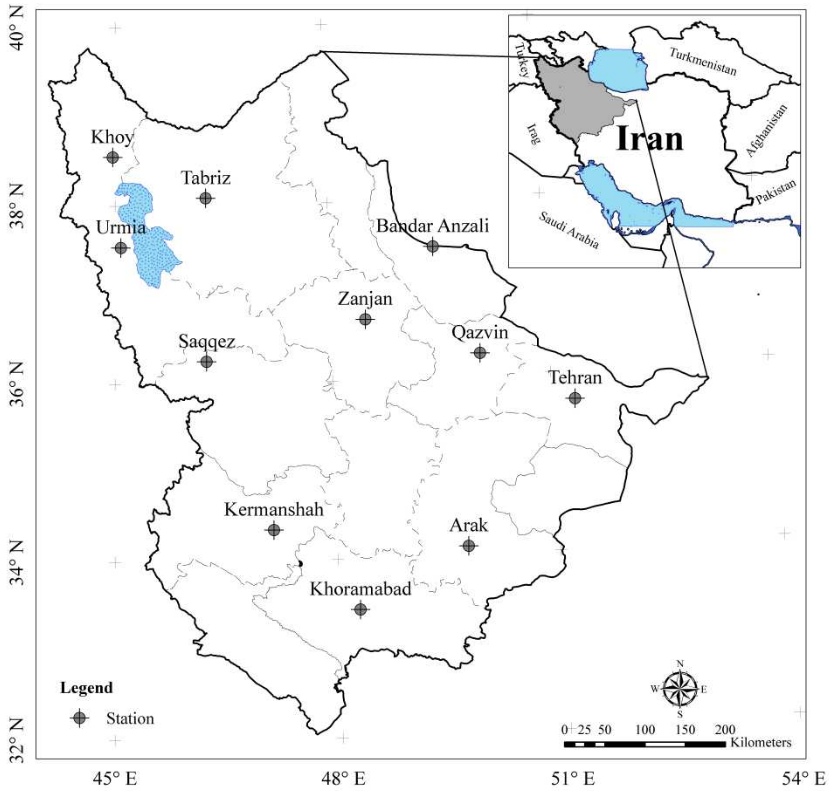

Fig. 1 Geographical location of the rainfall stations.

Table 2 Annual rainfall characteristics of eleven recording sites

Sign	Station	Elevation (m)	Geographic coordinates		Statistical properties of annual rainfall series (1961-2010)			
			Latitude (N)	Longitude (E)	Mean (mm)	CV	Skew	Lag-1 serial correlation (ρ)
Site-1	Arak	1708	34° 06′	49° 46′	332	0.30	0.31	-0.10
Site-2	Urmia	1316	37° 32′	45° 05′	332	0.30	0.83	0.25
Site-3	Bandaranzali	-26	37° 28′	49° 28′	1755	0.19	0.73	-0.15
Site-4	Tabriz	1361	38° 05′	46° 17′	283	0.30	0.91	0.36
Site-5	Tehran	1191	35° 41′	51° 19′	235	0.30	0.11	-0.15
Site-6	Khoramabad	1148	33° 26′	48° 17′	503	0.24	0.07	0.05
Site-7	Khoy	1103	38° 33′	44° 58′	292	0.28	0.40	0.24
Site-8	Zanjan	1663	36° 41′	48° 29′	305	0.26	0.00	-0.04
Site-9	Sagez	1523	36° 15′	46° 16′	488	0.27	0.52	0.16
Site-10	Gazvin	1279	36° 15′	50° 03′	317	0.27	0.31	-0.27
Site-11	Kermanshah	1319	34° 21′	47° 09′	450	0.27	0.56	0.11

2.2 Data Analysis and Generation

Stationarity and randomness of historical data should be evaluated so that the rainfall data for drought may be analyzed (Adeloye and Montaseri, 2002). Nonparametric Spearman's rank correlation, the modified Mann-Kendal procedure (Hamed and Rao 1998; Amirataee et al. 2015) and the Run test methods were used to test the stationarity and randomness, respectively (Adeloye and Montaseri 2002; McGhee 1985). The results indicated that the tests' statistics of stationarity and randomness of historical data were placed between the critical points (5% statistic level) for all the stations.

Selecting an appropriate probability distribution for rainfall data is an essential step in the stochastic drought monitoring investigations. For this purpose, Probability Plot Correlation

Coefficient (PPCC) test was applied to determine the most appropriate probability distribution of rainfall data among the five appropriate probability distributions (i.e. normal, two-parameter lognormal, three-parameter lognormal, Pearson type III and log Pearson type III) (Vogel and Kroll 1989; Vogel and Wilson 1996; Aksoy et al. 2008; McMahon et al. 2007; Yue and Hashino, 2007). The analysis revealed that the Pearson type III probability distribution was the best probability distribution of annual and monthly rainfall data for all the selected stations.

The ‘Run Theory’ proposed by Yevjevich (1967) to define hydrologic drought characteristics, was used to perform drought analysis (Fig. 2). In this concept, five main components of a hydrologic drought event have been identified as listed below (Yevjevich 1967; Dracup et al. 1980; Mishra et al. 2009) and shown in Fig. 2.

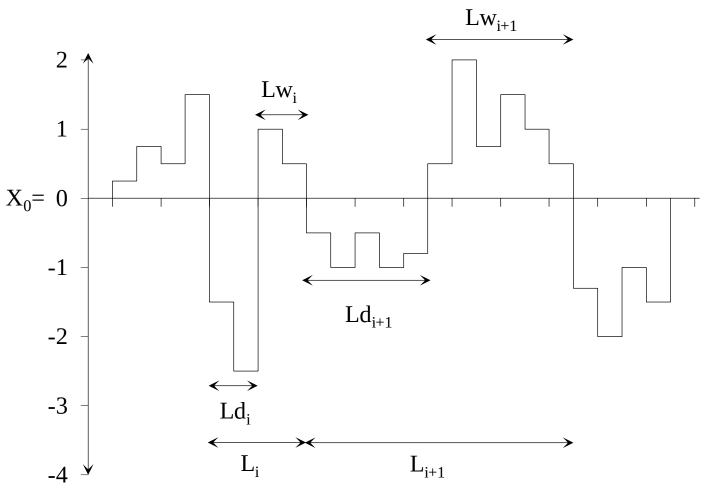

Fig. 2 Drought characteristics using the run theory.

- Drought duration (Ldi): It is a time period between the beginning (Td_b) and end (Td_e) of a consecutive drought event. A mathematical expressions of drought duration is presented as follow:

$$Ld_i = Td_e - Td_b \tag{4}$$

- Drought severity (Sd_i): It indicates a cumulative deficiency of a consecutive drought parameter below the critical level. For convenience, severity of drought events is taken to be positive and is given by the Equation 5.

$$Sd_i = -\sum_{k=Td_b}^{Td_e} SPI_k \tag{5}$$

where SPI_k is drought index value in period k.

- Drought intensity (Id_i): It is the average value of a consecutive drought parameter below the critical level. It is measured as the drought severity (Sd_i) divided by the duration (Ld_i) as Equation 6:

$$Id_i = \frac{Sd_i}{Ld_i} \tag{6}$$

- Wet or non-drought duration (Lw_i): It is a time period between the end and beginning of a consecutive drought event or time period between the beginning (Tw_e) and end (Tw_b) of a consecutive wet event as follow:

$$Lw_i = Tw_e - Tw_b \tag{7}$$

- Drought interarrival time (L_i): It is a time period between the beginnings of two consecutive droughts, e.g. $L_i = Ld_i + Lw_i$.

2.3 Monte Carlo Simulation

The parameters of the data generation model were estimated from the available historical data. Annual rainfalls were generated according to the Lag-1 Auto Regressive, AR(1), process as follows (McMahon and Adeloye 2005):

$$\mathrm{P_{i+1}} = \overline{\mathrm{P}} + \rho(\mathrm{P_i} - \overline{\mathrm{P}}) + \upsilon_i \delta\sqrt{1-\rho^2} \tag{8}$$

where, P_{i+1} and P_i are the annual rainfalls for the years i+1 and i, respectively. $\overline{P}$ is the average of annual rainfalls, δ is the standard deviation of annual rainfalls, ρ is Lag-1 serial correlation coefficient of annual rainfalls; and υ_i is the standardized normal random variate.

The Valencia–Schaake model was used for disaggregation of annual rainfalls into monthly rainfalls as follows (Valencia and Schaake 1973):

$$X_i = AZ_i + BV_i \tag{9}$$

where, X_i is the (12×1) vector of transformed zero mean monthly rainfalls for year i, Z_i is the transformed zero mean annual rainfall for year i, V_i is a (12×1) vector of independent standard normal random variables for year i and independent of Z_i, and A and B are (12×1) and (12×12) coefficient vector and matrix, respectively.

The elements of the A and B for a single site case are determined using the following matrices:

$$S_{ZZ} = E\left[Z_i Z_i^T\right] \tag{10}$$

$$S_{XX} = E\left[X_i X_i^T\right] \tag{11}$$

$$S_{XZ} = E\left[X_i Z_i^T\right] \tag{12}$$

$$S_{ZX} = E\left[Z_i X_i^T\right] \tag{13}$$

where, the matrix S_{zz} consists of the variance of annual data, S_{XX} and S_{XZ} are (12×12) and (12×1) matrices, respectively, and the elements of matrix S_{XX} are the variances and covariances for the same month and between two different months, respectively. Matrix S_{XZ} contains the lag-zero cross-covariances between annual and different months. S_{ZX} is a transposed matrix of S_{XZ}. Using the term of S_{ZZ}, S_{XX}, S_{XZ} and S_{ZX} the values of A and B are obtained by:

$$A = S_{XZ} S_{ZZ}^{-1} \tag{14}$$

$$BB^{T} = S_{XX} - AS_{ZX} = S_{XX} - S_{XZ}S_{ZZ}^{-1}S_{ZX} \tag{15}$$

The AR(1) and Valencia–Schaake models require the annual and monthly rainfalls to be normalized and, accordingly, the researchers used the Wilson-Hilferty transformation (Loucks et al. 1981).

Given each of the selected rainfall stations, 1000 possible sequences of annual and monthly rainfall totals were generated using Equation (8) Equation (9), respectively. It should be mentioned that each generated sequence was equal the historical data record length.

The statistical properties such as Average (Ave), Standard deviation (SD), Skewness (Skew) and Lag-1 serial correlation (ρ) of historical and generated annual rainfall data (for all stations) and monthly (Urmia Station) rainfall data are shown in Table 3 and Table 4, respectively. The results indicated that both the AR(1) and Valencia–Schaake model had satisfactory performance in reproducing annual and monthly characteristics of historical rainfall data records across all the stations.

Table 3 The statistical properties of historical and generated annual rainfall data across all the stations

		Stations										
	Parameters	Site-1	Site-2	Site-3	Site-4	Site-5	Site-6	Site-7	Site-8	Site-9	Site-10	Site-11
Historical data	Ave (mm)	332.3	332.2	1755.5	283.2	235.4	503.1	292.0	304.8	487.6	317.0	449.9
	SD (mm)	98.3	99.0	331.4	84.5	70.6	121.4	81.6	78.6	131.0	86.2	122.5
	Skew	0.31	0.83	0.73	0.91	0.11	0.07	0.40	0.00	0.52	0.31	0.56
	ρ	-0.10	0.25	-0.15	0.36	-0.15	0.05	0.24	-0.04	0.16	-0.27	0.11
Generated data	Ave (mm)	333.1	332.2	1755.5	283.3	235.5	503.1	292.0	304.8	487.6	317.0	450.0
	SD (mm)	97.2	96.6	326.1	82.0	69.7	119.5	79.9	77.5	128.4	85.1	120.3
	Skew	0.28	0.80	0.72	0.86	0.11	0.06	0.39	0.00	0.51	0.31	0.56
	ρ	-0.07	0.19	-0.15	0.29	-0.18	0.02	0.20	-0.09	0.13	-0.28	0.03

Table 4 The statistical properties of historical and generated monthly rainfall data in Urmia Station (Site-2)

		Month											
	Parameters	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Historical data	Ave (mm)	29.7	31.8	48.4	58.7	44.9	12.9	5.3	2.5	4.3	27.0	38.1	28.8
	SD	18.5	16.2	32.5	30.8	37.3	12.9	11.0	5.4	7.5	34.0	31.8	23.6
	Skew	0.92	0.60	1.14	0.42	1.08	1.22	3.13	3.48	2.46	1.77	1.36	1.97
	Correlation between month and year	-0.16	0.12	-0.17	0.20	0.16	0.24	0.11	-0.05	-0.11	0.10	0.02	-0.14
Generated data	Ave (mm)	29.5	31.8	48.5	58.1	44.3	13.2	5.6	2.6	4.7	28.4	37.2	28.3
	SD	17.7	16.3	31.6	29.7	33.9	12.1	9.4	4.8	7.0	31.6	28.7	21.1
	Skew	0.81	0.63	1.03	0.50	1.01	1.32	2.66	2.93	2.36	1.66	1.11	1.55
	Correlation between month and year	-0.22	0.16	-0.07	0.18	0.14	0.23	0.13	-0.01	-0.05	0.16	-0.01	0.04

Each of the 1000 sequences (annual and monthly) was then routed through a Stochastic Drought Simulation Model (SDSM) based on the Run Theory to generate the runs of drought indices and other drought characteristics. Thus, routing all 1000 replicates resulted in a “population” of these drought characteristics. Consequently, a total of 1000 possible drought event values sequences were considered for each of the eleven rainfall synoptic stations which led to a suitable sample of outputs for a comprehensive investigation of the performance of drought indices.

In summary, having used three metrological drought indices, the stochastic and historical rainfall data from 11 selected rainfall synoptic stations located in Northwest of Iran at the annual and monthly time-steps, it was attempted to conduct a drought characteristics analysis.

3 Results and Discussion

It should be noted that the present results were provided based on the historical and generated stochastic rainfall data ‘as population” and those were named the historical and generated results in the following section, respectively. Both groups of results were required to evaluate the capability of historical and stochastic generated data in the long-term drought characteristics investigation and inherent performance of drought indices analysis.

3.1 The Probability Density Function (PDF) of Wet and Dry Periods

The historical and generated probability density function of wet and dry periods were considered for three drought indices on the basis of the CQC drought events classification (Table 1).

To establish the generated PDF of wet and dry periods, average probability values of each category based on the stochastic population of drought index values (i.e. 1000 sequences) were employed. Fig. 3 shows the PDF of three drought indices values based on stochastic population and historical values at Urmia Station (Site-2). Results for other stations demonstrated similar results to Urmia Station. Comparison of the PDF based on historical and stochastic population values (i.e. based on generated data) did not show the same behaviors and, thus, realistic and accurate monitoring of long-term drought events using only historical data would be problematic.

According to Fig. 3, the PDF of wet and dry periods for the SPI was similar to the PDF of the standard normal distribution. Thus, the probability of normal state (0) was approximately 0.68 and the sum of different wet and dry states were equivalent to each other (about 0.16). The PDF of wet and dry periods for the SPI was perfectly symmetrical and the ranges of variations for different stations were minimal.

The PDF of wet and dry periods for the RAI was not symmetric and it also had a significant range of variation across different stations. Moreover, it did not follow the PDF of the standard normal distribution and the probability of normal state (0) for this index was equivalent to 0.40 and the sum of wet and dry periods was equivalent to 0.30. Regarding the RAI, the probability of wet or dry periods was nearly two times more than the probability of wet or dry periods for the SPI.

Regarding the PNPI, the PDF of wet and dry periods had significant ranges of variation and the PDF of wet and dry periods were a function of average (or coefficient variation (CV)) of rainfalls; actually, any given increase in average (or decrease in CV) of rainfall would increase normal state (0) and the probability of wet and dry states would decrease thereof. For example, the probability of normal state in Tehran Station (Site-5) was less than 0.68 with the minimum average (or maximum CV) of annual rainfall (235 mm, 0.30) but this probability was determined as 0.79 and 0.90 for Khorramabad (Site-6) Station and Bandaranzali (Site-3) Station with the average rainfall of 503 and 1755 mm (or the CV of 0.24 and 0.19), respectively. According to the results, the SPI had relatively higher priority for drought monitoring than the RAI and PNPI indices because wet and dry events, as the expected events of a normalized natural phenomenon, would better fit to a normal distribution.

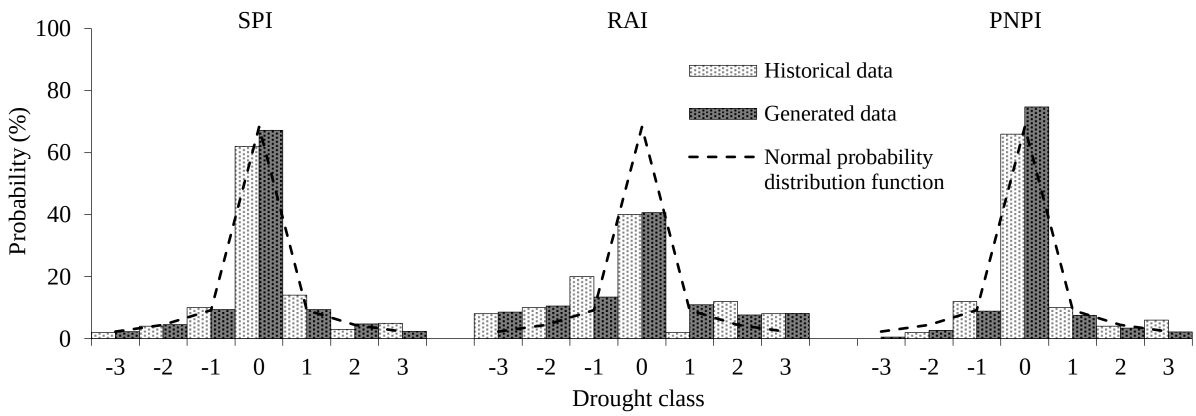

Fig. 3 Comparison of the PDF of wet and dry periods based on generated and historical rainfall data for three drought indices at Urmia (Site-2) Station with normal probability density function.

3.2 Relationship between the Drought Indices

Two data set of 55,000 values for RAI and PNPI (100 sequences of each 11 stations with 50-year record length) against SPI values were considered as testifier to investigate the relationship between the drought indices.

Figure 4 shows the generated and historical linear correlation between the values of SPI and two other indices for all stations on the basis of historical and generated annual rainfall data. Figure 4 clarifies that there is no linear relationship between RAI and PNPI drought indices values with SPI values and that each relationship demonstrated a different behavior. Also, both relationships for the historical and generated data demonstrated similar behavior.

The results showed that the relationship between RAI and SPI had a similar symmetrical behavior for wet and dry states. Accordingly, the RAI estimated different wet and dry states more intensely than SPI, because the RAI classified only about 60% of normal states as equal as SPI and the remainder (40%) was classified into wet and dry states. Deviation in the linear relation of PNPI dataset with SPI was a function of the mean (or CV) annual rainfall across stations and it did not follow a clear trend for all stations.

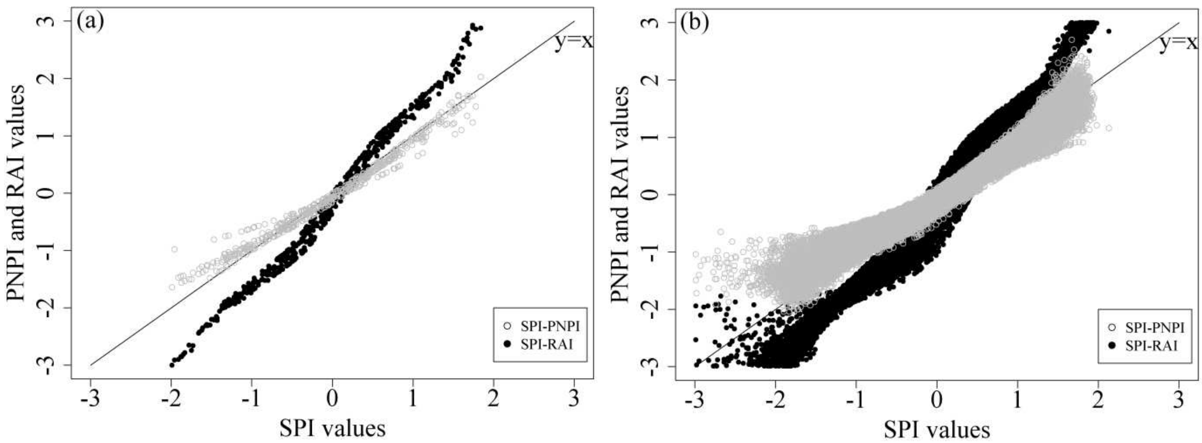

Fig. 4 Linear correlation between SPI values and two other indices (RAI and PNPI) values based on (a) historical and (b) generated annual rainfall data in all stations

3.3 The Main Characteristics of Drought

Having used the historical and generated rainfall data, it was attempted to examine three main drought characteristics of (i) duration, (ii) intensity and (iii) the interarrival times between two droughts. Regarding the prediction of long-term drought characteristics, it was indicated that the aforementioned characteristics were determined for each of the 1000 generated annual rainfall sequences and then they were averaged in every rainfall station. Given the SPI, RAI and PNPI indices, the mean and variation boundary of probability of duration, intensity and drought interarrival times for all stations and their historical and generated values in Urmia station have been presented in the Fig. 5, 6 and 7, respectively.

The results of three main drought characteristics, duration, intensity and drought interarrival times could be evaluated in terms of two major aspects: a) the performance of historical and generated rainfall data in terms of investigating long-term prediction of drought events, b) inherent performance of three drought indices. The results of Urmia Station indicated that the drought characteristics based on the historical rainfall data were established with very large irregular differences from the results of Monte Carlo procedure. Those large irregular deviations were completely different and unique at each selected station. Therefore, these results (Fig. 5, 6 and 7) confirmed that the historical rainfall data, as a single sample record from rainfall population, were absolutely insufficient for long-term prediction of drought events. However, the latter data were appropriate to define drought characteristics on the historical record length (i.e. 1961-2010).

On the other hand, the long-term expected drought characteristics (Fig. 5, 6 and 7) indicated that the inherent performances of SPI, RAI and PNPI were definitely separable and each of them represented a different class of long-term drought events. Given the former subsections of 3.1 and 3.2, it was attempted to discuss the essential differences of inherent performances of SPI, RAI and PNPI in regards to long-term prediction of aforementioned three main drought characteristics.

The probability of drought duration in the SPI was less than the probability of drought duration in the RAI. Accordingly, the probability of a one-year, two-year and three-year drought duration in the RAI (30%, 11% and 4%) was estimated to be about two, four and eight times larger than SPI (16%, 2.7% and 0.5) (Fig. 5). In addition, the range of variation in drought duration in SPI was very low across all stations and this variation considerably increased in the PNPI and RAI, respectively.

Given the SPI, RAI and PNPI, it was shown that one-year drought intensity was less than two-year drought intensity for major the selected stations (Fig. 6). However, it seems unlikely while the lag-1 serial correlation (ρ) of annual rainfall data of the selected stations yielded contrary results. Thus, one-year drought intensity value of Qazvin Station (Site-10) with a ρ=-0.27 was more than its two-year drought duration value. However, as ρ increased, higher levels of drought intensity for two-year than one-year drought intensity were observed. Besides, the mathematical relationship between two-year drought intensity and ρ was $I_2=1.86+4.2\rho$ with R = 0.6.

The occurrence probability of two-year interarrival time between droughts in the SPI (80%) was less than the occurrence probability of two-year interarrival time between droughts in the RAI (90%). However, the occurrence probability for a period of a four-year or longer drought in SPI was more than that of RAI (Fig. 7). In addition, the range of variation for the interarrival time between droughts in SPI and RAI indices was approximately equal but its value for PNPI was high (Fig. 7).

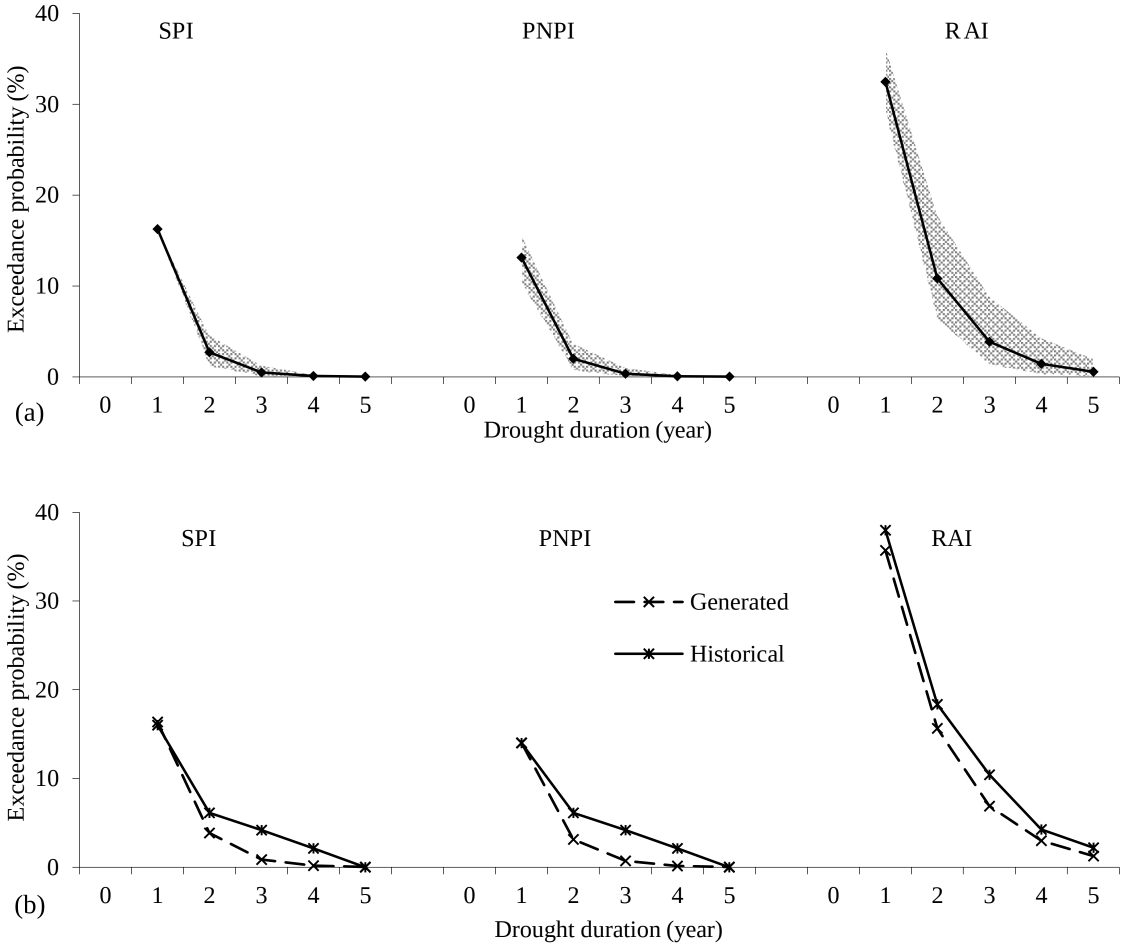

Fig. 5 Mean and variation boundary of probability of drought duration (a) based on generated data across all stations, (b) based on historical and generated data in Urmia Station

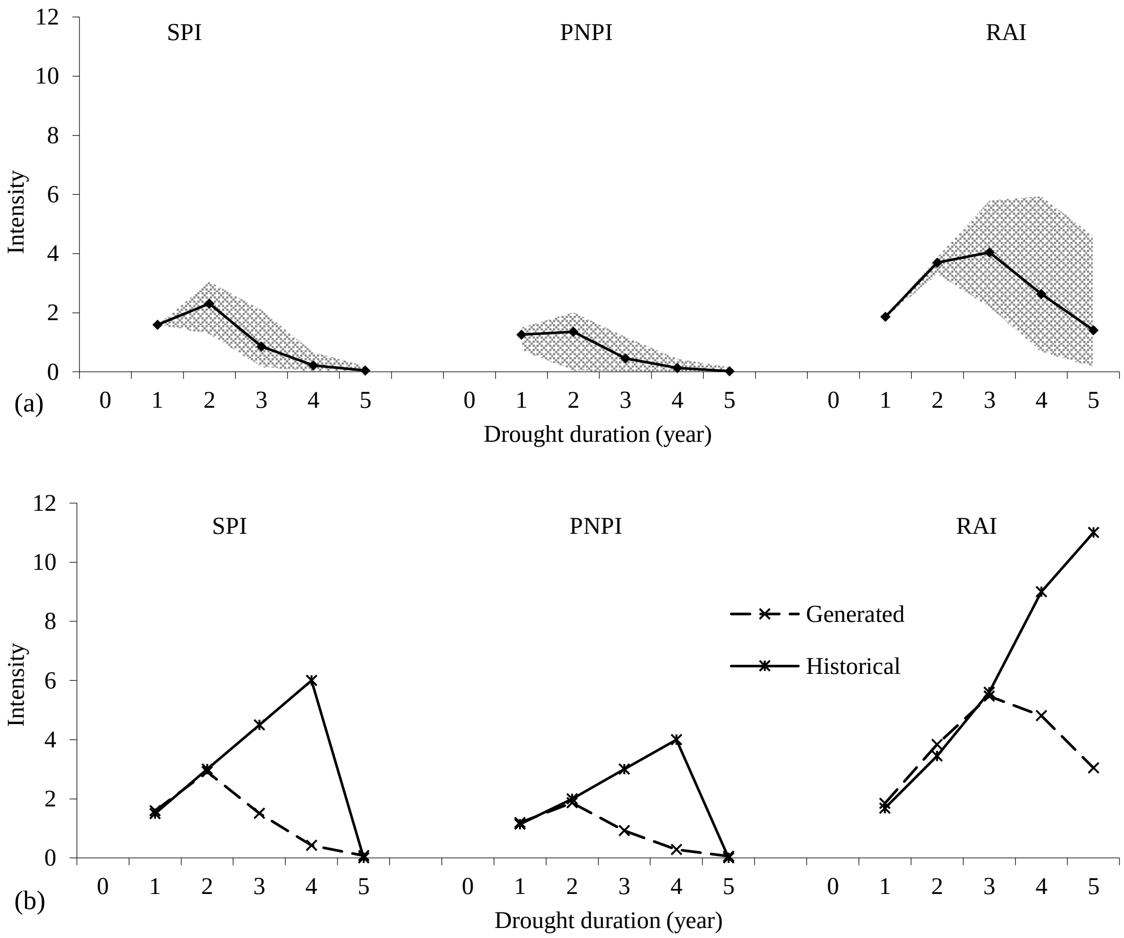

Fig. 6 Mean and variation boundary of probability of drought intensity (a) based on generated data across all stations, (b) based on historical and generated data in Urmia Station

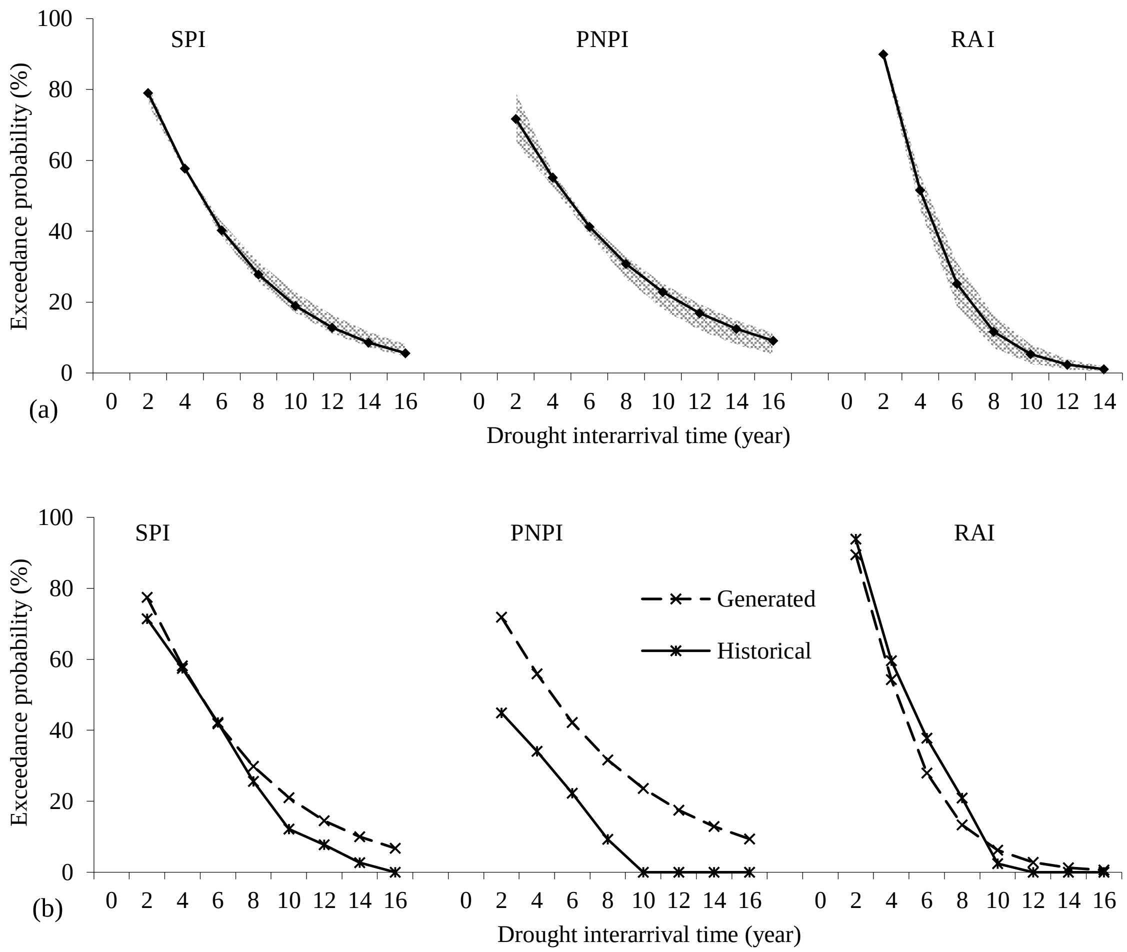

Fig. 7 Mean and variation boundary of probability of drought interarrival time (a) based on generated data across all stations, (b) based on historical and generated data in Urmia Station

3.4 Transition Probability Matrix

The transition probability matrix was applied on the basis of seven classified drought states to evaluate the conditional probability of different states of drought (see Table 1). The historical and generated steady state condition vectors of Urmia Station (Site-2) for three drought indices have been presented in Fig. 8. It should be noted that the results for the other 10 rainfall stations were similar to the results of Urmia Station. Interestingly, both historical and generated results (Fig. 8) were quite similar to the outcomes of PDF of wet and dry states (Fig. 3). As mentioned in section 3.1, the behavior of historical results was quite different from the typical behavior of stochastic population outcomes (see Table 5) and this was due to this fact that the historical rainfall data record is a sample of rainfall population over the past and future several thousand years. Therefore, a single historical rainfall data was insufficient and unrealistic for a long-term drought characteristics investigation and it was necessary to utilize the Monte Carlo procedure for such studies. Furthermore, we found that the annual conditional probability of various dry and wet states strongly correlated with lag-1 serial correlation of annual rainfall data. The three dry and wet states of moderate, severe and extreme were combined in two general classes of dry and wet states (D: Dry, W: Wet) to focus more clearly on the relationship. Table 5 presents the generated and historical transition matrix of various dry and wet states across two stations in terms of three drought indices. Consequently, it was observed that both historical and generated results yielded that the conditional probability of the DD (a dry year occurs after a dry year), DW (a wet year occurs after a dry year), WD, and WW states were a function of Lag-1 serial correlation of annual rainfall data. Thus, the probability of occurrence of dry after a dry year (DD) and wet after a wet year (WW) increased parallel with increase in lag-1 serial correlation and conditional probability of DW and WD decreased parallel with increase in lag-1 serial correlation.

Table 5 Three-state transition probability matrix: Drought (D), Normal (N) and Wet (W) across two stations in terms of three drought indices

Gazvin (ρ =-0.27)								**Urmia (ρ =0.25)**							
Generated				**Historical**				**Generated**				**Historical**			
SPI	D	N	W	**SPI**	D	N	W	**SPI**	D	N	W	**SPI**	D	N	W
D	0.07	0.15	0.27	D	0.00	0.15	0.44	D	0.24	0.16	0.10	D	0.38	0.09	0.18
N	0.65	0.68	0.66	N	0.89	0.59	0.56	N	0.67	0.67	0.66	N	0.50	0.68	0.54
W	0.28	0.15	0.07	W	0.11	0.25	0.00	W	0.10	0.17	0.24	W	0.13	0.22	0.27
PNPI	D	N	W	**PNPI**	D	N	W	**PNPI**	D	N	W	**PNPI**	D	N	W
D	0.06	0.12	0.24	D	0.00	0.12	0.43	D	0.22	0.13	0.08	D	0.43	0.09	0.10
N	0.69	0.75	0.71	N	0.86	0.72	0.57	N	0.69	0.72	0.70	N	0.57	0.70	0.60
W	0.25	0.13	0.04	W	0.14	0.17	0.00	W	0.08	0.14	0.22	W	0.00	0.21	0.30
RAI	D	N	W	**RAI**	D	N	W	**RAI**	D	N	W	**RAI**	D	N	W
D	0.20	0.31	0.44	D	0.13	0.27	0.54	D	0.44	0.34	0.26	D	0.47	0.30	0.37
N	0.41	0.42	0.39	N	0.73	0.36	0.23	N	0.39	0.40	0.41	N	0.37	0.45	0.36
W	0.38	0.26	0.17	W	0.13	0.37	0.23	W	0.18	0.25	0.33	W	0.16	0.25	0.27

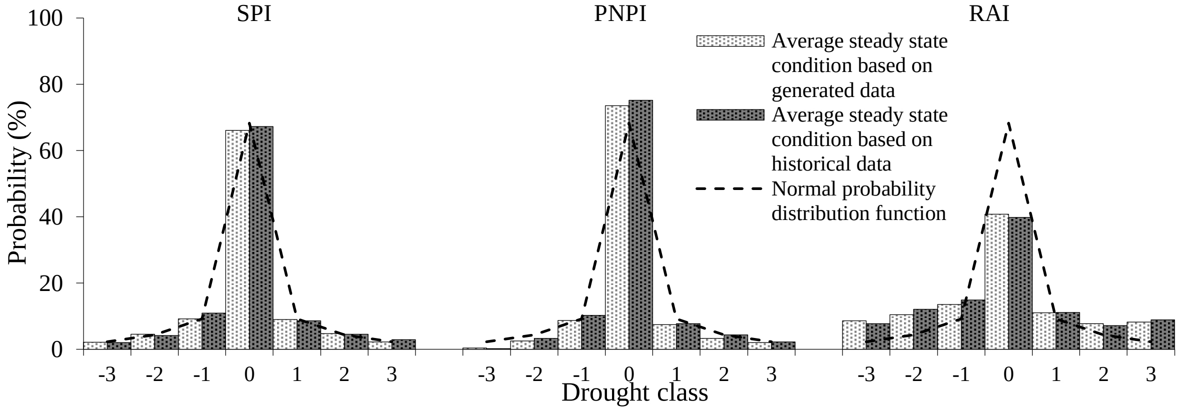

Fig. 8 The historical and generated steady state condition vector in Urmia Station in terms of three drought indices

3.5 Convergence of Annual and Monthly Drought Events

Having used the generated rainfall data in the selected stations and the SPI, RAI and PNPI, it was attempted to implement drought monitoring at both monthly and annual levels to evaluate the effect of different months on annual drought events. Given each 1000 sequences of generated rainfall data at each 11 station, the probability of simultaneous monthly and annual drought events was determined on the basis of each three indices. When the obtained values in each selected station were averaged to draw a Box-and-whisker plot diagram, Box-and-whisker plot diagrams were produced to show the maximum, minimum, 25%, median and 75% ranges for concurrency of annual and monthly drought events in the study area (Fig. 9).

Regarding the SPI, Fig. 9 indicates that the rainy months, i.e. November until the end of May, play a major role in determining annual drought status and the rest of the months of the year (embedded with relatively low rainfall amount) play an insignificant role in this regard. As the rainfall increases/decreases at any given month, its influence similarly increases/decreases thereof. Regarding the RAI and PNPI, low rainfall months play a major role in determining the annual drought position. Thus, for example, if low rainy months are located in the drought state, RAI and PNPI can predict that the same year will be affected by drought with 80% probability.

Therefore, making use of RAI and PNPI to investigate both short and long term periods of drought events or using prolonged drought analysis results for the extraction of short-term drought or vice versa will reveal the largest error.

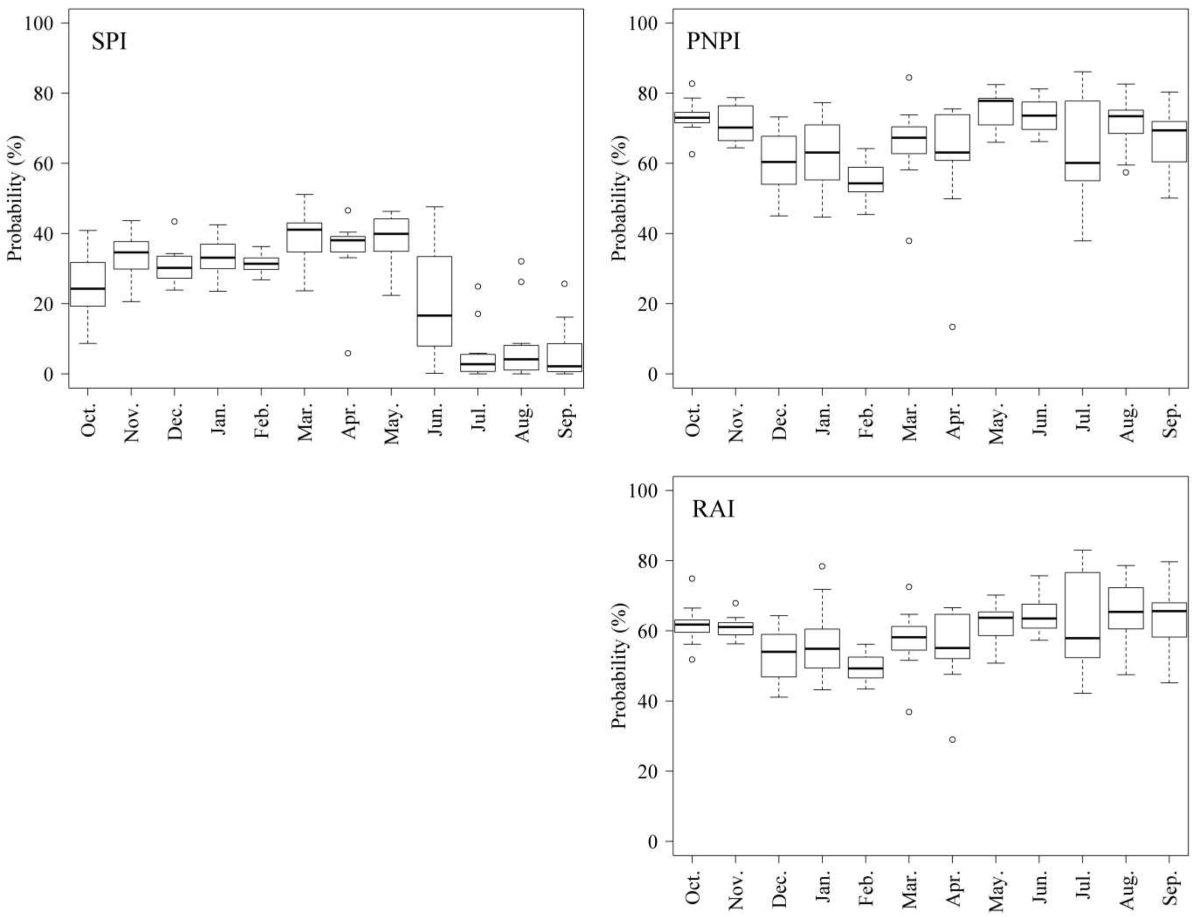

Fig. 9 Effect of different months of a year in drought situation

However, it appears that this conclusion will be true for a meteorological drought monitoring which is based solely on rainfall, and also where rainfall can define drought conditions by its own. Since the agriculture drought monitoring is essentially affected by evapotranspiration, it seems that those drought indices that based on rainfall and evapotranspiration such as may be preferably employed in such studies. In this case, any increase in evapotranspiration leads to more severe drought as well. As stated by Tsakiris et al. 2007, the RDI can be effectively associated with agricultural drought.

4. Conclusion

This study focused on emphasizing the advantages of application of Monte Carlo procedure in long-term drought events prediction and the inherent performance of three basically different types of drought indices.

It was found that a long-term drought events characterizing on the basis of historical rainfall time series was insufficient and erroneous. However, the results extracted on the basis of a historical rainfall data were quite accurate for the data record length. Hence, a Monte Carlo simulation approach as a powerful tool could be adopted to generate stochastic rainfall time series and result in a ‘population’ for a long-term drought characteristics investigation.

The results indicated that the performance of each of aforementioned three drought indices was completely distinct and it seemed that the inherent performance of each examined index was unique.

The results of subsections 3.1 and 3.4 showed that the PDF of wet and dry periods for SPI perfectly matched with the PDF of the normal distribution. Interestingly, the PDF of wet and dry periods for RAI was completely different from the PDF of normal distribution, e.g. the probability of normal state was 40%. On the other hand, the PDF of wet and dry periods for PNPI in terms of each station was unique and it was certainly dependent on the mean (or CV) of annual rainfall. Thus, the behavior and performance of PNPI in drought monitoring was non-spatial. Accordingly, this index would be untrustworthy and unreliable in terms of drought prediction and it could be concluded that the performance of the SPI in monitoring drought and determining the expected occurrence of the normalized natural processes with normal distribution was superior to other two indices.

The results of subsection 3.2 also showed that the RAI estimated the various wet and dry states more strictly than the SPI and it could be argued that this was due to the assumption of probability of approximate (RAI) fold of three different event statues of wet and dry periods (extreme, severe and moderate). As such, it was inconsistent with the expected occurrences of natural phenomena.

Subsection 3.3 represented the results of estimating the possibility of longer drought duration and severer drought severity for RAI than the SPI. This was likely because the normal state probability in this index (40 percent) was lower compared to that in the SPI (68%) and the probability of wet and dry periods in the RAI was almost twice that of the SPI.

The results of the conditional probabilities of different states of wet and dry periods showed the conditional probability of DD, WW, DW, and WD for all indices and it could be described as functions of Lag-1 serial correlation of rainfall data. So, Lag-1 serial correlation played an important role in the conditional probability for monitoring droughts events.

The results of section 3.5 showed that given the SPI, a rainy month played a major role in determining the position of annual drought but there was not any convergence in the RAI and PNPI. So, applying the RAI and PNPI for simultaneous analysis of short-term and long-term drought events or relying on the results of short-term drought to forecast or exploit long-term droughts or vice versa would be associated with high error levels.

Finally, the results of this study indicated that the application of the SPI would result in relatively high advantage for a comprehensive and accurate analysis. Accordingly, the application of RAI in drought events had a significant deviation from the expected events of normalized natural processes and its results could not be trusted to predict the events of drought. Besides, the PNPI was the most inefficient index in drought monitoring and it was misleading and erroneous to rely on the results of this index to predict the characteristics of drought.

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